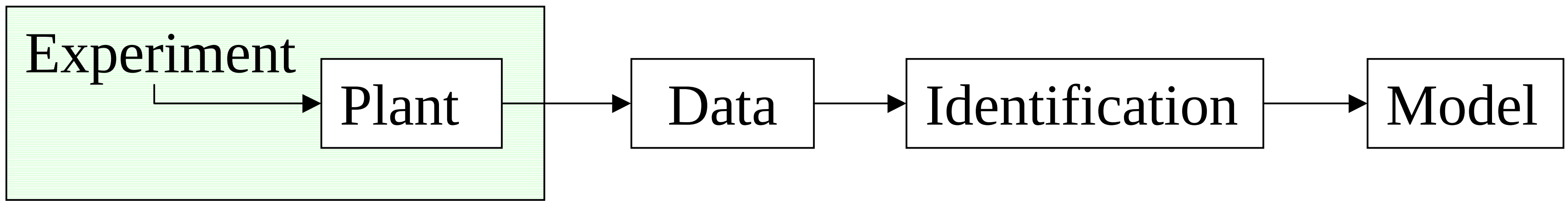
Lecture 8 - Model Identification

- What is system identification?
- Direct pulse response identification
- Linear regression
- Regularization
- Parametric model ID, nonlinear LS

What is System Identification?



- White-box identification
  - estimate parameters of a physical model from data
  - Example: aircraft flight model
- Gray-box identification
  - given generic model structure estimate parameters from data
  - Example: neural network model of an engine
- Black-box identification
  - determine model structure and estimate parameters from data
  - Example: security pricing models for stock market

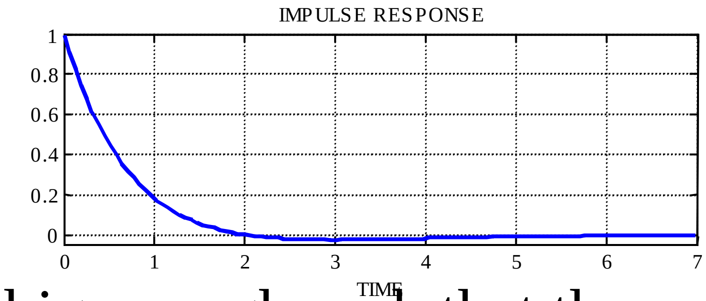
Rarely used in real-life control

Industrial Use of System ID

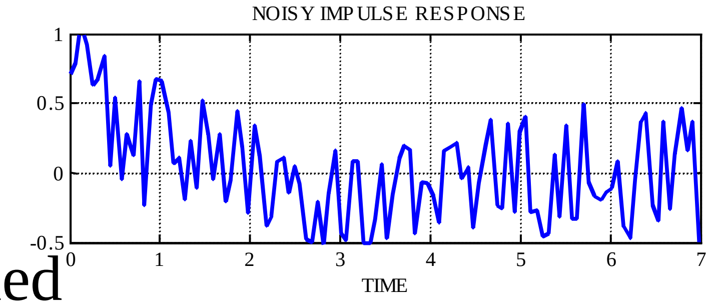
- Process control - most developed ID approaches
  - all plants and processes are different
  - need to do identification, cannot spend too much time on each
  - industrial identification tools
- Aerospace
  - white-box identification, specially designed programs of tests
- Automotive
  - white-box, significant effort on model development and calibration
- Disk drives
  - used to do thorough identification, shorter cycle time
- Embedded systems
  - simplified models, short cycle time

