E40M

RC Circuits and Impedance

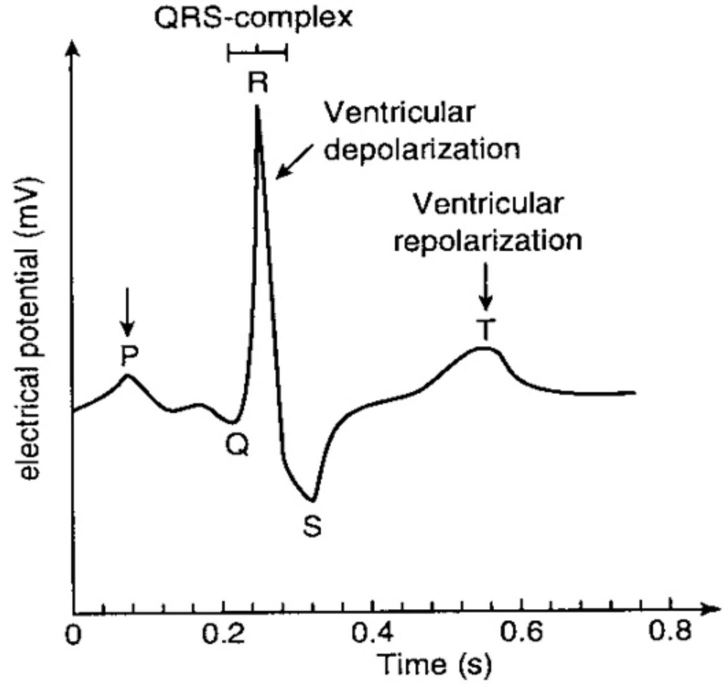
Reading

- Reader:
  - Chapter 6 – Capacitance (if you haven't read it yet)
  - Section 7.3 – Impedance
    - You should skip all the parts about inductors
    - We will talk about them in a lecture at the end of the quarter

EKG (Lab 4)

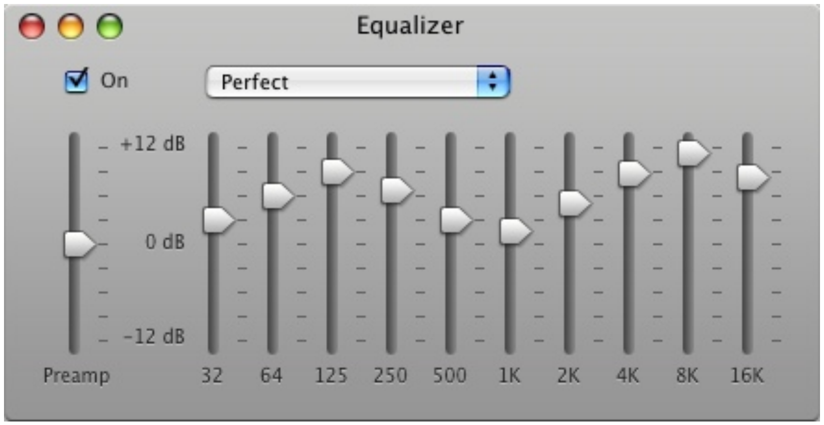
- Concepts
  - Amplifiers
  - Impedance
  - Noise
  - Safety
  - Filters

- Components
  - Capacitors
  - Inductors
  - Instrumentation and Operational Amplifiers

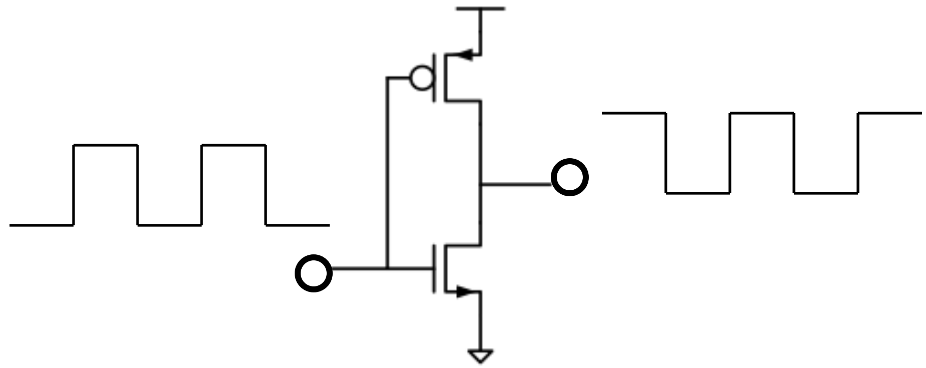


In this project we will build an electrocardiagram (ECG or EKG). This is a noninvasive device that measures the electrical activity of the heart using electrodes placed on the skin.

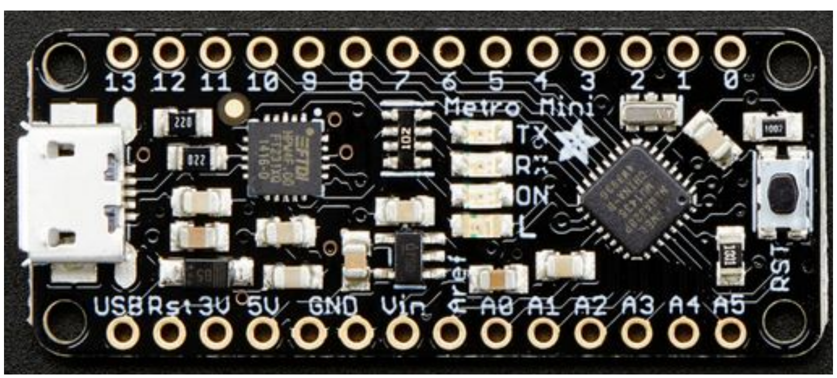
Why Are Capacitors Useful/Important?



