E40M
Capacitors

Midterm - Wed in class, this room

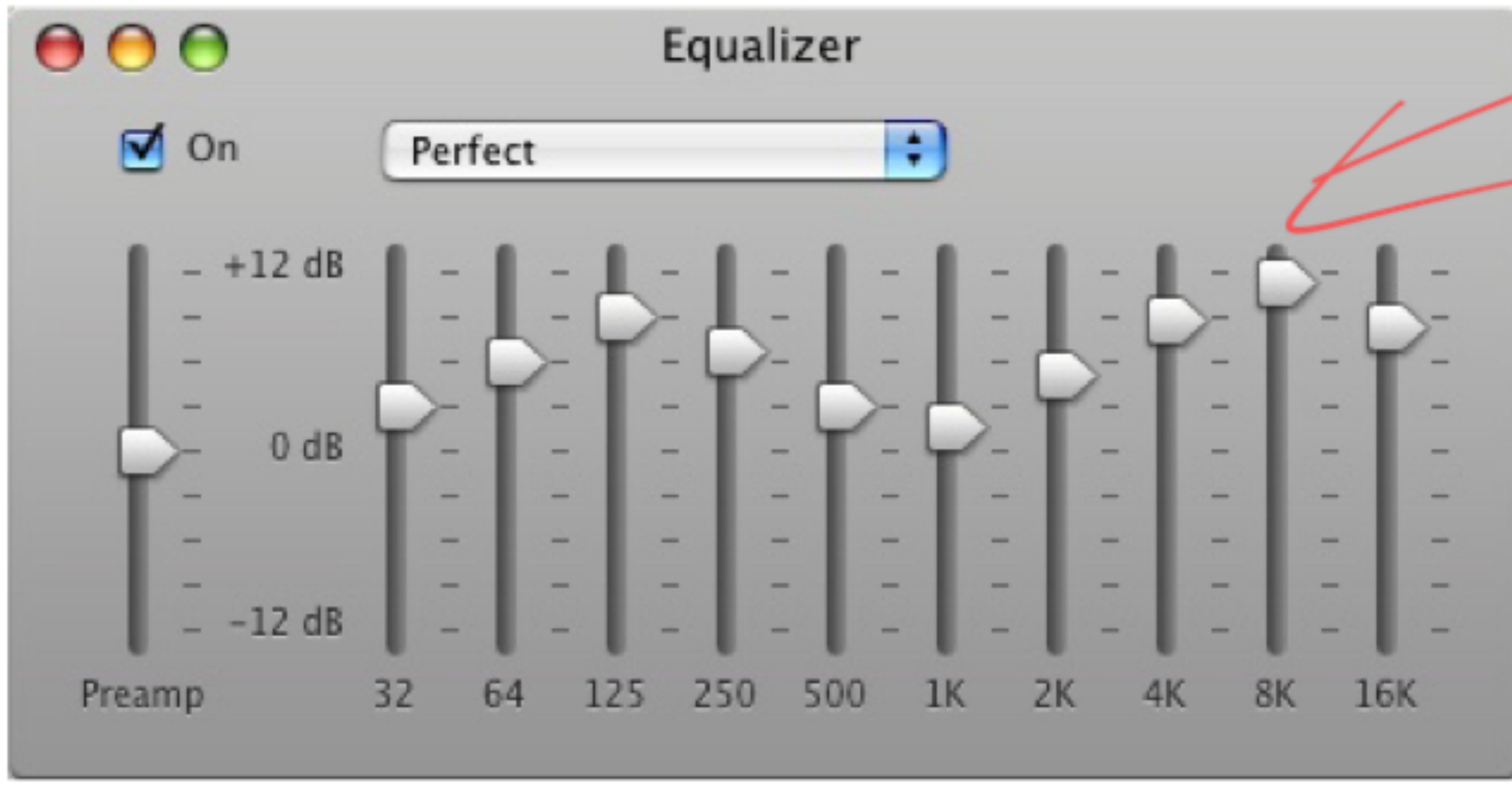
Cover through lecture #9, lab 2b, HW #4

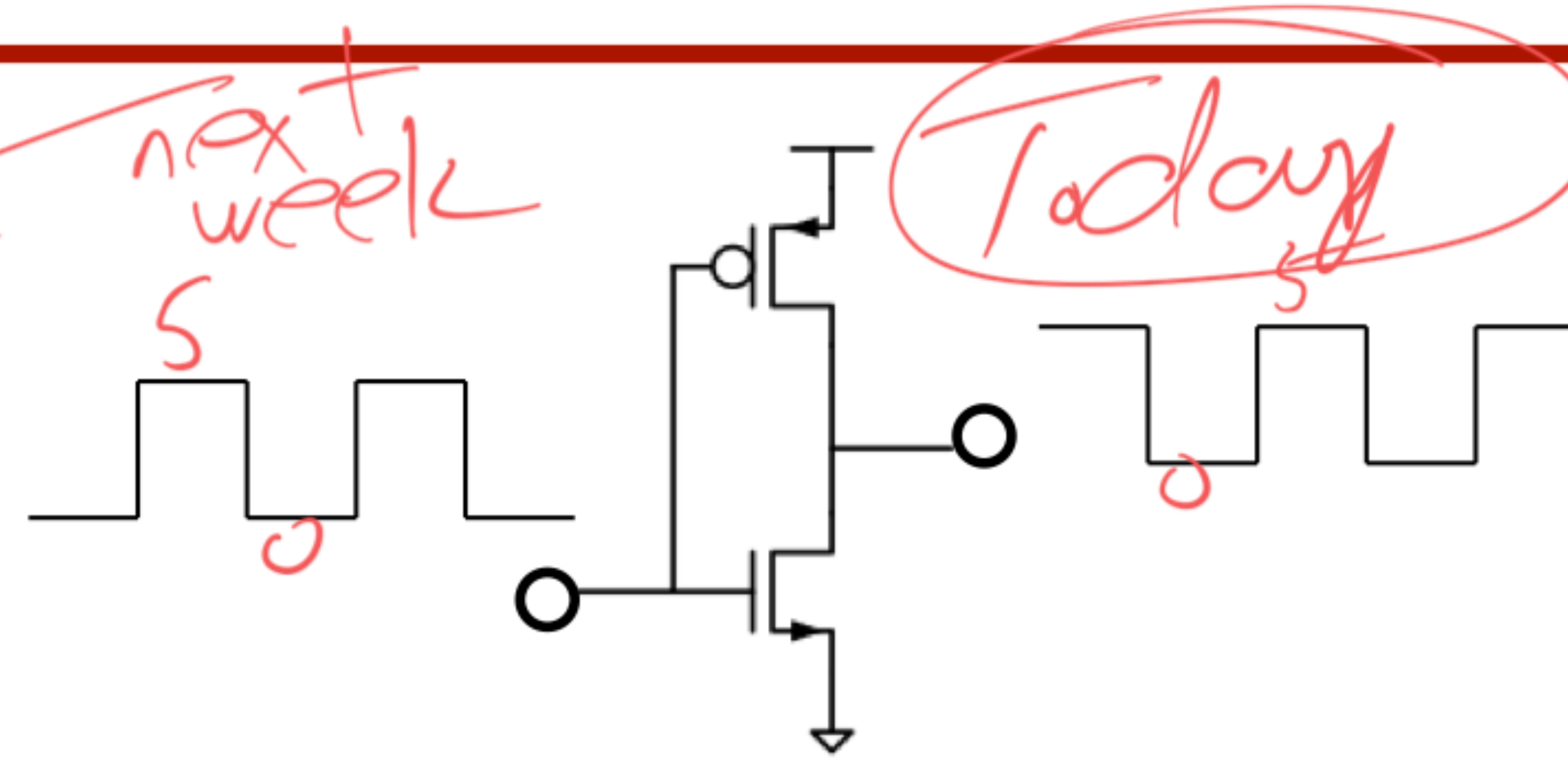
Reading

- Reader:
  - Chapter 6 – Capacitance

- A & L:
  - 9.1.1, 9.2.1

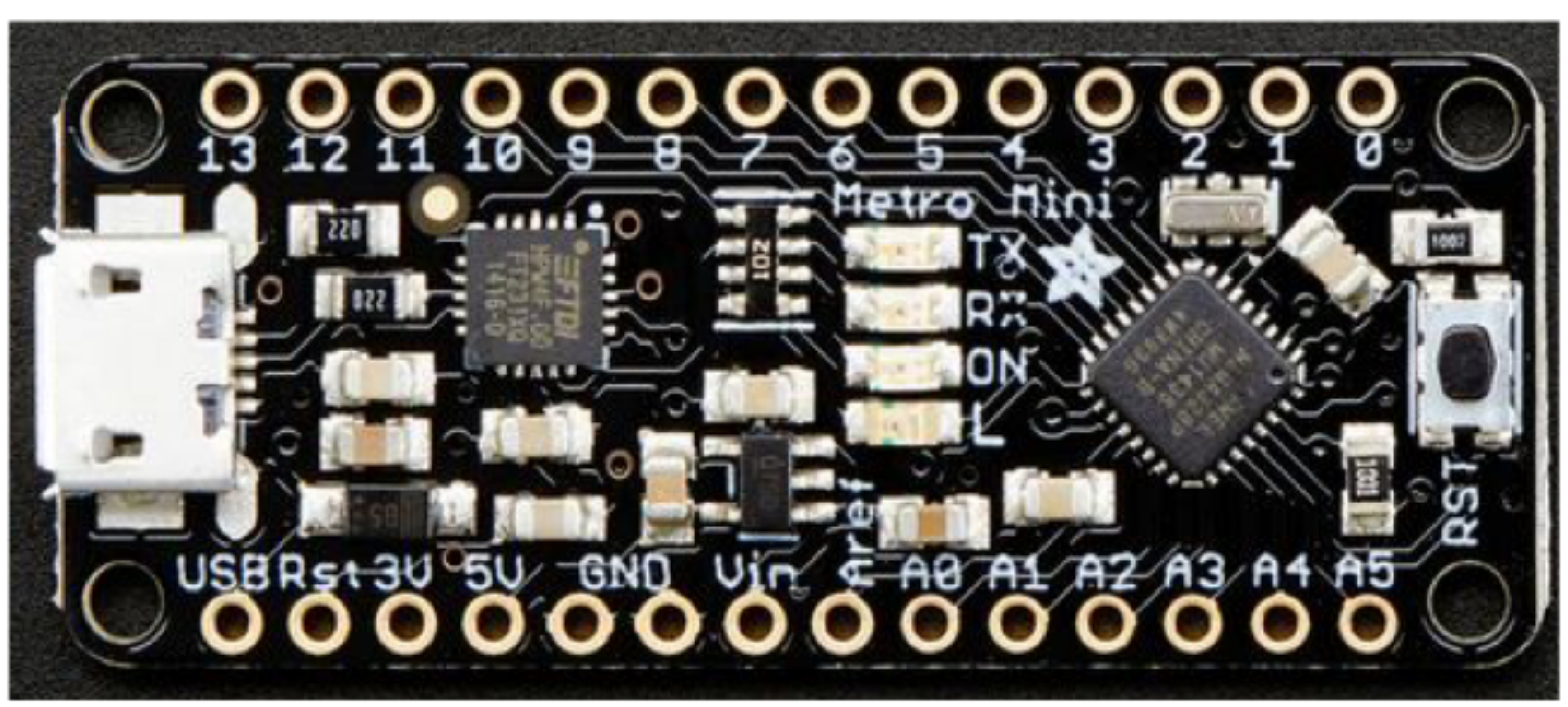
Why Are Capacitors Useful/Important?





How do we design circuits that respond to certain frequencies?

What determines how fast CMOS circuits can work?

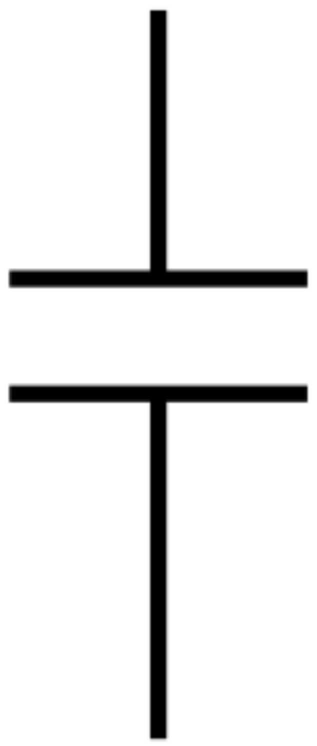


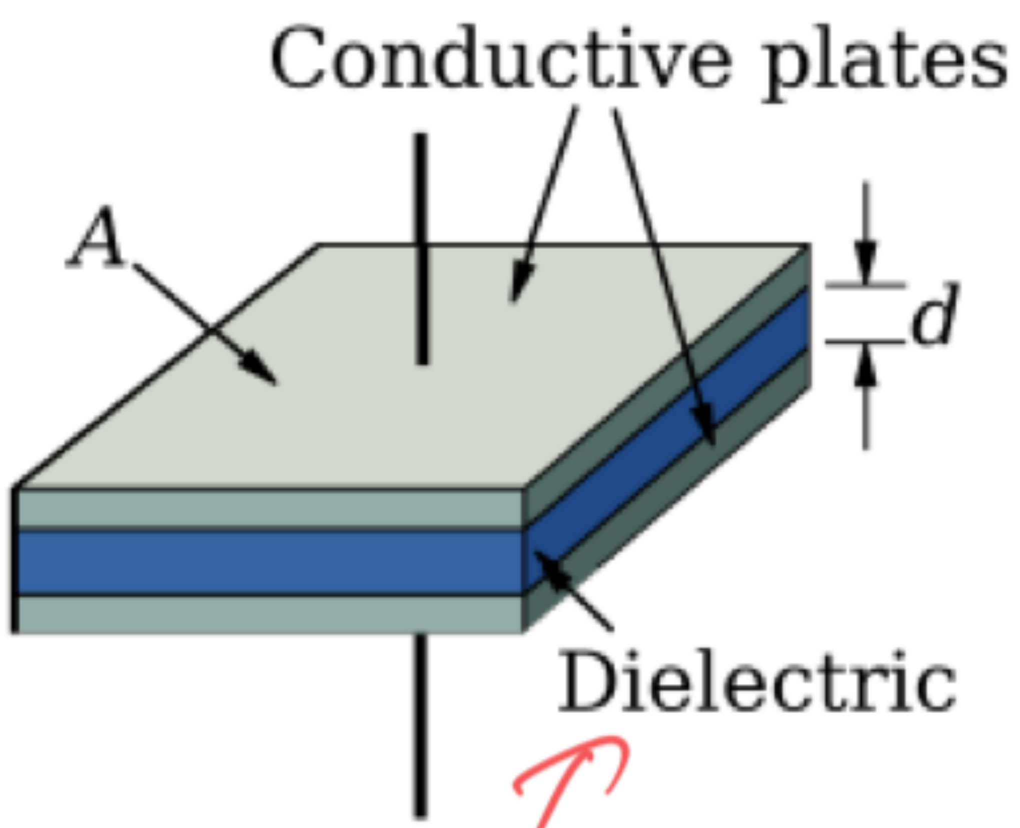
Why did you put a 200μF capacitor between Vdd and Gnd on your Arduino?

