Capacitors and inductors

ENGR 40M lecture notes — July 21, 2017
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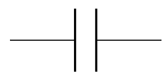
Unlike the components we've studied so far, in *capacitors* and *inductors*, the relationship between current and voltage doesn't depend only on the present. Capacitors and inductors *store* electrical energy—capacitors in an electric field, inductors in a magnetic field. This enables a wealth of new applications, which we'll see in coming weeks.

Quick reference

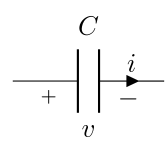
	Capacitor	**Inductor**
Symbol		
Stores energy in	electric field	magnetic field
Value of component (unit)	capacitance, C (farad, F)	inductance, L (henry, H)
I–V relationship	$i = C\dfrac{dv}{dt}$	$v = L\dfrac{di}{dt}$
At steady state, looks like	open circuit	short circuit

General behavior

In order to describe the voltage–current relationship in capacitors and inductors, we need to think of voltage and current as *functions of time*, which we might denote $v(t)$ and $i(t)$. It is common to omit (t) part, so v and i are implicitly understood to be functions of time.

The voltage v across and current i through a capacitor with capacitance C are related by the equation

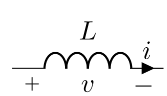
$$i = C\frac{dv}{dt},$$



where $\frac{dv}{dt}$ is the rate of change of voltage with respect to time.[1] From this, we can see that an sudden change in the voltage across a capacitor—however minute—would require infinite current. This isn't physically possible, so a capacitor's voltage *can't change instantaneously*. More generally, capacitors *oppose* changes in voltage—they tend to "want" their voltage to change "slowly".

Similarly, in an inductor with inductance L,

$$v = L\frac{di}{dt}.$$



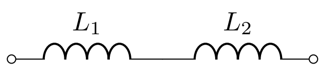
An inductor's current can't change instantaneously, and inductors oppose changes in current.

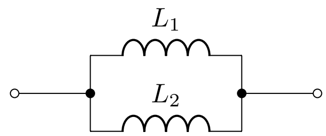
Note that we're following the *passive sign convention*, just like for resistors.

[1] That is, the derivative of voltage with respect to time. If you haven't studied calculus, think of this as the slope of the curve at a given time t, if you draw a graph of voltage against time.

Combinations in series and parallel

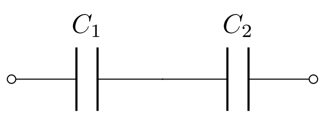
Inductors combine similarly to resistors:

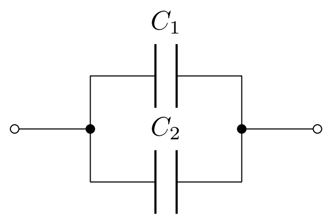




$$L_s = L_1 + L_2 \qquad L_p = \left(\frac{1}{L_1} + \frac{1}{L_2}\right)^{-1} = \frac{L_1 L_2}{L_1 + L_2}$$

Capacitors, however, are the other way round:



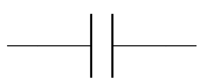


$$C_s = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = \frac{C_1 C_2}{C_1 + C_2} \qquad C_p = C_1 + C_2$$

Steady state analysis

Steady state refers to the condition where voltage and current are no longer changing. Most circuits, left undisturbed for sufficiently long, eventually settle into a steady state. In a circuit that is in steady state, $\frac{dv}{dt} = 0$ and $\frac{di}{dt} = 0$ for all voltages and currents in the circuit—including those of capacitors and inductors.

Thus, at steady state, in a capacitor, $i = C\frac{dv}{dt} = 0$, and in an inductor, $v = L\frac{di}{dt} = 0$. That is, in steady state, *capacitors look like open circuits*, and *inductors look like short circuits*, regardless of their capacitance or inductance.

 in steady state, looks like

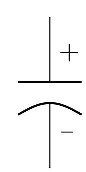
 in steady state, looks like

(This might seem trivial now, but we'll use this fact repeatedly in more complex situations later.)

Practical matters

Manufacturers typically specify a *voltage rating* for capacitors, which is the maximum voltage that is safe to put across the capacitor. Exceeding this can break down the dielectric in the capacitor.

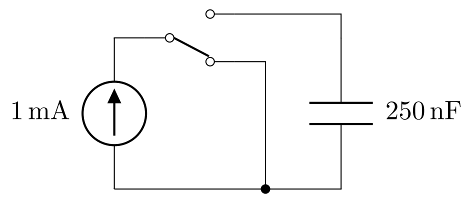
Capacitors are not, by nature, *polarized*: it doesn't normally matter which way round you connect them. However, some capacitors *are* polarized—in particular, electrolytic capacitors, where the insulator is a very thin oxide layer that would be depleted if a negative voltage is applied. In schematics, if a capacitor is polarized, we use a special symbol for it, as shown right.



