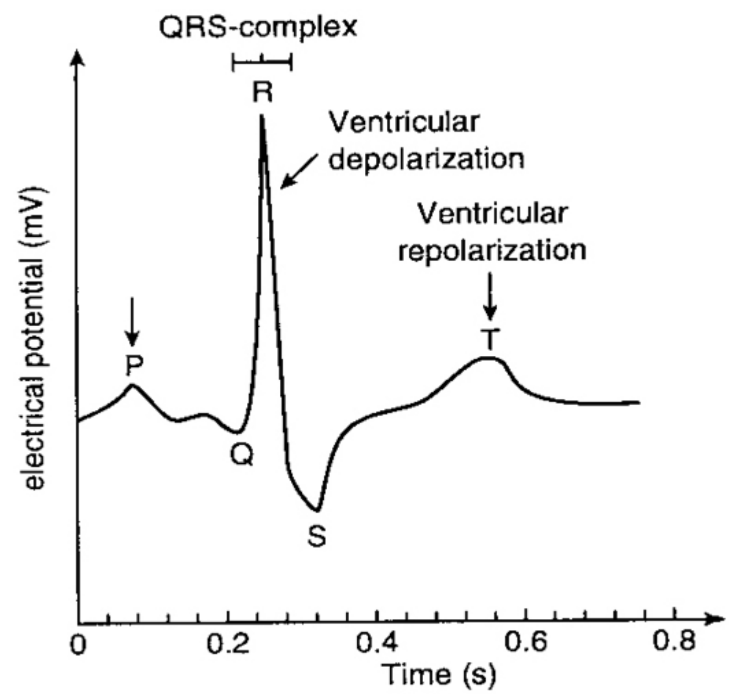
E40M
Bode Plots, dB

Reading

- Reader
  - 7.1-7.2 – Bode Plots

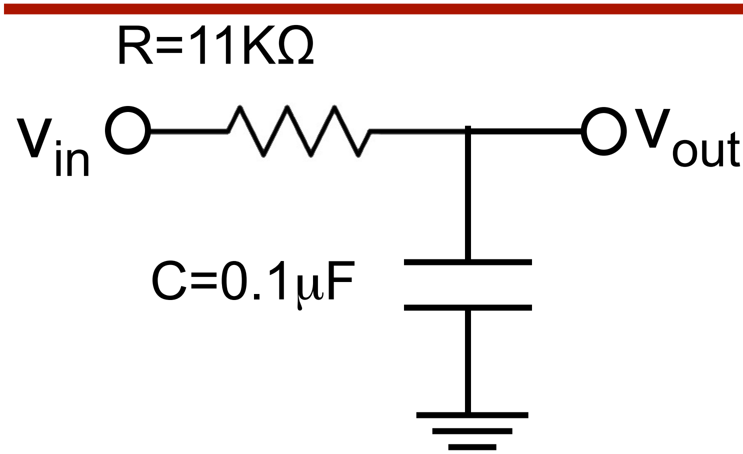
EKG Lab

- Concepts
  - Amplifiers
  - Impedance
  - Noise
  - Safety
  - Filters
- Components
  - Capacitors
  - Inductors
  - Instrumentation and Operational Amplifiers