CAPACITORS

Capacitors

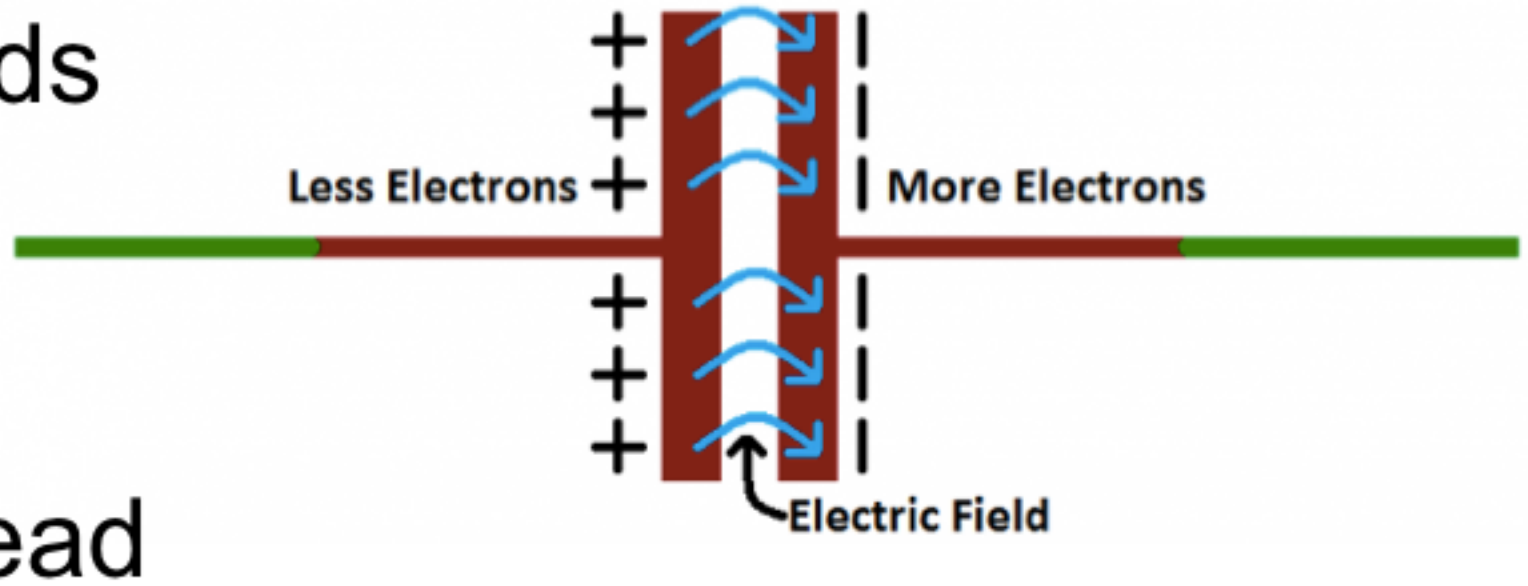
- What is a capacitor?
  - It is a new type of two terminal device
  - It is linear
    - Double V, you will double I
  - We will see it doesn't dissipate energy
    - Stores energy



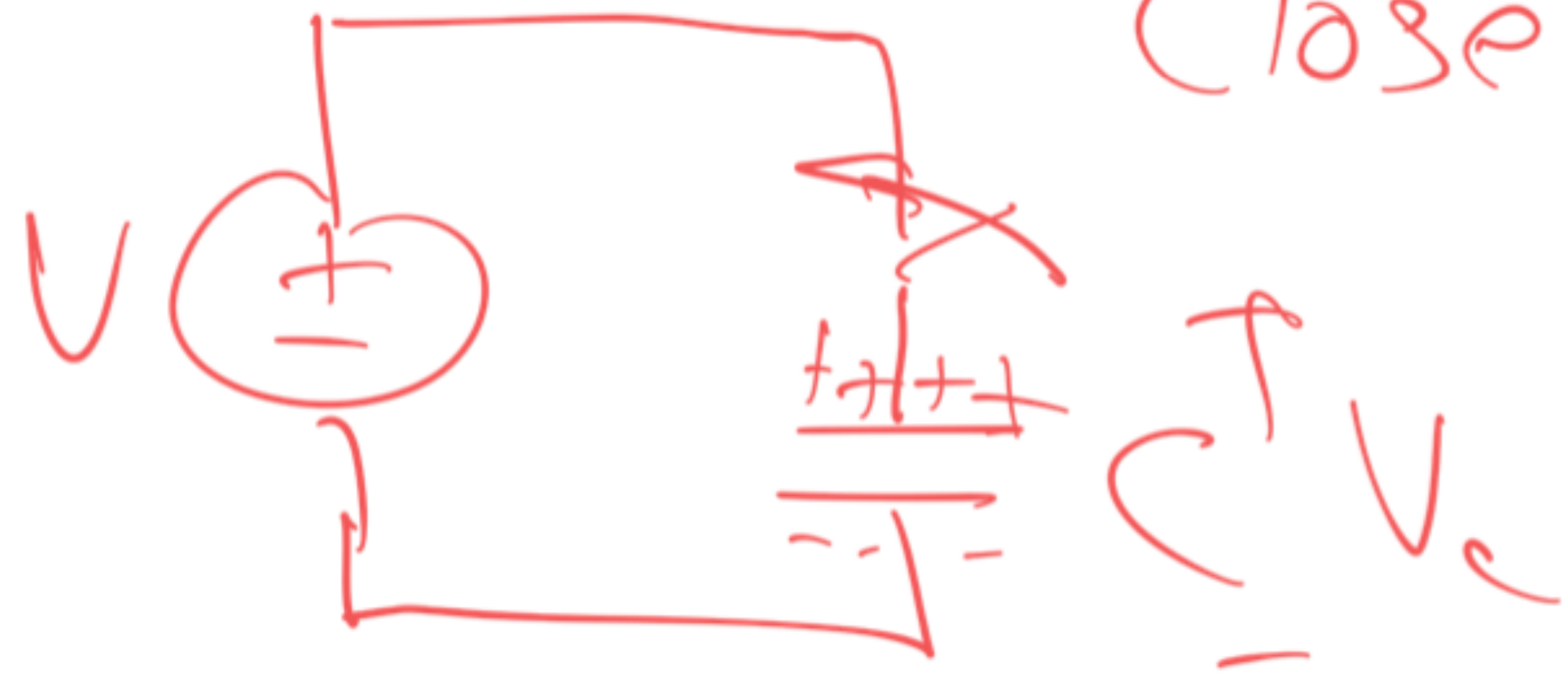


Insulator

- Rather than relating i and V
  - Relates Q, the charge stored on each plate, to Voltage
  - Q = CV
  - Q in Coulombs, V in Volts, and C in Farads



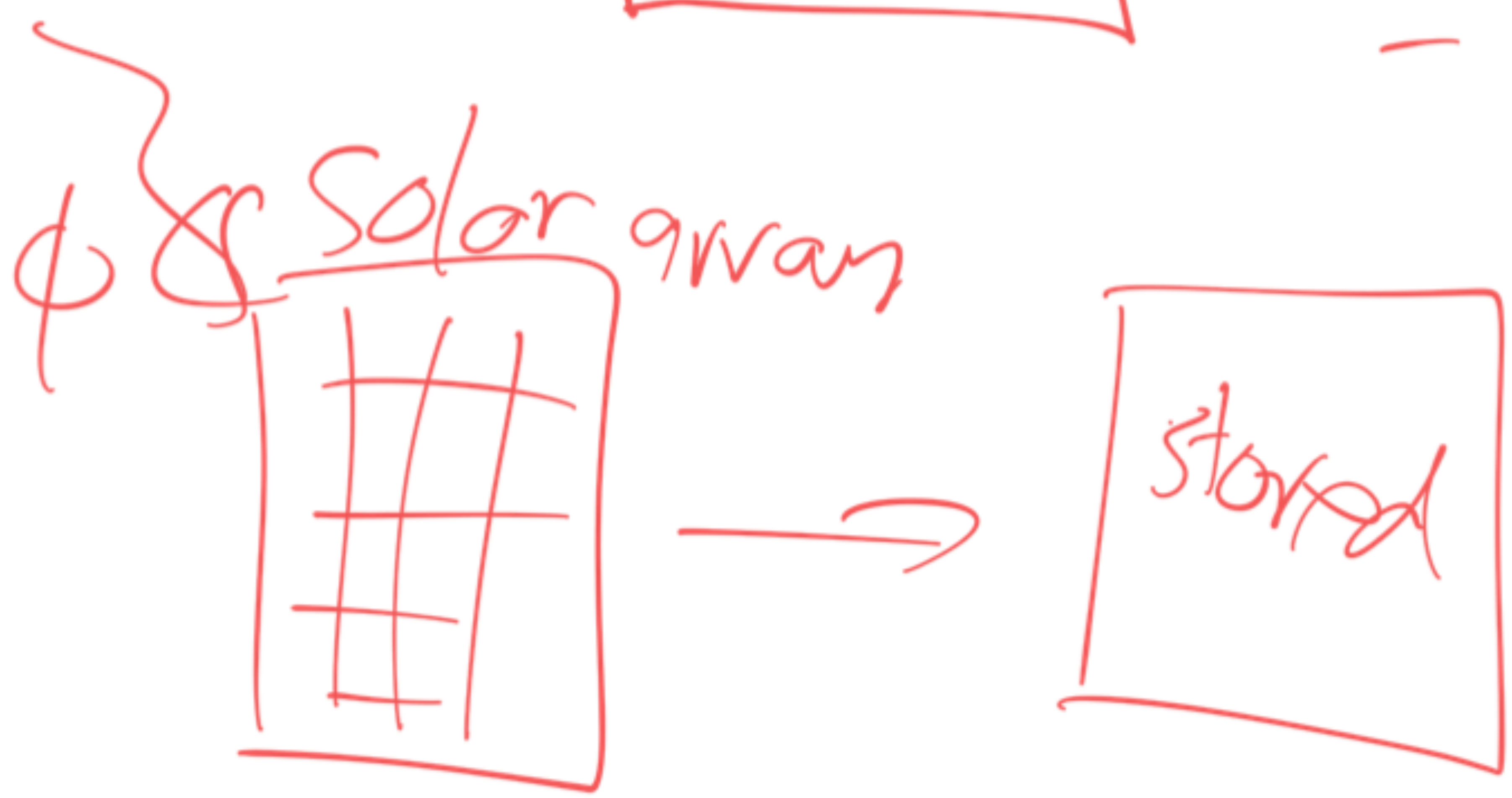
- Like all devices, it is always charge neutral
  - Stores +Q on one lead, -Q on the other lead



Close switch @ $t=0$

once $V_c = V$

then $i \to 0$



① Battery – solar cell project

② ~~Cap~~

much less energy density

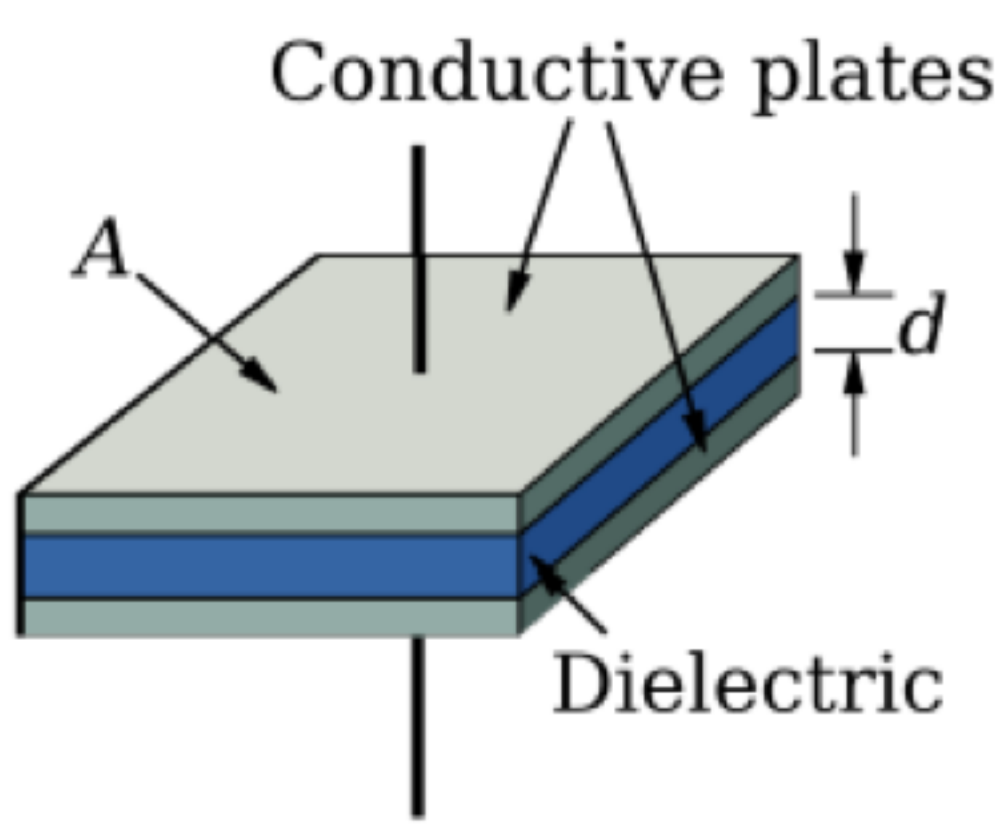
capacitors have many applications in circuits

iV for a Capacitor

- We generally don't work in Q, we like i and V
  - But current is charge flow, or dQ/dt

- So if Q = CV, and i=dQ/dt
  - i= C dV/dt

- This is a linear equation but between i and dV/dt. If you double i for all time, dV/dt will also double and hence V will double.



