Lecture 14 - Model Predictive Control Part 1: The Concept

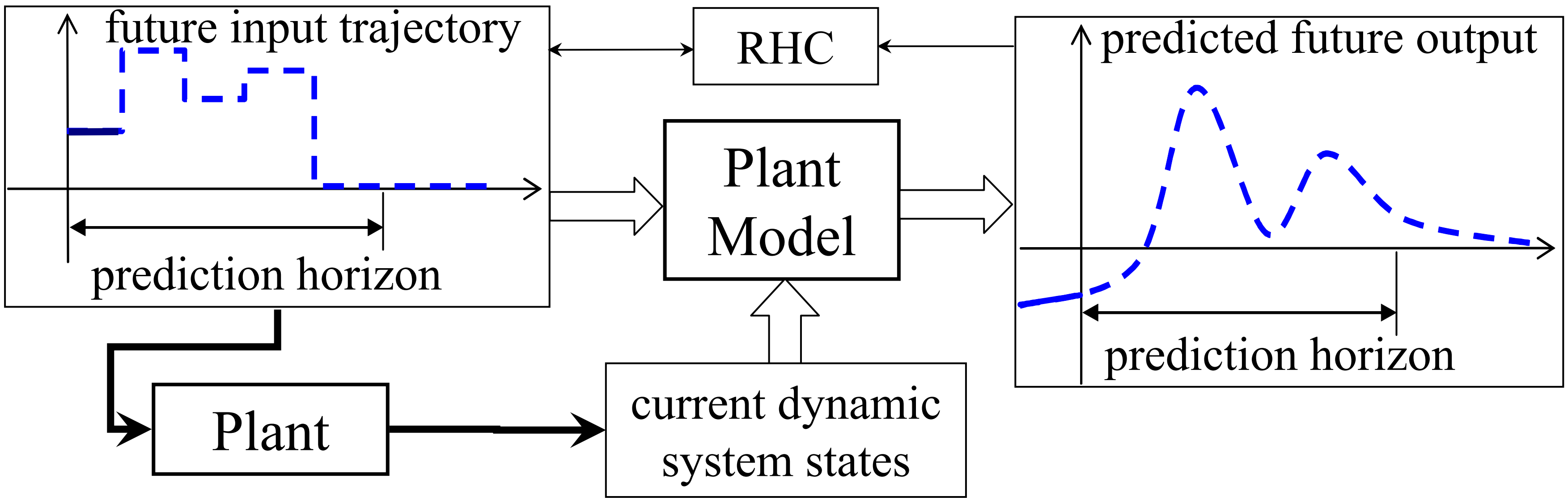
- History and industrial application resource:
  - Joe Qin, survey of industrial MPC algorithms
  - http://www.che.utexas.edu/~qin/cpcv/cpcv14.html
- Emerging applications
- State-based MPC
  - Conceptual idea of MPC
  - Optimal control synthesis
- Example
  - Lateral control of a car
- Stability
- Lecture 15: Industrial MPC

MPC concept

- MPC = Model Predictive Control
- Also known as
  - DMC = Dynamical Matrix Control
  - GPC = Generalized Predictive Control
  - RHC = Receding Horizon Control
- Control algorithms based on
  - Numerically solving an optimization problem at each step
  - Constrained optimization – typically QP or LP
  - Receding horizon control
- More details need to be worked out for implementation

Receding Horizon Control

- Receding Horizon Control concept



- At each time step, compute control by solving an open-loop optimization problem for the prediction horizon
- Apply the first value of the computed control sequence
- At the next time step, get the system state and re-compute

Current MPC Use

- Used in a majority of existing multivariable control applications
- Technology of choice for many new advanced multivariable control application
- Success rides on the computing power increase
- Has many important practical advantages

MPC Advantages

- Straightforward formulation, based on well understood concepts
- Explicitly handles constraints
- Explicit use of a model
- Well understood tuning parameters
  – Prediction horizon
  – Optimization problem setup
- Development time much shorter than for competing advanced control methods
- Easier to maintain: changing model or specs does not require complete redesign, sometimes can be done on the fly

History

- First practical application:
  - DMC – Dynamic Matrix Control, early 1970s at Shell Oil
  - Cutler later started Dynamic Matrix Control Corp.
- Many successful industrial applications
- Theory (stability proofs etc) lagging behind 10-20 years.
- See an excellent resource on industrial MPC
  - Joe Qin, Survey of industrial MPC algorithms
  - history and formulations
  - http://www.che.utexas.edu/~qin/cpcv/cpcv14.html