Impulse response identification

- Simplest approach: apply control impulse and collect the data



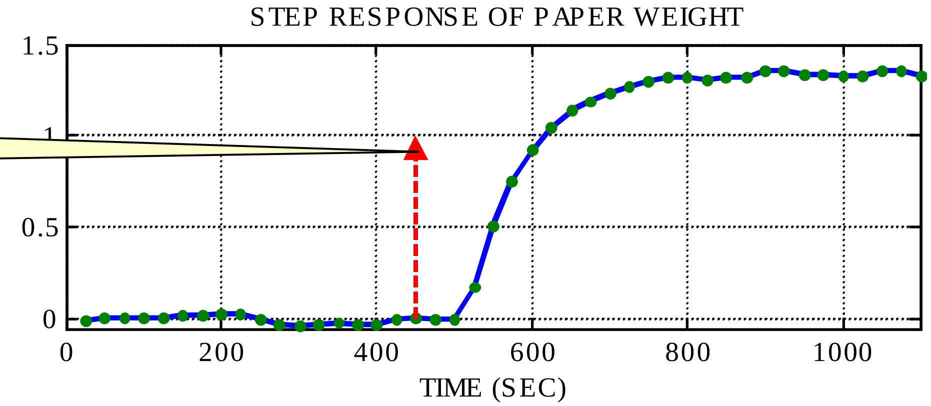
- Difficult to apply a short impulse big enough such that the response is much larger than the noise



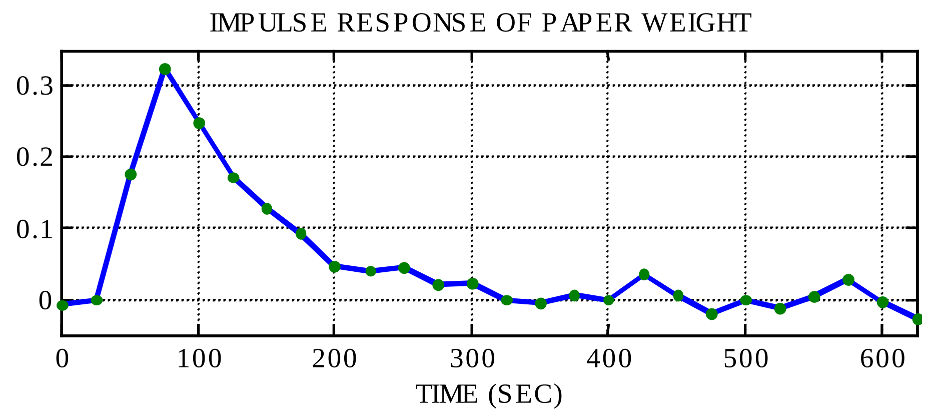
- Can be used for building simplified control design models from complex sims

Step response identification

- Step (bump) control input and collect the data
  - used in process control



- Impulse estimate still noisy: impulse(t) = step(t)-step(t-1)



Noise reduction

Noise can be reduced by statistical averaging:

- Collect data for mutiple steps and do more averaging to estimate the step/pulse response
- Use a parametric model of the system and estimate a few model parameters describing the response: dead time, rise time, gain
- Do both in a sequence
  - done in real process control ID packages
- Pre-filter data

Linear regression

- Mathematical aside
  - linear regression is one of the main System ID tools

Data — Regression weights — Regressor — Error

$$y(t) = \sum_{j=1}^{N} \theta_j \varphi_j(t) + e(t)$$

$$y = \Phi \theta + e$$

$$y = \begin{bmatrix} y(1) \\ \vdots \\ y(N) \end{bmatrix}, \Phi = \begin{bmatrix} \varphi_1(1) & \dots & \varphi_K(1) \\ \vdots & \ddots & \vdots \\ \varphi_1(N) & \dots & \varphi_K(N) \end{bmatrix}, \theta = \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_K \end{bmatrix}, e = \begin{bmatrix} e(1) \\ \vdots \\ e(N) \end{bmatrix}$$

Linear regression

- Makes sense only when matrix Φ is tall, $N > K$, more data available than the number of unknown parameters.
  - Statistical averaging
- Least square solution: $\|e\|^2 \to \min$
  - Matlab `pinv` or left matrix division `\`
- Correlation interpretation:

$$y = \Phi\theta + e$$

$$\hat{\theta} = \left(\Phi^T\Phi\right)^{-1}\Phi^T y$$

$$\hat{\theta} = R^{-1}c$$

$$R = \frac{1}{N}\begin{bmatrix} \sum_{t=1}^{N}\varphi_1^2(t) & \dots & \sum_{t=1}^{N}\varphi_K(t)\varphi_1(t) \\ \vdots & \ddots & \vdots \\ \sum_{t=1}^{N}\varphi_1(t)\varphi_K(t) & \dots & \sum_{t=1}^{N}\varphi_K^2(t) \end{bmatrix}, \qquad c = \frac{1}{N}\begin{bmatrix} \sum_{t=1}^{N}\varphi_1(t)y(t) \\ \vdots \\ \sum_{t=1}^{N}\varphi_K(t)y(t) \end{bmatrix}$$

