E40M

Solving Circuits using Nodal Analysis, Part II and EveryCircuit™

The Key Idea from Last Lecture
Systematic Nodal Analysis

1. Label all the nodes (V_A, V_B, or V_1, V_2, etc.), after selecting the node you choose to be Gnd.

2. Label all the branch currents (i_1, i_2, etc.) and choose directions for each of them

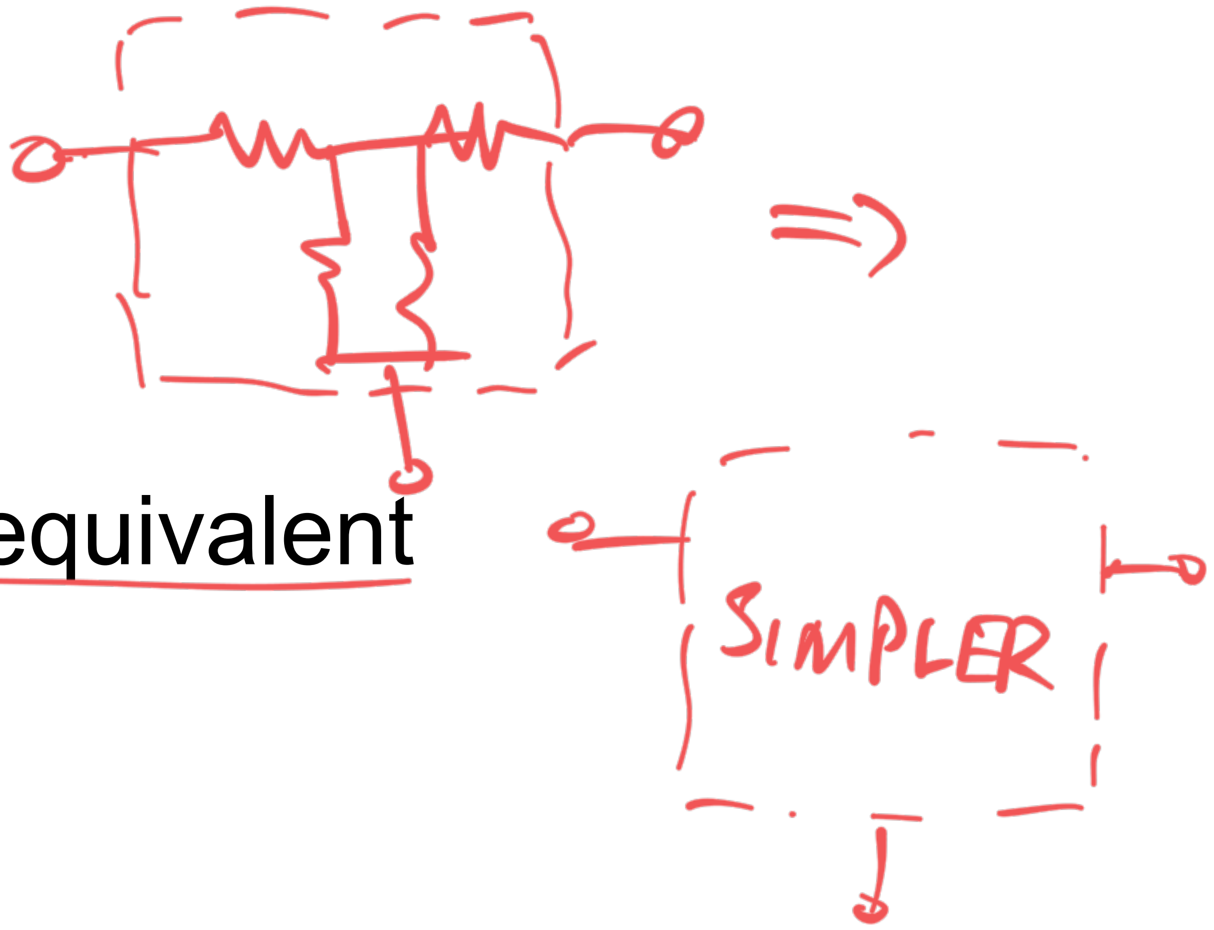
3. Write the KCL equations for every node except the reference (Gnd)
   - Sum of the device currents at each node must be zero
   algebraically.

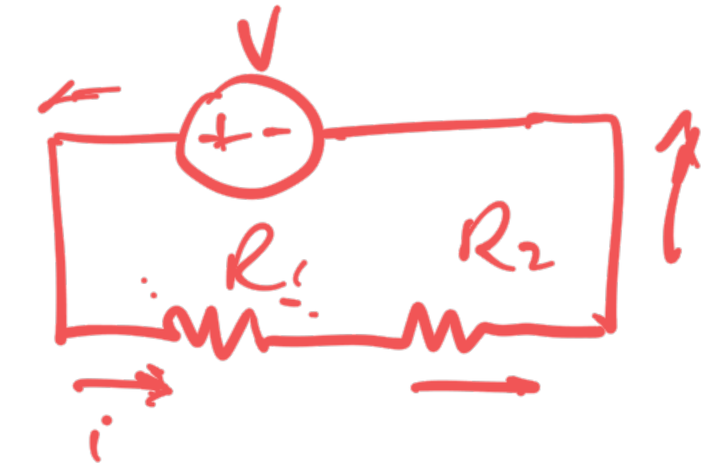
4. Substitute the equations for each device's current as a function of the node voltages, when possible

5. Solve the resulting set of equations

Today: Reducing Circuit Complexity

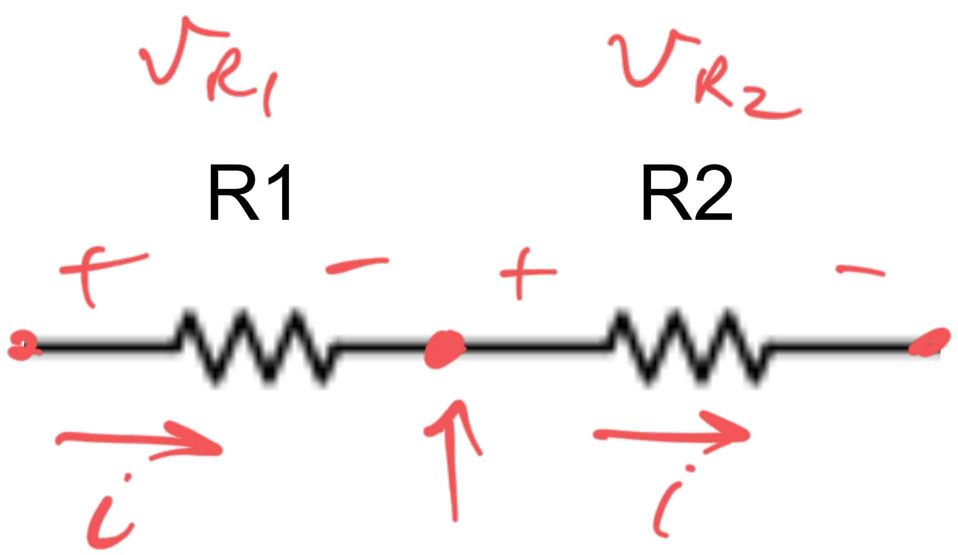
- Fewer variables is better
  - Could be fewer nodes
  - Could be fewer devices
- Can we break the circuit into pieces
  - Look at a **sub-circuit**
  - Replace that sub-circuit with a simpler equivalent
- We'll look at several examples

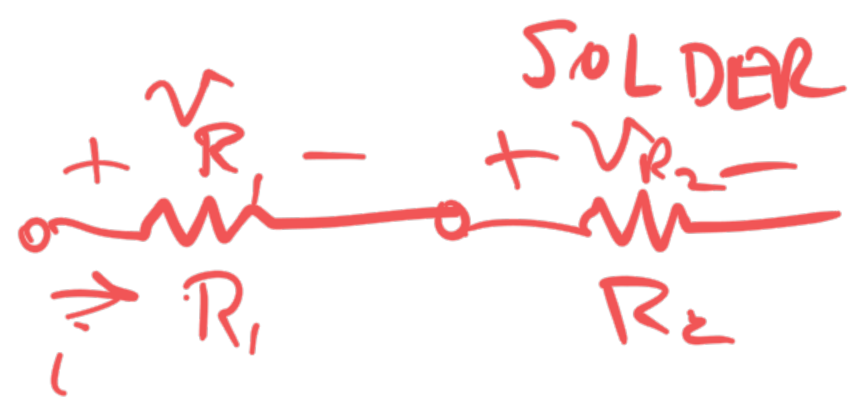




Series Combinations

- Two resistors in series ("share a current")
  - The voltage across the combination is the sum of the device voltages
  - The current through the devices is the same
  - So the effective resistance of the series is R = R1 + R2