Some Major Applications

Area	DMC Corp.	Setpoint Inc.	Honeywell Profimatics	Adersa	Treiber Controls	Total
Refining	360	320	290	280	250	1500
Petrochemicals	210	40	40	-	-	290
Chemicals	10	20	10	3	150	193
Pulp and Paper	10	-	30	-	5	45
Gas	-	-	5	-	-	5
Utility	-	-	2	-	-	2
Air Separation	-	-	-	-	5	5
Mining/Metallurgy	-	2	-	7	6	15
Food Processing	-	-	-	41	-	41
Furnaces	-	-	-	42	-	42
Aerospace/Defense	-	-	-	13	-	13
Automotive	-	-	-	7	-	7
Other	10	20	-	45	-	75
Total	600	402	377	438	416	2233
First App	DMC:1985	IDCOM-M:1987 SMCA:1993	PCT:1984 RMPCT:1991	IDCOM:1973 HIECON:1986	OPC:1987	
Largest App	603x283	35x28	28x20	-	24x19	

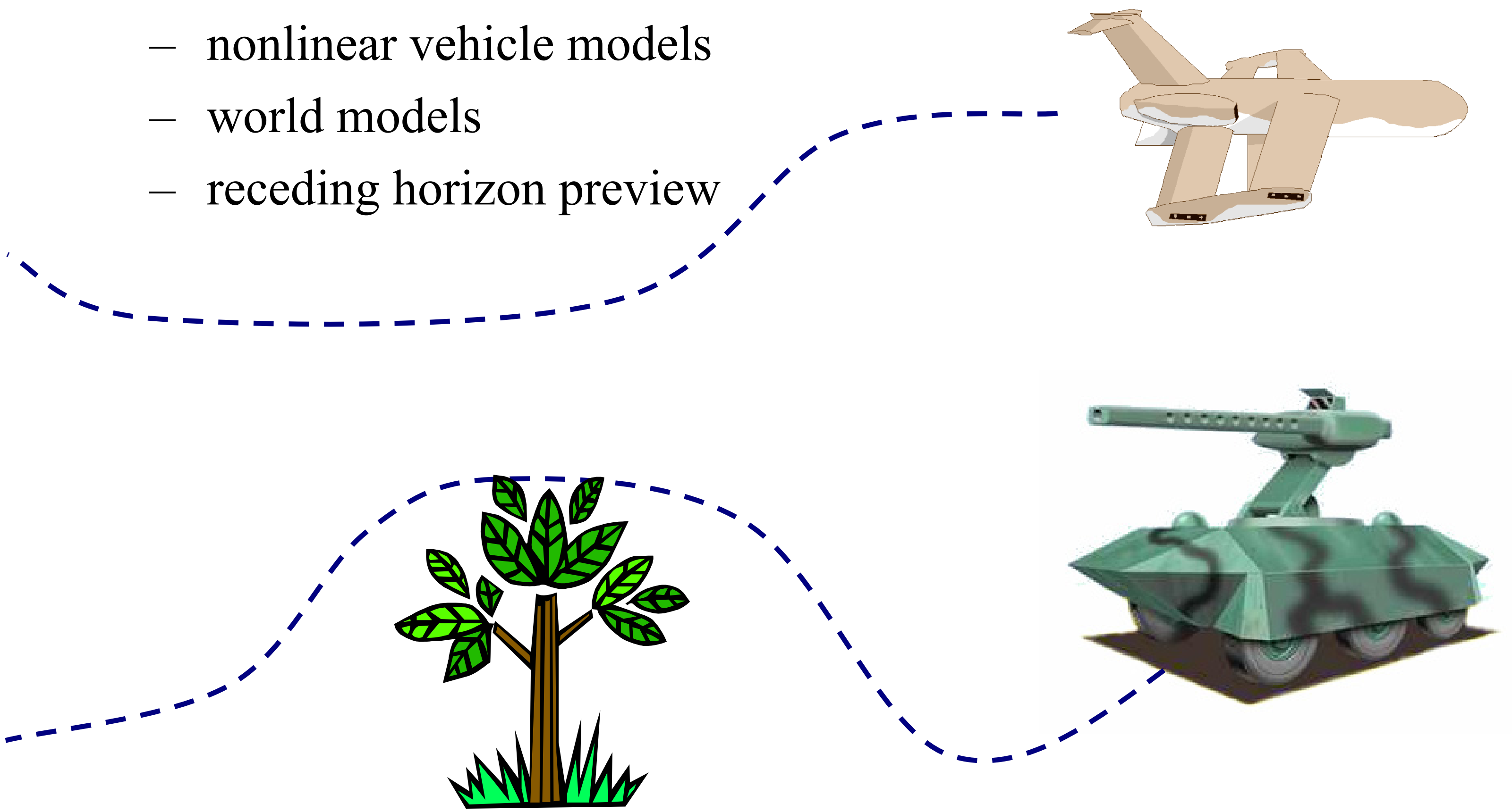
- From Joe Qin, http://www.che.utexas.edu/~qin/cpcv/cpcv14.html
- 1995 data, probably 1-2 order of magnitude growth by now