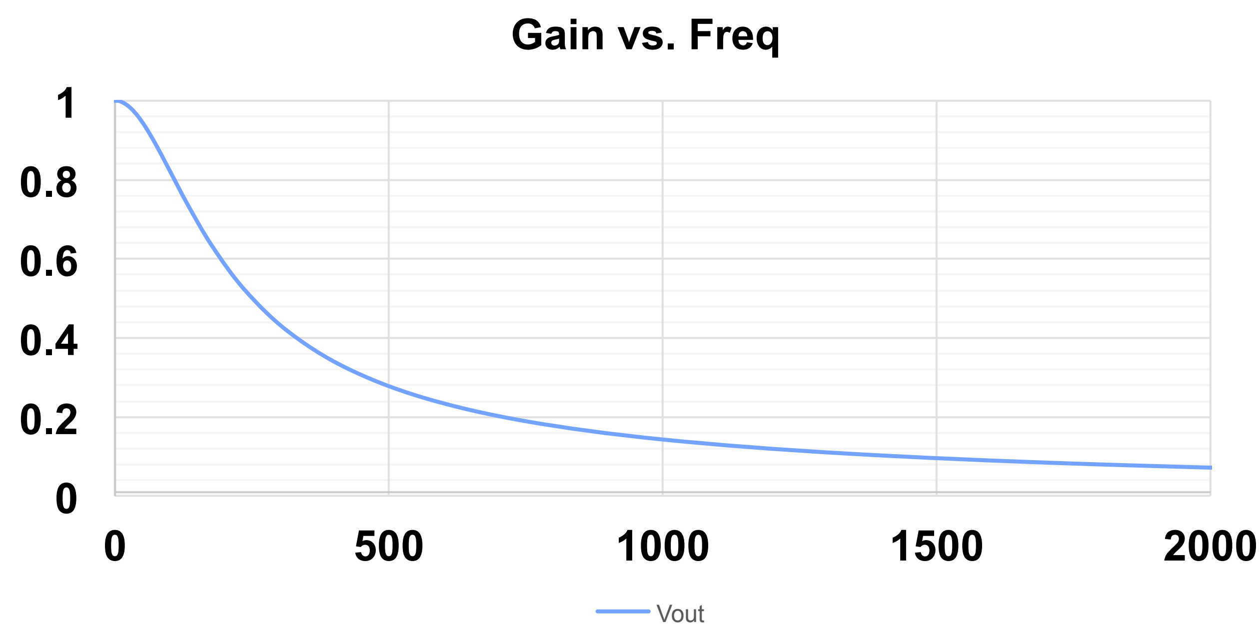


In this project we will build an electrocardiagram (ECG or EKG). This is a noninvasive device that measures the electrical activity of the heart using electrodes placed on the skin.

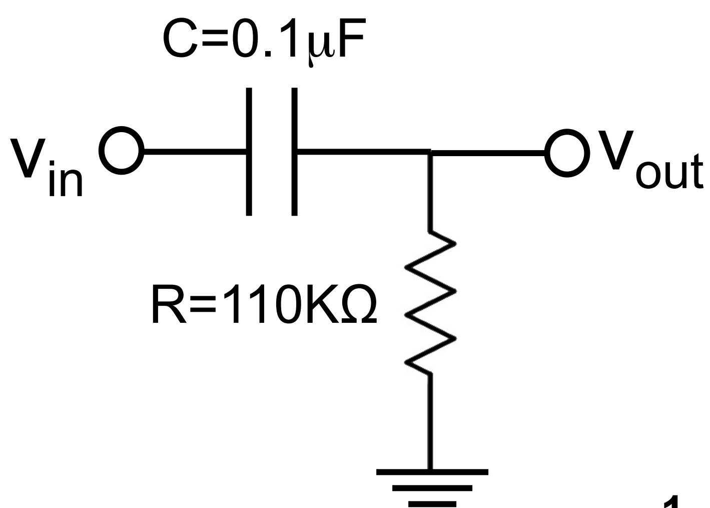
Analyzing RC Circuits Using Impedance – Review (Low Pass Filter)



$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{j*2\pi FC}}{R+\frac{1}{j*2\pi FC}} = \frac{1}{1+j*2\pi FRC}$$

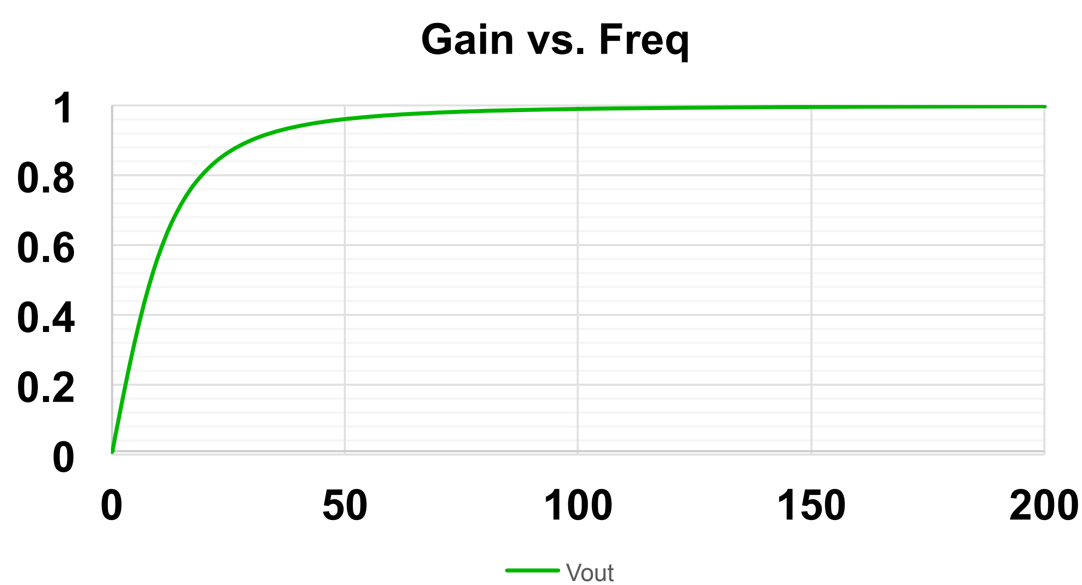


Analyzing RC Circuits Using Impedance – Review (High Pass Filter)