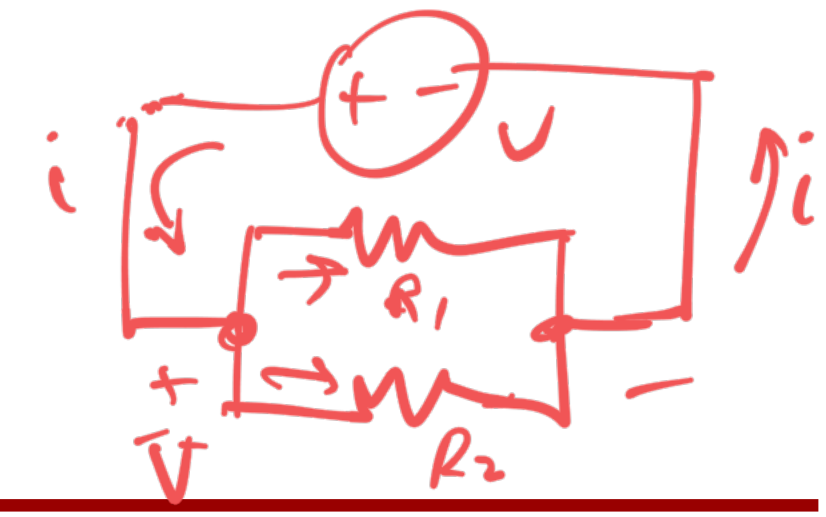
$$v_{R_1} = i R_1 \; ; \; v_{R_2} = i R_2$$

$$V = v_{R_1} + v_{R_2} = i R_1 + i R_2$$

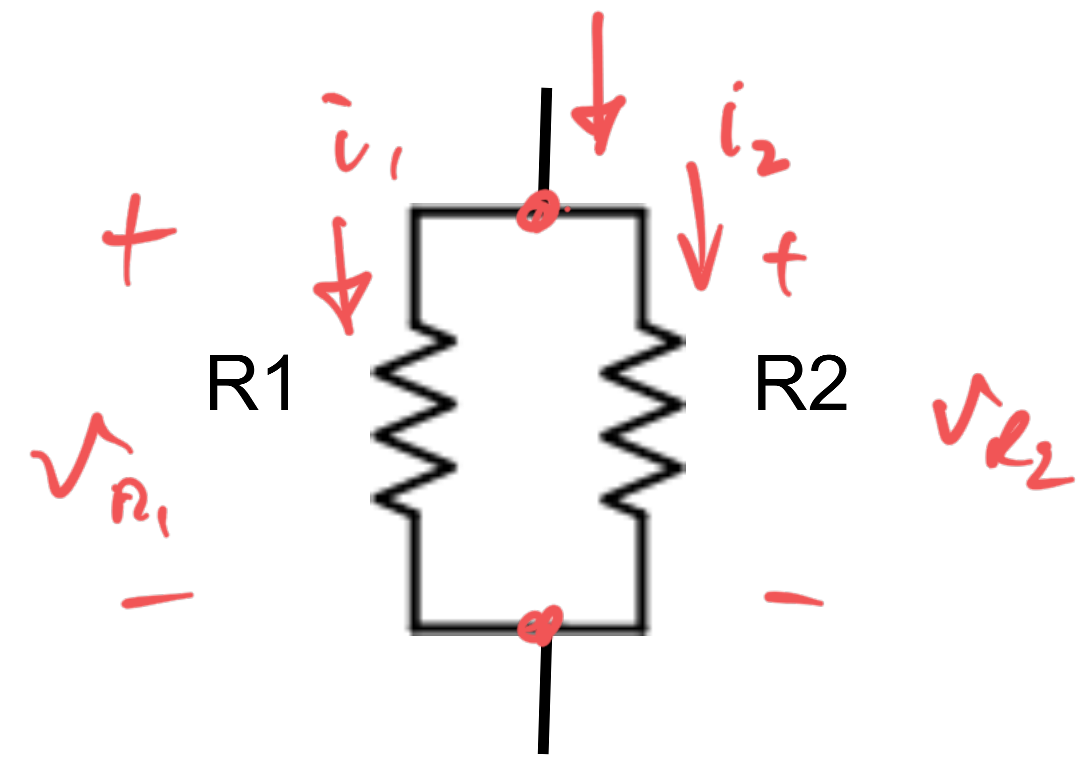
$$V = i(R_1 + R_2)$$

- So we can replace series resistors
  - With a single equivalent resistor
  - Removes a node voltage and device from our equations!

Parallel Combinations



- Two resistors in parallel
  - The total current through parallel resistors is the sum of the currents through the two resistors
  - The voltage across each resistor is the same … they "share a voltage"
  - So the effective resistance of parallel resistors is:

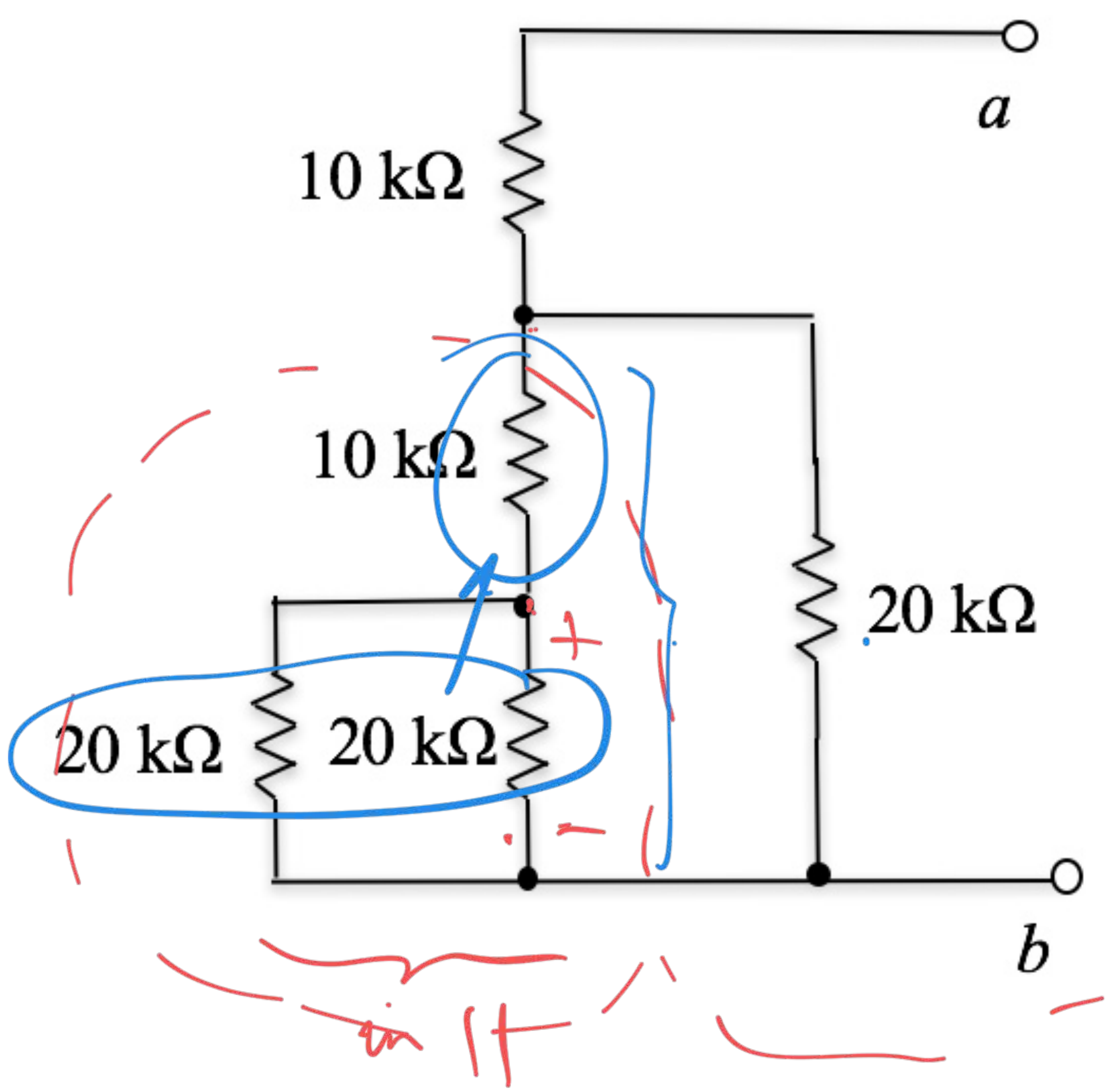


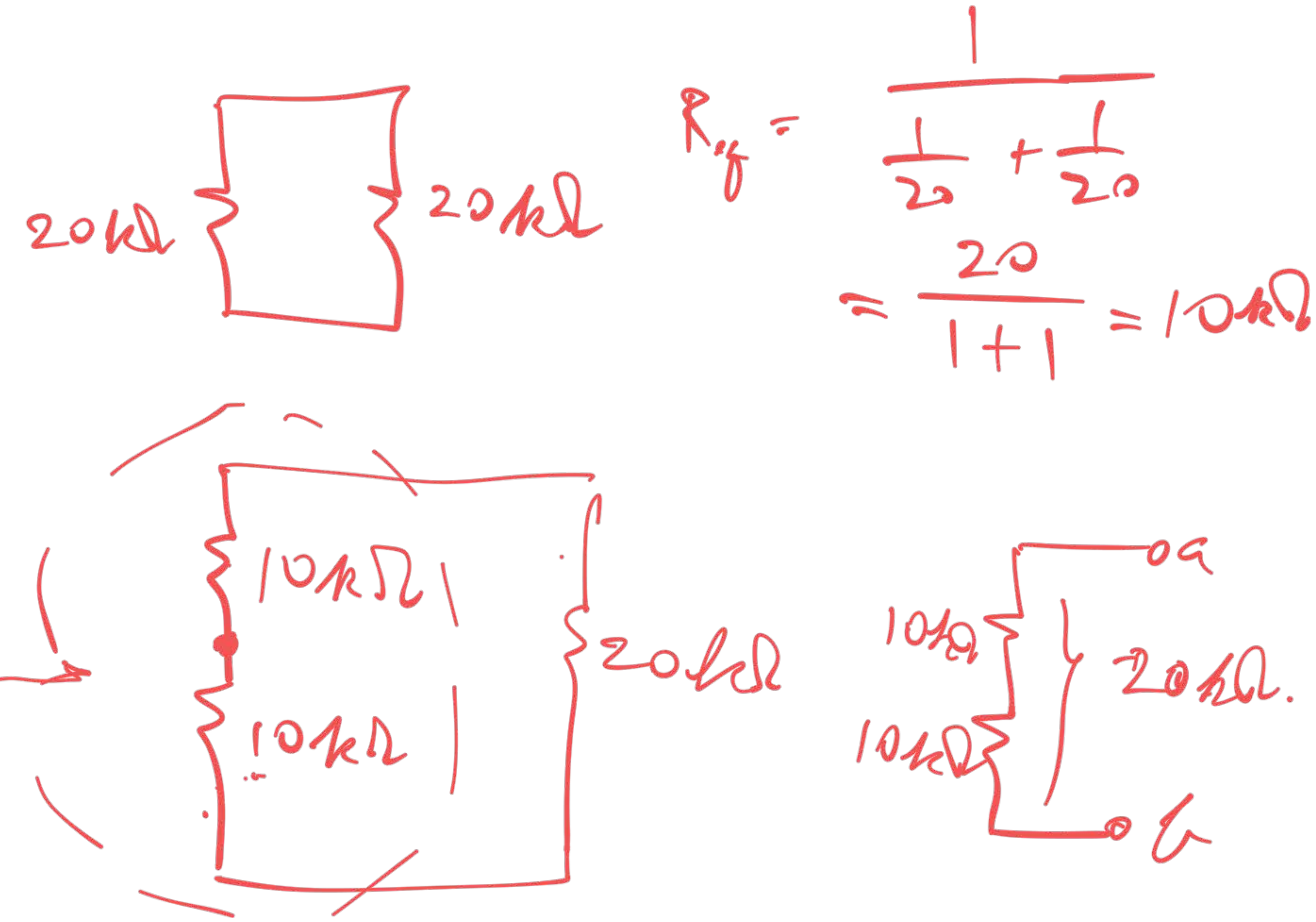
$$1/R = 1/R1 + 1/R2$$

$$R = (R1 \cdot R2) / (R1+R2)$$

Using Series and Parallel Combinations to Simplify Circuits

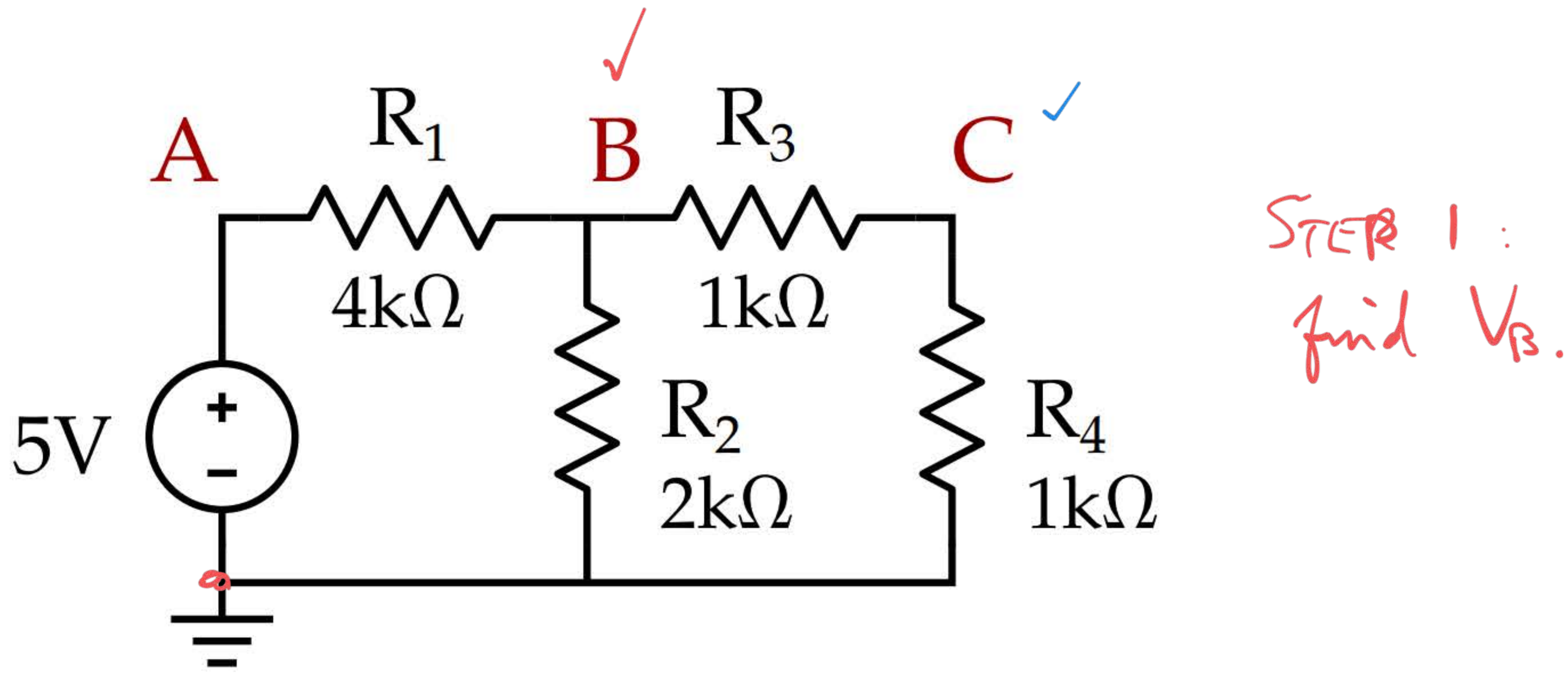
Example: Find the resistance between node a and node b



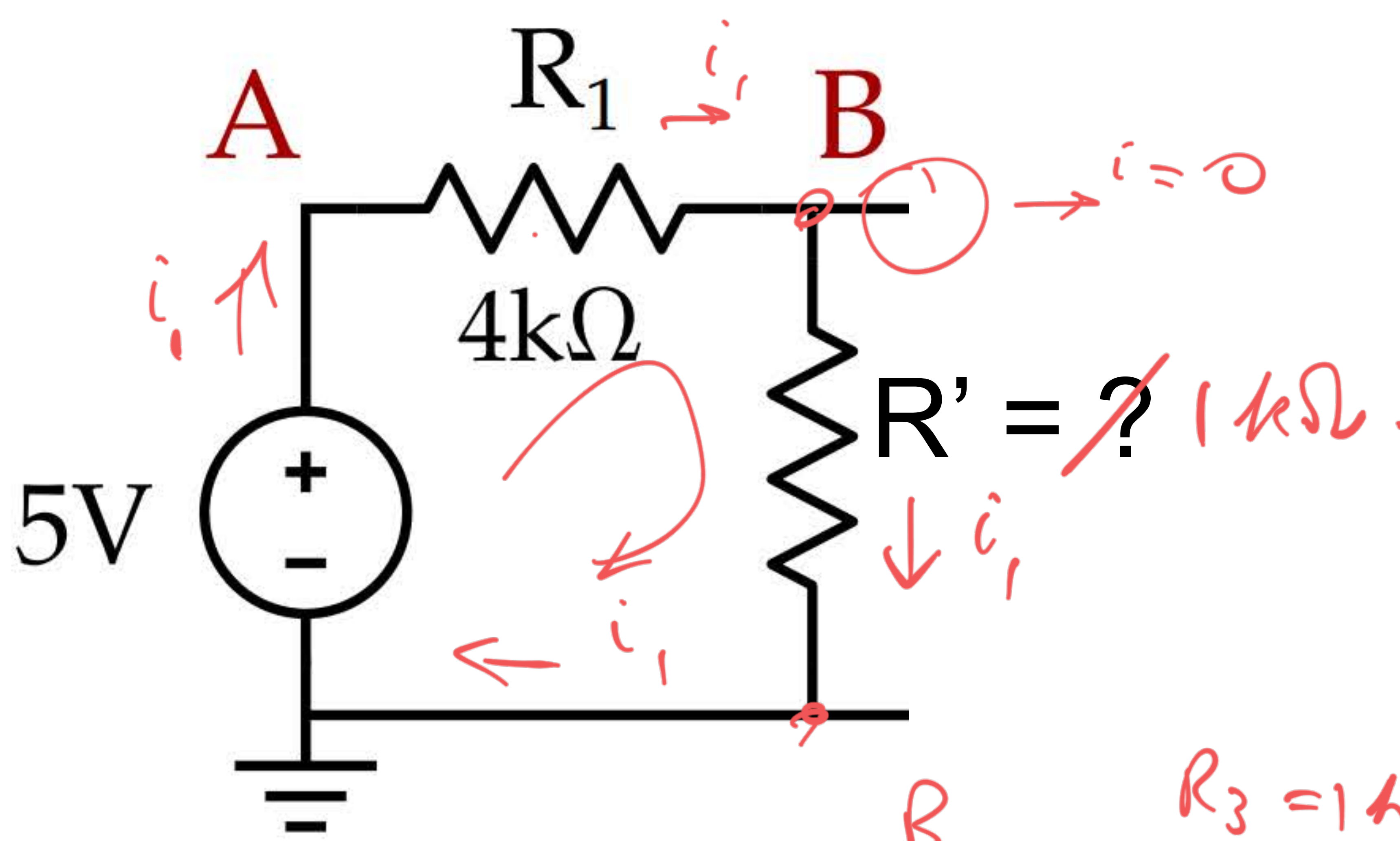


But What About This Circuit?

