Lecture 8 - Model Identification

- What is system identification?
- Direct pulse response identification
- Linear regression
- Regularization
- Parametric model ID, nonlinear LS
What is System Identification?

• **White-box identification**
  – estimate parameters of a physical model from data
  – Example: aircraft flight model

• **Gray-box identification**
  – given generic model structure estimate parameters from data
  – Example: neural network model of an engine

• **Black-box identification**
  – determine model structure and estimate parameters from data
  – Example: security pricing models for stock market
Industrial Use of System ID

- Process control - most developed ID approaches
  - all plants and processes are different
  - need to do identification, cannot spend too much time on each
  - industrial identification tools
- Aerospace
  - white-box identification, specially designed programs of tests
- Automotive
  - white-box, significant effort on model development and calibration
- Disk drives
  - used to do thorough identification, shorter cycle time
- Embedded systems
  - simplified models, short cycle time
Impulse response identification

- Simplest approach: apply control impulse and collect the data

- Difficult to apply a short impulse big enough such that the response is much larger than the noise

- Can be used for building simplified control design models from complex sims
Step response identification

- Step (bump) control input and collect the data
  - used in process control

- Impulse estimate still noisy: \( \text{impulse}(t) = \text{step}(t) - \text{step}(t-1) \)
Noise reduction

Noise can be reduced by statistical averaging:

• Collect data for multiple steps and do more averaging to estimate the step/pulse response

• Use a parametric model of the system and estimate a few model parameters describing the response: dead time, rise time, gain

• Do both in a sequence
  – done in real process control ID packages

• Pre-filter data
Linear regression

- Mathematical aside
  - linear regression is one of the main System ID tools

\[ y(t) = \sum_{j=1}^{N} \theta_j \varphi_j(t) + e(t) \]

\[ y = \Phi \theta + e \]

\[
\begin{bmatrix}
y(1) \\
\vdots \\
y(N)
\end{bmatrix}, \quad
\Phi = \begin{bmatrix}
\varphi_1(1) & \ldots & \varphi_K(1) \\
\vdots & \ddots & \vdots \\
\varphi_1(N) & \ldots & \varphi_K(N)
\end{bmatrix}, \quad
\theta = \begin{bmatrix}
\theta_1 \\
\vdots \\
\theta_K
\end{bmatrix}, \quad
\begin{bmatrix}
e(1) \\
\vdots \\
e(N)
\end{bmatrix}
\]
Linear regression

- Makes sense only when matrix $\Phi$ is tall, $N > K$, more data available than the number of unknown parameters.
  - Statistical averaging
- Least square solution: $\|e\|^2 \rightarrow \min$
  - Matlab `pinv` or left matrix division
- Correlation interpretation:

$$y = \Phi \theta + e$$

$$\hat{\theta} = (\Phi^T \Phi)^{-1} \Phi^T y$$

$$\hat{\theta} = R^{-1} c$$

$$R = \frac{1}{N} \begin{bmatrix}
\sum_{t=1}^{N} \phi_1^2(t) & \ldots & \sum_{t=1}^{N} \phi_K(t) \phi_1(t) \\
\vdots & \ddots & \vdots \\
\sum_{t=1}^{N} \phi_1(t) \phi_K(t) & \ldots & \sum_{t=1}^{N} \phi_K^2(t)
\end{bmatrix},$$

$$c = \frac{1}{N} \begin{bmatrix}
\sum_{t=1}^{N} \phi_1(t) y(t) \\
\vdots \\
\sum_{t=1}^{N} \phi_K(t) y(t)
\end{bmatrix}.$$
Example: linear first-order model

\[ y(t) = ay(t-1) + gu(t-1) + e(t) \]

- Linear regression representation
  \[
  \begin{align*}
  \varphi_1(t) &= y(t-1) \\
  \varphi_2(t) &= u(t-1)
  \end{align*}
  \]

\[ \theta = \begin{bmatrix} a \\ g \end{bmatrix} \quad \hat{\theta} = (\Phi^T\Phi)^{-1} \Phi^T y \]

- This approach is considered in most of the technical literature on identification


- Matlab Identification Toolbox
  - Industrial use in aerospace mostly
  - Not really used much in industrial process control

- Main issue:
  - small error in \( a \) might mean large change in response
Regularization

• Linear regression, where $\Phi^T \Phi$ is ill-conditioned
• Instead of $\|e\|^2 \rightarrow \min$ solve a regularized problem
  $$\|e\|^2 + r\|\theta\|^2 \rightarrow \min$$
  $r$ is a small regularization parameter
• Regularized solution
  $$\hat{\theta} = (\Phi^T \Phi + rI)^{-1} \Phi^T y$$
• Cut off the singular values of $\Phi$ that are smaller than $r$
Regularization