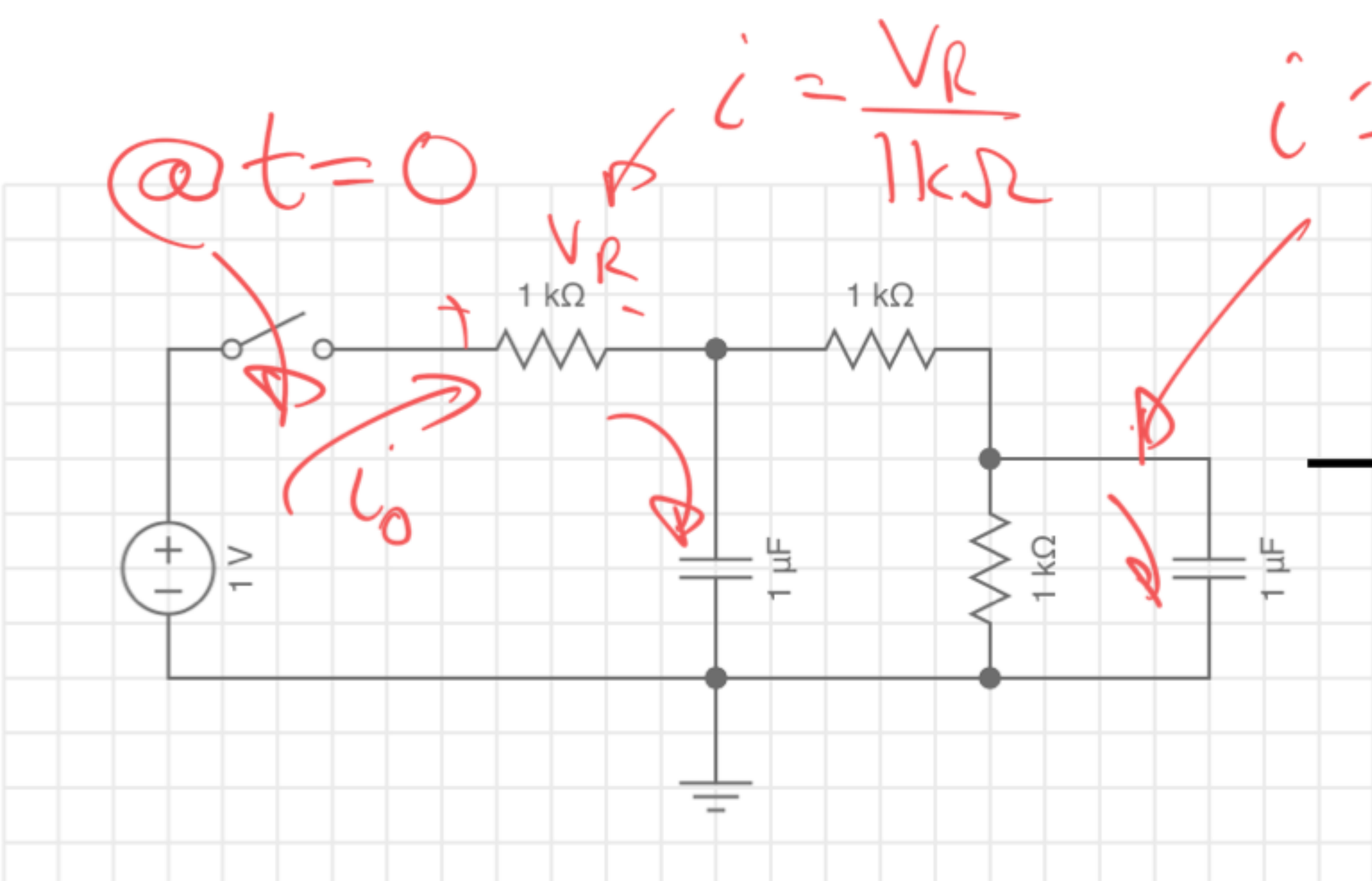
$$C = \frac{\varepsilon A}{d}$$

$C \uparrow$ as $A \uparrow$
$C \uparrow$ as $d \downarrow$

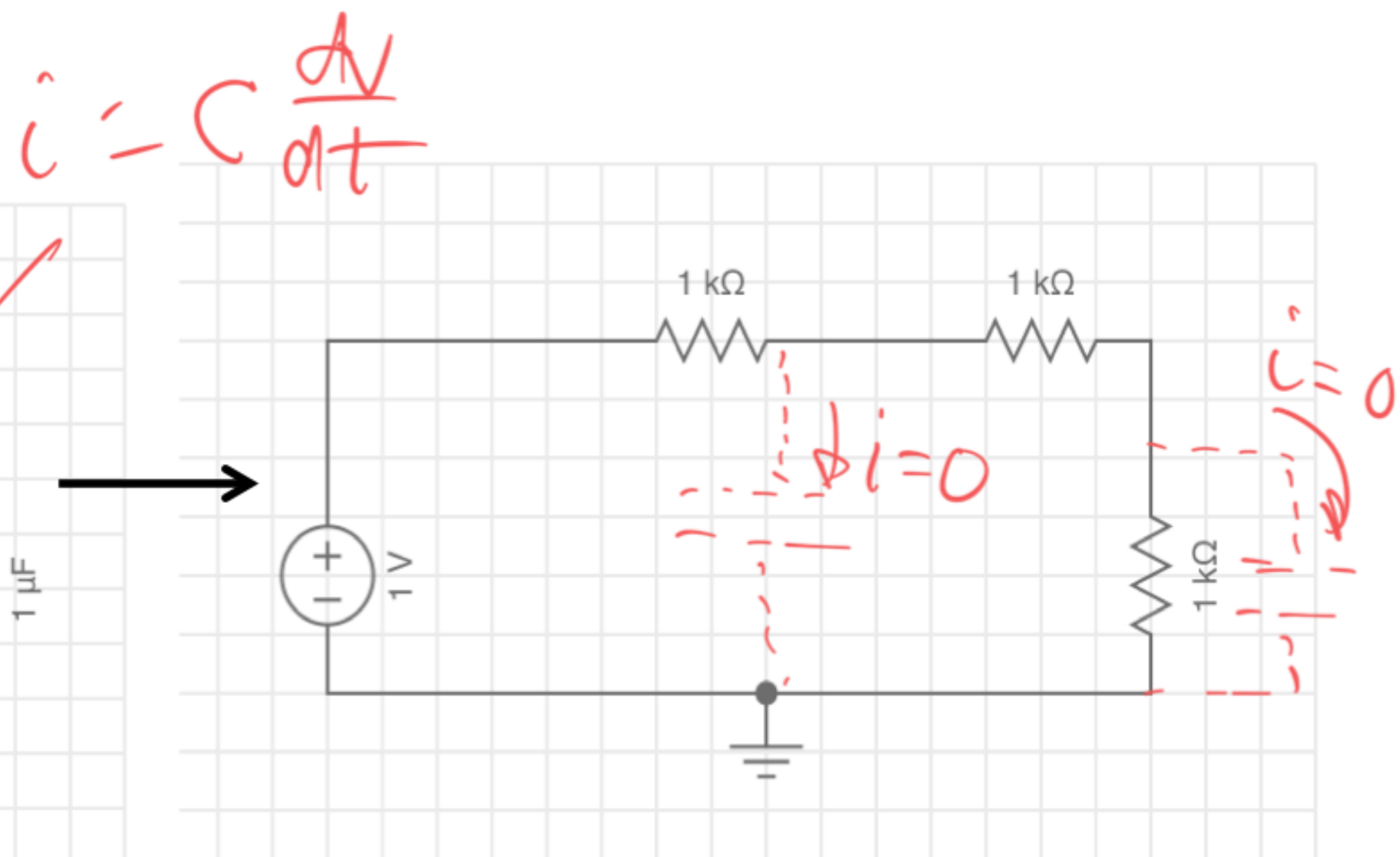
where ε is the dielectric constant

Capacitors Only Affect Time Response not Final Values

- Capacitors relate i to dV/dt
- This means if the circuit "settles down" and isn't changing with time, a capacitor has no effect (looks like an open circuit).



@t = 0

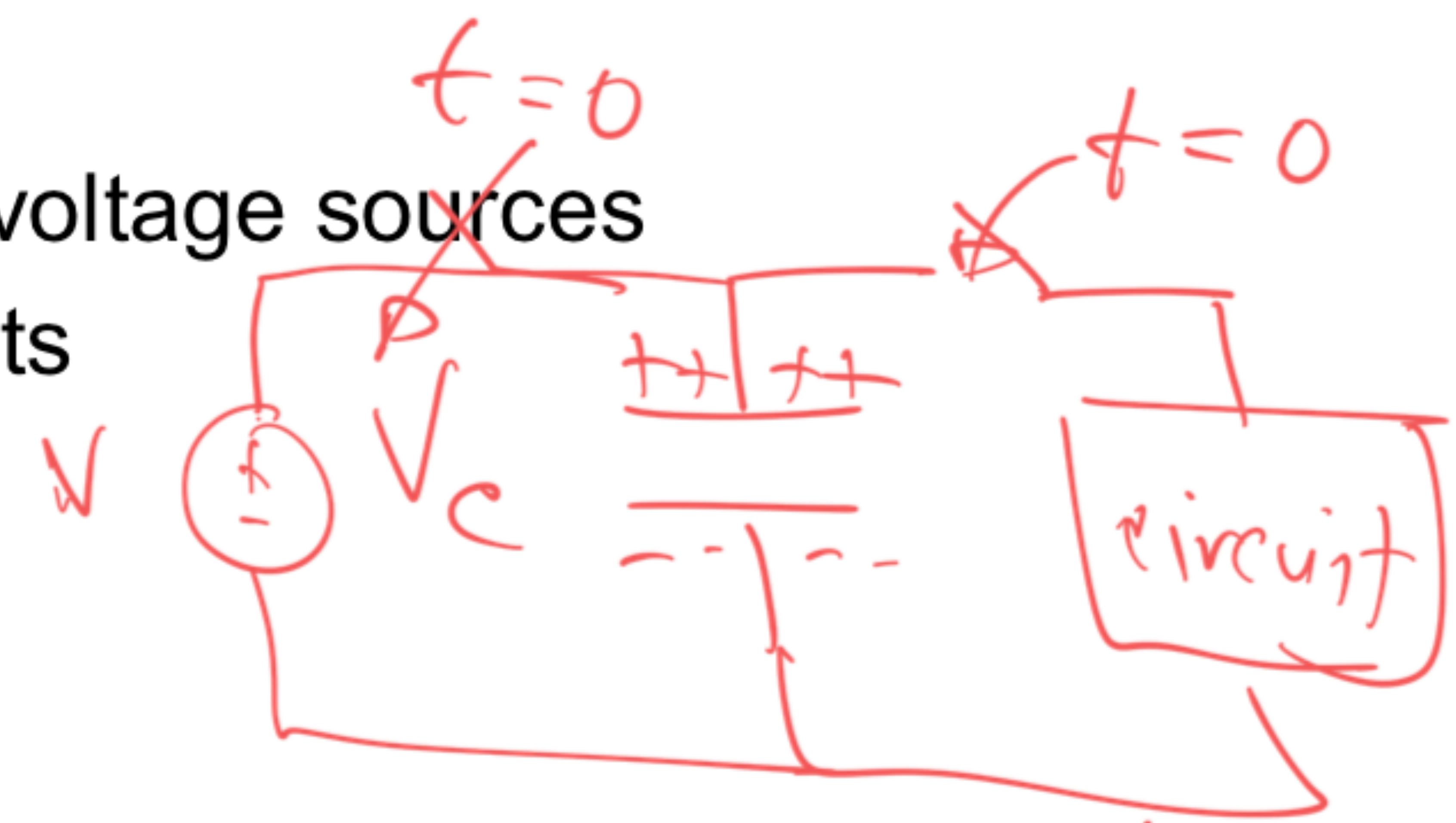


@t = ∞

So What Do Capacitors Do?

- It affects how fast a voltage can change
  - Current sets dV/dt, and not V
  - Fast changes require lots of current
- For very small Δt capacitors look like voltage sources
  - They can supply very large currents
  - And not change their voltage
- But for large Δt
  - Capacitors look like open circuits (they don't do anything)