- R_3 and R_4 are in series
  - But I need to find the voltage at the node I will eliminate
  - “collapse and then expand”
- First eliminate the node to simplify the circuit



First Solve for the Voltage at Node B



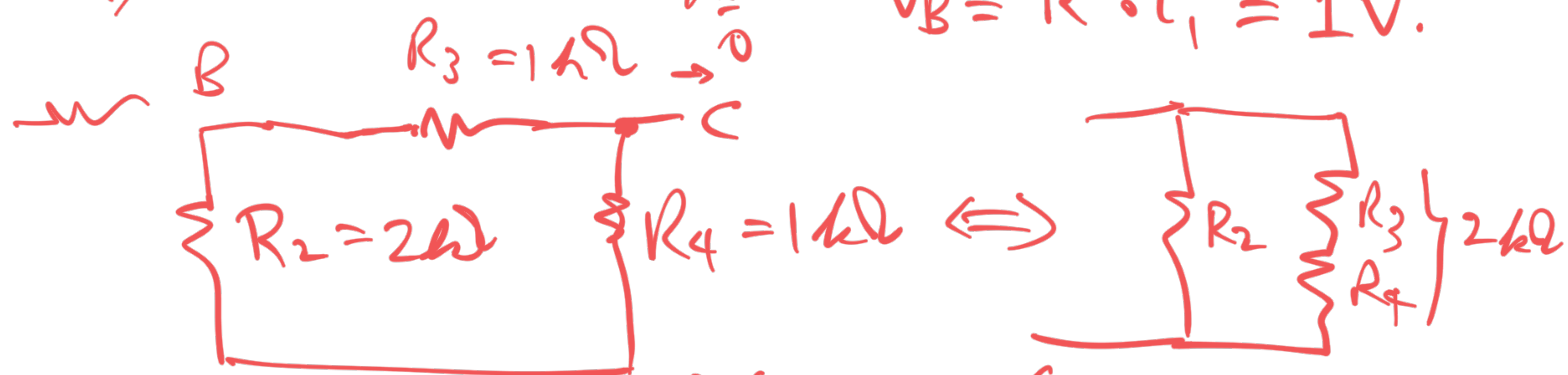
Node A : $V_A = 5V$

$-5V + i_1 R_1 + i_1 R_2 = 0$

$i_1 = \frac{5V}{5k\Omega} = 1mA$

Node B

$V_B = R' \cdot i_1 = 1V$



$R' = R_2 \| (R_3 + R_4)$

Then Solve for the Voltage at Node C

Find V_C :

R_3 & R_4 are in series

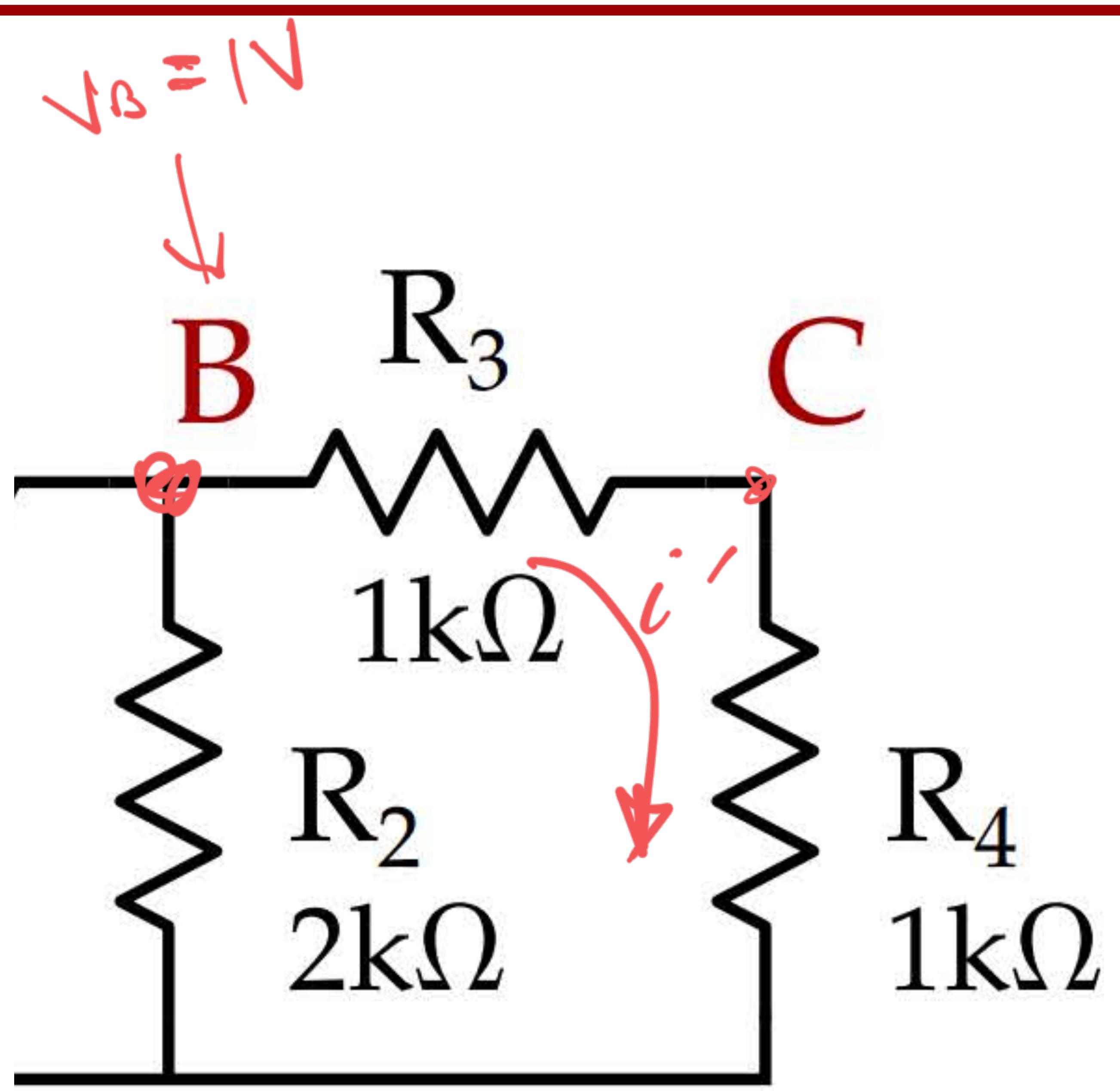
$$i' = \frac{V_B}{R_3 + R_4} = \frac{1V}{2k\Omega}$$

$$= \frac{1}{2} mA$$

$$V_C = R_4 \cdot i'$$

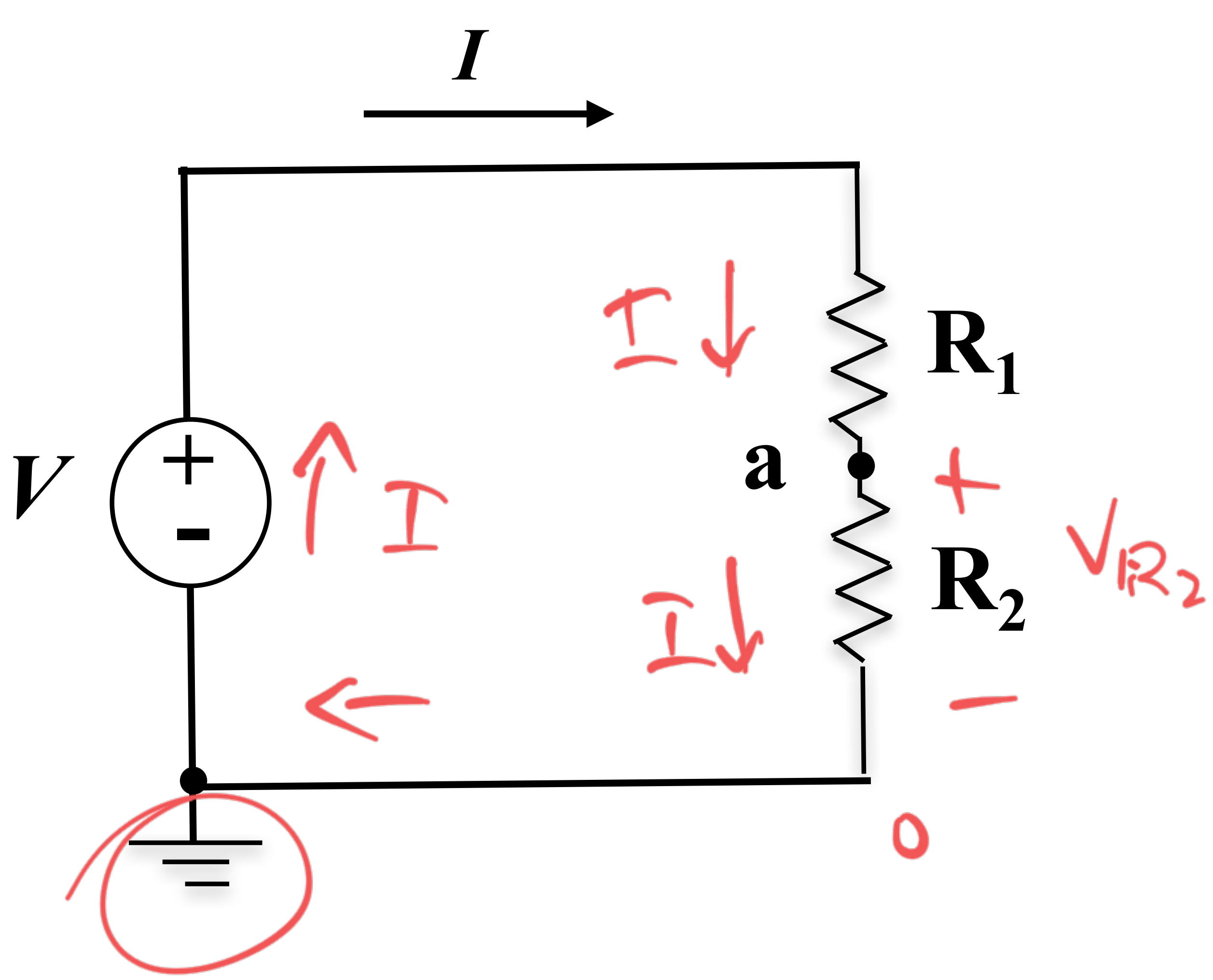
$$= (1k\Omega)(1/2\ mA)$$

$$= \underline{1/2\ V}.$$



Voltage Divider

- First simplify circuit to a single resistor and find the current I
- Then use the current to find the voltage V_a