Example: linear first-order model

$$y(t) = ay(t-1) + gu(t-1) + e(t)$$

- Linear regression representation

$$\varphi_1(t) = y(t-1) \qquad \theta = \begin{bmatrix} a \\ g \end{bmatrix} \qquad \hat{\theta} = \left(\Phi^T \Phi\right)^{-1} \Phi^T y$$

$$\varphi_2(t) = u(t-1)$$

- This approach is considered in most of the technical literature on identification

Lennart Ljung, *System Identification: Theory for the User*, 2nd Ed, 1999

- Matlab Identification Toolbox
  - Industrial use in aerospace mostly
  - Not really used much in industrial process control
- Main issue:
  - small error in *a* might mean large change in response

Regularization

- Linear regression, where $\Phi^T\Phi$ is ill-conditioned
- Instead of $\|e\|^2 \rightarrow \min$ solve a regularized problem

$$\|e\|^2 + r\|\theta\|^2 \rightarrow \min$$

$$\boxed{y = \Phi\theta + e}$$

  r is a small regularization parameter
- Regularized solution

$$\hat{\theta} = \left(\Phi^T\Phi + rI\right)^{-1}\Phi^T y$$

- Cut off the singular values of Φ that are smaller than r

Regularization

- Analysis through SVD (singular value decomposition)

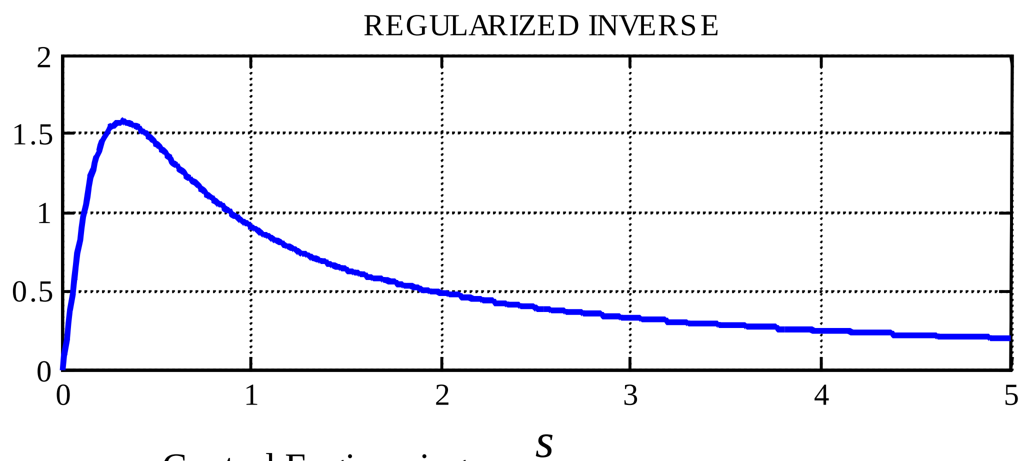
$$\Phi = USV^T; \qquad V \in R^{n,n}; U \in R^{m,m}; S = \mathrm{diag}\{s_j\}_{j=1}^{n}$$

- Regularized solution

$$\hat{\theta} = \left(\Phi^T\Phi + rI\right)^{-1}\Phi^T y = V\left[\mathrm{diag}\left\{\frac{s_j}{s_j^2 + r}\right\}_{j=1}^{n}\right]U^T y$$

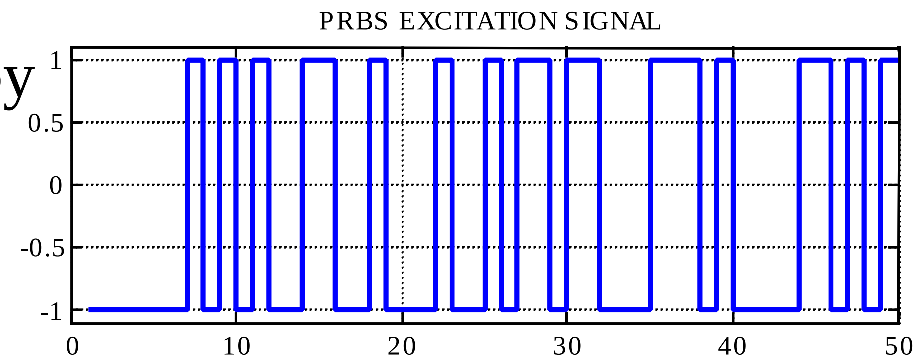
- Cut off the singular values of Φ that are smaller than r

