E40M
Capacitors

Midterm - Wed in class, this room
Cover through lecture #9, lab 2b, HW #4
Reading

• Reader:
  – Chapter 6 – Capacitance

• A & L:
  – 9.1.1, 9.2.1
Why Are Capacitors Useful/Important?

How do we design circuits that respond to certain frequencies? What determines how fast CMOS circuits can work?

Why did you put a 200μF capacitor between Vdd and Gnd on your Arduino?
CAPACITORS
Capacitors

- What is a capacitor?
  - It is a new type of two terminal device
  - It is linear
    - Double V, you will double I
  - We will see it doesn’t dissipate energy
    - Stores energy

- Rather than relating i and V
  - Relates Q, the charge stored on each plate, to Voltage
    - $Q = CV$
  - Q in Coulombs, V in Volts, and C in Farads

- Like all devices, it is always charge neutral
  - Stores +Q on one lead, -Q on the other lead
Close switch \( @ t = 0 \)
Once \( V_c = V \)
then \( i \to 0 \)
- Battery - solar cell project
- Cap
  - much less energy density

Capacitors have many applications in circuits
iV for a Capacitor

- We generally don’t work in Q, we like i and V
  - But current is charge flow, or dQ/dt

- So if Q = CV, and i=dQ/dt
  - \( i = C \frac{dV}{dt} \)

- This is a linear equation but between i and dV/dt. If you double i for all time, dV/dt will also double and hence V will double.

\[
C = \frac{\varepsilon A}{d}
\]

where \( \varepsilon \) is the dielectric constant
Capacitors Only Affect Time Response not Final Values

- Capacitors relate $i$ to $dV/dt$
- This means if the circuit “settles down” and isn’t changing with time, a capacitor has no effect (looks like an open circuit).

@ $t = 0$

@ $t = \infty$
So What Do Capacitors Do?

- It affects how fast a voltage can change
  - Current sets $dV/dt$, and not $V$
  - Fast changes require lots of current

- For very small $\Delta t$ capacitors look like voltage sources
  - They can supply very large currents
  - And not change their voltage

- But for large $\Delta t$
  - Capacitors look like open circuits (they don’t do anything)
Capacitor Energy

- The Power that flows into a charging capacitor is
  \[ P = iV = \left( C \frac{dV}{dt} \right) V \]
- And the energy stored in the capacitor is
  \[ E = \int P \, dt \]
  \[ \therefore E = \int_0^V P \, dt = \int_0^V CV \, dV = \frac{1}{2} CV^2 \]
- This energy is stored and can be released at a later time. No energy is lost.
REAL CAPACITORS
Capacitor Types

- There are many different types of capacitors
  - Electrolytic, tantalum, ceramic, mica, . . .

- They come in different sizes
  - Larger capacitance
    - Generally larger size
  - Higher voltage compliance
    - Larger size

- Electrolytic have largest cap/volume
  - But they have limited voltage
  - They are polarized
    - One terminal must be + vs. other

Gate of MOS Transistor

- Is a capacitor between Gate and Source

- To change the gate voltage
  - You need a current pulse (to cause $dV/dt$)

- If the current is zero (floating)
  - $dV/dt = 0$, and the voltage remains what it was!
All Real Wires Have Capacitance

- It will take some charge to change the voltage of a wire
  - Think back to our definition of voltage
    - Potential energy for charge
  - To make a wire higher potential energy
    - Some charge has to flow into the wire, to make the energy higher for the next charge that flows into it

- This capacitance is what sets the speed of your computer
  - And determines how much power it takes!
\[ C = \sum C_{wire} + C_{in} \]
Capacitor Info, If You Know Physics E&M…

- Models the fact that energy is stored in electric fields
  - Between any two wires that are close to each other

- A capacitor is formed by two terminals that are not connected
  - But are close to each other
  - The closer they are, the larger the capacitor

- To create a voltage between the terminals
  - Plus charge is collected on the positive terminal
  - Negative charge is collected on the negative terminal

- This creates an electric field (Gauss’s law)
  - Which is what creates the voltage across the terminals
  - There is energy stored in this electric field
Capacitors in Parallel and Series

\[ i_T = i_1 + i_2 + i_3 = C_1 \frac{dV}{dt} + C_2 \frac{dV}{dt} + C_3 \frac{dV}{dt} \]

\[ = (C_1 + C_2 + C_3) \frac{dV}{dt} \]

\[ \therefore C_{eqv} = C_1 + C_2 + C_3 \]