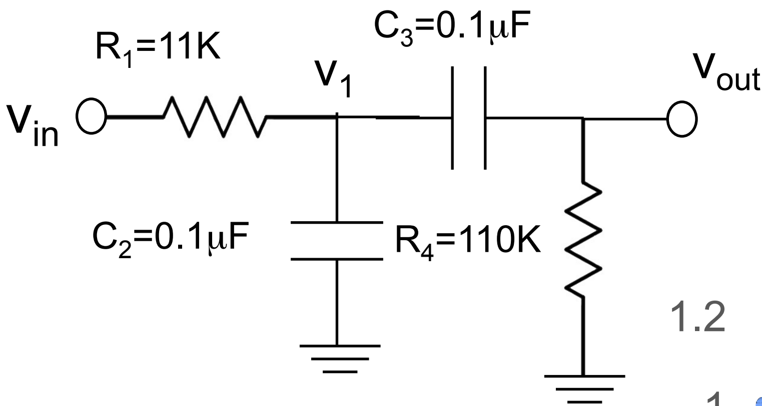


$$\frac{V_{out}}{V_{in}} = \frac{R}{R + \frac{1}{j*2\pi FC}} = \frac{j*2\pi FRC}{1 + j*2\pi FRC}$$

RC = 11ms; 2πRC is about 70ms

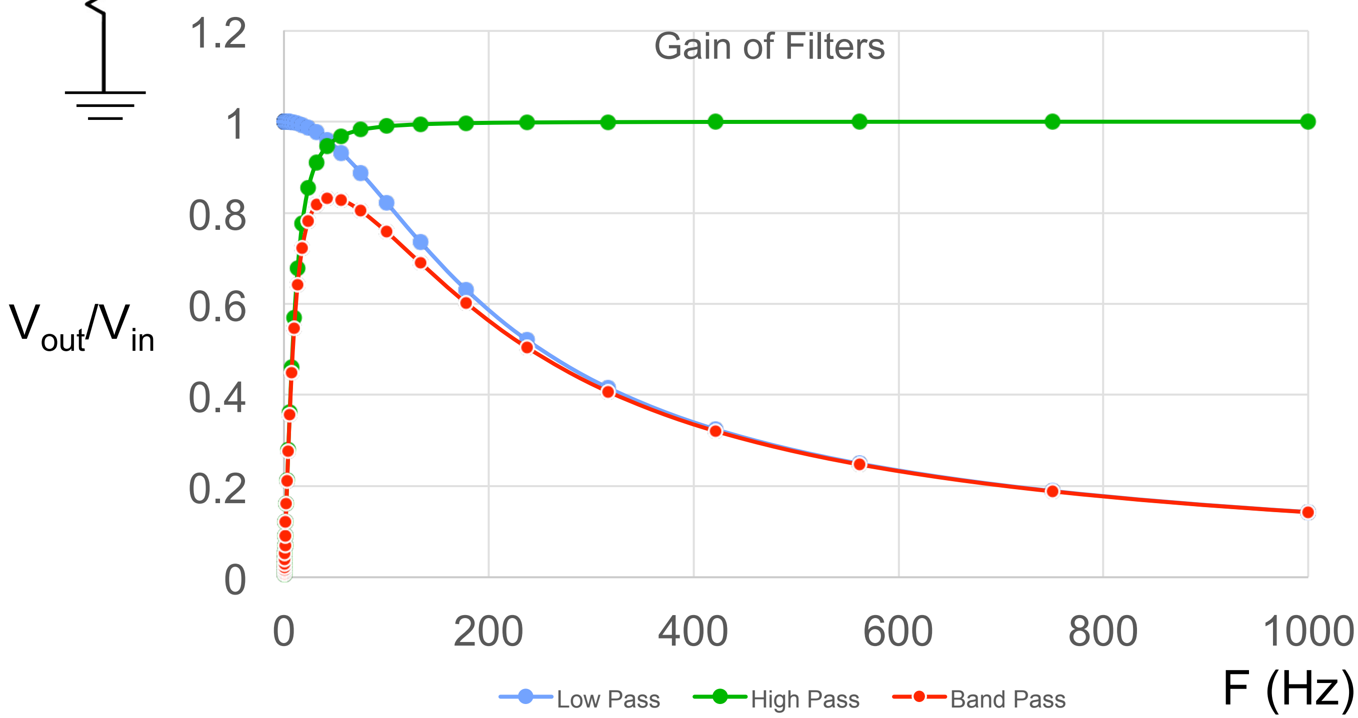


Analyzing RC Circuits Using Impedance – Review (Band Pass Filter)



$$\frac{V_{out}}{V_{in}} = \frac{j * 2\pi F R_4 C_3}{1 + j * 2\pi F (R_4 C_3 + R_1 C_2 + R_1 C_3) + \left(j * 2\pi F\right)^2 R_1 C_2 R_4 C_3}$$