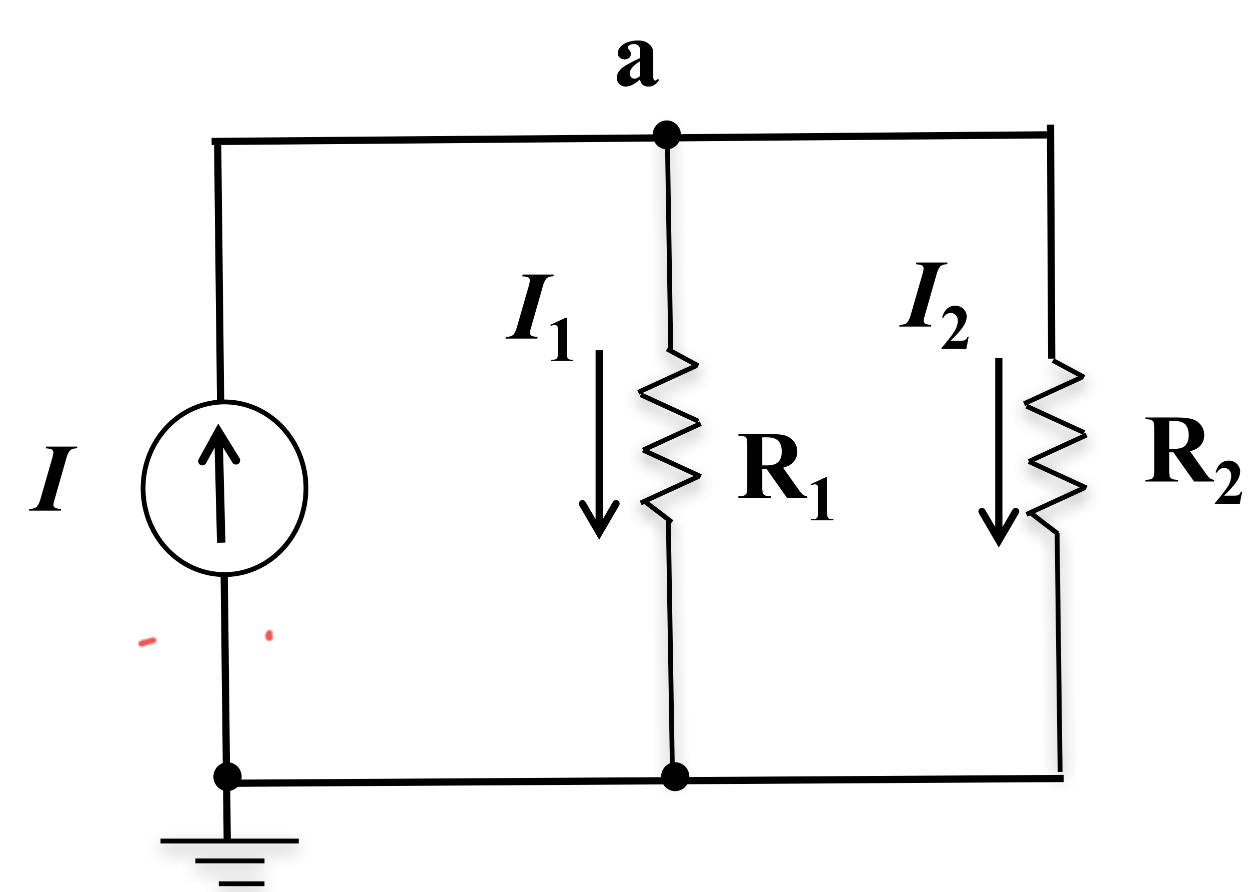
$$I = \frac{V}{R_1 + R_2} \leftarrow \text{SERIES}$$

$$V_a = I \cdot R_2 = \left(\frac{V}{R_1 + R_2}\right) \cdot R_2$$

$$V_a = R_2 \left[\frac{1}{R_1 + R_2}\right] \cdot V \leftarrow$$

Current Divider

- In this case simplify the circuit to a single resistor, then find voltage across each resistor and use it to find the current through each resistor



- FIND VOLTAGE V_a : $\frac{R_1 R_2}{R_1+R_2}$

$R' = R_1 \| R_2$

$V_a = I \cdot (R_1 \| R_2)$

- FIND I_1 (OR I_2) :

$$I_1 = \frac{V_a}{R_1} = I \left[\frac{R_1 \| R_2}{R_1} \right] = I \left[\frac{\frac{R_1 R_2}{R_1+R_2}}{R_1} \right]$$

$$I_1 = I \left(\frac{R_2}{R_1+R_2} \right)$$

Intuition on Dividers

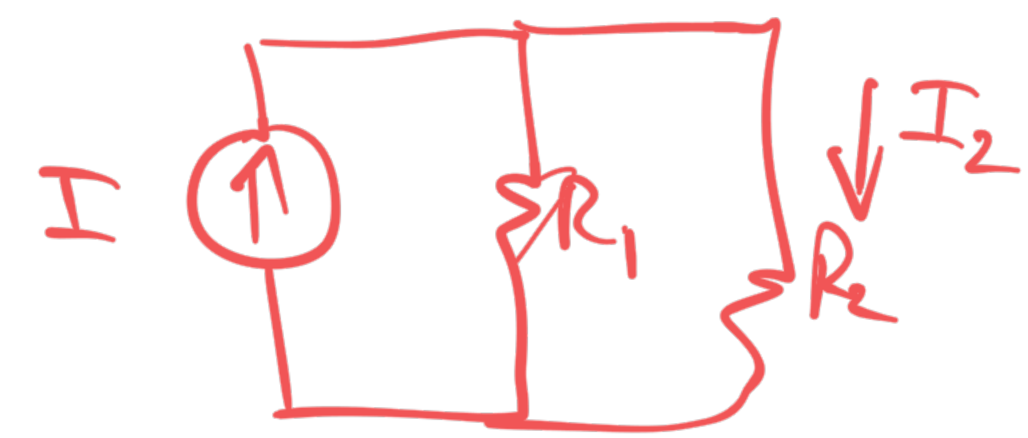
- Voltage divider: $R_2 = 10\ R_1$ … what is $V_a = V_{R2}$ in terms of V?

$$V_a = V\frac{R_2}{R_1+R_2} = V\left(\frac{10R_1}{11R_1}\right) \approx 9/10$$

LARGE ⇒ HOGS THE VOLTAGE

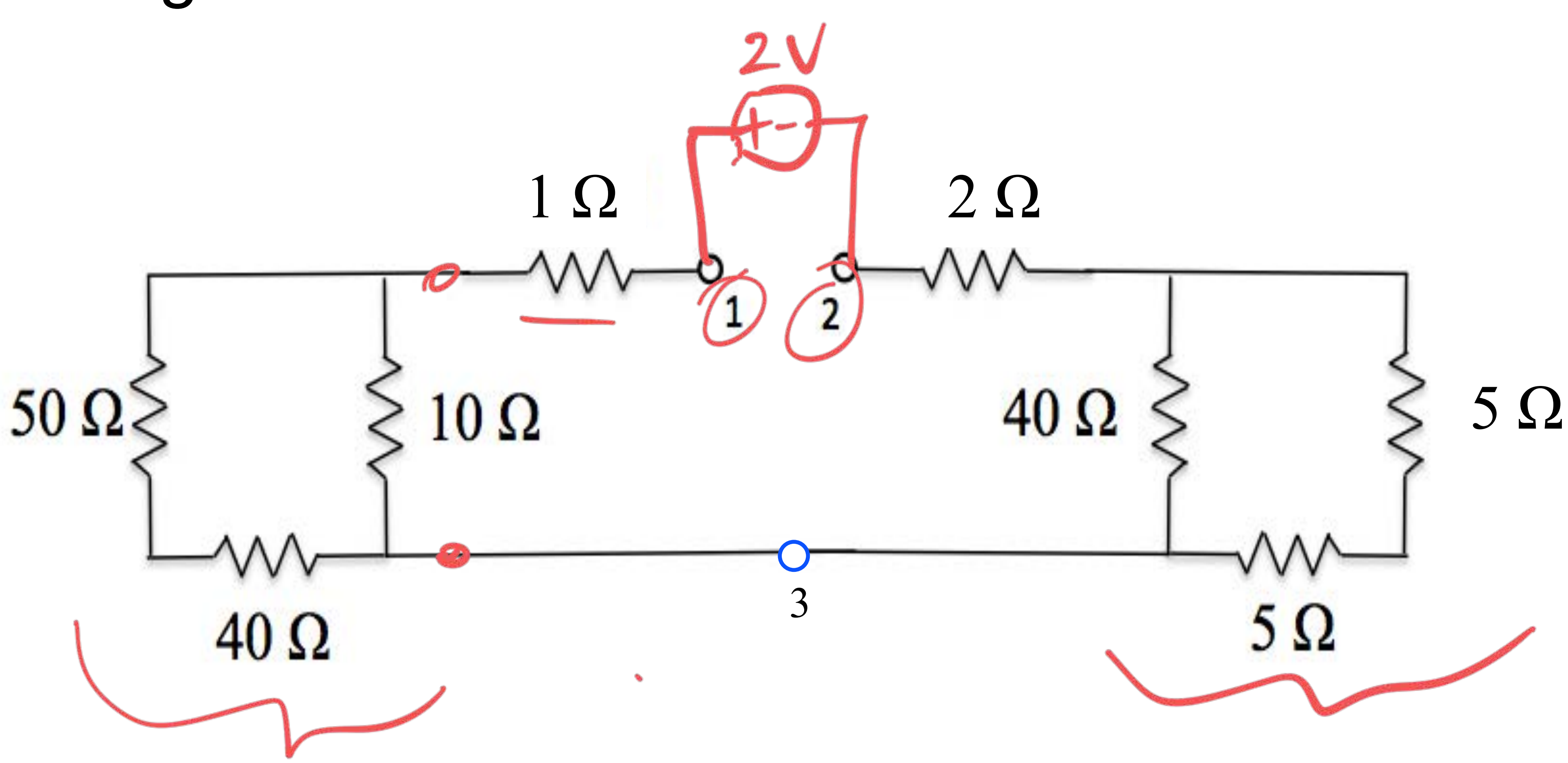
- Current divider: $R_2 = 10\ R_1$ … what is I_2 in terms of I?