$$i = C\frac{dV}{dt}$$

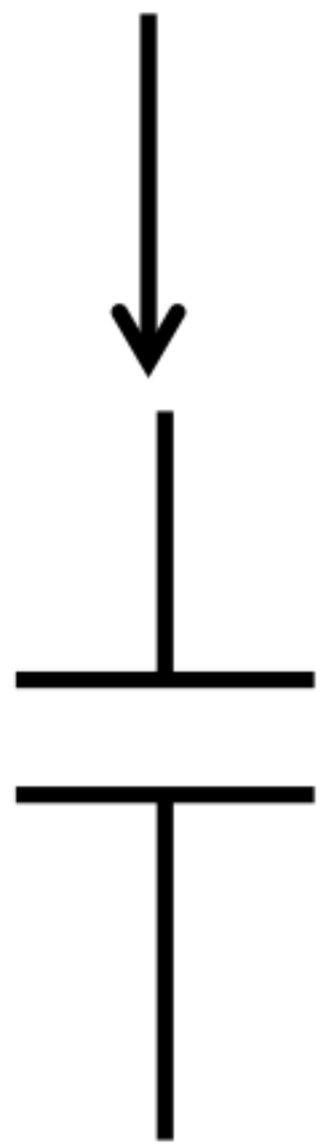


t=∞
charge all gone

Capacitor Energy

- The Power that flows into a charging capacitor is

$$P = iV = \left(C\frac{dV}{dt}\right)V$$



- And the energy stored in the capacitor is

$$E = \int P\,dt$$

$$\therefore E = \int P\,dt = \int_0^V CV\,dV = \frac{1}{2}CV^2$$

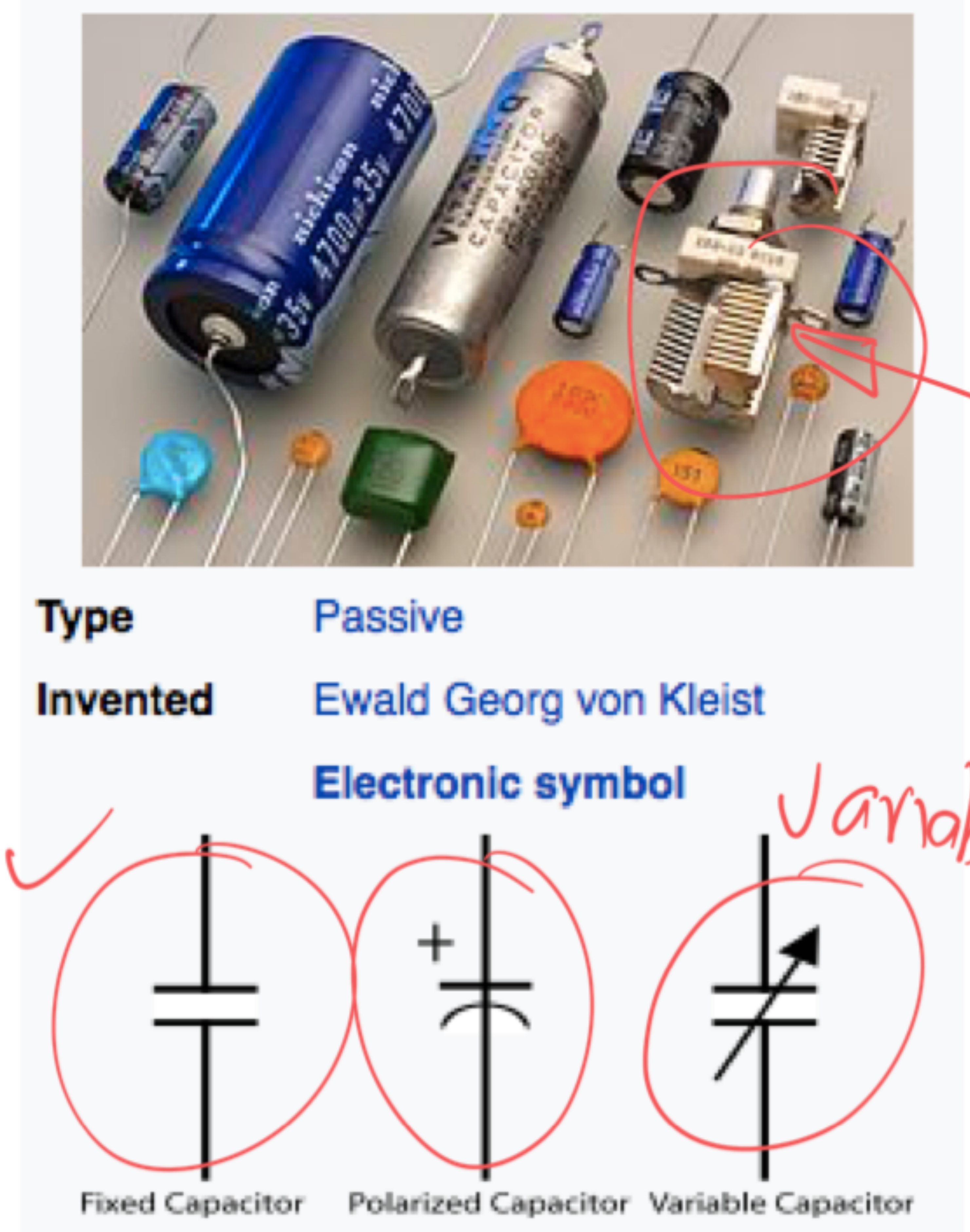
- This energy is stored and can be released at a later time. No energy is lost.

REAL CAPACITORS

Capacitor Types

- There are many different types of capacitors
  - Electrolytic, tantalum, ceramic, mica, . . .
- They come in different sizes
  - Larger capacitance
    - Generally larger size
  - Higher voltage compliance
    - Larger size
- Electrolytic have largest cap/volume
  - But they have limited voltage
  - They are polarized
    - One terminal must be + vs. other

Al_2O_3

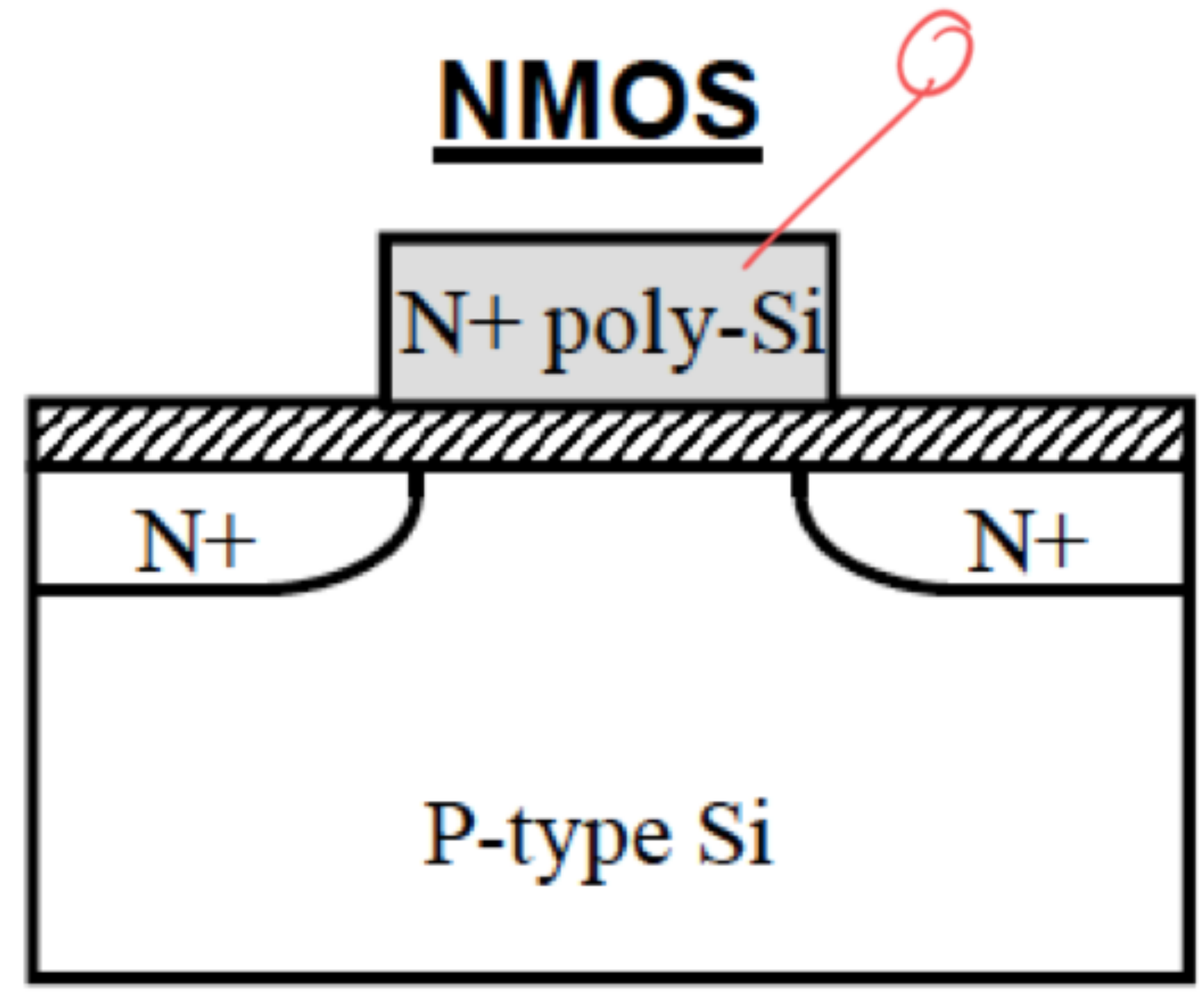


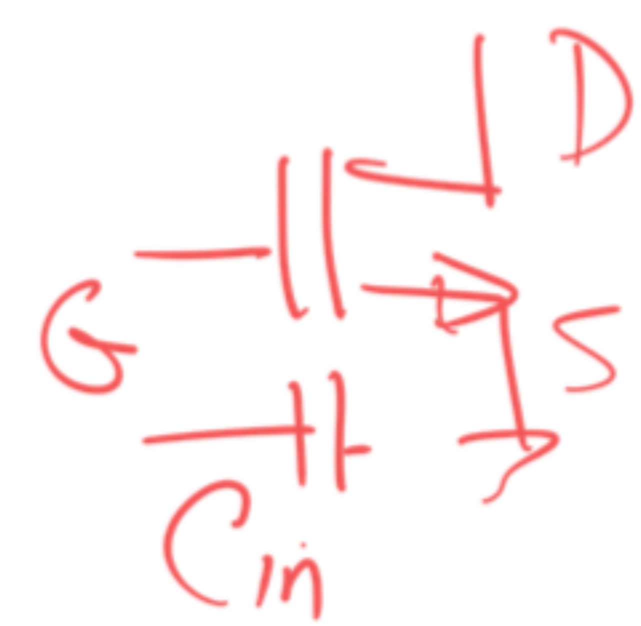
http://en.wikipedia.org/wiki/Types_of_capacitor

Gate of MOS Transistor

- Is a capacitor between Gate and Source
- To change the gate voltage
  - You need a current pulse (to cause dV/dt)
- If the current is zero (floating)
  - dV/dt = 0, and the voltage remains what it was!