$$\frac{s}{s^2 + 0.1}$$



Linear regression for FIR model

- Identifying impulse response by applying multiple steps
- PRBS excitation signal
- FIR (impulse response) model



$$y(t) = \sum_{k=1}^{K} h(k)u(t-k) + e(t)$$

PRBS =
Pseudo-Random Binary Sequence,
see `IDINPUT` in Matlab

- Linear regression representation

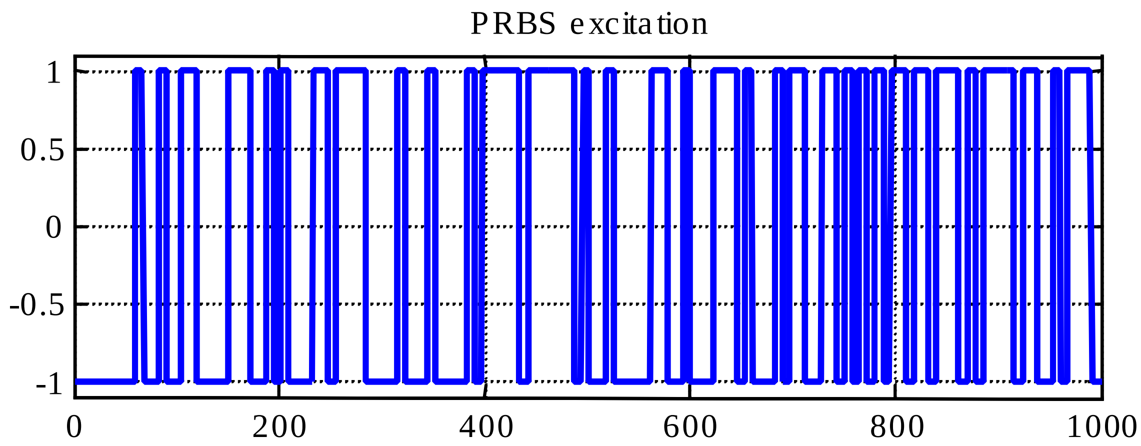
$$\varphi_1(t) = u(t-1)$$
$$\vdots$$
$$\varphi_K(t) = u(t-K)$$

$$\theta = \begin{bmatrix} h(1) \\ \vdots \\ h(K) \end{bmatrix}$$

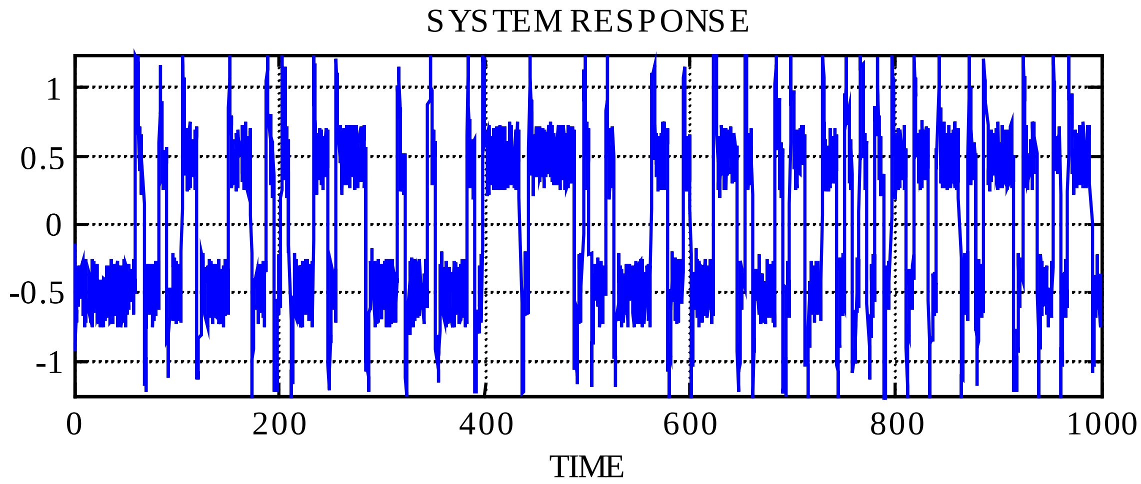
$$\hat{\theta} = \left(\Phi^T \Phi + rI\right)^{-1} \Phi^T y$$