How do we design circuits that respond to certain frequencies?



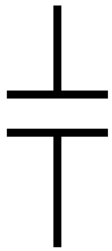
What determines how fast CMOS circuits can work?



Why did you put a 200 μF capacitor between Vdd and Gnd on your Arduino?

Key Ideas on Capacitors and RC Circuits - Review

- Capacitors store charge
  - The voltage across the capacitor is proportional to Q
    - V = Q/C; or Q = CV
    - Q in Coulombs, V in Volts, and C in Farads
  - But like all devices it is charge neutral
    - Stores +Q on one terminal; stores –Q on the other



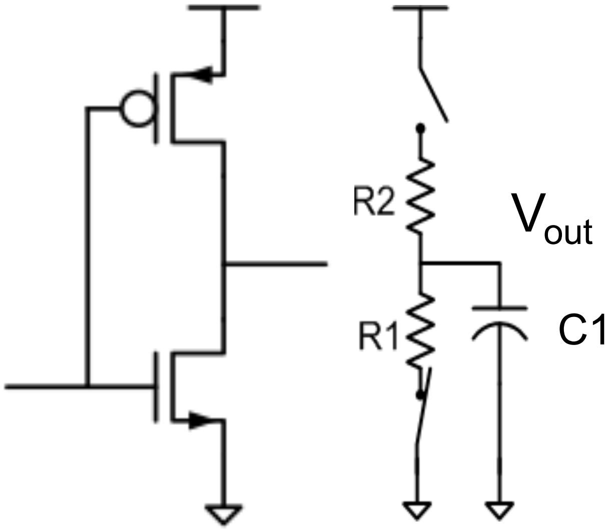
- Sometimes we purposely use capacitors in circuits;
  - Other time we use them to model the capacitance of wires
    - These are sometime called parasitic capacitance

- Resulting i-V relation:

$$i = C(dV/dt)$$

Key Ideas on Capacitors and RC Circuits - Review

- The voltage across a capacitor can't change instantaneously
  - That means the voltage across a capacitor won't change the instant after any switches/transistors flip
- Want to find the capacitor voltage verses time



- Just write the nodal equations:
  - We just have one node voltage, V_{out}

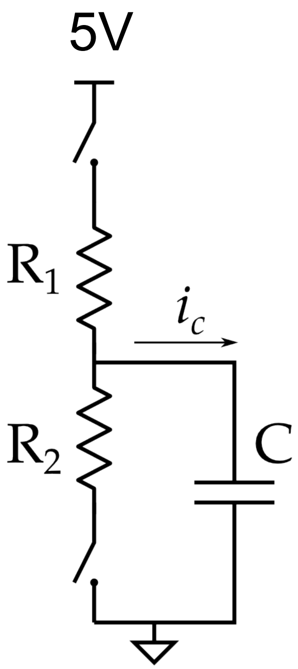
    $i_{RES} = V_{out}/R_1$

    $i_{CAP} = CdV_{out}/dt$

- From KCL, the sum of the currents must be zero, so

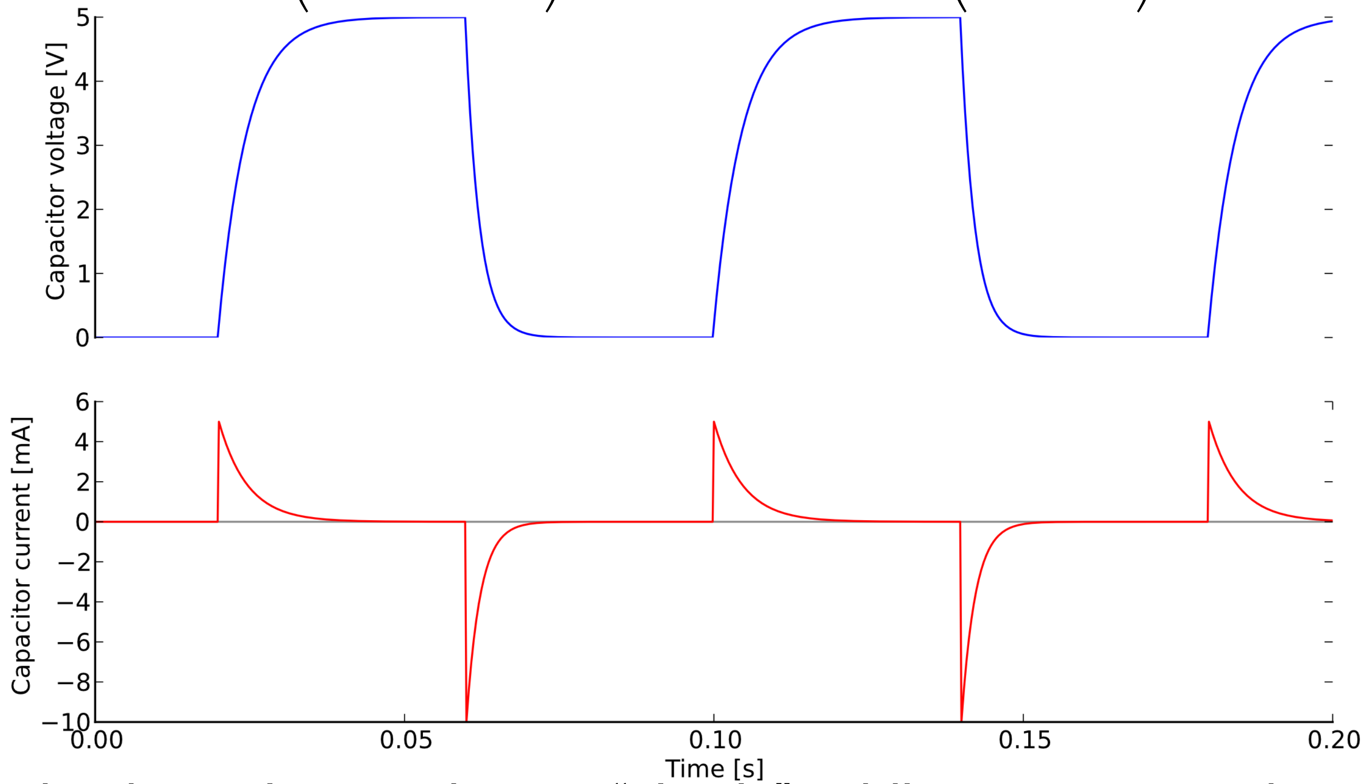
$$\frac{dV_{out}}{dt} = -\frac{V_{out}}{R_1 C}$$