$i = C\frac{dv}{dt}$

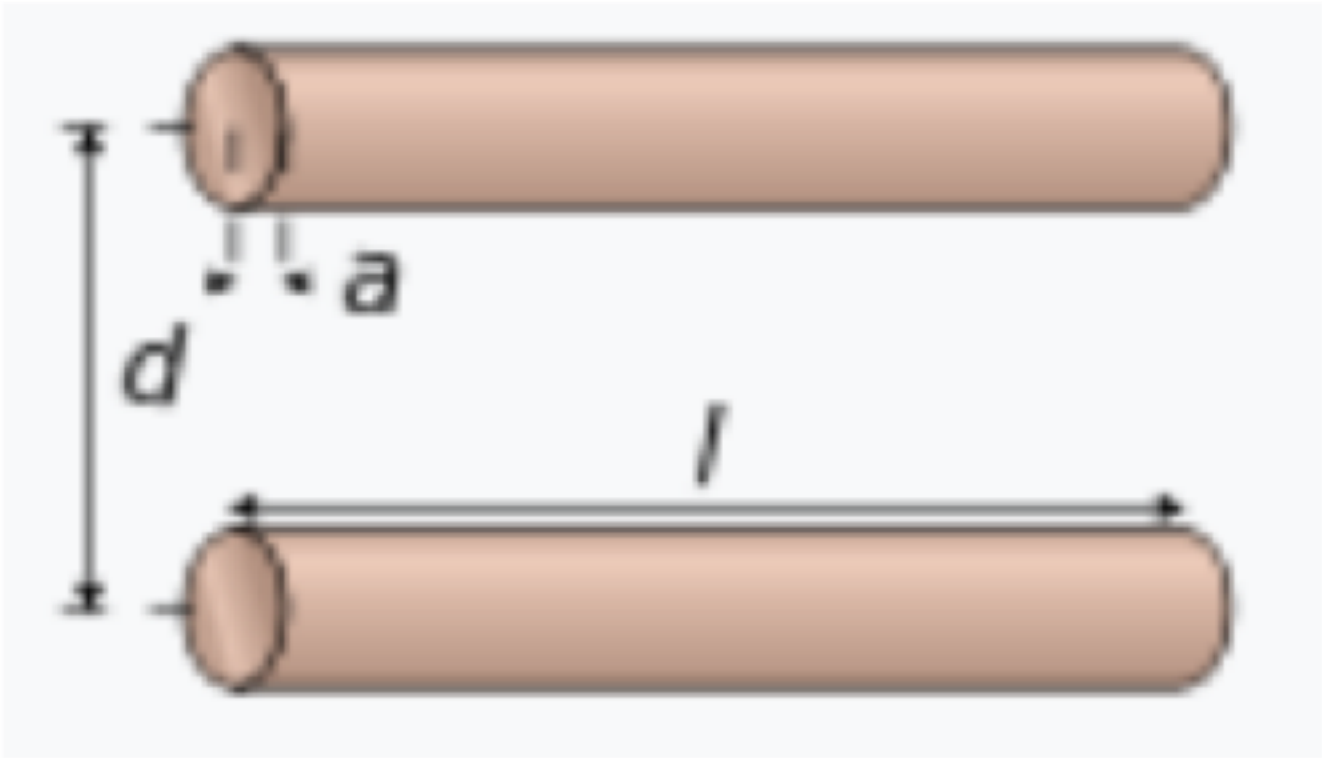


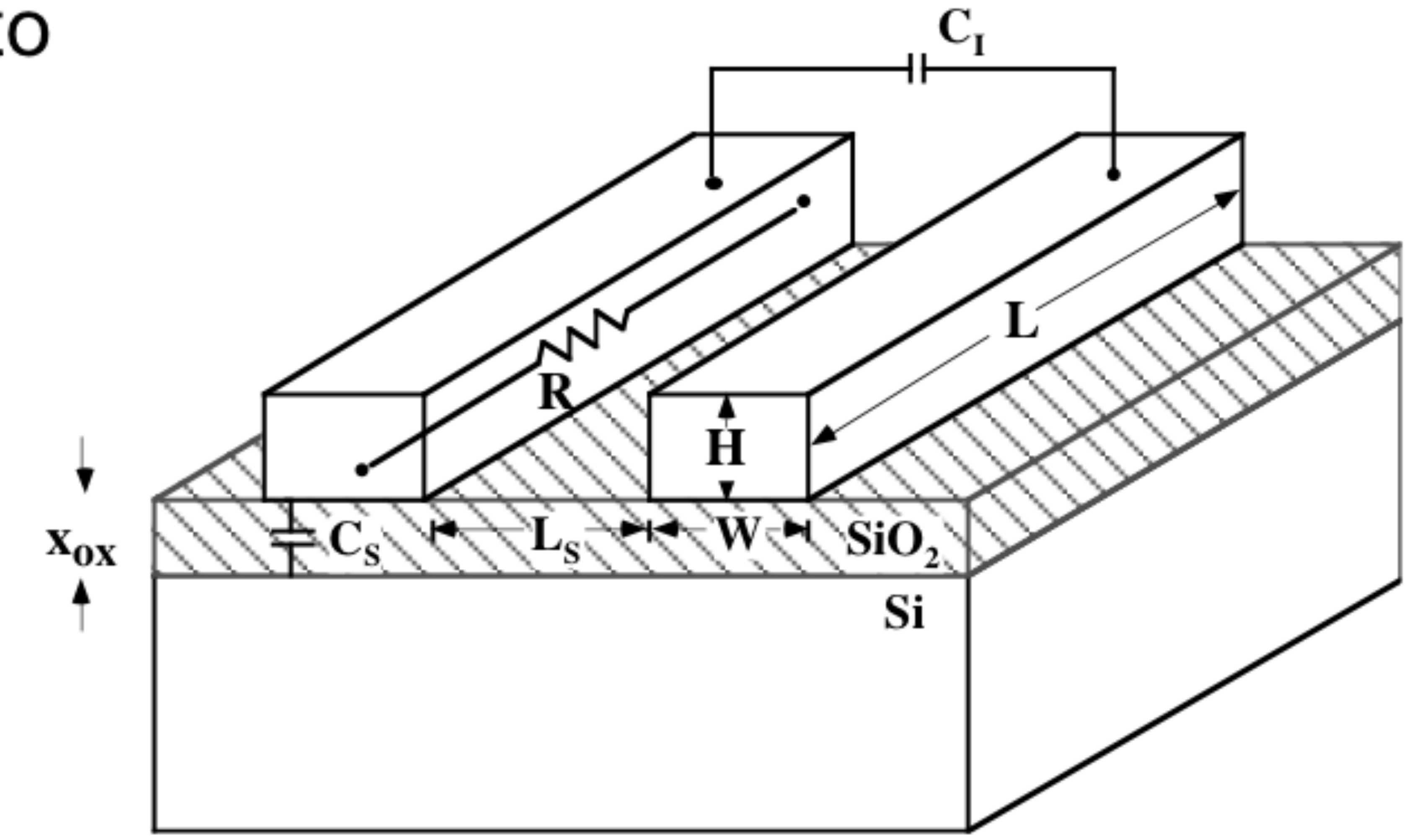


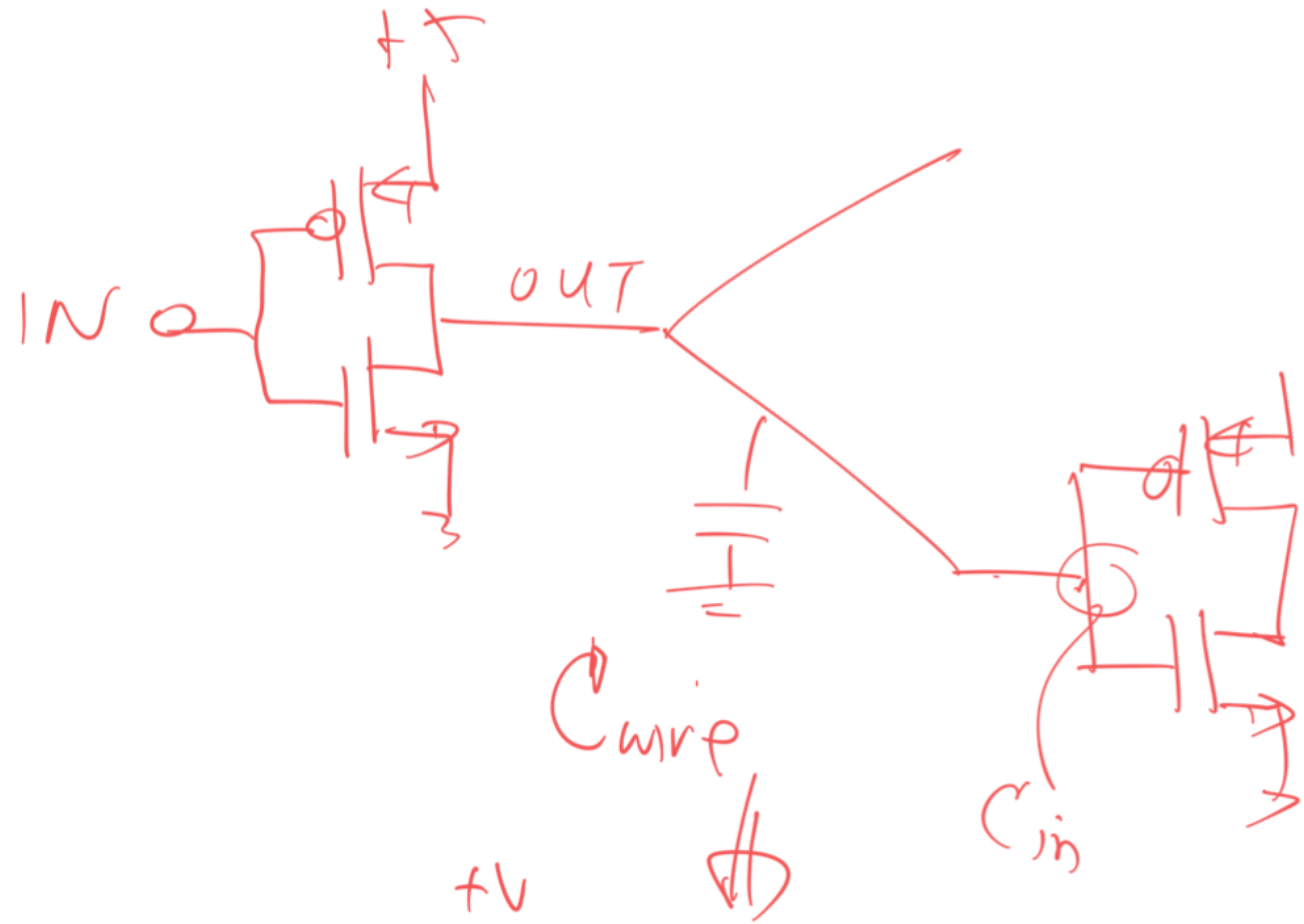
All Real Wires Have Capacitance

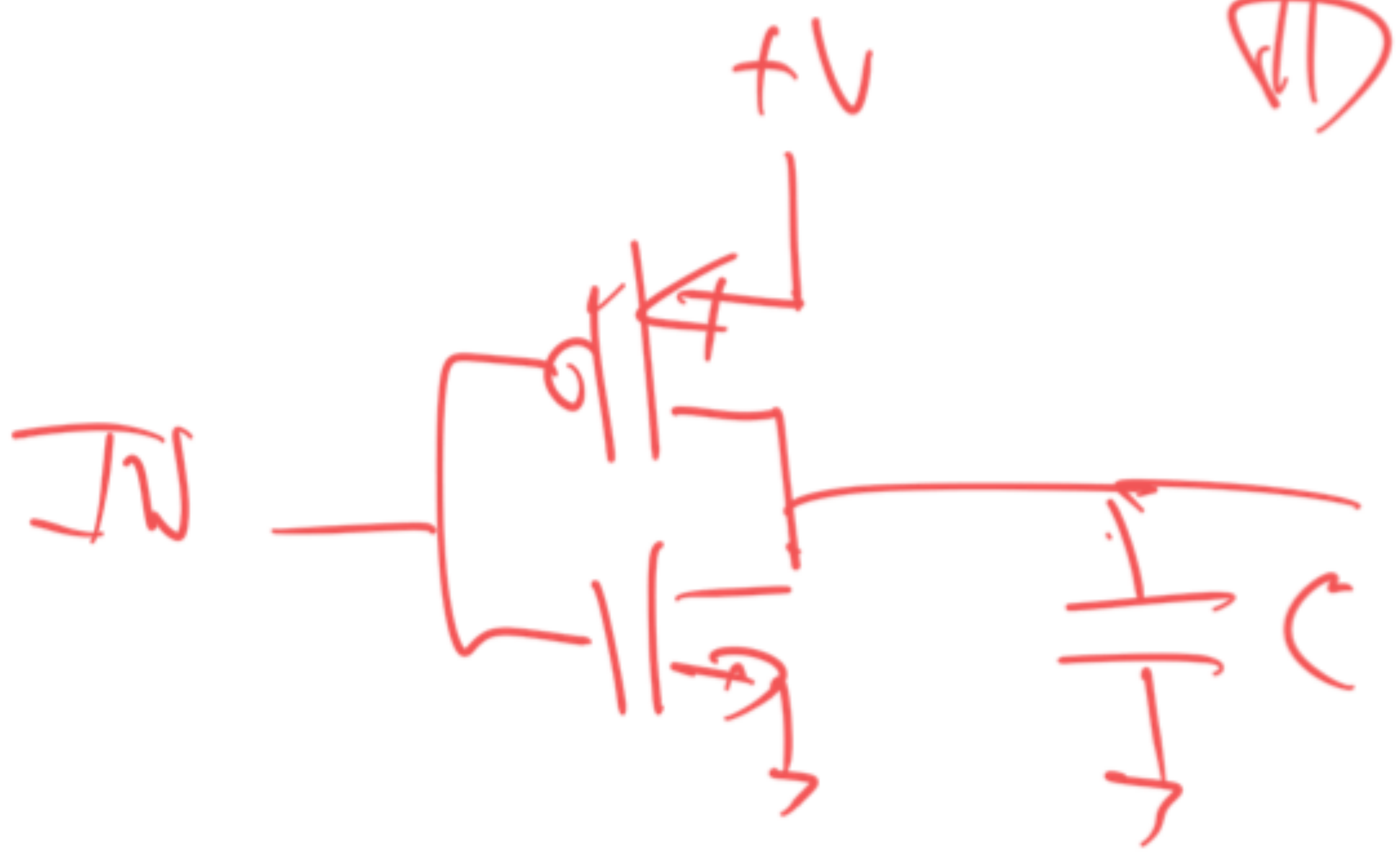
- It will take some charge to change the voltage of a wire
  - Think back to our definition of voltage
    - Potential energy for charge
  - To make a wire higher potential energy
    - Some charge has to flow into the wire, to make the energy higher for the next charge that flows into it

- This capacitance is what sets the speed of your computer
  - And determines how much power it takes!





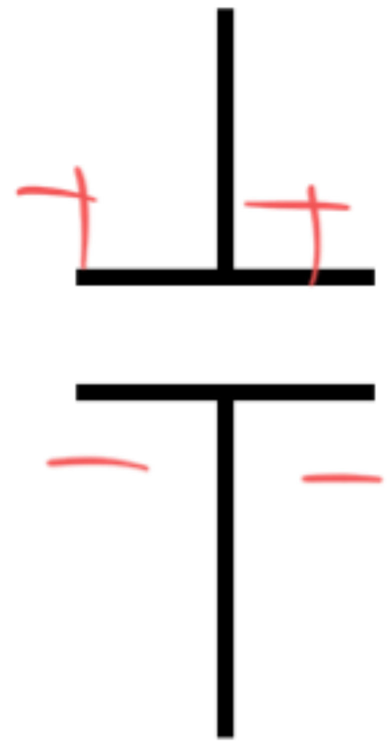




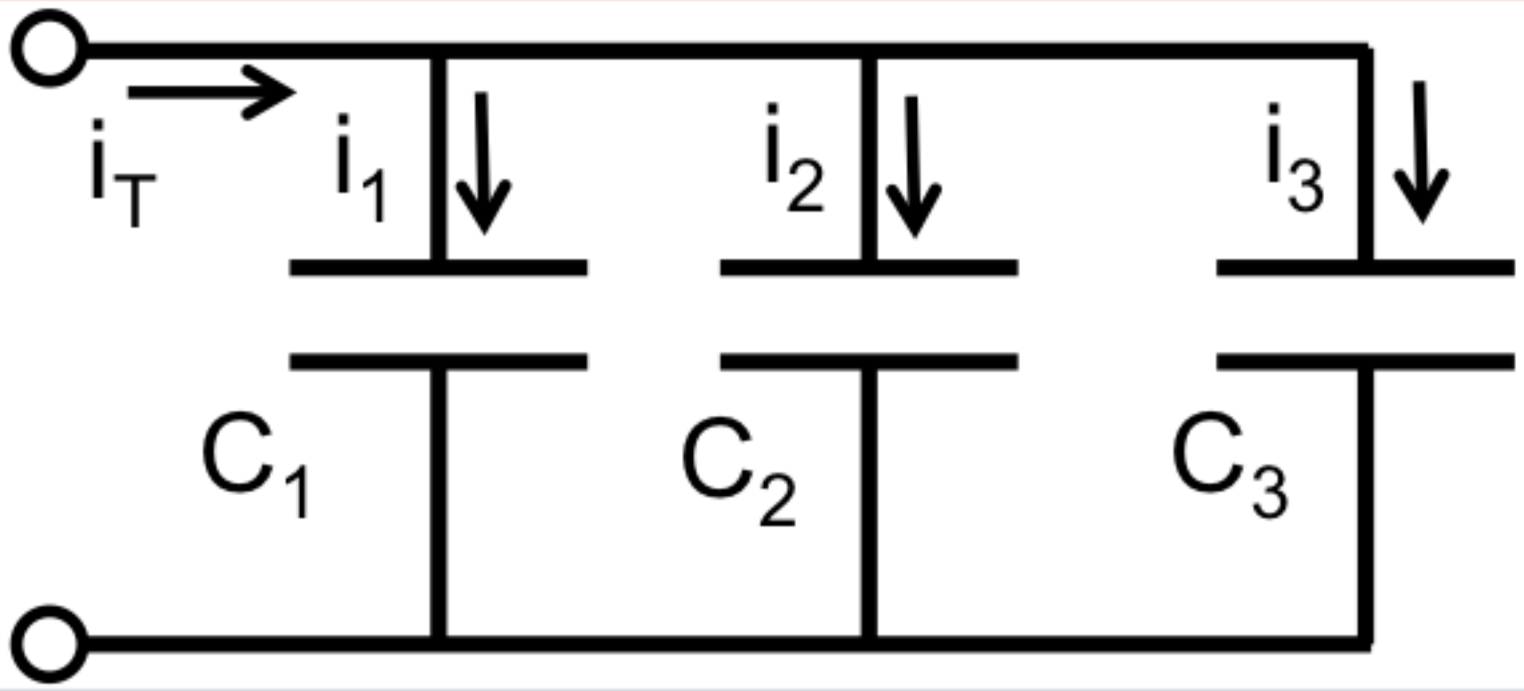
$$C = \sum C_{wire} + C_{IN}$$

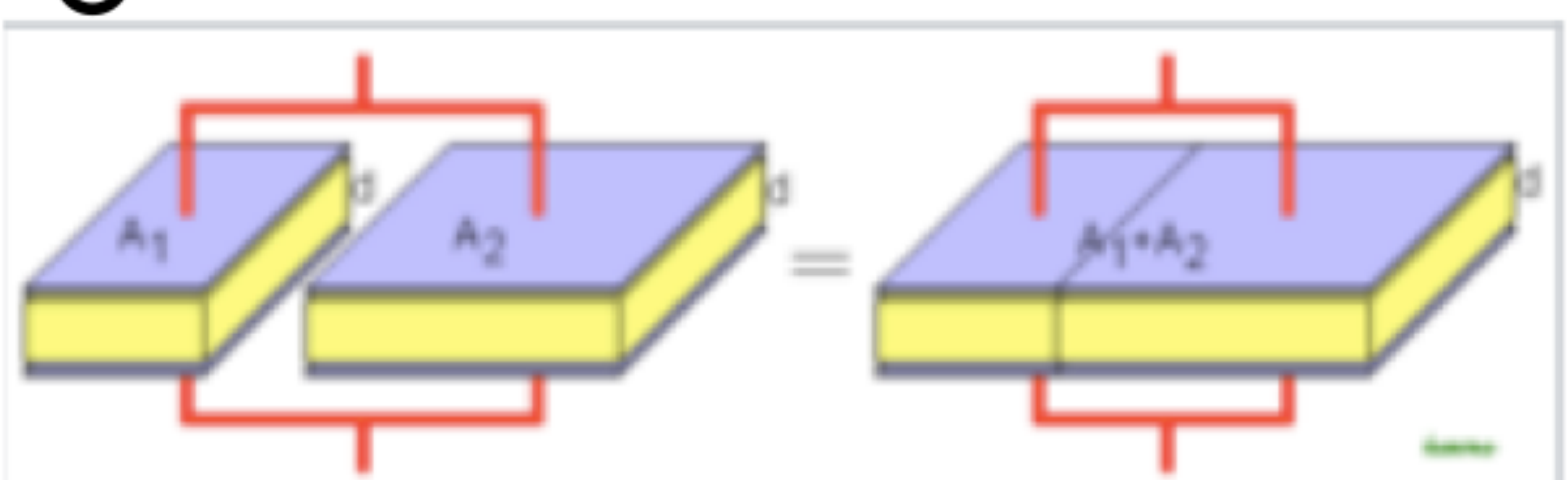
Capacitor Info, If You Know Physics E&M…