Example: FIR model ID

- PRBS excitation input



- Simulated system output: 4000 samples, random noise of the amplitude 0.5

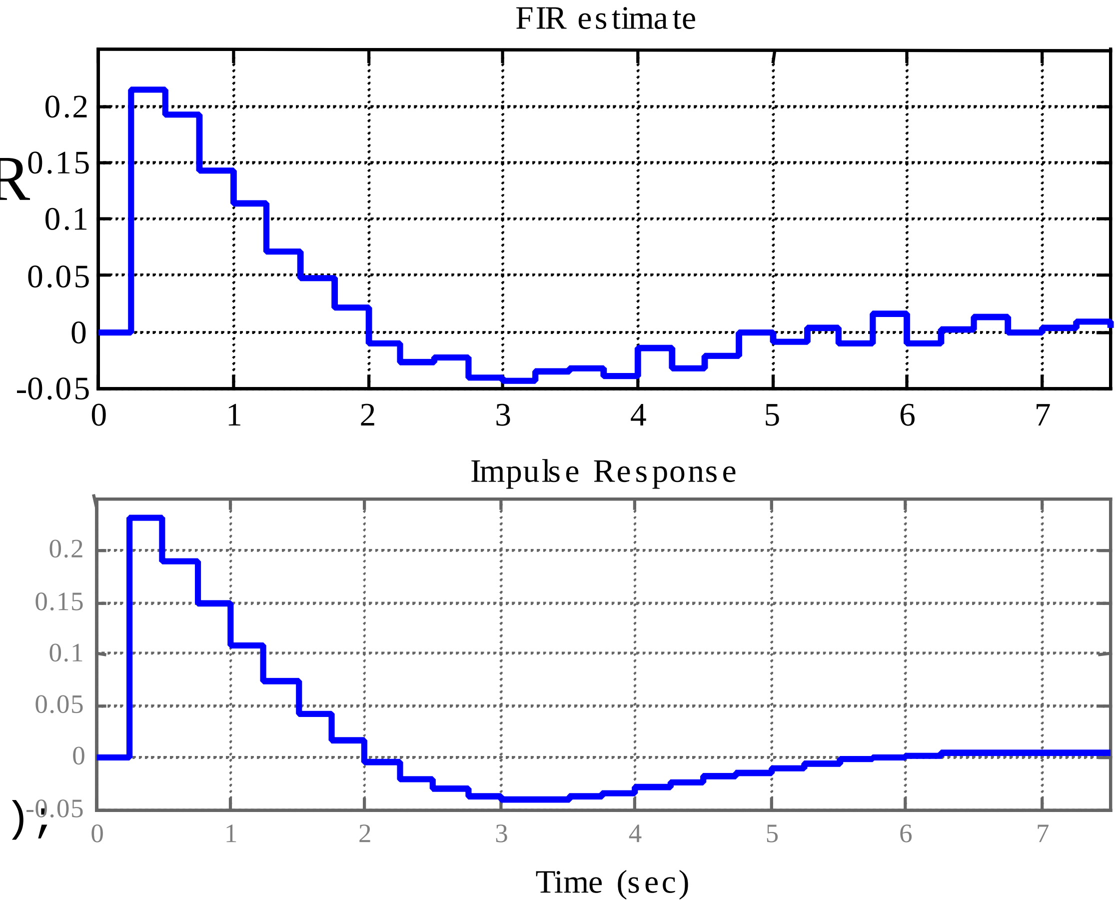


Example: FIR model ID

- Linear regression estimate of the FIR model

- Impulse response for the simulated system:

```
T=tf([1 .5],[1 1.1 1]);
P=c2d(T,0.25);
```