Key Ideas on Capacitors and RC Circuits - Review



$$V_{out} = 5V\left(1 - e^{-t/R_1C}\right)$$

$$V_{out} = 5V\left(e^{-t/R_1C}\right)$$



In capacitor circuits, voltages change "slowly", while currents can be instantaneous.

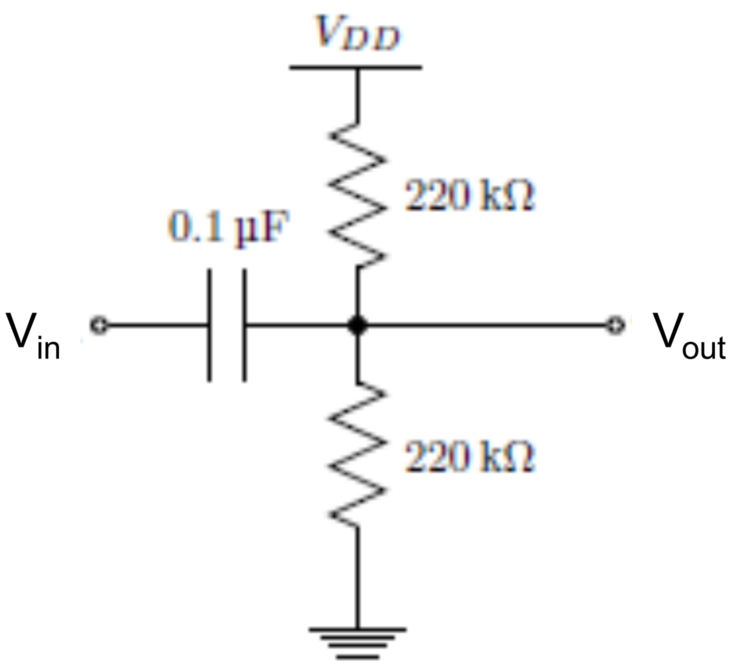
RC Circuit Analysis Approaches

- For finding voltages and currents as functions of time, we solve linear differential equations or run EveryCircuit.

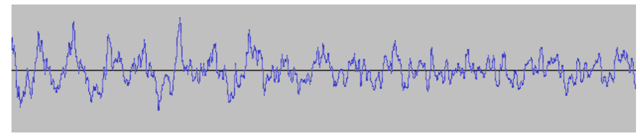
- There’s a new and very different approach for analyzing RC circuits, based on the “frequency domain.” This approach will turn out to be very powerful for solving many problems.

How Can We Solve This Circuit?

- The input is sound from your computer; the output is going to go to your Arduino
- Now V_{in} is a complicated waveform
  - How are we going to find V_{out}?
- Two approaches
  - EveryCircuit
  - Decompose the input into sine waves: frequency analysis