- Models the fact that energy is stored in electric fields
  - Between any two wires that are close to each other
- A capacitor is formed by two terminals that are not connected
  - But are close to each other
  - The closer they are, the larger the capacitor
- To create a voltage between the terminals
  - Plus charge is collected on the positive terminal
  - Negative charge is collected on the negative terminal
- This creates an electric field (Gauss's law)
  - Which is what creates the voltage across the terminals
  - There is energy stored in this electric field



Capacitors in Parallel and Series





$$C = \frac{\varepsilon A}{d}$$

$$i_T = i_1 + i_2 + i_3 = C_1\frac{dV}{dt} + C_2\frac{dV}{dt} + C_3\frac{dV}{dt}$$

$$= \left(C_1 + C_2 + C_3\right)\frac{dV}{dt}$$

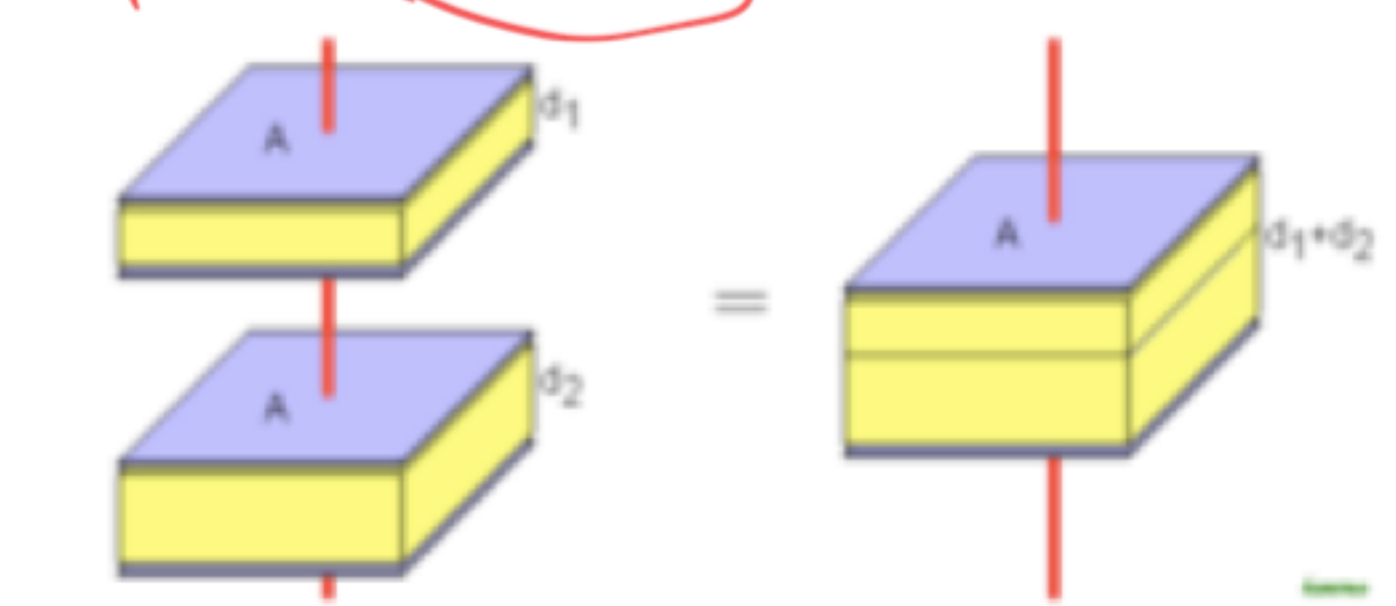
$$\therefore C_{eqv} = C_1 + C_2 + C_3$$

like R's in series

Capacitors in Parallel and Series

$$C_{eqv} = \frac{C_1 C_2}{C_1 + C_2}$$

if 2 caps



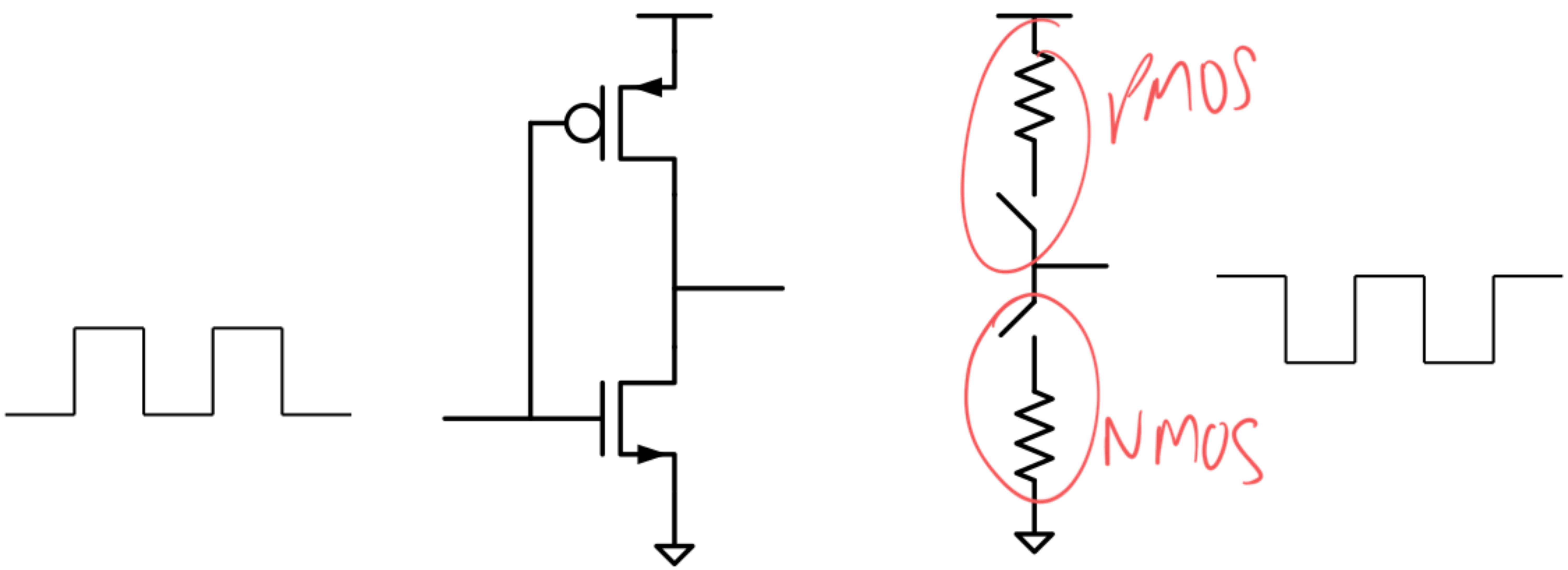
$$V_T = V_1 + V_2 + V_3 = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = \frac{Q}{C_{eqv}}$$

$$\therefore \frac{1}{C_{eqv}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

CAPACITOR RESISTOR CIRCUITS

Capacitors and Logic Gate Speeds

- When the input changes from low to high
  - The pMOS turns off, and the nMOS turns on
  - The output goes from high to low
- But in this model
  - The output changes as soon as the input changes

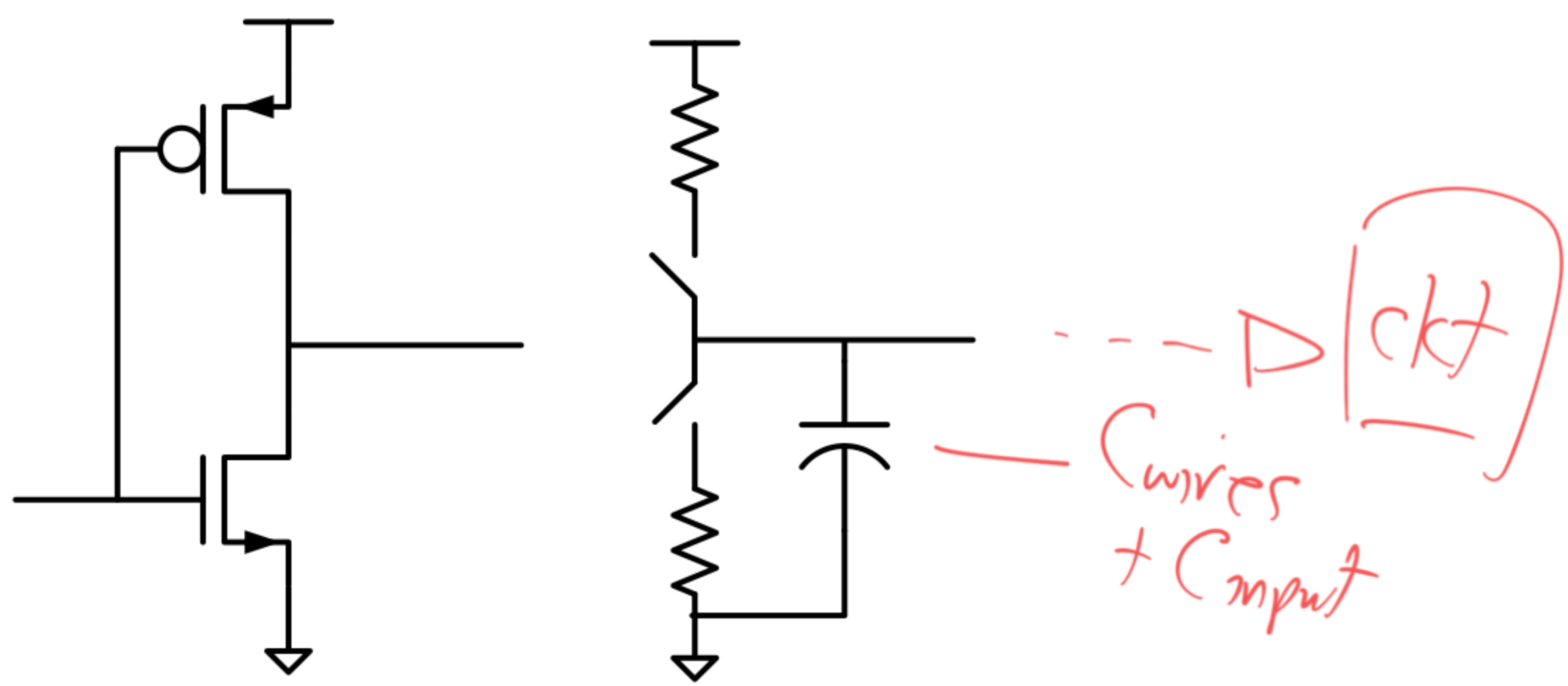


Gates Are NOT Zero Delay

- It would be great if logic gates had zero delay
  - But they don't
- Fortunately, it is easy to figure out the delay of a gate
  - It is just caused by the transistor resistance
    - Which we know about already
  - And the transistor and wire capacitance