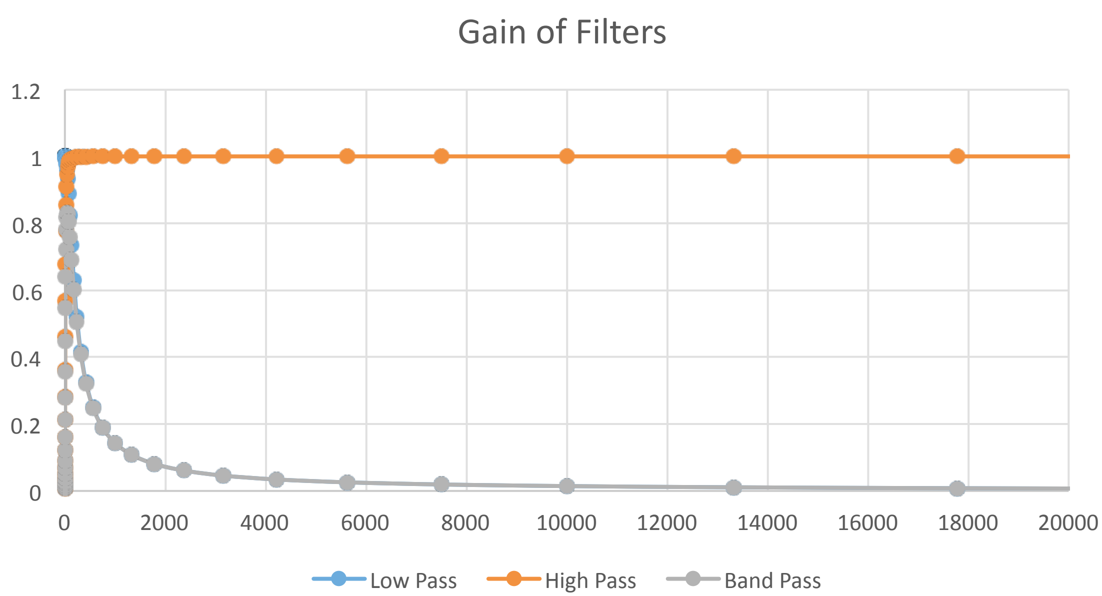
- We'll use a filter that operates like this in the ECG lab project.



BODE PLOTS

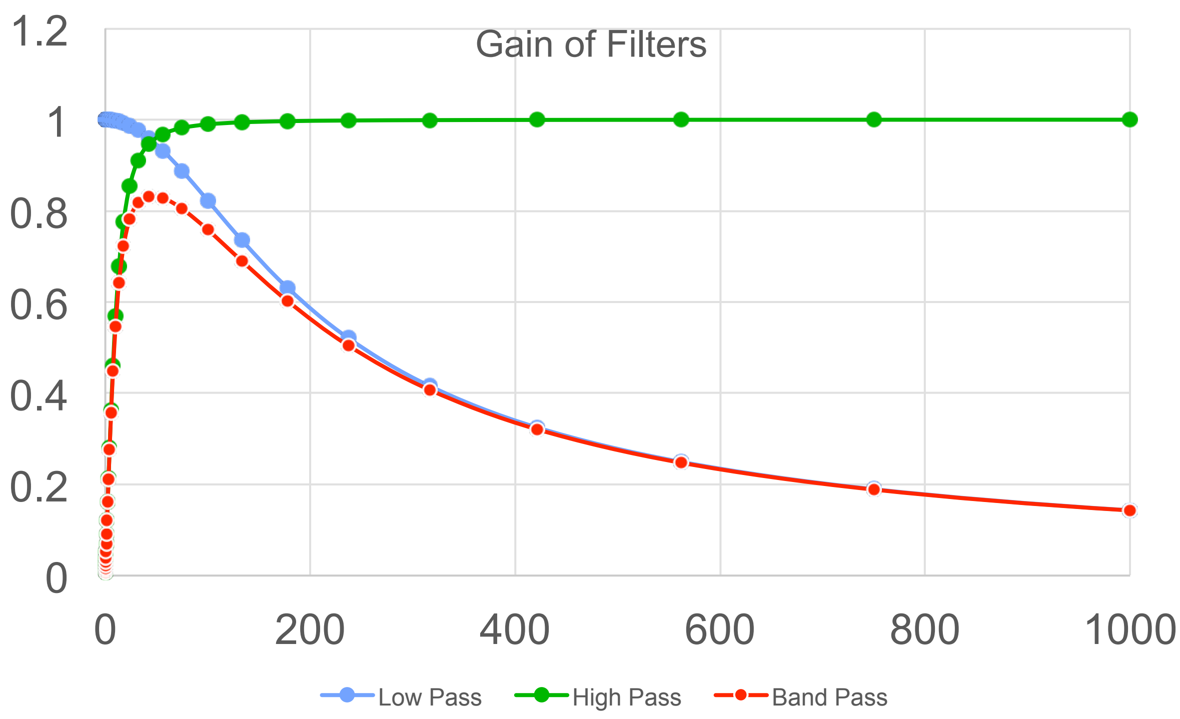
Our Plots Are Not Very Good

- Most of the plot is for the "high frequency"
  - Your ear is very interested in each octave (2x) in freq
  - If you plot the full audio spectrum (20-20kHz)
    - 50% of the plot will be from 10-20kHz
      - And that is only one octave of ten!
    - You won't be able to see the first five octaves!