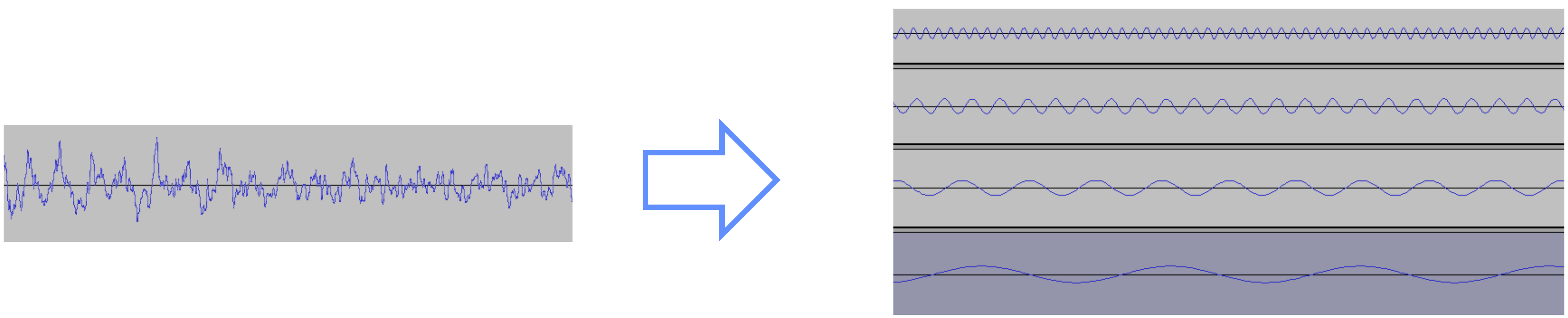


Time Domain vs. Frequency Domain



- Directly solving for the output to this:
  - Requires a computer
  - And the output will just be another squiggly line

- But
  - This waveform is the sum of sinewaves



Superposition To The Rescue

- We know that sound can be represented by
  - A sum of sinewaves
- We also know that R, C are linear elements
  - So superposition holds
- Superposition says
  - The output is the sum of the response from each source
- So the output from a sound waveform
  - Is the sum of the outputs generated from each sinewave

Properties of Sinewaves

- The problem with capacitors is that they take derivatives
  - This makes the problem solution a differential equation

- Exponential waveforms are nice since

$$\frac{d}{dt}\left(e^{-\frac{t}{\tau}}\right) = -\frac{1}{\tau}\left(e^{-\frac{t}{\tau}}\right)$$

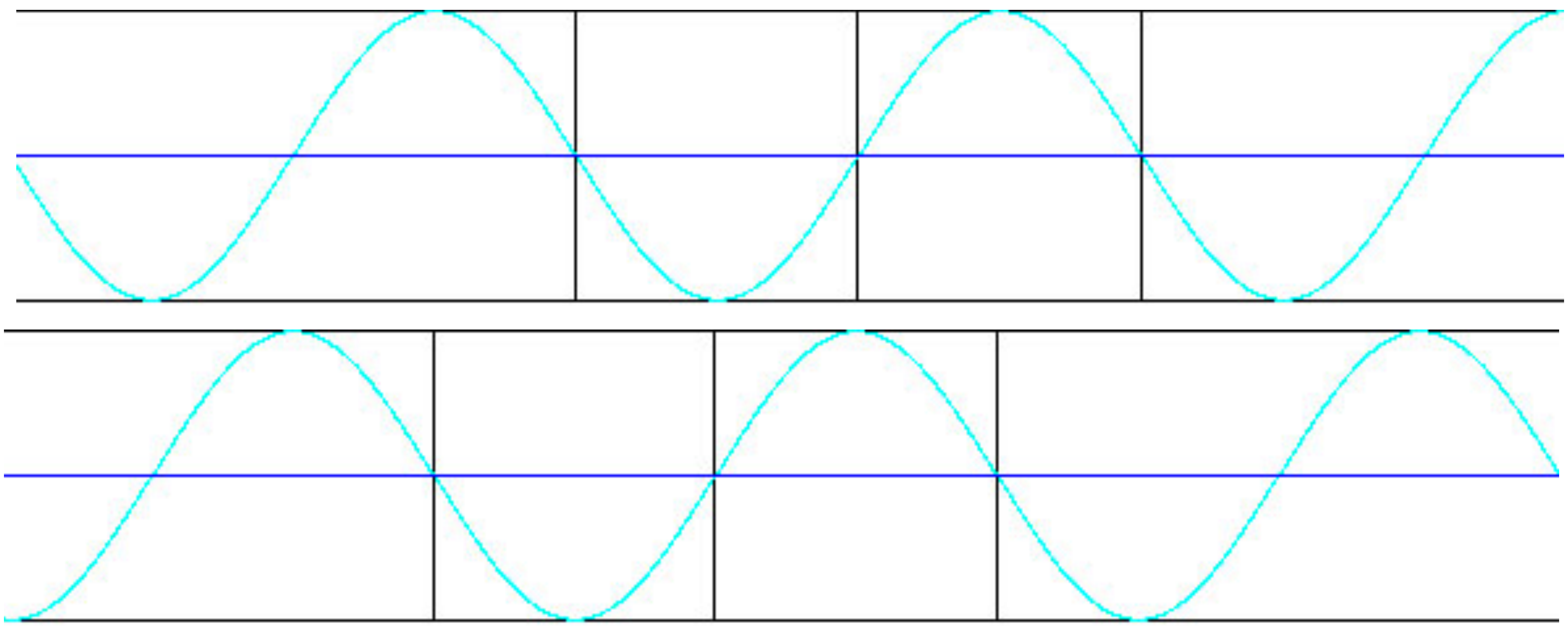
- Sine waves have a similar property

$$\frac{d}{dt}\left[\sin(2\pi F t)\right] = 2\pi F \cos(2\pi F t)$$

$$\frac{d}{dt}\left[\cos(2\pi F t)\right] = -2\pi F \sin(2\pi F t)$$

What This Means

- If you drive a R, C, circuit with sin(2πF t)
  - All the waveforms in the circuits will be sin(2πF t)
    - At different amplitudes, and with a phase shift
    - We will mark terms that are phase shifted by a ‘j’.
      [“j” actually has a deeper meaning – explained in the reader.]