Improved Model

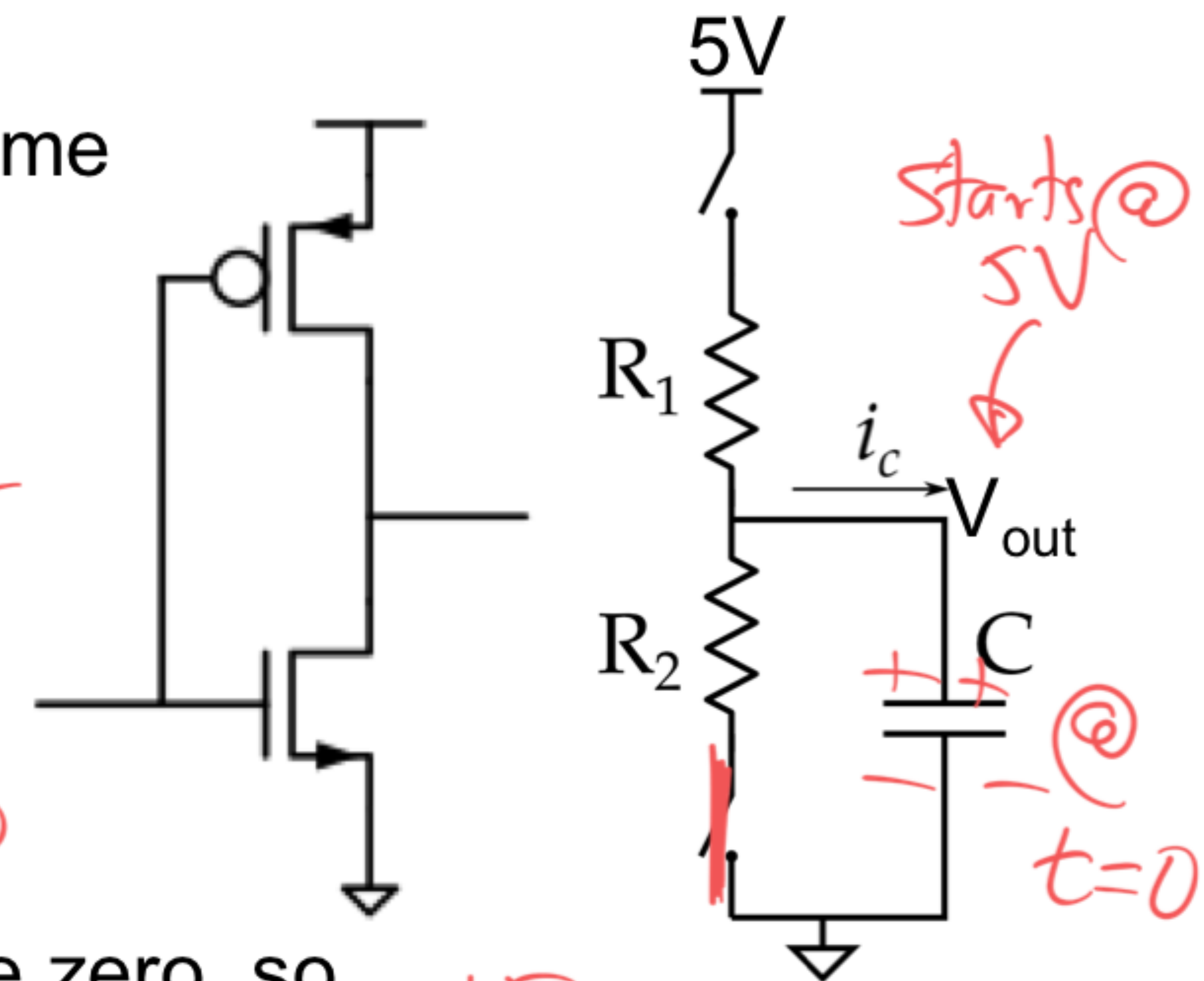
- Just add a capacitor to the output node
  - Its value is equal to the capacitance of the gates driven
  - Plus the capacitance of the wire itself



RC Circuit Equation

- When the input to the inverter is low, the output will be at V_{dd}
  – Right after the input rises, here is the circuit
- Want to find the capacitor voltage verses time
- Just write the nodal equations:
  – We just have one node voltage, V_{out} +5
  – $i_{RES} = V_{out}/R_2$
  – $i_{CAP} = CdV_{out}/dt$

0 t=0



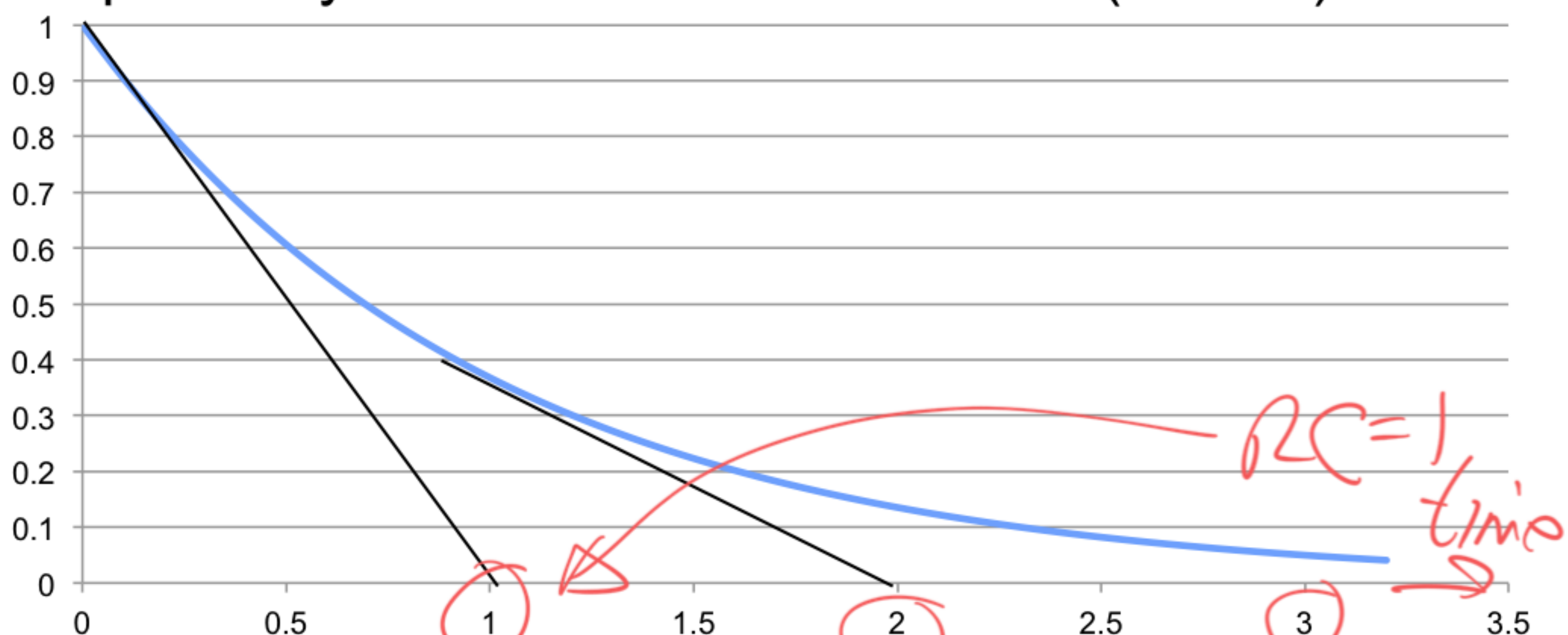
- From KCL, the sum of the currents must be zero, so

$$\frac{dV_{out}}{dt} = -\frac{V_{out}}{R_2 C}$$

RC Circuit Equations

- Solving, $$\int_5^{V} \frac{dV_{out}}{V_{out}} = -\int_0^{t} \frac{dt}{R_2C}$$ so that $$\ln\left(V_{out}\right) - \ln\left(5V\right) = -\frac{t}{R_2C}$$
- This is an exponential decay
  - The x axis is in time constants
  - The y axis has been normalized to 1
  - Slope always intersects 0 one tau later (τ = RC)

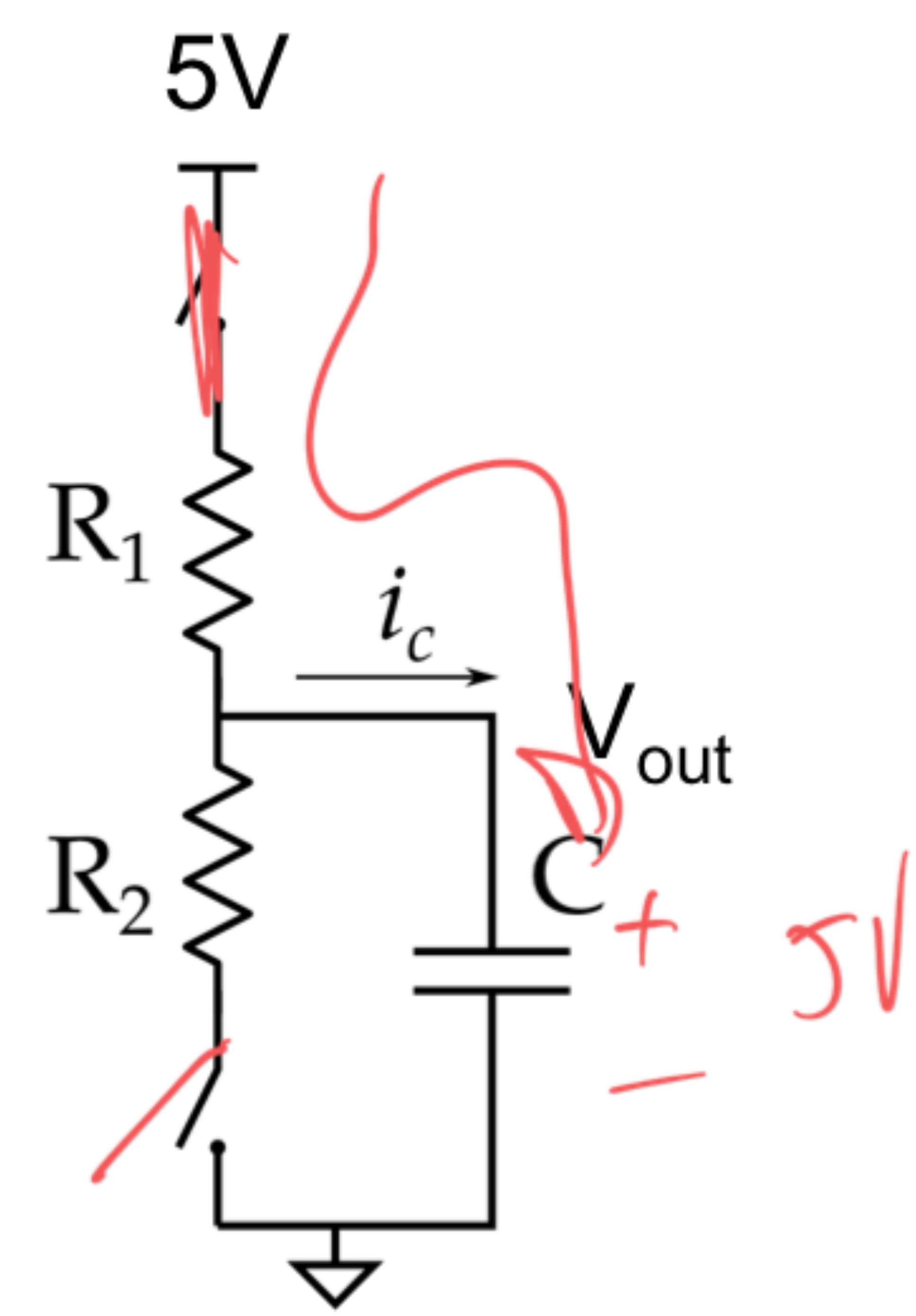
$$\therefore V_{out} = 5V\left(e^{-t/R_2C}\right)$$



What Happens When Input Falls?

- Now the voltage across the capacitor starts at 0V
  - $i = (V_{dd} - V_{out})/R_1$
  - $dV_{out}/dt = i/C$

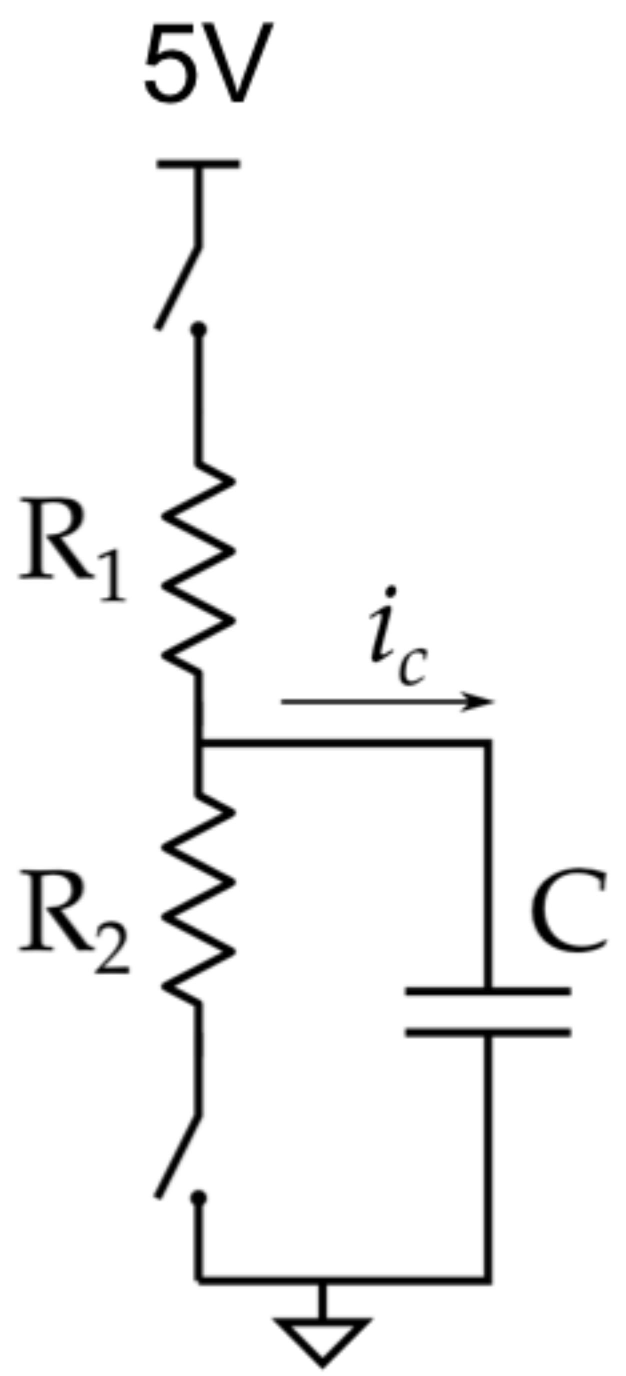
$$\frac{dV_{out}}{dt} = \frac{\left(V_{dd} - V_{out}\right)}{R_1 C}$$