- Analysis through SVD (singular value decomposition)
  \[ \Phi = USV^T; \quad V \in R^{n,n}; U \in R^{m,m}; S = \text{diag} \{ s_j \}_{j=1}^n \]
- Regularized solution
  \[ \hat{\theta} = (\Phi^T \Phi + rI)^{-1} \Phi^T y = V \left[ \text{diag} \left\{ \frac{s_j}{s_j^2 + r} \right\}_{j=1}^n \right] U^T y \]
- Cut off the singular values of \( \Phi \) that are smaller than \( r \)
Linear regression for FIR model

- Identifying impulse response by applying multiple steps
- PRBS excitation signal
- FIR (impulse response) model

\[ y(t) = \sum_{k=1}^{K} h(k)u(t - k) + e(t) \]

- Linear regression representation

\[ \varphi_1(t) = u(t - 1) \]
\[ \vdots \]
\[ \varphi_K(t) = u(t - K) \]

\[ \theta = \begin{bmatrix} h(1) \\ \vdots \\ h(K) \end{bmatrix} \]

\[ \hat{\theta} = \left( \Phi^T \Phi + rI \right)^{-1} \Phi^T y \]

PRBS = Pseudo-Random Binary Sequence, see IDINPUT in Matlab
Example: FIR model ID

- PRBS excitation input

- Simulated system output: 4000 samples, random noise of the amplitude 0.5
Example: FIR model ID

- Linear regression estimate of the FIR model

\[
T = \text{tf}([1, 0.5], [1, 1, 1, 1])
\]

\[
P = \text{c2d}(T, 0.25);
\]
Nonlinear parametric model ID

- Prediction model depending on the unknown parameter vector $\theta$
  
  $u(t) \rightarrow \text{MODEL}(\theta) \rightarrow \hat{y}(t \mid \theta)$

- Loss index
  
  $J = \sum \| y(t) - \hat{y}(t \mid \theta) \|^2$

- Iterative numerical optimization. Computation of $V$ as a subroutine

IEEE Conf. on Decision and Control, Las Vegas, NV, 2002
Parametric ID of step response

- First order process with deadtime
- Most common industrial process model
- Response to a control step applied at $t_B$

$$y(t \mid \theta) = \gamma + \left\{ \begin{array}{ll} g \left(1 - e^{(t-t_B-T_D)/\tau} \right), & \text{for } t > t_B - T_D \\ 0, & \text{for } t \leq t_B - T_D \end{array} \right.$$
Gain estimation

• For given $\tau, T_D$, the modeled step response can be presented in the form

$$y(t \mid \theta) = \gamma + g \cdot y_1(t \mid \tau, T_D)$$

• This is a linear regression

$$y(t \mid \theta) = \sum_{k=1}^{2} w_k \phi_k(t) \quad w_1 = g \quad \phi_1(t) = y_1(t \mid \tau, T_D)$$
$$w_2 = \gamma \quad \phi_2(t) = 1$$

• Parameter estimate and prediction for given $\tau, T_D$

$$\hat{w}(\tau, T_D) = \left( \Phi^T \Phi \right)^{-1} \Phi^T y \quad \hat{y}(t \mid \tau, T_D) = \hat{y} + \hat{g} \cdot y_1(t \mid \tau, T_D)$$
Rise time/dead time estimation

- For given \( \tau, T_D \), the loss index is
  \[
  V = \sum_{t=1}^{N} |y(t) - \hat{y}(t | \tau, T_D)|^2
  \]

- Grid \( \tau, T_D \) and find the minimum of \( V = V(\tau, T_D) \)
Examples: Step response ID

- Identification results for real industrial process data
- This algorithm works in an industrial tool used in 500+ industrial plants, many processes each

![Graph showing step response with linear and nonlinear regression models](image_url)

**Process parameters:** Gain = 0.134; Tdel = 0.00; Trise = 119.8969
Linear filtering

- A trick that helps: pre-filter data
- Consider data model
  \[ y = h^* u + e \]

- \( L \) is a linear filtering operator, usually LPF

\[
L_y = L(h^* u) + Le \\
y_f \quad e_f
\]

\[
L(h^* u) = (Lh)^* u = h^* (Lu)
\]

- Can estimate \( h \) from filtered \( y \) and filtered \( u \)
- Or can estimate filtered \( h \) from filtered \( y \) and ‘raw’ \( u \)
- Pre-filter bandwidth will limit the estimation bandwidth
Multivariable ID

• Apply SISO ID to various input/output pairs
• Need $n$ tests - excite each input in turn

• Step/pulse response identification is a key part of the industrial Multivariable Predictive Control packages.