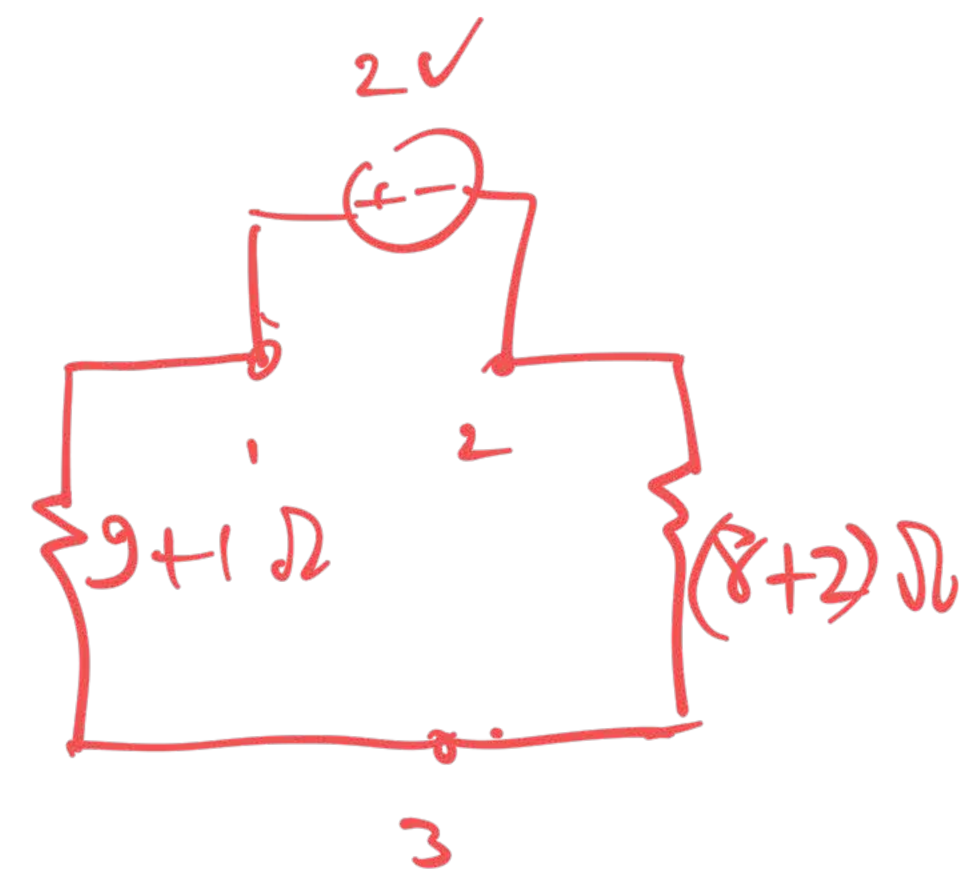
$$I_2 = \left(\frac{R_1}{R_1+R_2}\right)\cdot I = \left(\frac{R_1}{11R_1}\right)\cdot I \approx 1/10$$

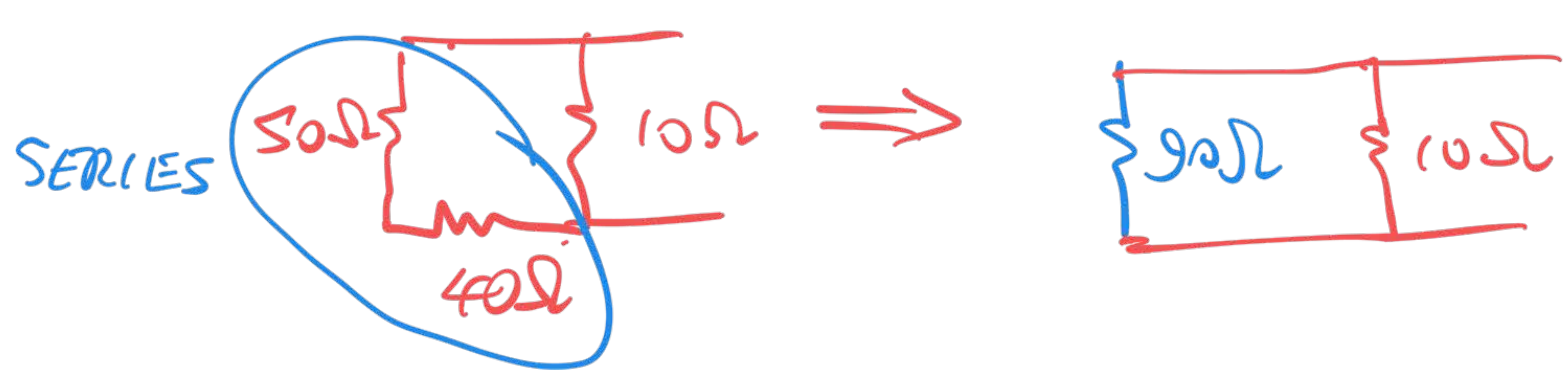


Series-Parallel Reduction

We connect a 2 V battery between nodes 1(+) and 2 (-).
What current flows through the latter?
What is the voltage difference between node 2 and node 3?