More Plot Issues

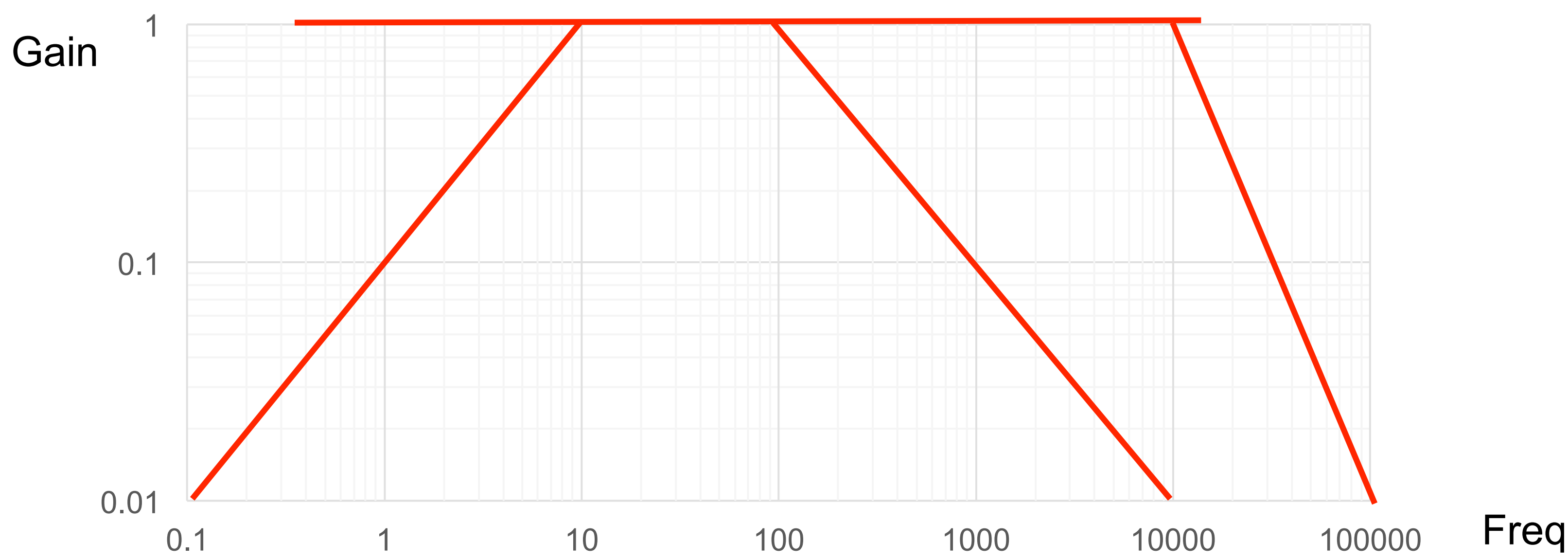
- The plots usually are proportional to:
  - Constant, or F or F^{-1} or F^2 or F^{-2}
  - It would be great if these were easy to see on a plot



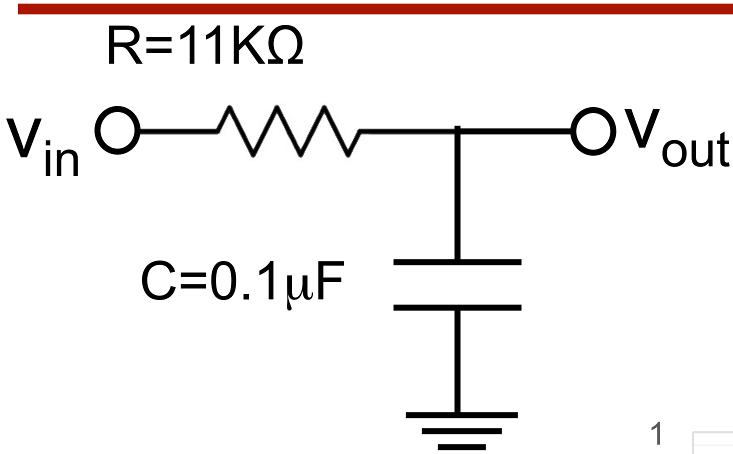
There is an Easy Way to Fix Both Issues

- Use a log-log plot
  - That is plot the log(Gain) vs log(F)
  - Usually labeled with Gain and F
    - But the spacing between numbers is their log
- Any power of F is a straight line $\log(F^n) = n^* \log(F)$