Sinewave Driven Circuits

- All voltages and currents are sinusoidal
- So we really just need to figure out
  - What is the amplitude of the resulting sinewave
  - And sometimes we need the phase shift, too (but not always)
- These values don't change with time
  - This problem is very similar to solving for DC voltages/currents
- In fact we can solve it *exactly the same way* …

IMPEDANCE

Impedance

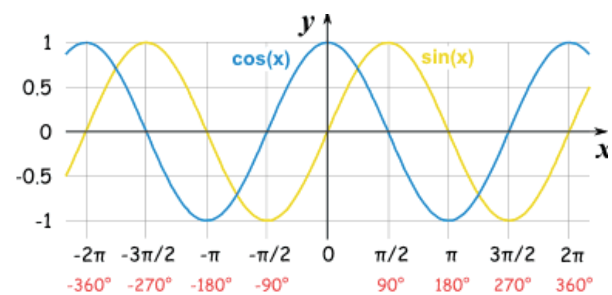
- Impedance is a concept that is a generalization of resistance:

$$R = \frac{V}{i}$$

R is simply a number with the units of Ohms.

- What about a capacitor? If V and i are sine waves, then

$$Z_C = \frac{V}{i} = \frac{V}{CdV/dt} = \frac{V_0 \sin(2\pi Ft)}{2\pi FCV_0 \cos(2\pi Ft)}$$

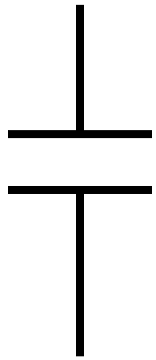


$$Z_C = \frac{V}{i} = \frac{1}{j * 2\pi FC}$$

… if we ignore phase shift, $Z_C = \frac{V}{i} = \frac{1}{2\pi FC}$

Impedance of a Capacitor

- The impedance of a capacitor depends on frequency
- At low frequencies (F ≈ 0) $Z_C \to \infty$ and a capacitor behaves like an open circuit. Thus, if we are doing a "DC" analysis of a circuit (voltages and currents), capacitors are modeled as open circuits.
- At very high frequencies (F ≈ infinity) $Z_C \to 0$ and a capacitor behaves like a short circuit.
- At intermediate frequencies, the capacitor has an impedance given by Z_C



$$Z_C = \frac{V}{i} = \frac{1}{j * 2\pi F C}$$

USING IMPEDANCE

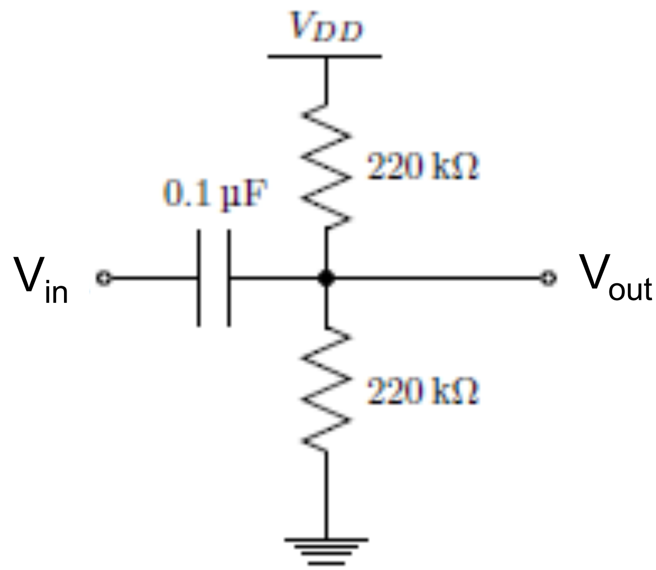
Using Impedance Makes Everything an R Circuit!

- Find V_{out} / V_{in}

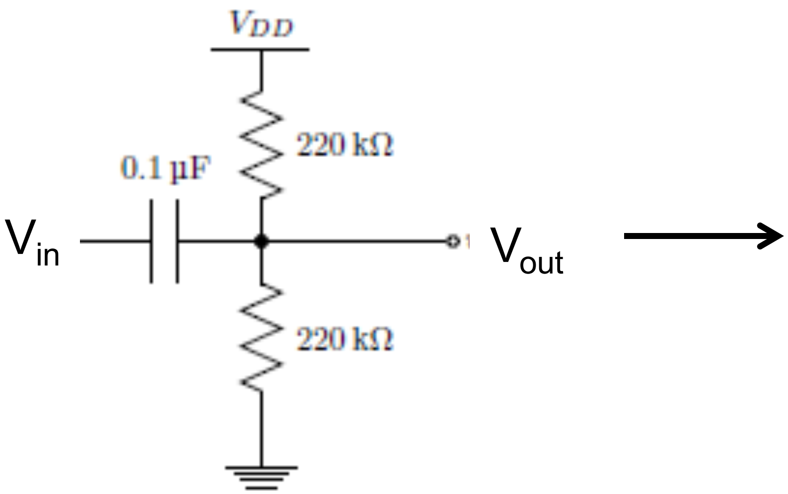
- First, note that the capacitor $Z_C = \infty$ at F = 0 (DC), so it becomes an open circuit.