Emerging MPC applications

- Nonlinear MPC
  - just need a computable model (simulation)
  - NLP optimization
- Hybrid MPC
  - discrete and parametric variables
  - combination of dynamics and discrete mode change
  - mixed-integer optimization (MILP, MIQP)
- Engine control
- Large scale operation control problems
  - Operations management (control of supply chain)
  - Campaign control

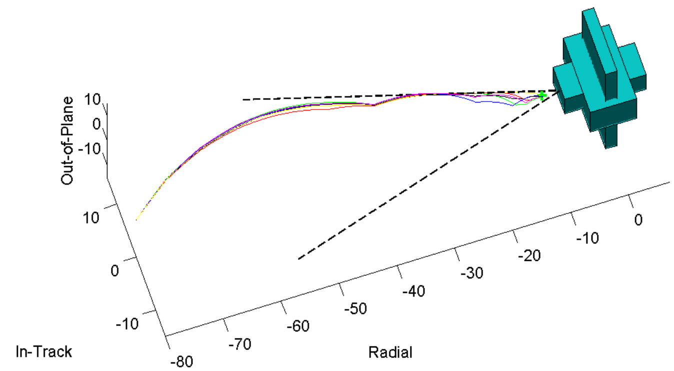
Emerging MPC applications

- Vehicle path planning and control
  - nonlinear vehicle models
  - world models
  - receding horizon preview



Emerging MPC applications

- Spacecraft rendezvous with space station
  - visibility cone constraint
  - fuel optimality