"like R's in series"
Capacitors in Parallel and Series

\[ C_{eq} = \frac{C_1 C_2}{C_1 + C_2} \]

\[ V_T = V_1 + V_2 + V_3 = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = \frac{Q}{C_{eqv}} \]

\[ \therefore \frac{1}{C_{eqv}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \]
CAPACITOR RESISTOR CIRCUITS
Capacitors and Logic Gate Speeds

• When the input changes from low to high
  – The pMOS turns off, and the nMOS turns on
  – The output goes from high to low

• But in this model
  – The output changes as soon as the input changes
Gates Are NOT Zero Delay

- It would be great if logic gates had zero delay
  - But they don’t

- Fortunately, it is easy to figure out the delay of a gate
  - It is just caused by the transistor resistance
    - Which we know about already
  - And the transistor and wire capacitance
Improved Model

- Just add a capacitor to the output node
  - Its value is equal to the capacitance of the gates driven
  - Plus the capacitance of the wire itself
RC Circuit Equation

- When the input to the inverter is low, the output will be at $V_{dd}$
  - Right after the input rises, here is the circuit

- Want to find the capacitor voltage verses time

- Just write the nodal equations:
  - We just have one node voltage, $V_{out}$
  - $i_{RES} = V_{out}/R_2$
  - $i_{CAP} = C \frac{dV_{out}}{dt}$

- From KCL, the sum of the currents must be zero, so
  $$\frac{dV_{out}}{dt} = -\frac{V_{out}}{R_2C}$$
RC Circuit Equations

- Solving,

\[ \frac{\int_{V} \frac{dV_{\text{out}}}{V_{\text{out}}}}{5} = -\int_{0}^{t} \frac{dt}{R_{2}C} \]

so that \[ \ln(V_{\text{out}}) - \ln(5V) = -\frac{t}{R_{2}C} \]

- This is an exponential decay
  - The x axis is in time constants
  - The y axis has been normalized to 1
  - Slope always intersects 0 one tau later (\( \tau = RC \))

\[ \therefore V_{\text{out}} = 5V \left( e^{-\frac{t}{R_{2}C}} \right) \]
What Happens When Input Falls?

• Now the voltage across the capacitor starts at 0V
  \[ i = \frac{(V_{dd} - V_{out})}{R_1} \]
  \[ \frac{dV_{out}}{dt} = \frac{V_{dd} - V_{out}}{R_1C} \]

• Not quite the right form
  – Need to fix it by changing variables
  – Define \( V_{\text{new}} = V_{dd} - V_{out} \)
  – \( \frac{dV_{out}}{dt} = -\frac{dV_{\text{new}}}{dt} \), since \( V_{dd} \) is fixed

\[
\int_{\frac{V}{5}}^{\frac{V_{\text{new}}}{V}} \frac{dV_{\text{new}}}{V_{\text{new}}} = -\int_{0}^{t} \frac{dt}{R_1C} \quad \text{so that} \quad \ln\left(\frac{V_{\text{new}}}{5}\right) = -\frac{t}{R_1C} \quad \therefore \quad V_{out} = 5V \left(1 - e^{-t/R_1C}\right)
\]
RC Circuits in the Time Domain

\[ V_{\text{out}} = 5V \left(1 - e^{-t/R_1C}\right) \]

\[ V_{\text{out}} = 5V \left(e^{-t/R_2C}\right) \]

In capacitor circuits, voltages change “slowly”, while currents can be instantaneous.
Simple RC Circuit Demo

EveryCircuit Demo – CMOS Inverter
Interesting Aside

- Exponentials “never” reach their final value
- So if this logic gate is driving another gate, when does the next gate think its input is 0 or 1?
- This is one of the reasons why logic levels are defined as a range of values.
Learning Objectives

- Understand what a capacitor is
  - \( i = C \frac{dV}{dt} \)
  - It is a device that tries to keep voltage constant
    - Will supply current (in either direction) to resist voltage changes

- Understand how voltages and current change in R C circuits
  - Voltage waveforms are continuous
    - Takes time for their value to change
    - Exponentially decay to final value (the DC value of circuit)
  - Currents can charge abruptly