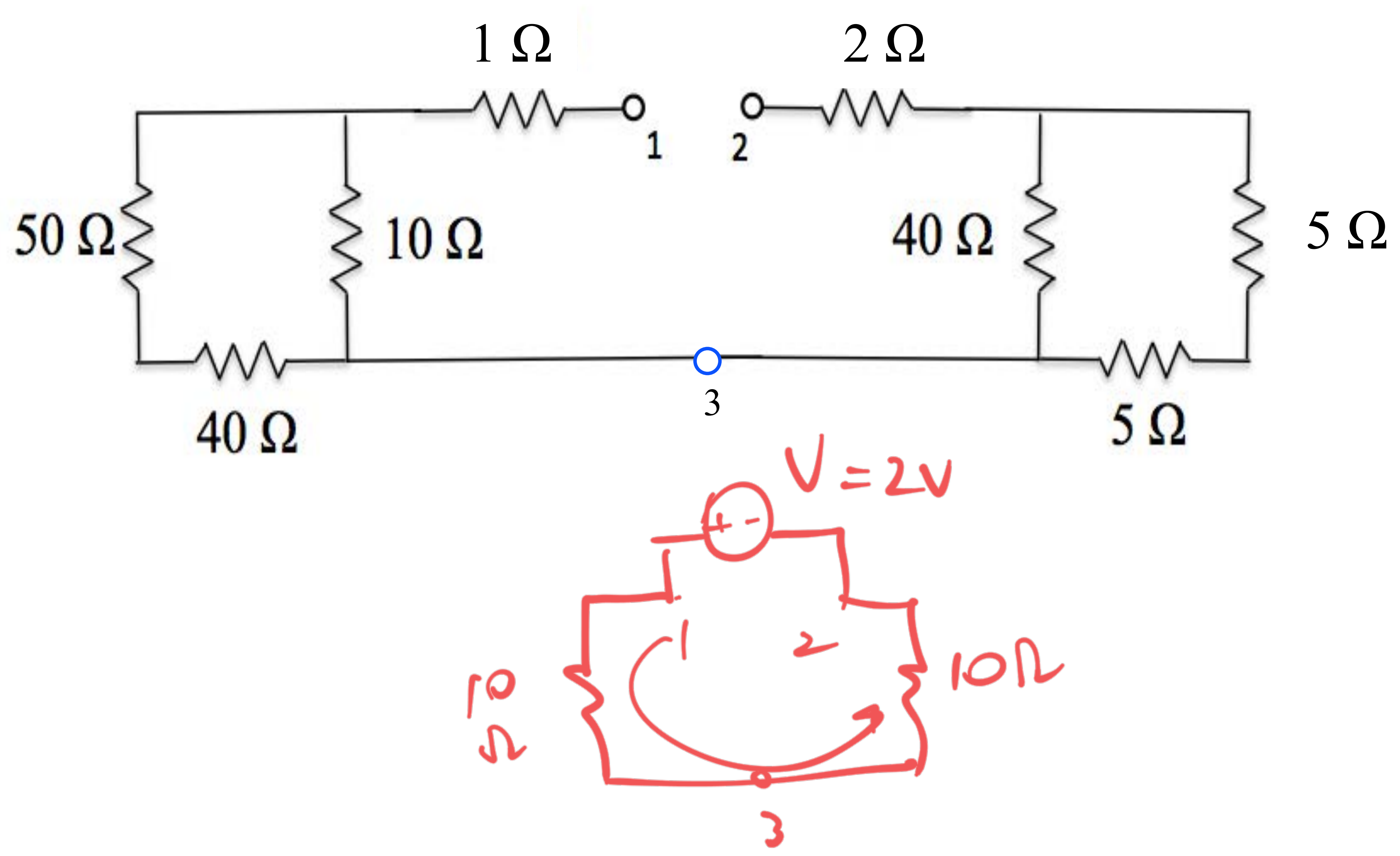






$$\Rightarrow \frac{90 \times 10}{90+10} = \frac{900}{100} = 9\Omega.$$

Series-Parallel Reduction



Superposition For Linear Circuits

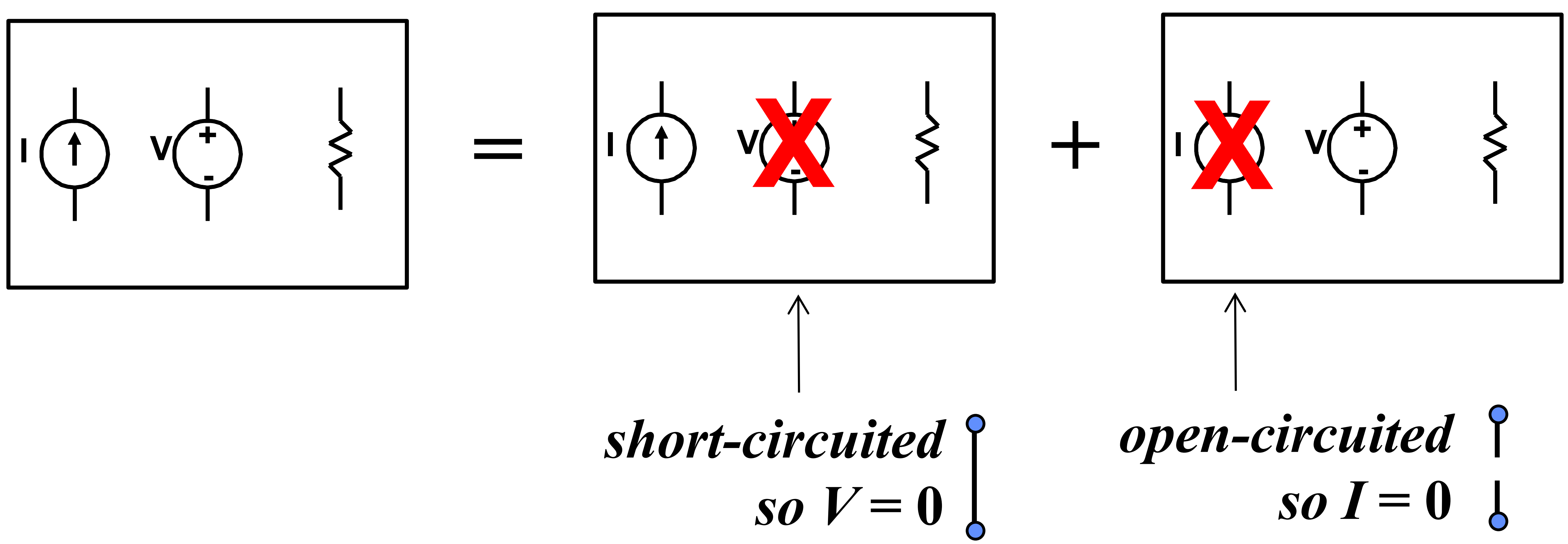
> If a circuit contains more than one independent source, the voltage (or current) response of any element in the circuit is equal to the algebraic sum of the individual responses associated with the individual independent sources, as if each had been acting alone.

- Reason:
  - Resistors, voltage, and current sources are linear
  - Resulting equations are linear

- What's the benefit?
  - Superposition enables the analysis of several simpler circuits in place of one complicated circuit

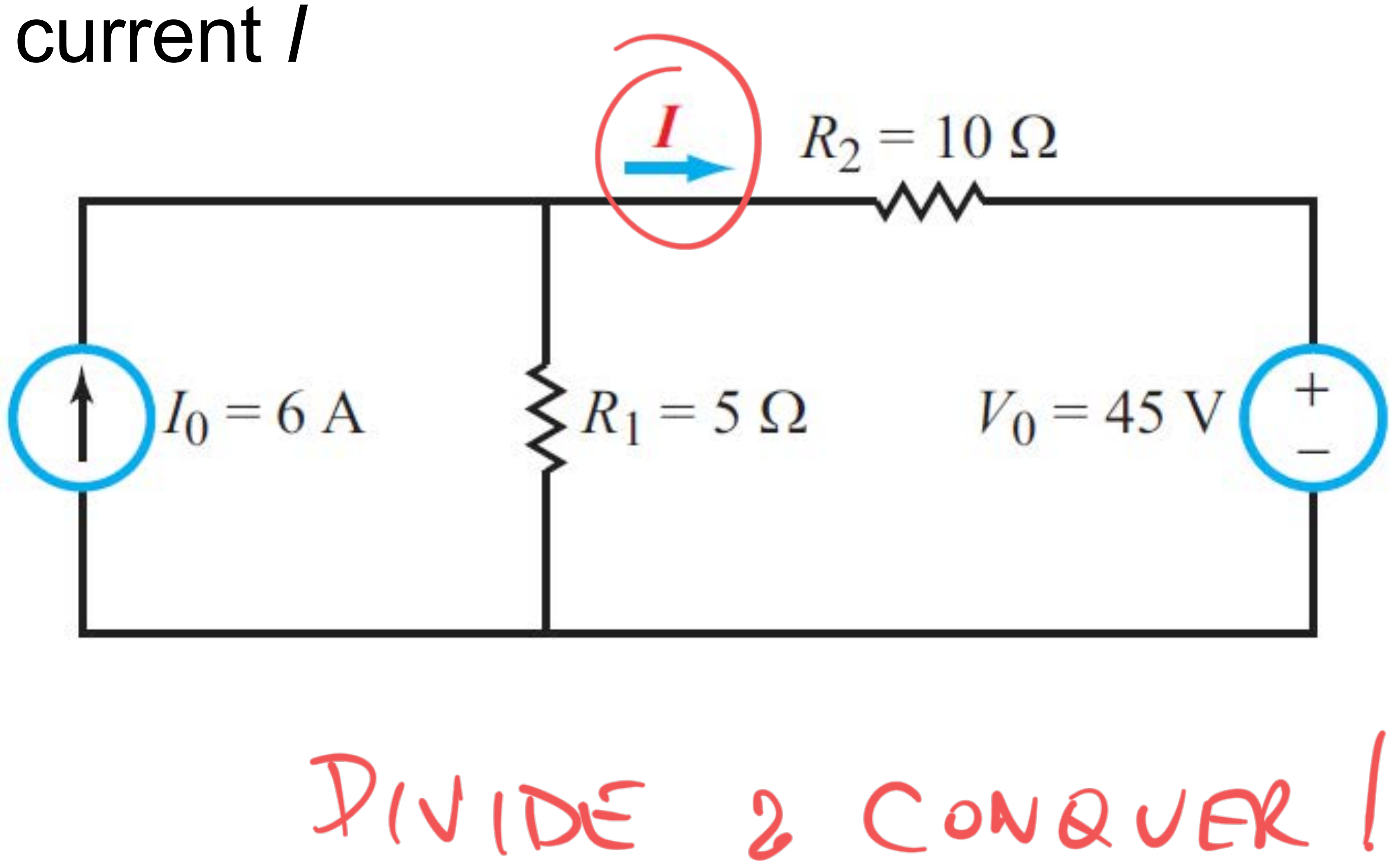
Applying Superposition

- Calculate the response of the circuit for each independent source at a time, with the other's turned off
- What happens when we turn off a source?
  - Voltage sources: have 0 V (are **shorted** … replace by a wire)
  - Current sources: have 0 current (are **opened** … replace by a *broken* wire)



Applying Superposition

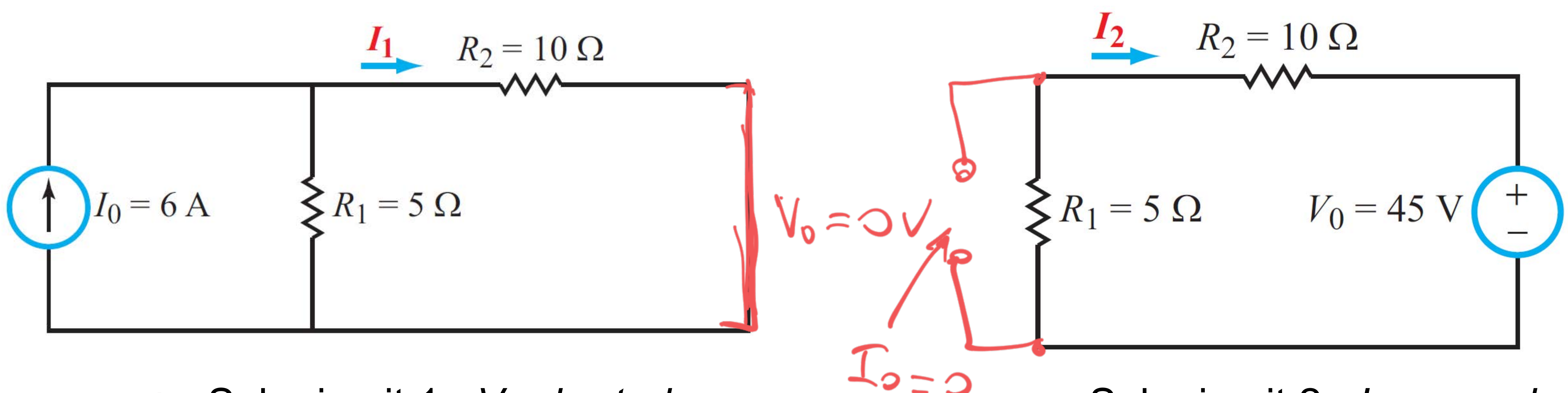
- We need to “zero-out” sources into order to find the sub-circuits (one per source)
- Find the current *I*



F. T. Ulaby and M. M. Maharbiz, *Circuits*, NSTP, 2009, p. 97.

Applying Superposition

- We need to "zero-out" sources into order to find the sub-circuits (one per source)



