E40M
Capacitors
Reading

• Reader:
  – Chapter 6 – Capacitance

• A & L:
  – 9.1.1, 9.2.1
Why Are Capacitors Useful/Important?

How do we design circuits that respond to certain frequencies? What determines how fast CMOS circuits can work?

Why did you put a 200μF capacitor between Vdd and Gnd on your Arduino?
CAPACITORS
Capacitors

• What is a capacitor?
  – It is a new type of two terminal device
  – It is linear
    • Double $V$, you will double $I$
  – We will see it doesn’t dissipate energy
    • Stores energy

• Rather than relating $i$ and $V$
  – Relates $Q$, the charge stored on each plate, to Voltage
  – $Q = CV$
  – $Q$ in Coulombs, $V$ in Volts, and $C$ in Farads

• Like all devices, it is always charge neutral
  – Stores $+Q$ on one lead, $-Q$ on the other lead
iV for a Capacitor

- We generally don’t work in Q, we like i and V
  - But current is charge flow, or \( \frac{dQ}{dt} \)

- So if \( Q = CV \), and \( i = \frac{dQ}{dt} \)
  - \( i = C \frac{dV}{dt} \)

- This is a linear equation but between \( I \) and \( \frac{dV}{dt} \). If you double \( i \) for all time, \( \frac{dV}{dt} \) will also double and hence \( V \) will double.

\[
C = \frac{\varepsilon A}{d}
\]

where \( \varepsilon \) is the dielectric constant
Capacitors Only Affect Time Response not Final Values

- Capacitors relate $I$ to $\frac{dV}{dt}$
- This means if the circuit “settles down” and isn’t changing with time, a capacitor has no effect (looks like an open circuit).

@ $t = 0$

@ $t = \infty$
So What Do Capacitors Do?

- It affects how fast a voltage can change
  - Current sets \( \frac{dV}{dt} \), and not \( V \)
  - Fast changes require lots of current

- For very small \( \Delta t \) capacitors look like voltage sources
  - They can supply very large currents
  - And not change their voltage

- But for large \( \Delta t \)
  - Capacitors look like open circuits (they don’t do anything)
Capacitor Energy

• The Power that flows into a charging capacitor is

\[ P = iV = \left( C \frac{dV}{dt} \right) V \]

• And the energy stored in the capacitor is

\[ E = \int P \, dt \]

\[ \therefore E = \int P \, dt = \int CV \, dV = \frac{1}{2} CV^2 \]

• This energy is stored and can be released at a later time. No energy is lost.
REAL CAPACITORS
Capacitor Types

- There are many different types of capacitors
  - Electrolytic, tantalum, ceramic, mica, . . .

- They come in different sizes
  - Larger capacitance
    - Generally larger size
  - Higher voltage compliance
    - Larger size

- Electrolytic have largest cap/volume
  - But they have limited voltage
  - They are polarized
    - One terminal must be + vs. other

Gate of MOS Transistor

• Is a capacitor between Gate and Source

• To change the gate voltage
  – You need a current pulse (to cause $dV/dt$)

• If the current is zero (floating)
  – $dV/dt = 0$, and the voltage remains what it was!
All Real Wires Have Capacitance

- It will take some charge to change the voltage of a wire
  - Think back to our definition of voltage
    - Potential energy for charge
  - To make a wire higher potential energy
    - Some charge has to flow into the wire, to make the energy higher for the next charge that flows into it

- This capacitance is what sets the speed of your computer
  - And determines how much power it takes!
Capacitor Info, If You Know Physics E&M…

• Models the fact that energy is stored in electric fields
  – Between any two wires that are close to each other

• A capacitor is formed by two terminals that are not connected
  – But are close to each other
  – The closer they are, the larger the capacitor

• To create a voltage between the terminals
  – Plus charge is collected on the positive terminal
  – Negative charge is collected on the negative terminal

