Name: ______________________________ Lab section/TA: ______________________________

ENGR 40M Problem Set 3

Due 1:30pm, July 21, 2017

Homework should be submitted on Gradescope, at `http://www.gradescope.com/`. The entry code to enroll in the course is available at `https://web.stanford.edu/class/engr40m/restricted/gradescope.html`.

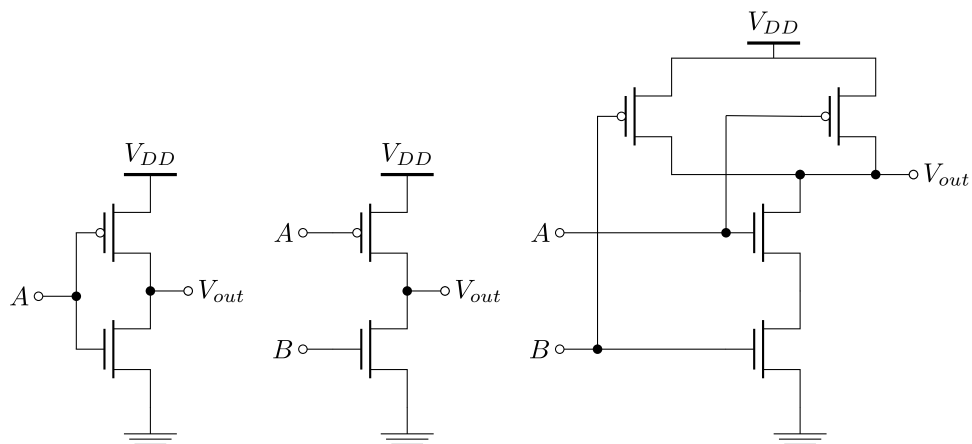
Problem 1: CMOS Logic Gates

(12 points) The output of a valid logic gate should always be connected to either V_{DD} or ground, and never both at the same time. For each of the three circuits below:

(i) State whether or not it is a valid logic gate.

(ii) If it does not form a valid logic gate, state why. If it is a valid logic gate, give a boolean expression for its function.

Assume the wires labelled A and B are digital logic signals that can be either V_{DD} or ground (“high” or “low”, respectively, and never both at the same time).



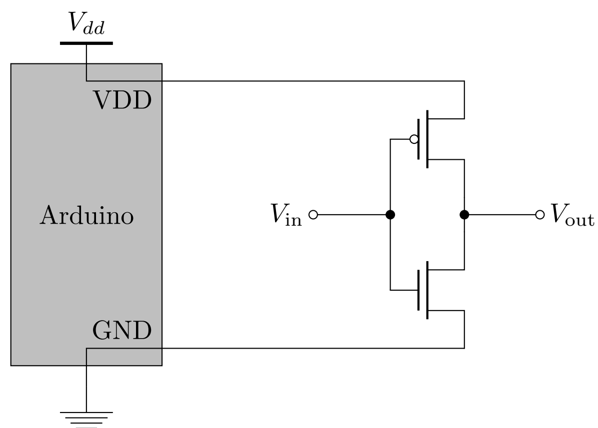
Problem 2: OR gate design with transistors

(10 points) Draw a schematic for a **CMOS** three input OR gate. The logic formula for the gate is $(A \mid\mid B \mid\mid C)$ i.e., the output is high if **any** of the inputs are high. Be sure to clearly label the inputs and output.

Hint: Try to construct a NOR gate first, where $\text{NOR}(A, B, C)$ *has logical formula* $!(A \mid\mid B \mid\mid C)$*. Note that* $\text{OR}(A, B, C) = \text{NOT}(\text{NOR}(A, B, C))$

Problem 3: When your transistors get hot!

(8 points) Below is a power inverter that will be used in lab 2b, when we add the Arduino to our useless box. Assume $V_{DD} = 5\,\mathrm{V}$. The transistor datasheets say the PMOS threshold voltage is $0.7\,\mathrm{V}$, and the NMOS threshold voltage is anywhere between 1 and $2\,\mathrm{V}$.



A team of students built their power inverter and connected it to the V_{dd} and ground pins of their Arduino. They wanted to test their code before connecting it to the Arduino, so they left V_{in} disconnected from the I/O pin on the Arduino. After a while, they noticed that the transistors became rather warm.

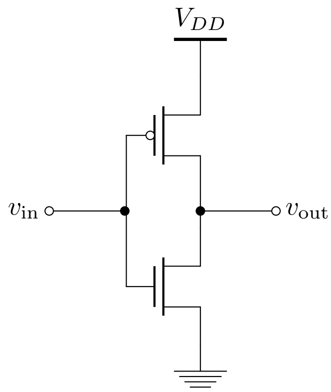
(a) Explain why a floating input can, but is not guaranteed to, cause the transistors to become warm.