Nonlinear parametric model ID

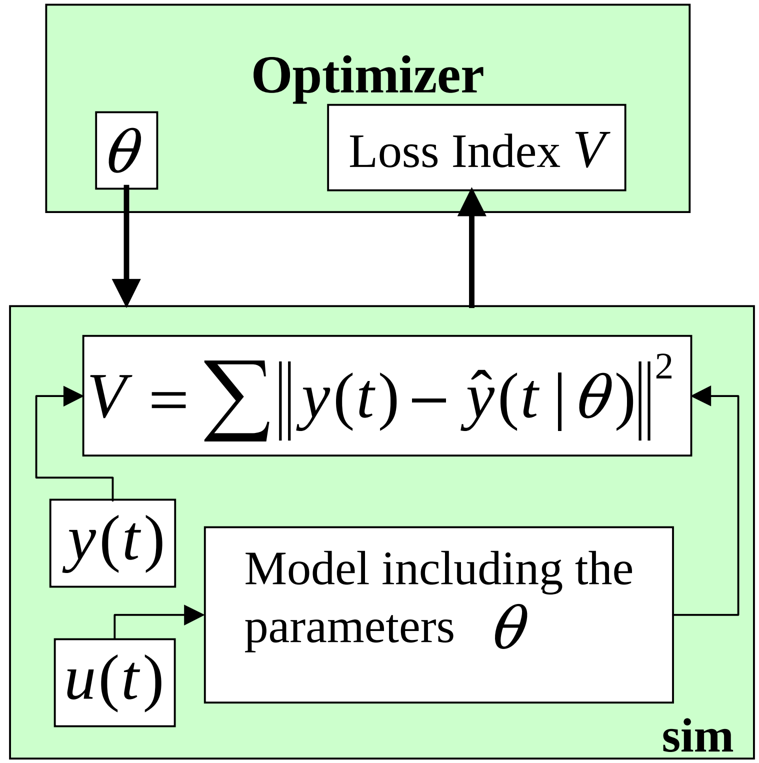
- Prediction model depending on the unknown parameter vector θ

$$u(t) \rightarrow \text{MODEL}(\theta) \rightarrow \hat{y}(t \mid \theta)$$

- Loss index

$$J = \sum \left\| y(t) - \hat{y}(t \mid \theta) \right\|^2$$

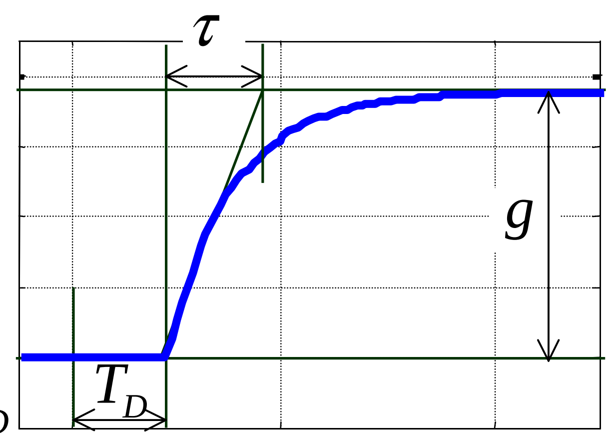
- Iterative numerical optimization. Computation of V as a subroutine



Lennart Ljung, "Identification for Control: Simple Process Models," *IEEE Conf. on Decision and Control*, Las Vegas, NV, 2002

Parametric ID of step response

- First order process with deadtime
- Most common industrial process model
- Response to a control step applied at t_B