From Richards & How, MIT

- Underwater vehicle guidance

- Missile guidance

State-based control synthesis

- Consider single input system for better clarity

$$x(t+1) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t)$$

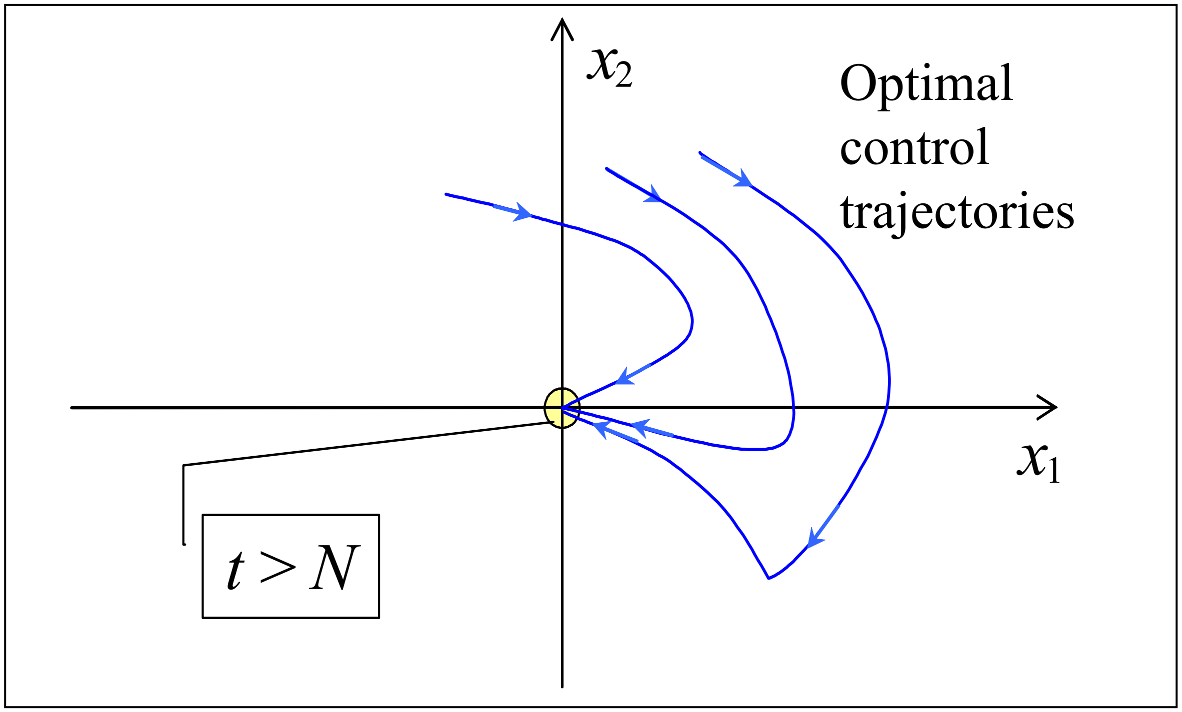
- Infinite horizon optimal control

$$\sum_{\tau=t+1}^{\infty} \left(y(\tau)\right)^2 + r\left(u(\tau) - u(\tau-1)\right)^2 \to \min$$

$$\text{subject to : } \quad |u(\tau)| \le u_0$$

- Solution = Optimal Control Synthesis

State-based MPC – concept



- Optimal control trajectories converge to (0,0)
- If N is large, the part of the problem for $t > N$ can be neglected
- Infinite-horizon optimal control ≈ horizon-N optimal control

State-based MPC

- Receding horizon control; *N*-step optimal

$$J = \sum_{\tau=t+1}^{t+N} \left(y(\tau)\right)^2 + r\left(u(\tau) - u(\tau-1)\right)^2 \rightarrow \min$$

$$\text{subject to: } \left|u(\tau)\right| \le u_0,$$

$$x(t+1) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t)$$

- Solution ≈ Optimal Control Synthesis

$$x(t) \rightarrow \left[\text{MPC Problem Solver}\right] \rightarrow u(t)$$

Predictive Model

- Predictive system model

$$Y = Gx + HU + Fu$$ initial condition response + control response

Predicted output

$$Y = \begin{bmatrix} y(t+1) \\ \vdots \\ y(t+N) \end{bmatrix}$$

Future control input

$$U = \begin{bmatrix} u(t+1) \\ \vdots \\ u(t+N) \end{bmatrix}$$

Current state (initial condition)