$$\therefore v_{out}\left(DC\right) =$$

- We can now use superposition. Assume we have a sine wave input at V_{in}

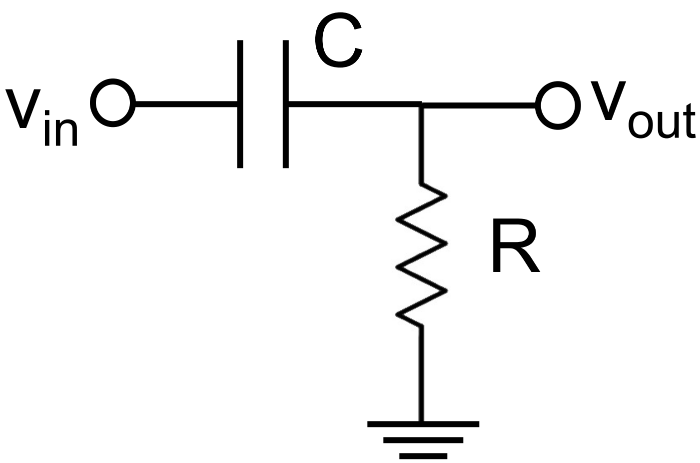


RC Circuit Analysis Using Impedance



- The circuit becomes just a voltage divider, and we can analyze it the same way we have analyzed resistor only circuits.
  - That's the power of using impedance!

Analyzing RC Circuits Using Impedance



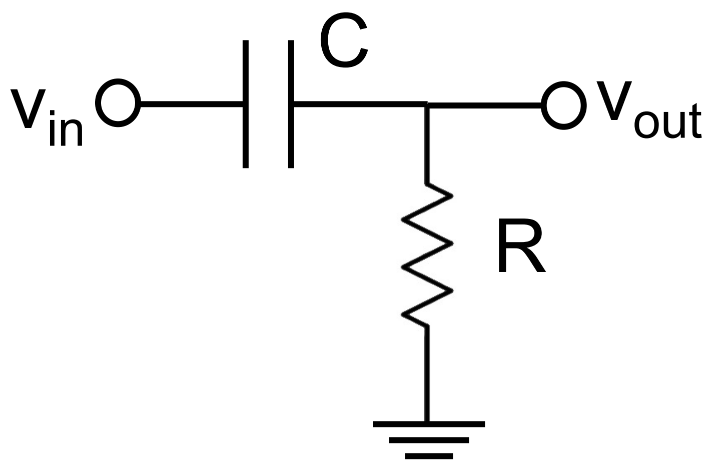
$$Z_C = j * \frac{1}{2\pi FC} \qquad Z_R = R$$

- If the circuit had two resistors then we would know how to analyze it

$$\frac{V_{out}}{V_{in}} = \frac{R_2}{R_1 + R_2} \quad \text{or more generally,} \quad \frac{V_{out}}{V_{in}} = \frac{Z_2}{Z_1 + Z_2}$$