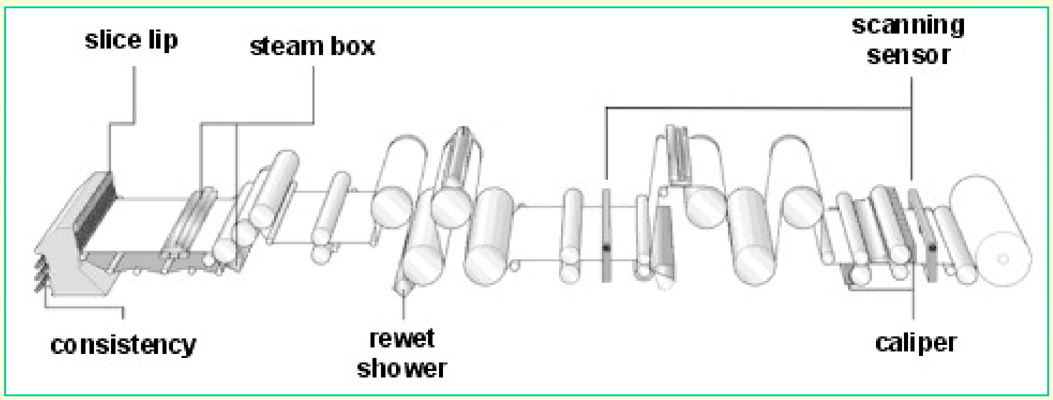


$$y(t\,|\,\theta) = \gamma + \begin{cases} g\left(1 - e^{(t - t_B - T_D)/\tau}\right), & \text{for } t > t_B - T_D \\ 0, & \text{for } t \le t_B - T_D \end{cases}$$

$$\theta = \begin{bmatrix} \gamma \\ g \\ \tau \\ T_D \end{bmatrix}$$

Example:

Paper machine process



Gain estimation

- For given τ, T_D, the modeled step response can be presented in the form

$$y(t\,|\,\theta) = \gamma + g \cdot y_1(t\,|\,\tau, T_D)$$

- This is a linear regression

$$y(t\,|\,\theta) = \sum_{k=1}^{2} w_k \varphi_k(t) \qquad \begin{aligned} w_1 &= g \qquad & \varphi_1(t) &= y_1(t\,|\,\tau, T_D) \\ w_2 &= \gamma \qquad & \varphi_2(t) &= 1 \end{aligned}$$

- Parameter estimate and prediction for given τ, T_D

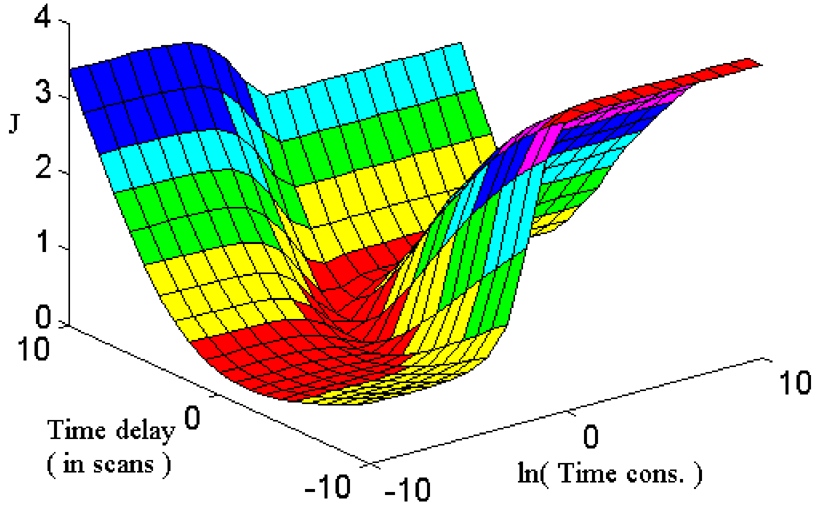
$$\hat{w}(\tau, T_D) = \left(\Phi^T \Phi\right)^{-1} \Phi^T y \qquad \hat{y}(t\,|\,\tau, T_D) = \hat{\gamma} + \hat{g} \cdot y_1(t\,|\,\tau, T_D)$$