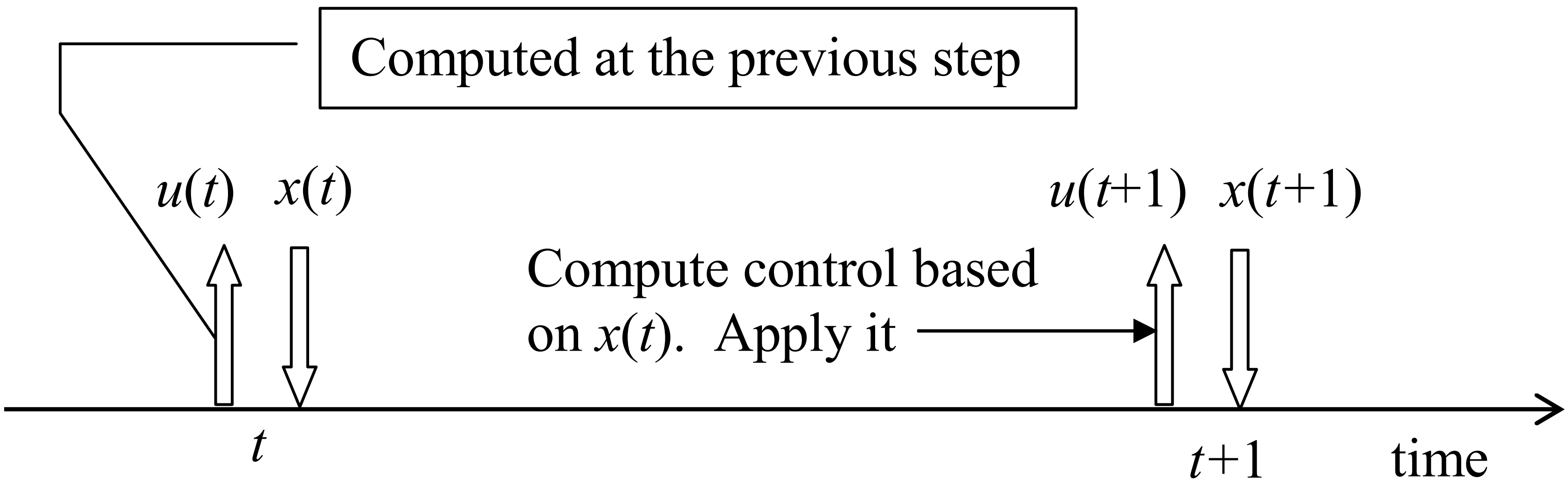
$$x = x(t)$$

$$u = u(t)$$ →computed at the previous step

- Model matrices

$$G = \begin{bmatrix} CA \\ \vdots \\ CA^n \end{bmatrix} \quad H = \begin{bmatrix} 0 & 0 & \dots & 0 \\ CB & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{N-2}B & CA^{N-3}B & \dots & 0 \end{bmatrix} = \begin{bmatrix} h(1) & 0 & \dots & 0 \\ h(2) & h(1) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ h(N) & h(N-1) & \dots & h(1) \end{bmatrix} \quad F = \begin{bmatrix} h(2) \\ h(3) \\ \vdots \\ h(N+1) \end{bmatrix}$$

Computations Timeline



- Assume that control u is applied and the state x is sampled at the same instant t
- Entire sampling interval is available for computing u

MPC Optimization Problem Setup

- MPC optimization problem

$$J = Y^T Y + rU^T D^T DU \to \min$$

$$\text{subject to : } \; |U| \le u_0,$$

$$Y = Gx + HU + Fu$$

1st difference matrix

$$D = \begin{bmatrix} 1 & -1 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix}$$

- This is a QP problem

- Solution

$$x(t) \to \left[\text{MPC Problem, QP Solver}\right] \to U \to u(t+1) = U(1)$$

QP solution

- QP Problem:

$$AU \le b$$

$$J = \frac{1}{2}U^T Q U + f^T U \to \min$$

$U = U(t)$ Predicted control sequence

$$Q = rD^T D + H^T H$$

$$f = H^T\left(Gx + Fu\right)$$

$$A = \begin{bmatrix} I \\ -I \end{bmatrix} \qquad b = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \cdot u_0$$

- Standard QP codes can be used

Linear MPC

- Nonlinearity is caused by the constraints
- If constraints are inactive, the QP problem solution is

$$U = Q^{-1} f \qquad\qquad u = l^T U \qquad\qquad l = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

- This is linear state feedback

$$u(t+1) = l^T \left(rD^T D + H^T H \right)^{-1} H^T \left(Gx(t) + Fu(t) \right)$$

$$u = z^{-1} Kx + z^{-1} Su$$

$$K = l^T \left(rD^T D + H^T H \right)^{-1} H^T G \qquad\qquad S = l^T \left(rD^T D + H^T H \right)^{-1} H^T F$$

- Can be analyzed as a linear system, e.g., check eigenvalues

$$u = \frac{z^{-1}}{1 - Sz^{-1}} Kx \qquad\qquad zx = Ax + Bu$$

Nonlinear MPC Stability

- **Theorem -** from Bemporad et al (1994)

  Consider a MPC algorithm for a linear plan with constraints. Assume that there is a terminal constraint

  $x(t+N)=0$ for predicted state x and

  $u(t+N)=0$ for computed future control u

  If the optimization problem is feasible at time t,

  then the coordinate origin is stable.