  So the slope of the line is the power of F

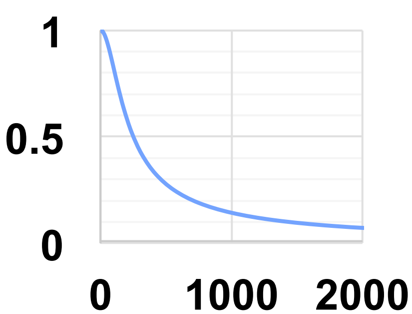


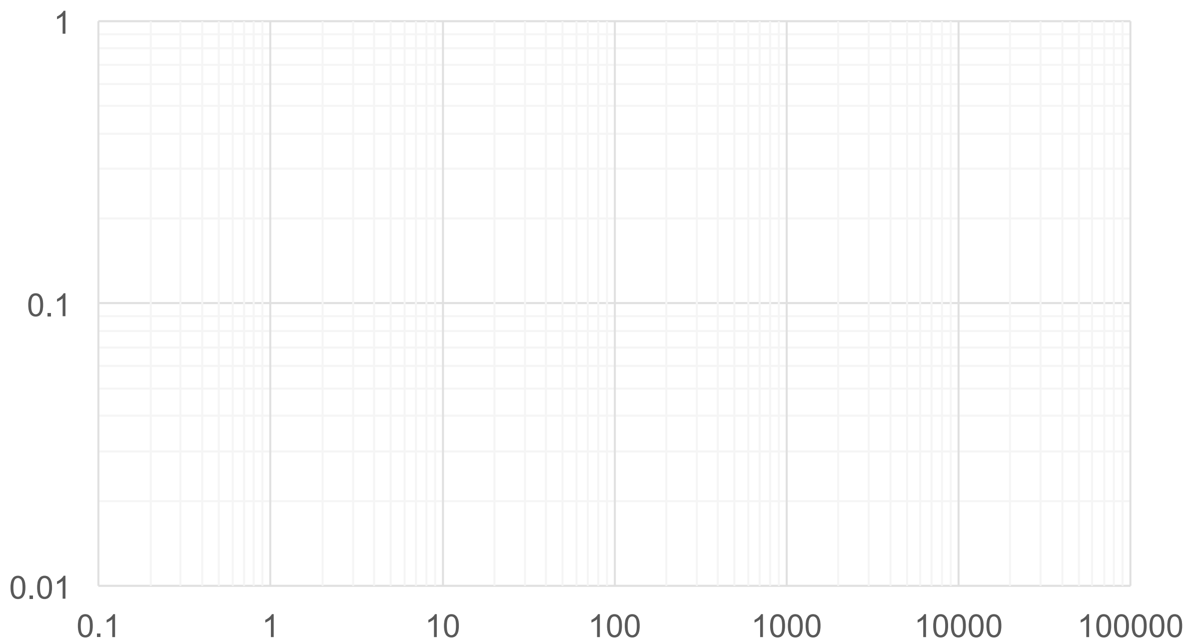
Example – Low Pass Filter



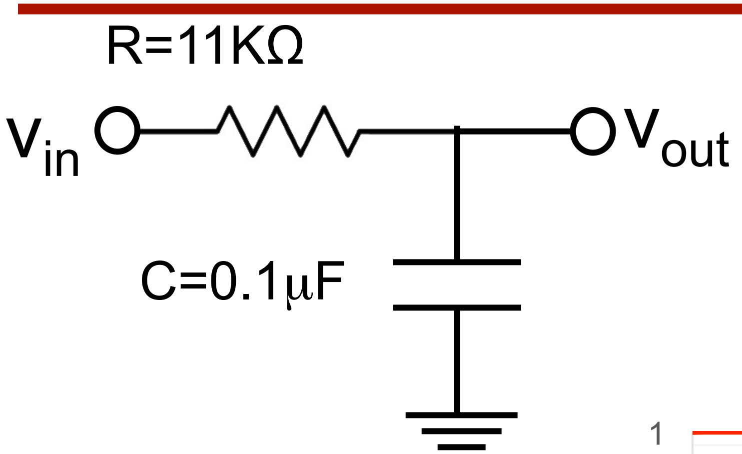
$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{j * 2\pi FC}}{R + \frac{1}{j * 2\pi FC}} = \frac{1}{1 + j * 2\pi FRC} = \frac{1}{1 + jF / F_c}$$