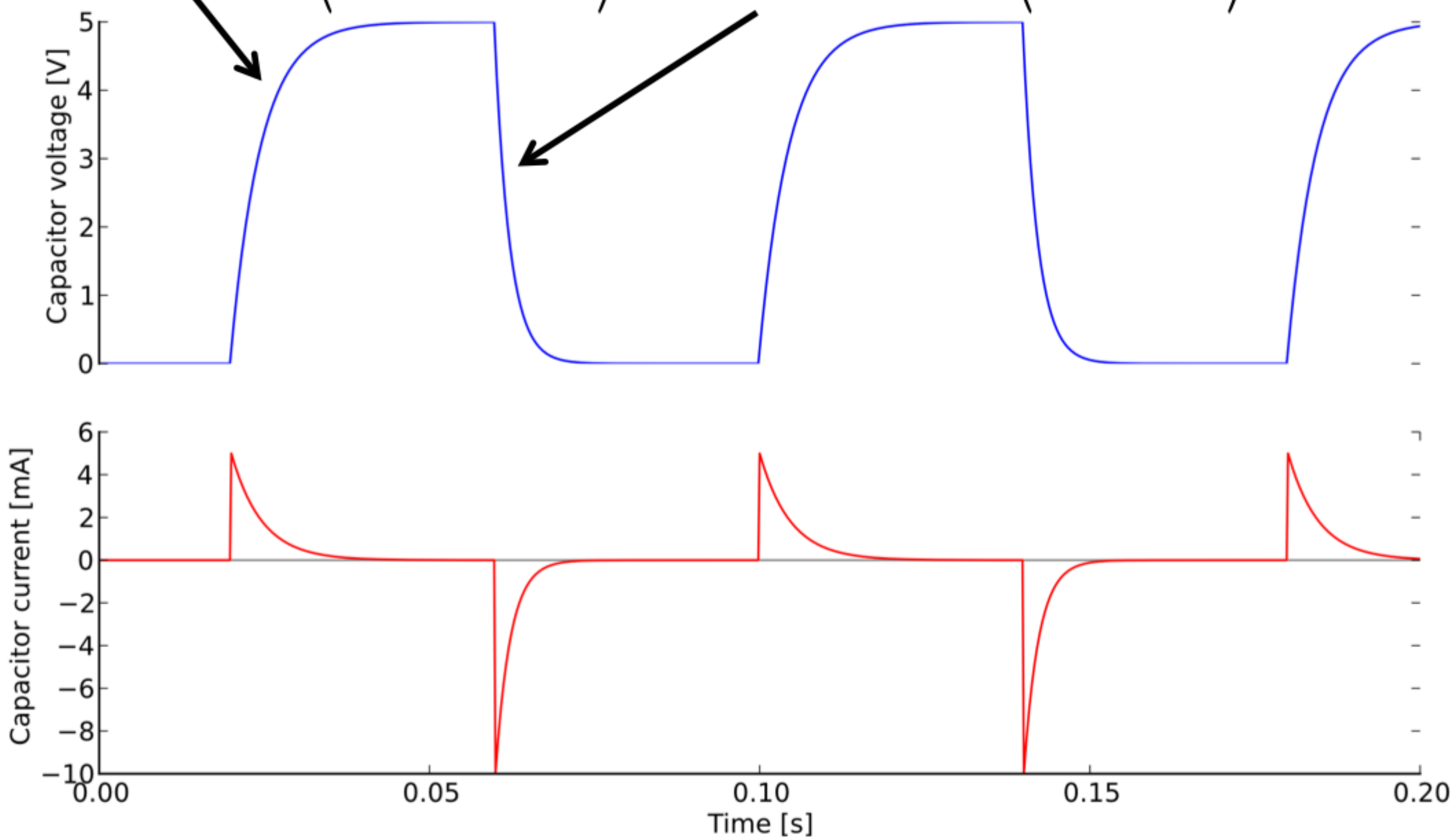
- Not quite the right form
  - Need to fix it by changing variables
  - Define $V_{new} = V_{dd} - V_{out}$
  - $dV_{out}/dt = = - dV_{new}/dt$, since V_{dd} is fixed

$$\int_5^V \frac{dV_{new}}{V_{new}} = -\int_0^t \frac{dt}{R_1 C} \quad \text{so that} \quad \ln\left(V_{new}\right) - \ln\left(5\right) = -\frac{t}{R_1 C} \quad \therefore V_{out} = 5V\left(1 - e^{-t/R_1 C}\right)$$

RC Circuits in the Time Domain

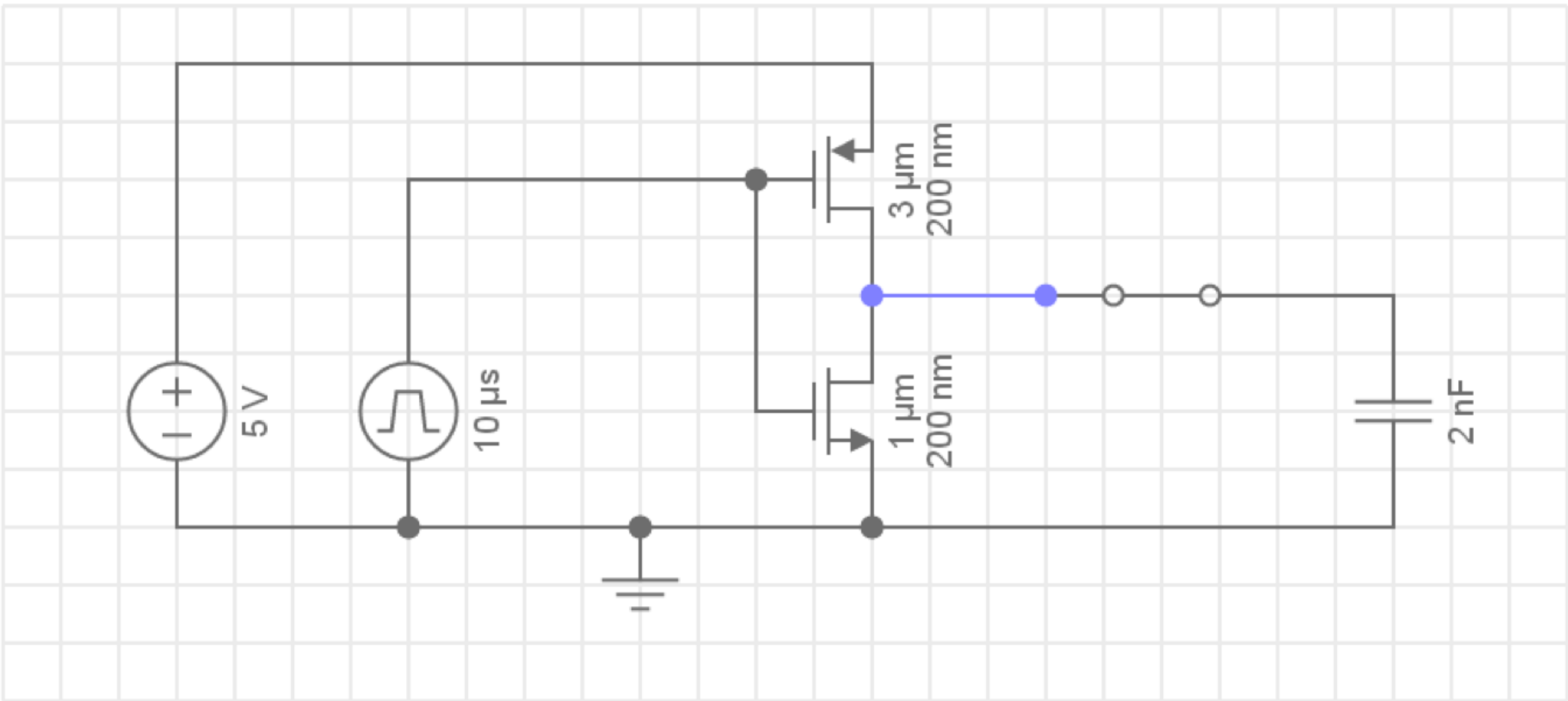


$$V_{out} = 5V\left(1 - e^{-t/R_1C}\right) \qquad V_{out} = 5V\left(e^{-t/R_2C}\right)$$



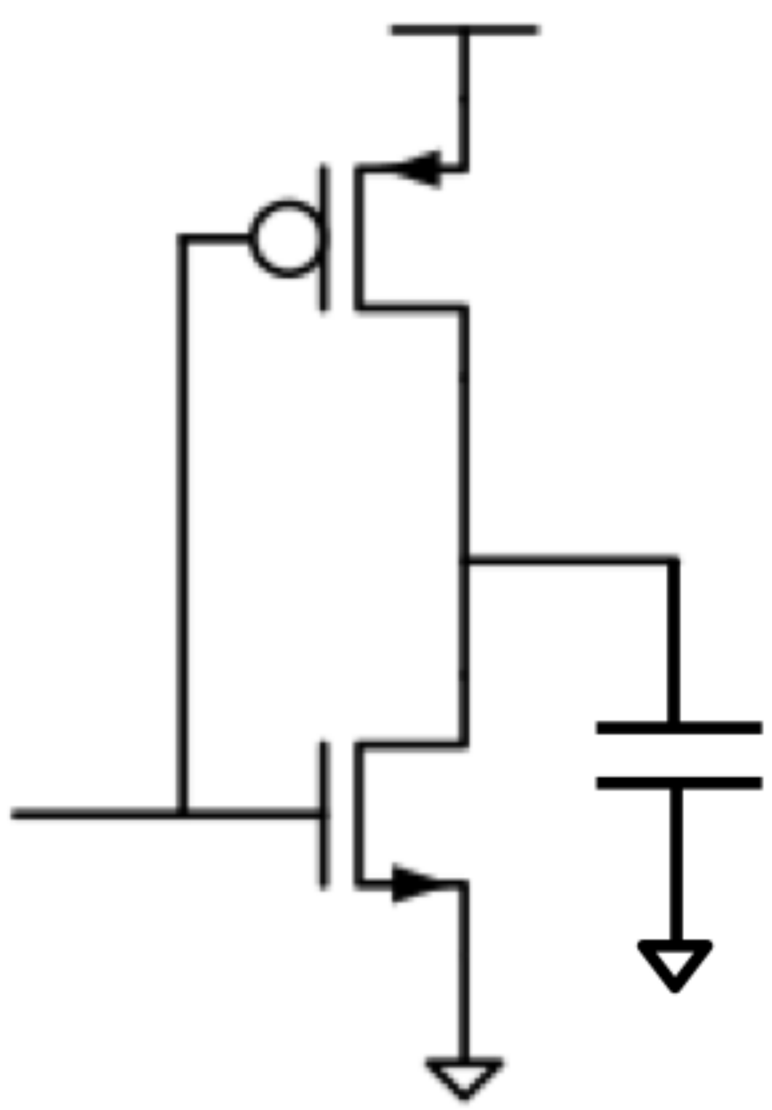
In capacitor circuits, voltages change “slowly”, while currents can be instantaneous.

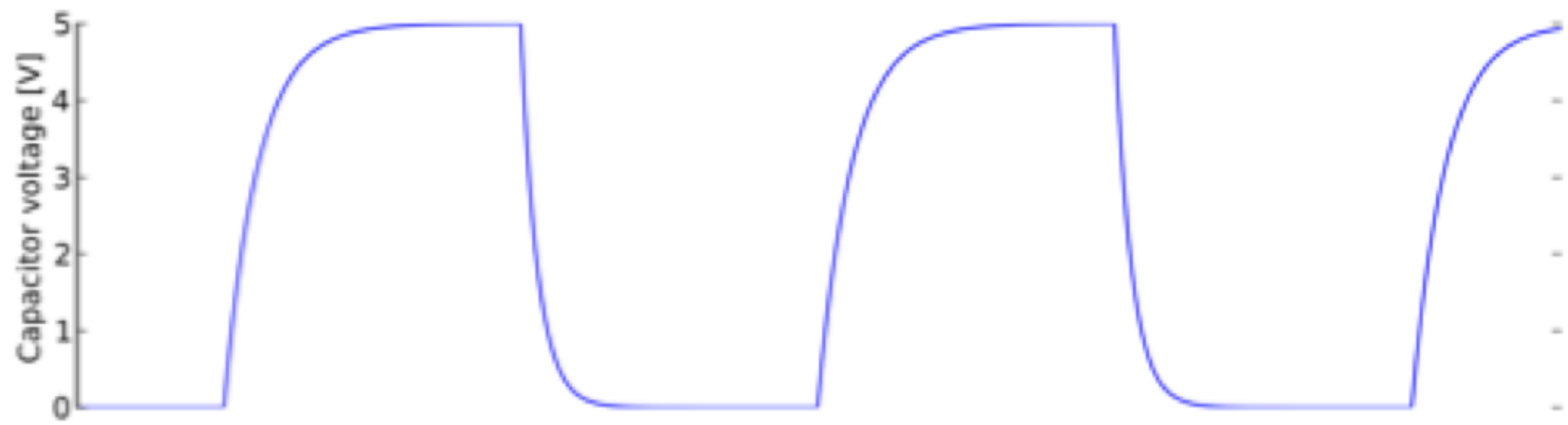
Simple RC Circuit Demo



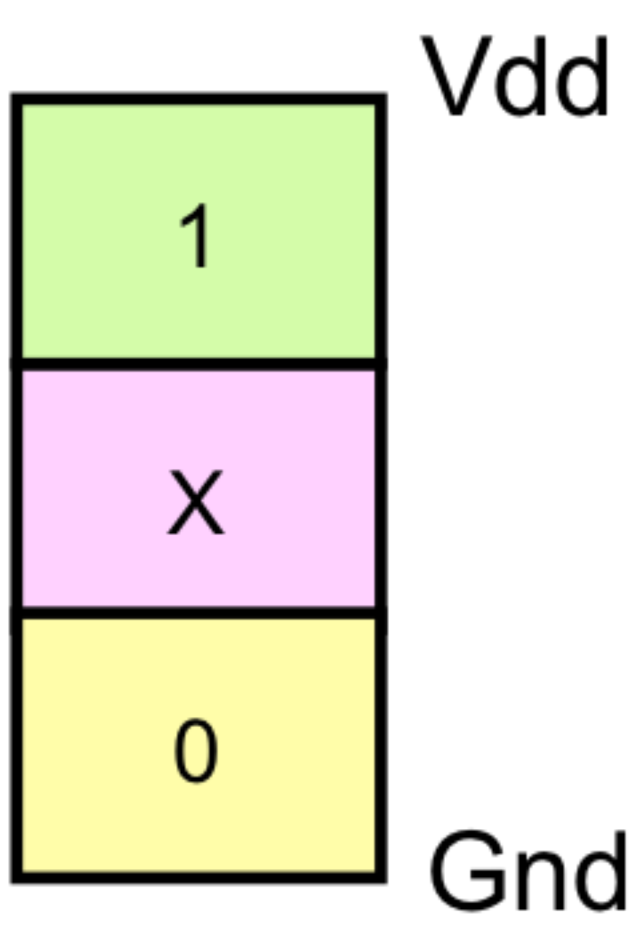
EveryCircuit Demo – CMOS Inverter

Interesting Aside





- Exponentials “never” reach their final value
- So if this logic gate is driving another gate, when does the next gate think its input is 0 or 1?
- This is one of the reasons why logic levels are defined as a range of values.



Learning Objectives

- Understand what a capacitor is
  - i=C dV/dt
  - It is a device that tries to keep voltage constant
    - Will supply current (in either direction) to resist voltage changes

- Understand how voltages and current change in R C circuits
  - Voltage waveforms are continuous
    - Takes time for their value to change
    - Exponentially decay to final value (the DC value of circuit)
  - Currents can charge abruptly