Sub-circuit 1: V_0 *shorted*

Sub-circuit 2: I_0 *opened*

$$I_1 = I_0\left(\frac{R_1}{R_1+R_2}\right) = 6A\left(\frac{5\Omega}{15\Omega}\right) = 2A$$

$$I_2 = \left(\frac{-V_0}{R_1+R_2}\right) = \frac{-45V}{15\Omega} = -3A$$

$$I = I_1 + I_2 = 2 - 3 = -1A.$$

F. T. Ulaby and M. M. Maharbiz, *Circuits*, NSTP, 2009, p. 97.

EveryCircuit

Circuit Debugging

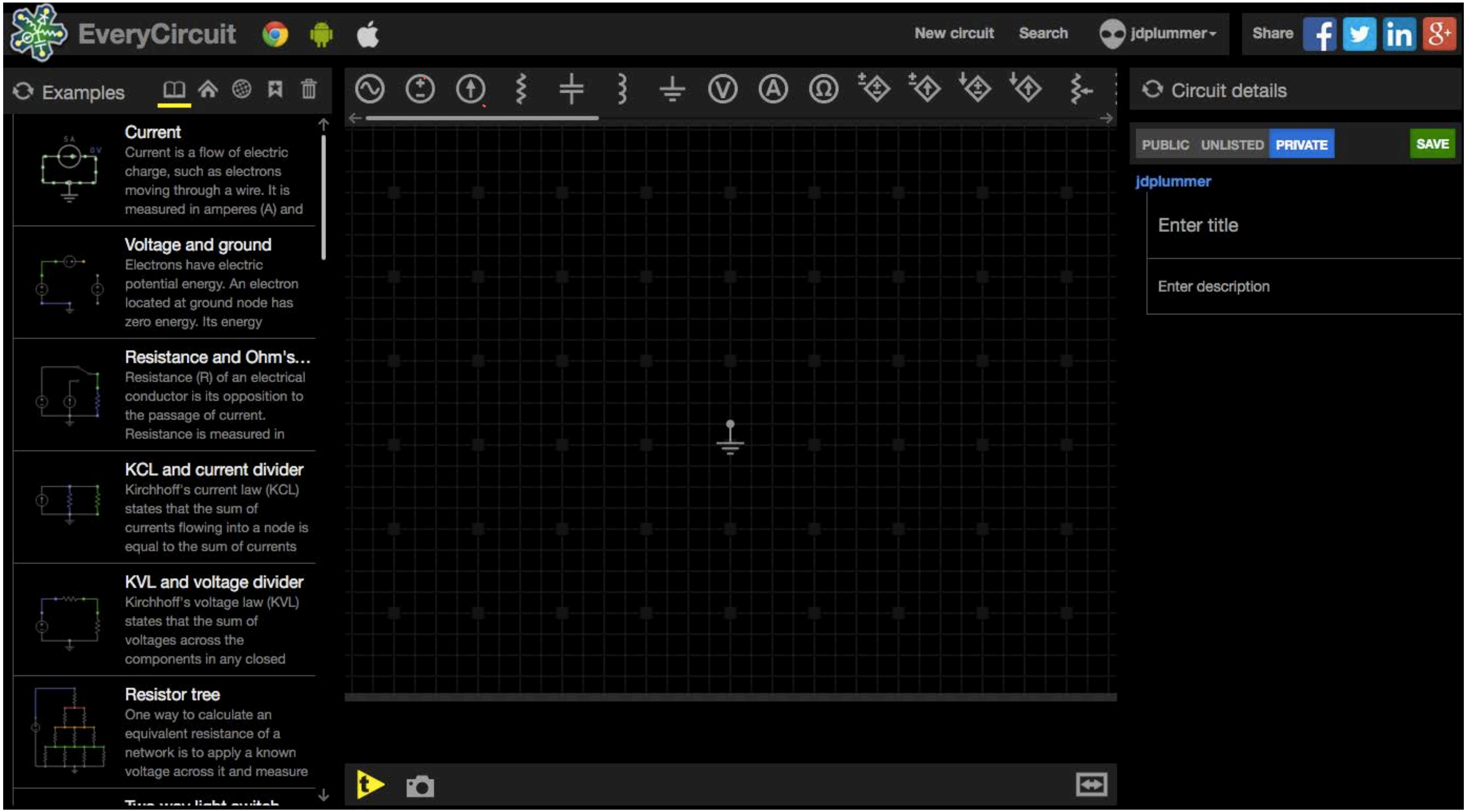
- For future labs you will be building more complex circuits
  - You will build these circuits using breadboards
- These circuits will contain many different components
  - Including transistors with three connections
- Sometimes these circuits won't work the way you expect
  - Perhaps your circuit is wrong
  - Or perhaps you just connected it up wrong
  - How do you debug it in either case?

Circuit Simulator

- We create a program to estimate how our circuit will behave
- The program shows the wiring in a nice way
  - and makes it easy to probe the voltage and current
  - It has built-in voltage and current meters
- It also makes it easy to change component values
  - So you can tune/play with your circuit
- You are going to use an easy-to-use simulator: EveryCircuit

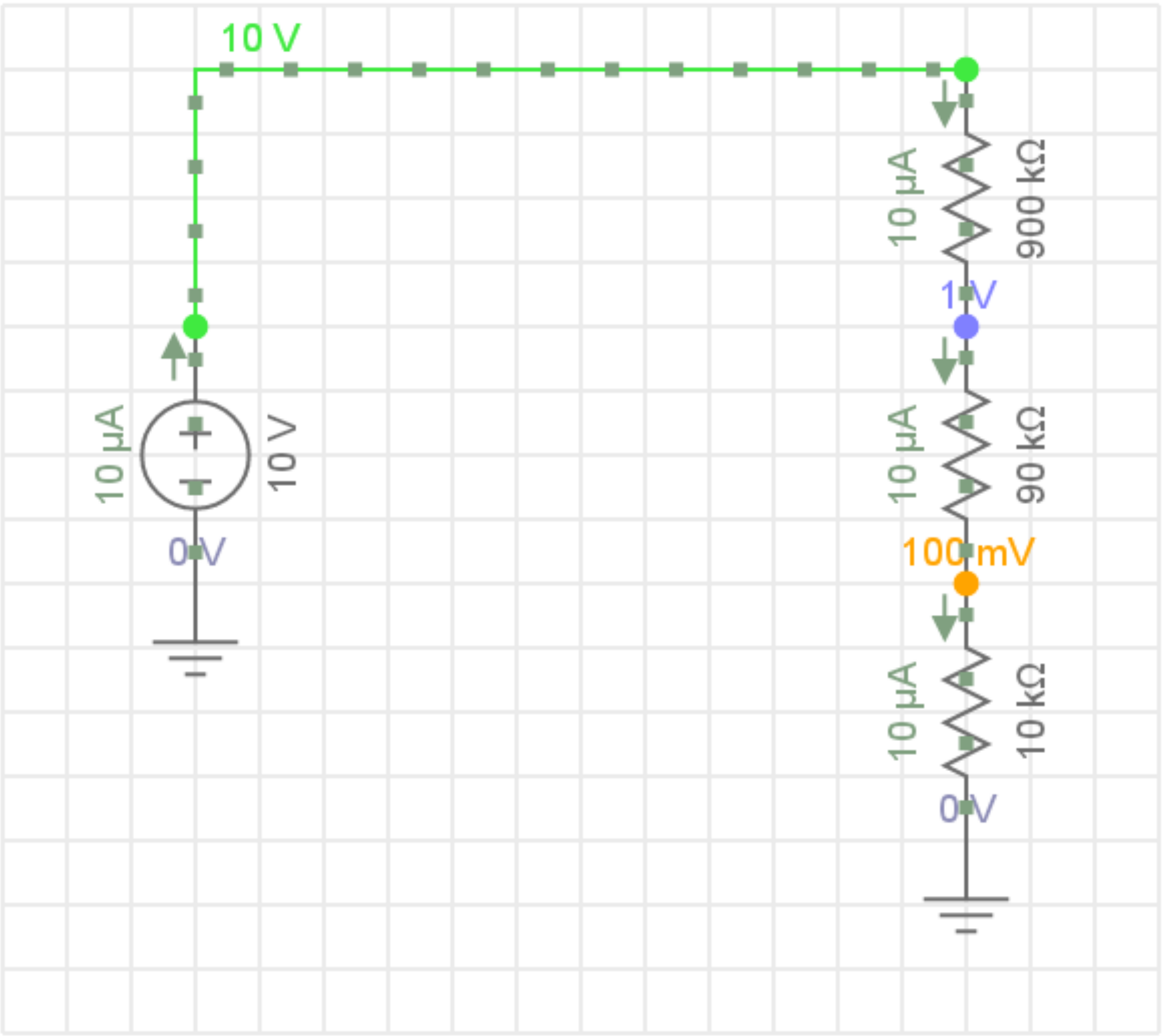
Every Circuit

http://everycircuit.com/app/



Every Circuit
http://everycircuit.com/app/

- Simple simulator that we will use for circuits



Quick Use Notes

- To connect two nodes, select one node, then select another node.
- To delete a single wire in a node, select the node, then select the wire, then press Delete.
- To maximize schematic area in browser window (remove circuit explorer on the left and circuit details on the right) click the right-most icon in the menu below the schematic.

Every Circuit's Keyboard Shortcuts

- R : Rotate selected device
- F : Flip selected device
- A : Adjust parameter of a selected device
- T : Toggle selected switch
- W : Add / remove voltage of selected node or current of selected device to / from oscilloscope
- S : Adjust simulation speed
- Esc : deselect all
- Arrows : move selected component or workspace
- Plus / Minus : zoom in / out
- Space : start or pause simulation
- Delete : delete selected device or cut selected wire
- Ctrl + Z : Undo
- Ctrl + Y : Redo

Activate Your License

- **http://everycircuit.com/licensekeyactivation**

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(Good during spring quarter)

Learning Objectives

- Understand how to solve for device voltage and currents
  - First label node voltages (KVL)
  - Solve current equations at each node (KCL)
  - Called nodal analysis
- Be able to break a large circuit into smaller circuits
  - This is standard “divide and conquer” approach
- Recognize some common circuit patterns
  - Which reduce the complexity of the circuit you need to solve
  - Start with series and parallel resistors
- Superposition is a powerful tool for handling multiple sources
- EveryCircuit can solve your circuits, so you can be sure your homework and prelab answers are correct!