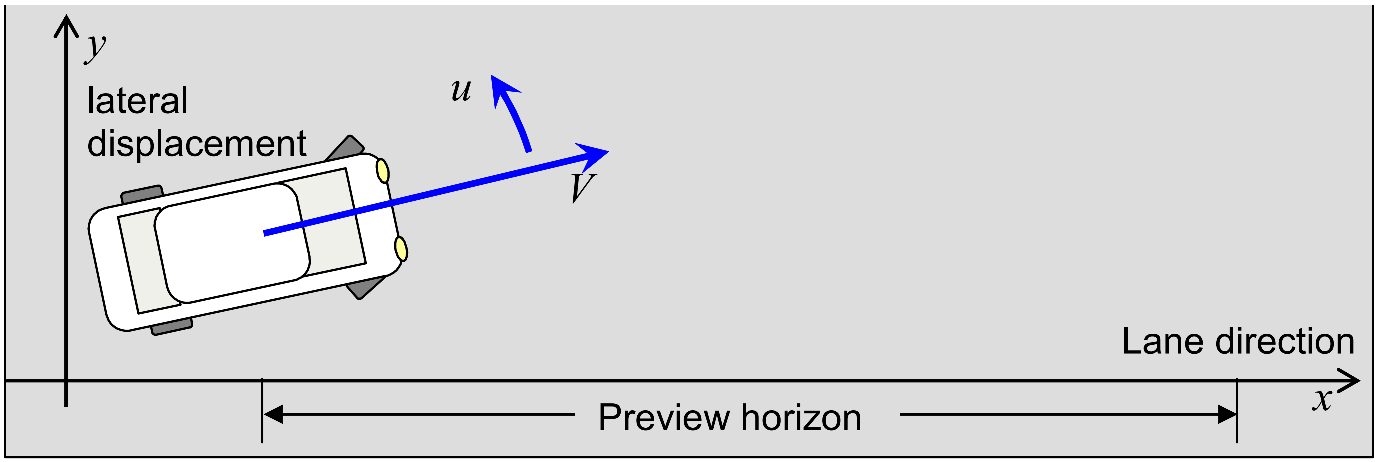
Proof.

  Use the performance index J as a Lyapunov function. It decreases along the finite feasible trajectory computed at time t. This trajectory is suboptimal for the MPC algorithm, hence J decreases even faster.

MPC Stability

- The analysis could be useful in practice
  - Theory says a terminal constraint is good
- MPC stability formulations
  (Mayne et al, *Automatica*, 2000)
- Terminal equality constraint
- Terminal cost function
  - Dual mode control – infinite horizon
- Terminal constraint set
  - Increase feasibility region
- Terminal cost and constraint set

Example: Lateral Control of a Car



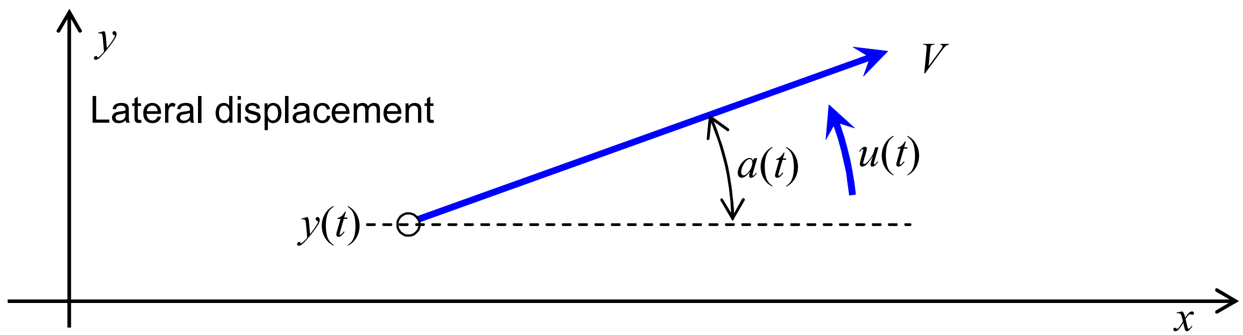
- Preview Control – MacAdam's driver model (1980)
- Consider predictive control design
- Simple kinematical model of a car driving at speed V

$$\dot{x} = V\cos a$$

$$\dot{y} = V\sin a \qquad \text{lateral displacement}$$

$$\dot{a} = u \qquad \text{steering}$$

Lateral Control of a Car - Model



- Assume a straight lane – tracking a straight line
- Linearized system: assume $a << 1$

$$\sin a \approx a \qquad \dot{y} = Va$$

$$\cos a \approx 1 \qquad \dot{a} = u$$

- Sampled-time equations (sampling time T_s)

$$a(t+1) = a(t) + u(t)T_s$$

$$y(t+1) = y(t) + a(t)VT_s + u(t)\cdot 0.5VT_s^{\ 2}$$

Lateral Control of a Car - MPC

State-space system: $x(t+1) = Ax(t) + Bu(t)$

$$x(t) = \begin{bmatrix} a(t) \\ y(t) \end{bmatrix} \qquad A = \begin{bmatrix} 1 & 0 \\ VT_s & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} T_s \\ 0.5VT_s^{\,2} \end{bmatrix}$$