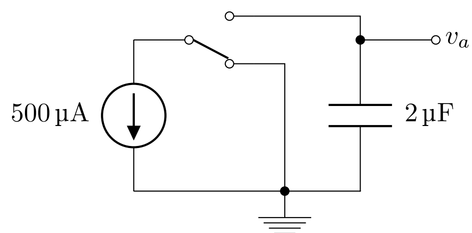
polarized capacitor

Examples

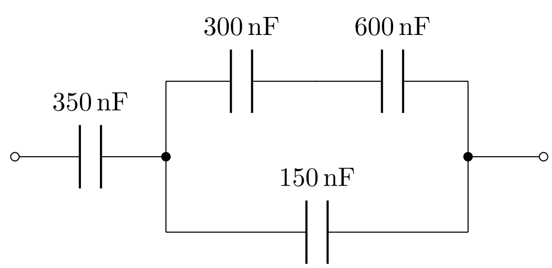
Example 1. In the (contrived) circuit below, at $t = 0$, the voltage across the capacitor is $0\,\mathrm{V}$. The switch begins on the lower throw, moves to the upper throw at $t = 0\,\mathrm{ms}$, then moves back to the lower throw at $t = 1\,\mathrm{ms}$. Draw a graph of the voltage across the capacitor as a function of time.



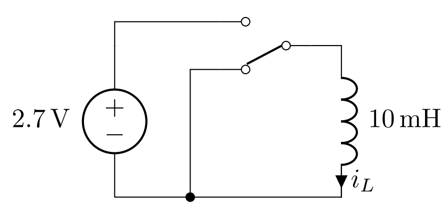
Example 2. In the (contrived) circuit below, at $t = 0\,\mathrm{ms}$, $v_a = 10\,\mathrm{V}$. The switch begins on the lower throw, and moves to the upper throw at $t = 0\,\mathrm{ms}$. When does the capacitor's voltage to reach $0\,\mathrm{V}$?



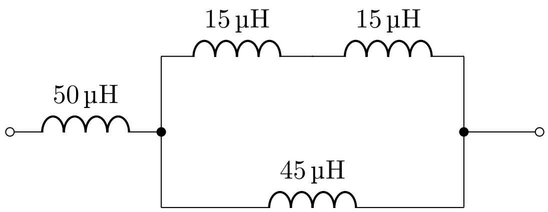
Example 3. You wish to replace the following network of capacitors with a single capacitor. What should its value be?



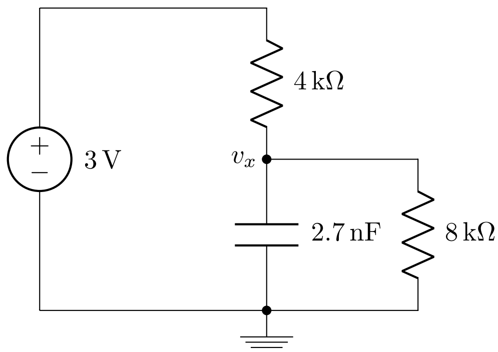
Example 4. In the (contrived) circuit below, at $t = 0$, the current through the inductor is $100\,\mathrm{mA}$ *upwards*. The switch begins on the lower throw, moves to the upper throw at $t = 0\,\mathrm{ms}$, then moves back to the lower throw at $t = 1\,\mathrm{ms}$. Draw a graph of the current through the inductor i_L, *with reference direction going downwards*, as a function of time.



Example 5. Find the total inductance of this inductor network.

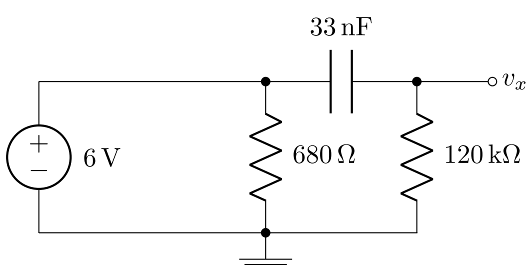


Example 6. The circuit below has reached steady state. What is the voltage v_x?



Example 7. The circuit below has reached steady state.

(a) What is the voltage v_x?
(b) What is the voltage across the capacitor? (Be sure to specify the direction.)



Example 8. The circuit below has reached steady state. What is the voltage v_x?