- So we can still use the voltage divider approach with impedances

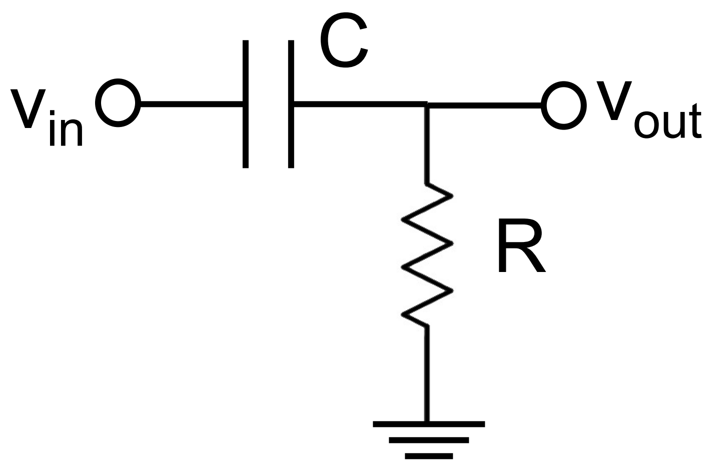
Analyzing RC Circuits Using Impedance

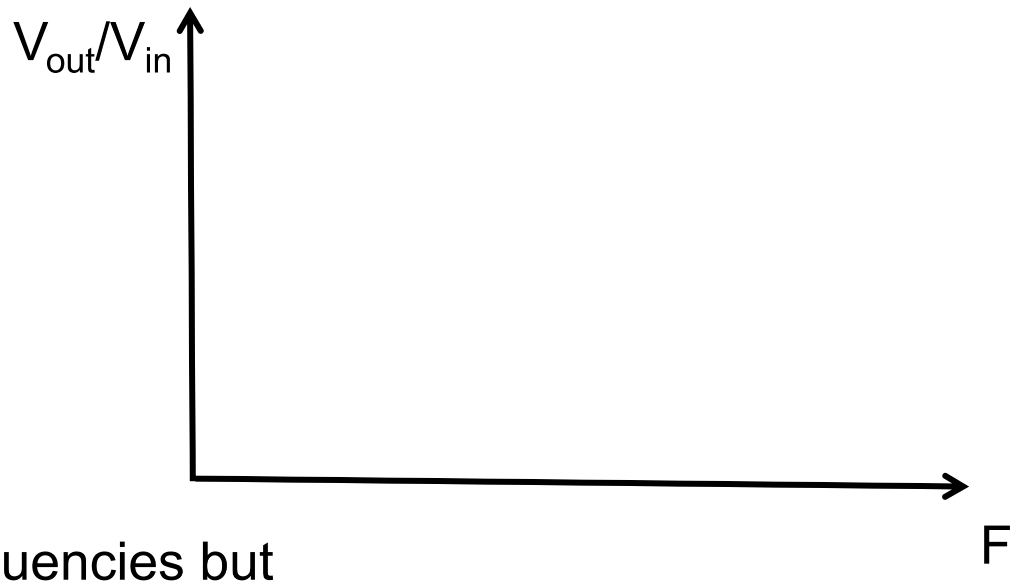


$$\frac{V_{out}}{V_{in}} = \frac{R}{R + \frac{1}{j * 2\pi FC}} = \frac{j * 2\pi FRC}{1 + j * 2\pi FRC}$$

- At low frequencies, (F ≈ 0), V_{out} = 0 which means that low frequencies are not passed to the output. The capacitor blocks them.
  - Recall that we used this idea earlier to calculate the DC voltage at the output.
- At high frequencies (F large), $V_{out} = V_{in}$

Frequency Dependence of RC Circuit



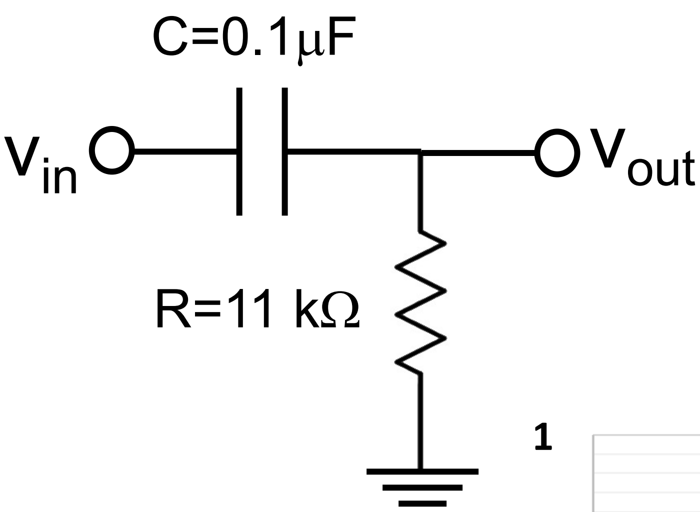


- This circuit passes high frequencies but blocks low frequencies.

- Sometimes called a “high pass filter”.

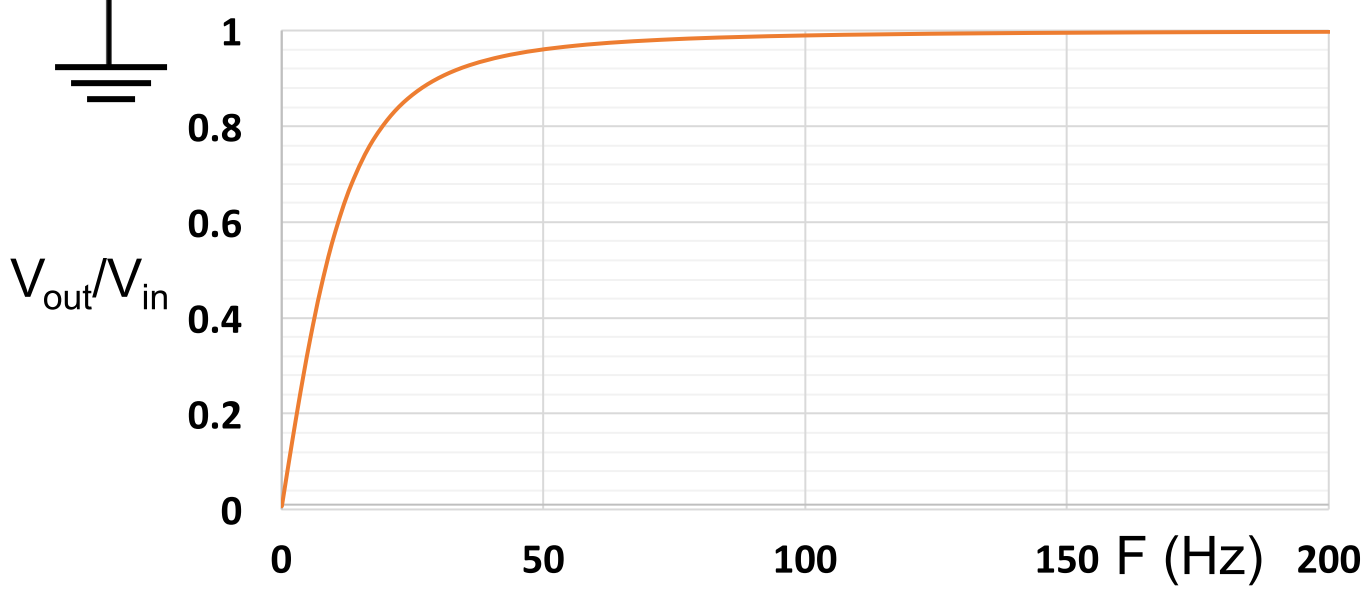
$$\frac{V_{out}}{V_{in}} = \frac{j * 2\pi FRC}{1 + j * 2\pi FRC}$$