• This creates an electric field (Gauss’s law)
  – Which is what creates the voltage across the terminals
  – There is energy stored in this electric field
Capacitors in Parallel and Series

\[ i_T = i_1 + i_2 + i_3 = C_1 \frac{dV}{dt} + C_2 \frac{dV}{dt} + C_3 \frac{dV}{dt} \]

\[ = (C_1 + C_2 + C_3) \frac{dV}{dt} \]

\[ \therefore C_{\text{eqv}} = C_1 + C_2 + C_3 \]

\[ V_T = V_1 + V_2 + V_3 = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = \frac{Q}{C_{\text{eqv}}} \]

\[ \therefore \frac{1}{C_{\text{eqv}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \]
CAPACITOR RESISTOR CIRCUITS
Capacitors and Logic Gate Speeds

- When the input changes from low to high
  - The pMOS turns off, and the nMOS turns on
  - The output goes from high to low

- But in this model
  - The output changes as soon as the input changes
Gates Are NOT Zero Delay

• It would be great if logic gates had zero delay
  – But they don’t

• Fortunately, it is easy to figure out the delay of a gate
  – It is just caused by the transistor resistance
    • Which we know about already
  
  – And the transistor and wire capacitance
Improved Model

- Just add a capacitor to the output node
  - Its value is equal to the capacitance of the gates driven
  - Plus the capacitance of the wire itself
When the input to the inverter is low, the output will be at $V_{dd}$
- Right after the input rises, here is the circuit

Want to find the capacitor voltage verses time

Just write the nodal equations:
- We just have one node voltage, $V_{out}$
  - $i_{RES} = V_{out}/R_2$
  - $i_{CAP} = Cdv_{out}/dt$

From KCL, the sum of the currents must be zero, so

$$\frac{dV_{out}}{dt} = -\frac{V_{out}}{R_2C}$$
RC Circuit Equations

- Solving, 
  \[ \int_{5}^{V_{out}} \frac{dV_{out}}{V_{out}} = - \int_{0}^{t} \frac{dt}{R_{2}C} \]  
  so that \[ \ln(V_{out}) - \ln(5V) = - \frac{t}{R_{2}C} \]

- This is an exponential decay
  - The x axis is in time constants 
  - The y axis has been normalized to 1 
  - Slope always intersects 0 one tau later \((\tau = RC)\)
What Happens When Input Falls?

• Now the voltage across the capacitor starts at 0V
  – \( i = \frac{(V_{dd} - V_{out})}{R_1} \)
  – \( \frac{dV_{out}}{dt} = \frac{i}{C} \)
  
  \[
  \frac{dV_{out}}{dt} = \frac{(V_{dd} - V_{out})}{R_1C}
  \]

• Not quite the right form
  – Need to fix it by changing variables
  – Define \( V_{new} = V_{dd} - V_{out} \)
  – \( \frac{dV_{out}}{dt} = = - \frac{dV_{new}}{dt} \), since \( V_{dd} \) is fixed

\[
\int_{0}^{5} \frac{dV_{new}}{V_{new}} = -\int_{0}^{t} \frac{dt}{R_1C} \quad \text{so that} \quad \ln(V_{new}) - \ln(5) = -\frac{t}{R_1C} \quad \therefore \quad V_{out} = 5V\left(1 - e^{-t/R_1C}\right)
\]
In capacitor circuits, voltages change “slowly”, while currents can be instantaneous.
Simple RC Circuit Demo

EveryCircuit Demo – CMOS Inverter
Interesting Aside

- Exponentials “never” reach their final value
- So if this logic gate is driving another gate, when does the next gate think its input is 0 or 1?
- This is one of the reasons why logic levels are defined as a range of values.
Learning Objectives

- Understand what a capacitor is
  - \( i = C \frac{dV}{dt} \)
  - It is a device that tries to keep voltage constant
    - Will supply current (in either direction) to resist voltage changes

- Understand how voltages and current change in R C circuits
  - Voltage waveforms are continuous
    - Takes time for their value to change
    - Exponentially decay to final value (the DC value of circuit)
  - Currents can charge abruptly