Note: *Do **not** make this mistake in the lab. Always ensure that the inputs to the inverters are connected to V_{dd}, ground or a* driven *output pin.*

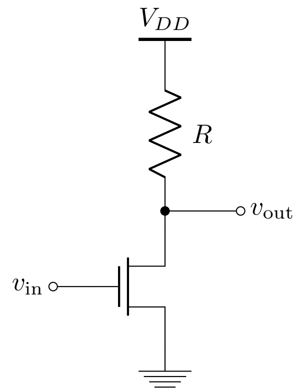
(b) From the transistor datasheets, the on resistance of a PMOS is $0.05\,\Omega$, and the on resistance of an NMOS is $0.2\,\Omega$. The off resistances are so big they don't bother to say what the values are. In the scenario above where the transistors are getting warm, how much power is each transistor dissipating?

Problem 4: Inverter power consumption

(12 points) In class, we studied the CMOS inverter, shown below on the left. It is also possible to implement an inverter using only an NMOS transistor and a resistor, a technology called *NMOS logic*. An NMOS logic implementation of an inverter is shown on the right. Assume that $V_{DD} = 5\,\mathrm{V}$.



CMOS design



NMOS design

In this problem, we will compare the power consumption in each of these circuits. Assume that all transistors are ideal MOS transistor switches.

(a) How much power is consumed by the CMOS design, when $v_{\mathrm{in}} = 0\,\mathrm{V}$?

(b) How much power is consumed by the CMOS design, when $v_{\mathrm{in}} = 5\,\mathrm{V}$?

(c) How much power is consumed by the NMOS design, when $v_{\mathrm{in}} = 0\,\mathrm{V}$? (Give your answer in terms of R.)

(d) How much power is consumed by the NMOS design, when $v_{\mathrm{in}} = 5\,\mathrm{V}$? (Give your answer in terms of R.)

(e) If the input to each inverter is high for half the time (and low for the other half), is there a value of R that could allow the *average* power consumed by the NMOS design to match that of the CMOS design, while still allowing the NMOS design to function correctly? If so, what can R be; or if not, why not?

Now, we will consider the internal resistance model of MOS transistors. Assume that for all transistors, $R_{\text{on}} = 10\,\Omega$ and $R_{\text{off}} = 1\,\text{M}\Omega$, and assume (when calculating averages) that the input is high for half the time.

(f) Find the average power consumed by the CMOS design.

(g) For reasons we'll discuss in a few weeks, we have to keep the value of R in the NMOS design relatively small. Suppose that R is $5\,\text{k}\Omega$. Find the average power consumed by the NMOS design, and compare this to the CMOS design.

Remark: There is also power consumed in the act *of switching a transistor, so there is more to power consumption than just resistive loss. We'll come back to this in a couple weeks.*

Problem 5: Binary arithmetic and negative numbers

(12 points) Representing positive numbers in binary is straightforward, but negative numbers are a little more tricky. One solution would be to use the most significant bit (MSB) to represent the sign, and to use the rest of the bits to represent the magnitude of the number. However, this requires some rather complex logic to detect the sign and handle addition and subtraction properly.

A better solution, and the one most commonly used, is known as "two's complement notation." In two's complement, positive numbers are represented just as usual, and the MSB of a positive number is always zero. To find the two's-complement representation for a negative number, take the binary representation of its absolute value, invert all of its bits, and add one.

As an example, to represent -1 with four bits, you would take the following steps:

1. The 4-bit binary representation of $|-1| = 1$ is 0001
2. Inverting all bits, we get 1110
3. Adding 1, we get 1111

(a) Find the 7-bit two's complement representation for the decimal number -21.

(b) Use your answer to (a) to find $-21 + 9$ in two's complement representation.

(c) Show that the sum of $-1 + 1 = 0$ using 7-bit two's complement math.

(d) For each of the following bits of code, what number will be printed? Briefly explain why. Assume that an `int` is 16 bits, and a `char` is 8 bits (which is the case in most Arduino boards).
Hint 1: Each of these is doing something a little bit weird.
Hint 2: If you're stuck or have doubts, try running the code on your Arduino.

```
char x = -33;
unsigned char y = x;
Serial.println(y);
```

```
char x = 128;
int y = x;
Serial.println(y);
```

```
unsigned int x = 0;
unsigned int y = x - 3;
Serial.println(y);
```

```
int x = 0;
int y = x - 2;
unsigned int z = y;
Serial.println(z);
```

Problem 6: Reflection

(2 points)

(a) How long did it take you to complete this assignment?

(b) Which problem was the most difficult, and why?