$2\pi RC$ is about 7ms; F_c= 140Hz



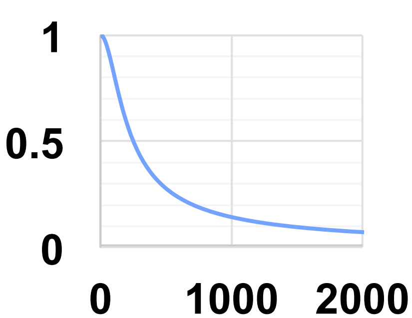


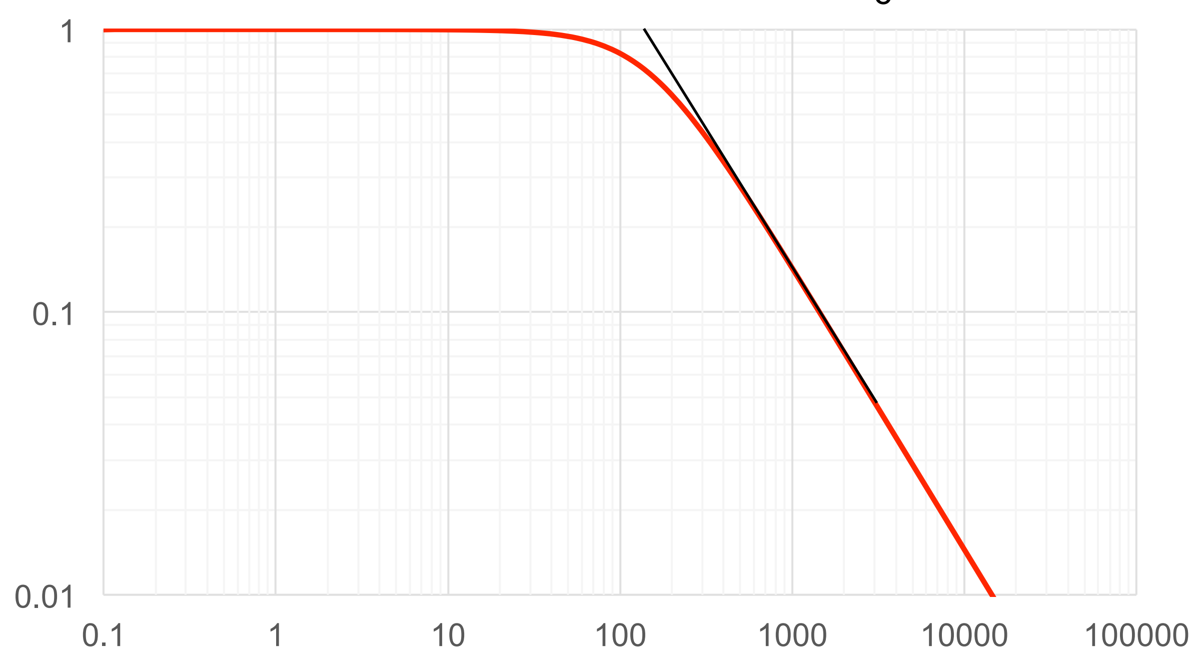
Low Pass



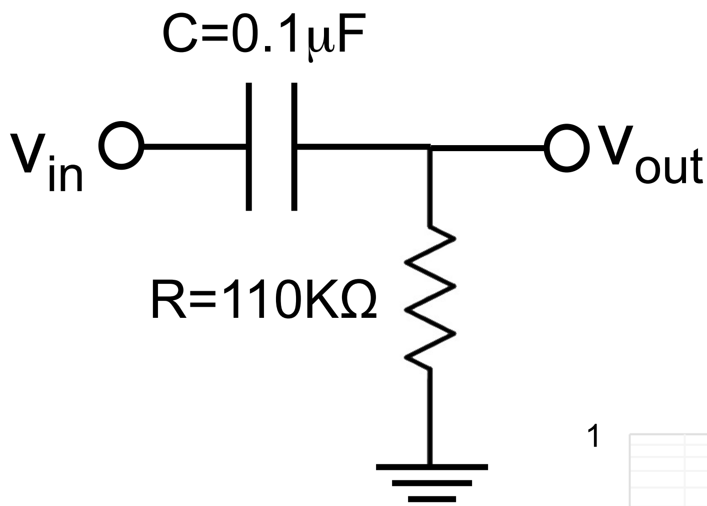
$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{j*2\pi FC}}{R+\frac{1}{j*2\pi FC}} = \frac{1}{1+j*2\pi FRC} = \frac{1}{1+jF/F_c}$$