Observation: $y(t) = Cx(t)$ $\qquad C = \begin{bmatrix} 0 & 1 \end{bmatrix}$

- Formulate predictive model

$$Y = Gx + HU + Fu$$

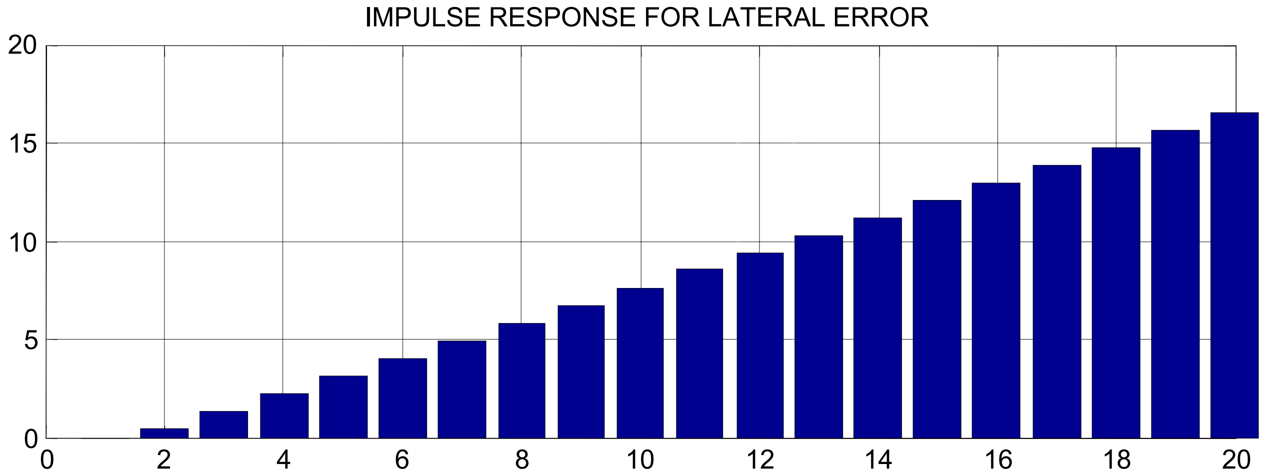
- MPC optimization problem

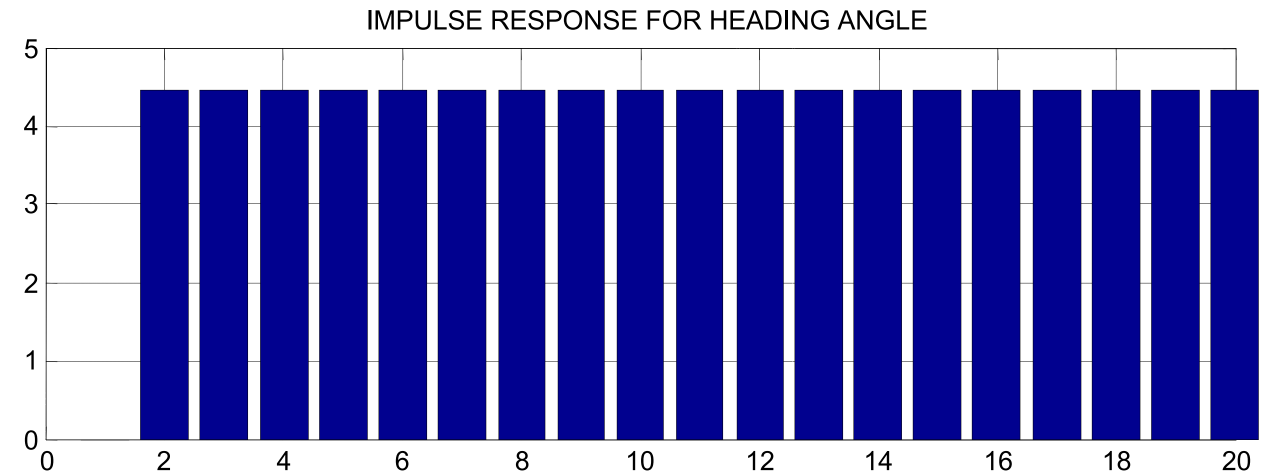
$$J = \left(Gx + HU + Fu\right)^T \left(Gx + HU + Fu\right) + rU^T D^T DU \to \min$$

