1. PLL loop controller design
Consider the PLL loop phase signal model (Lecture 4, Slides 10--11)
\[
\dot{\theta} = d - K_u u \\
e = K_d \sin(\theta)
\]
Assume that in this model \(K_u = 1; K_d = 1\). The loop (and its low-pass filters) is designed to work with a carrier signal around 2MHz. The PLL phase should converge in about 100\(\mu\)s leaving enough room for the low-pass filtering effects to be left outside of the closed-loop bandwidth.
(a) Design a PI controller (loop filter) for the PLL to satisfy the above specification for the linearized model and provide a transient with no oscillations. What are the feedback gains of the controller?
(b) Produce simulation results for PLL phase transient with the designed controller (a) when the carrier frequency experiences sudden 60KHz step change. Simulate and compare the transients for the linearized and nonlinear models.

*Hint*
You can use Matlab ode45 function for the nonlinear simulation

2. Disk drive
Consider the disk head control problem described in Matlab Control Systems Toolbox demo diskdemo.m.
Design a PID controller for this problem. To improve performance, the PID controller should work with the filtered plant output. Use the notch filter designed in the demo. The relevant code from the demo and the baseline controller designed in the demo are given in the files diskPID.m, diskdemo.mat posted at the course website. After running the script diskPID.m the following useful data is left in the workspace
\%
Ts - sampling time
\%
Gd - nominal sampled-time model
\%
Gdm - array of 16 perturbed models for Monte Carlo analysis
\%
notchd - notch filter to filter out the flexible resonance
\%
C2 - 4-th order controller designed in the demo
\%
- use C2 as a baseline to compare with your design

(a) Design a discrete time PID controller (using the notch filter notchd) that provides at least the same performance as the controller C2. The performance should be determined both for the nominal plant and the Monte Carlo perturbations as implemented in the script for C2. What are the PID gains? Show the simulation results. The performance should be better than for C2.
(b) Briefly describe the approach you took to find the PID gains and the reason for using this approach.
3. Feedforward
Consider a satellite attitude control problem discussed in detail in Powell Chapter 9, Section 9.2. The dynamics of the Line of Sight control system are linear and can be described by the following transfer function
\[ y = \frac{0.036(s + 25)}{s^2(s^2 + 0.15s + 1)}u \]
The control goal is to move the system from \( y=0 \) to \( y=1 \) in about 10 sec. This is an aggressive goal since 10 sec is just 1.5 times larger than the 6.5-sec system oscillation period. The desired acceleration profile for this motion is a half period of sinusoid. The desired acceleration starts at zero, is positive for 5 sec, then negative for 5 sec, then becomes zero again.
(a) Write system input-output representation for the acceleration as an output and an analytical expression the desired acceleration profile
(b) By using a regularized system inversion through frequency domain find a noncausal feedforward. As the regularization parameter, use 1% of the system maximal gain. Sample the system at 50Hz frequency at least when computing the Fourier transforms. Produce a plot of the feedforward.
(c) Produce plots for the system coordinate change: desired, with the computed feedforward, and the error. Explore how the error will change if the system vibration frequency would vary 2% in either direction.

4. PID for sideways heat transfer
A model for the sideways heat transfer was described in Lecture 2 (Slides 11 and 15) the step response data for the model were dealt with in Assignment 1, Problem 3. An IIR model of the impulse response is computed as described in Lecture 2, Slide 21 by sampling the continuous-time pulse response data at 1 sec interval. This model corresponds to the following sampled-time transfer function
\[
\frac{1.797e-009 z^4 + 0.00017 z^3 + 0.003674 z^2 + 0.006498 z + 0.001904}{z^5 - 2.338 z^4 + 1.97 z^3 - 0.7206 z^2 + 0.1008 z}
\]
(a) Design a continuous-time PID controller for the system that will ensure the heat flux settling to a setpoint in about 20 s with minimal overshoot. Use discrete time simulation with the above described model to validate the design and demonstrate the performance. Produce plots of closed-loop responses to the setpoint change, compare to open loop step response plots, step response for the control input to the setpoint change in the closed loop, and step response to an output disturbance. What are the PID gains? How much faster is the closed loop response to the setpoint compared to the open loop response? Explain how the closed-loop disturbance rejection compares to the open loop.

Hint
Using Matlab functions tf, step, and feedback might save you some time