Rise time/dead time estimation

- For given τ, T_D, the loss index is

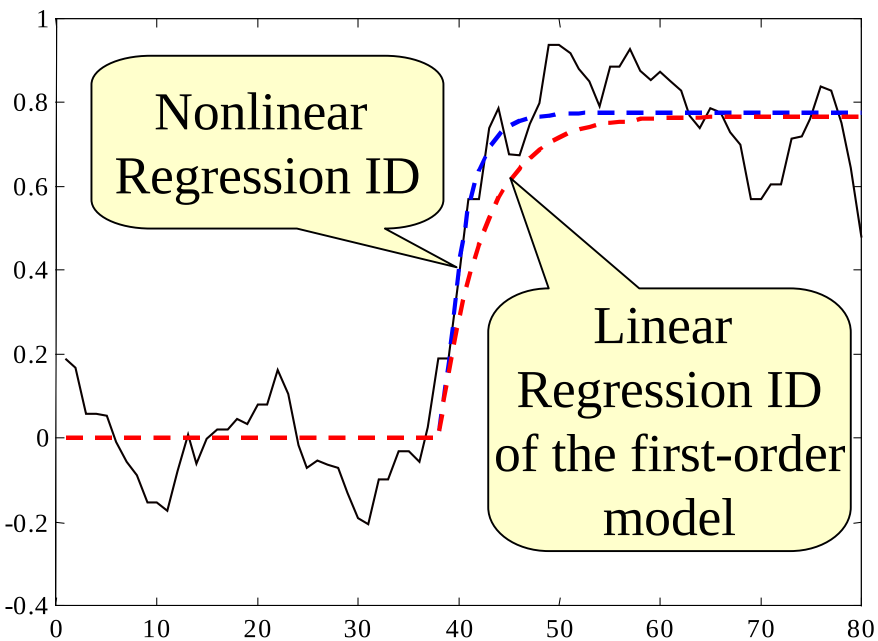
$$V = \sum_{t=1}^{N} \left| y(t) - \hat{y}(t \mid \tau, T_D) \right|^2$$

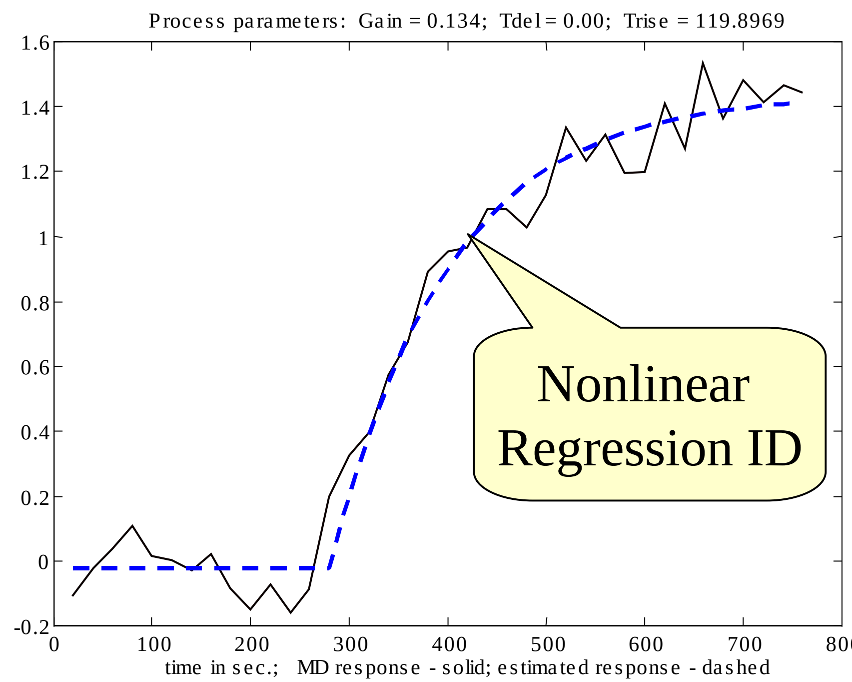
- Grid τ, T_D and find the minimum of $V = V(\tau, T_D)$



Examples: Step response ID

- Identification results for real industrial process data
- This algorithm works in an industrial tool used in 500+ industrial plants, many processes each





Linear filtering

- A trick that helps: pre-filter data
- Consider data model

$$y = h * u + e$$

- L is a linear filtering operator, usually LPF

$$\underbrace{Ly}_{y_f} = L(h * u) + \underbrace{Le}_{e_f}$$

$$L(h * u) = (Lh) * u = h * (Lu)$$

- Can estimate h from filtered y and filtered u
- Or can estimate filtered h from filtered y and 'raw' u
- Pre-filter bandwidth will limit the estimation bandwidth

Multivariable ID

- Apply SISO ID to various input/output pairs
- Need *n* tests - excite each input in turn

- Step/pulse response identification is a key part of the industrial Multivariable Predictive Control packages.