Analyzing RC Circuits Using Impedance (High Pass Filter)



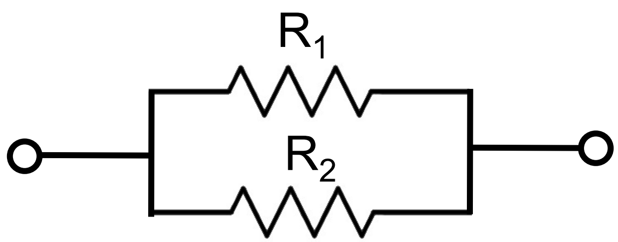
$$\frac{V_{out}}{V_{in}} = \frac{R}{R + \frac{1}{j*2\pi FC}} = \frac{j*2\pi FRC}{1 + j*2\pi FRC}$$

RC = 11ms; 2πRC about 70ms

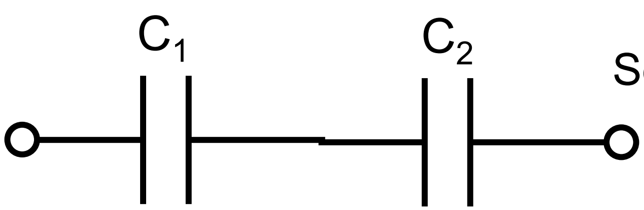


Impedance of Other RC Circuits

Series: $Z_{eq} = Z_1 + Z_2 = R_1 + R_2$

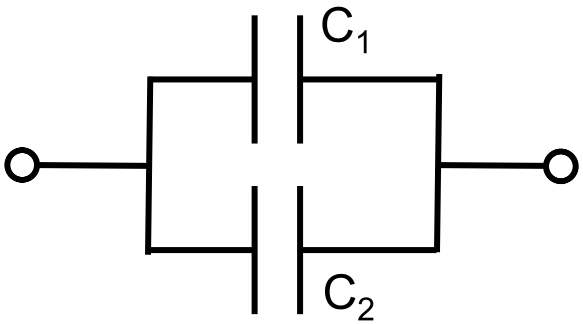


Parallel: $Z_{eq} = \dfrac{1}{\dfrac{1}{Z_1} + \dfrac{1}{Z_2}} = \dfrac{R_1 R_2}{R_1 + R_2}$



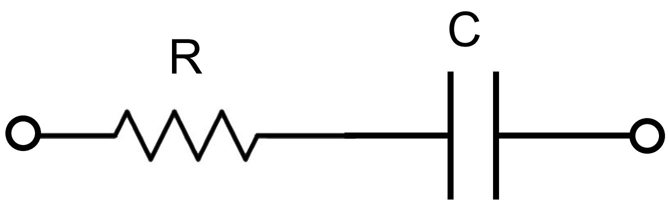
Series: $Z_{eq} = Z_1 + Z_2 = \dfrac{1}{j * 2\pi F C_1} + \dfrac{1}{j * 2\pi F C_2} = \dfrac{1}{j * 2\pi F}\left(\dfrac{1}{C_1} + \dfrac{1}{C_2}\right)$

$$= \frac{1}{j * 2\pi F}\left(\frac{C_1 C_2}{C_1 + C_2}\right)$$

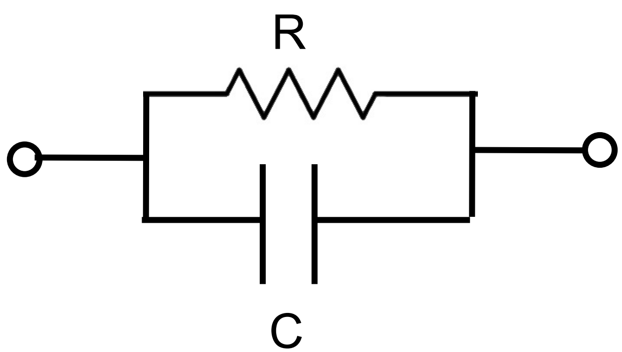


Parallel: $Z_{eq} = \dfrac{1}{\dfrac{1}{Z_1} + \dfrac{1}{Z_2}} = \dfrac{1}{j * 2\pi F C_1 + j * 2\pi F C_1} = \dfrac{1}{j * 2\pi F (C_1 + C_2)}$

Impedance of Other RC Circuits



$$\text{Series}: \quad Z_{eq} = Z_1 + Z_2 = R + \frac{1}{j * 2\pi FC} = \frac{1 + j * 2\pi FRC}{j * 2\pi FC}$$



$$\text{Parallel}: \quad Z_{eq} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2}} = \frac{1}{\frac{1}{R} + j * 2\pi FC} = \frac{R}{1 + j * 2\pi FRC}$$

Check limits on these expressions!

Learning Objectives for Today

- Generalize RC circuit analysis in the time domain
- Impedance is the relationship between voltage and current
  - For a sinusoidal input
  - $Z = V/I$ so for a capacitor, $Z = 1/2\pi FC$ or $1/j*2\pi FC$
- Understand how to use impedance to analyze RC circuits
  - Compute the "voltage divider" ratio to find output voltage
  - Calculate series and parallel effective impedances