subject to : $|U| \le u_0$,

- Solution: $x(t) \to [\text{MPC QP}] \to U \to u(t+1) = U(1)$

Impulse Responses

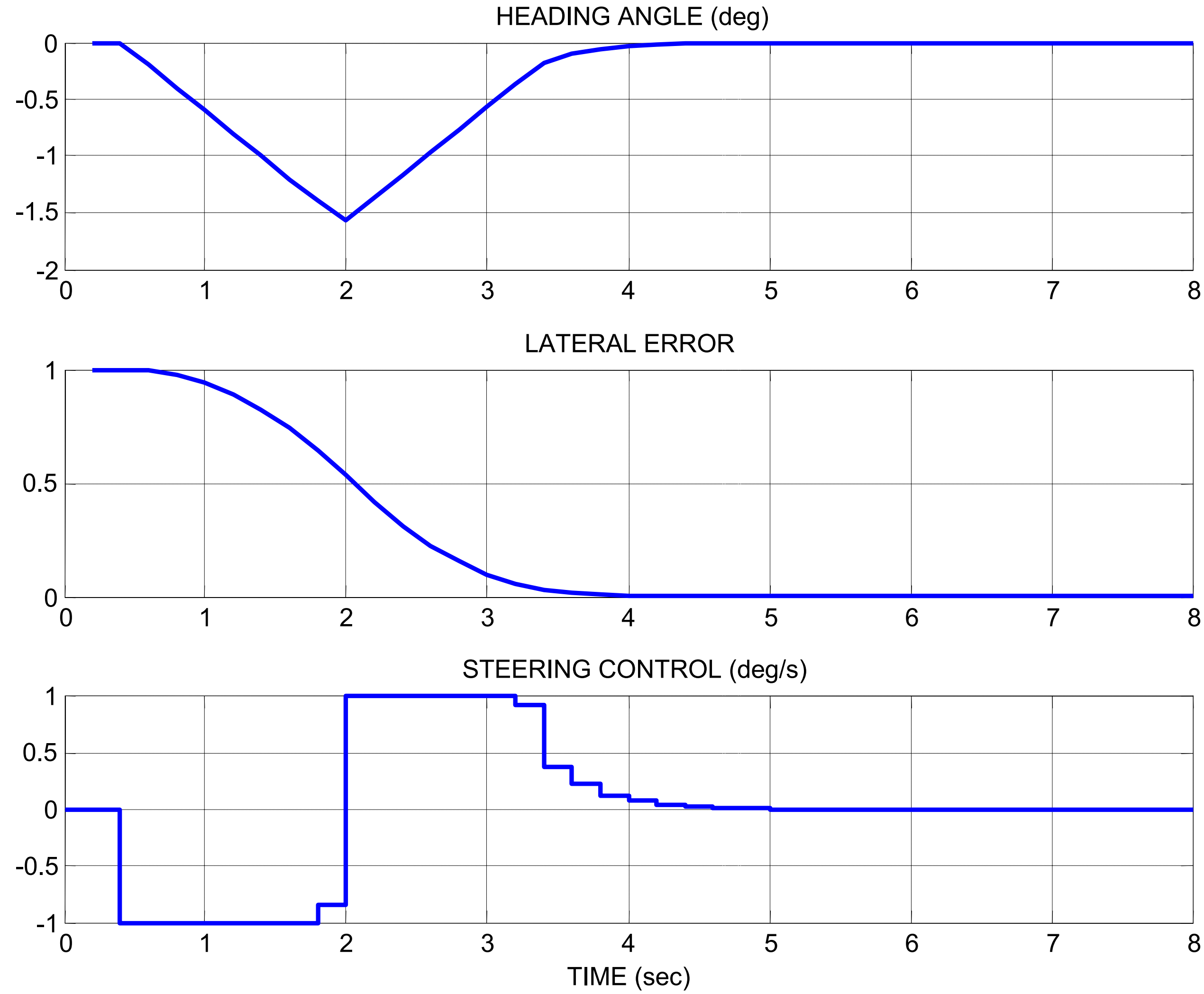




Lateral Control of a Car - Simulation

Simulation Results:

- $V = 50$ mph
- Sample time of 200ms
- $N = 20$
- All variables in SI units
- $r = 1$



Control Design Issues

- Several important issues remain
  - They are not visible in this simulation
  - Will be discussed in Lecture 15 (MPC, Part 2)
- All states might not be available
- Steady state error
  - Need integrator feedback
- Large angle deviation
  - linearized model deficiency
  - introduce soft constraint