$2\pi RC$ is about 7ms; F_c= 140Hz



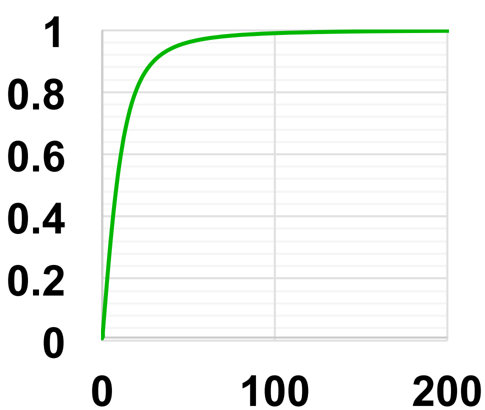


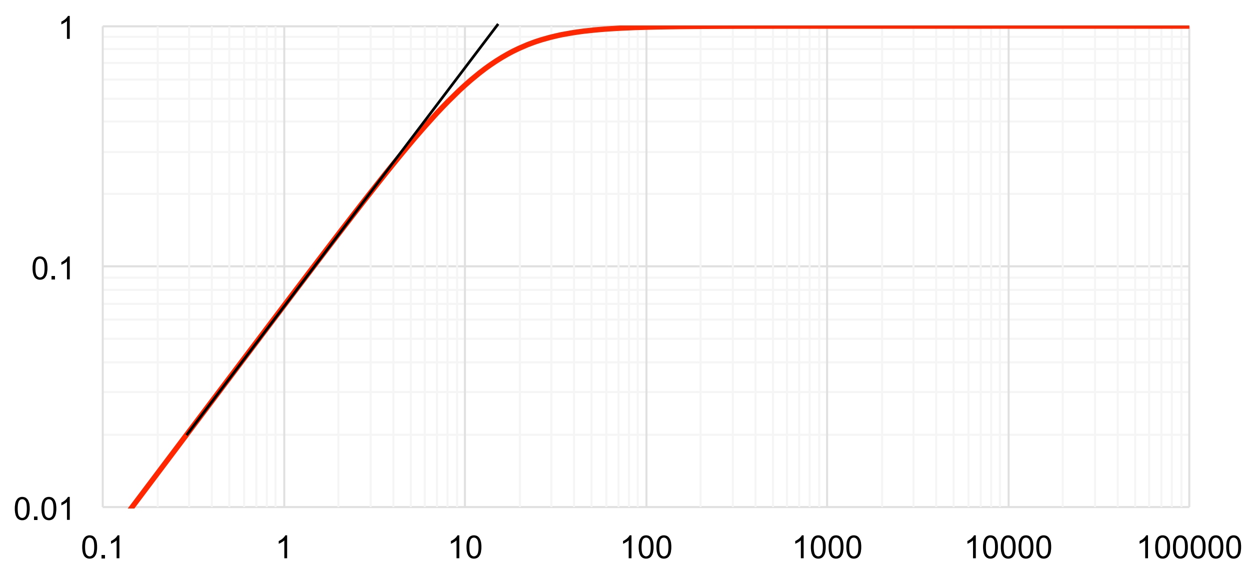
High-Pass



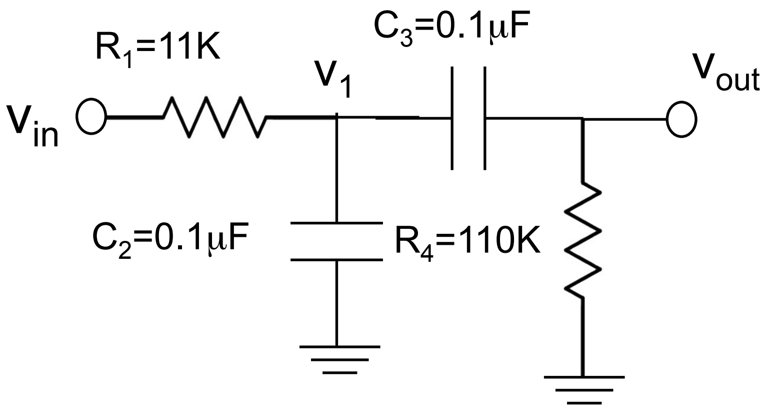
$$\frac{V_{out}}{V_{in}} = \frac{R}{R + \frac{1}{j * 2\pi FC}} = \frac{j * 2\pi FRC}{1 + j * 2\pi FRC}$$

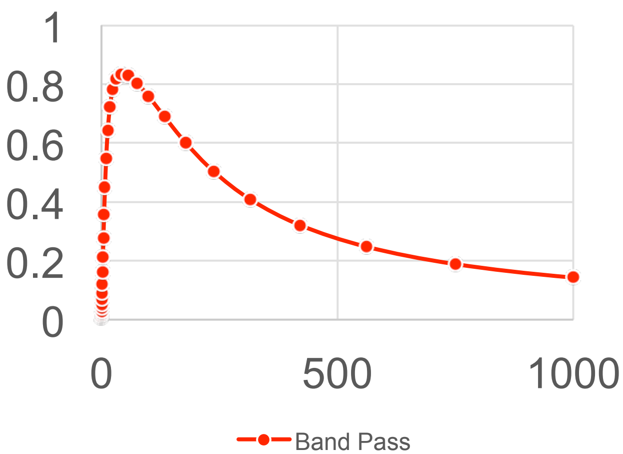
2πRC is about 70ms; F= 14Hz

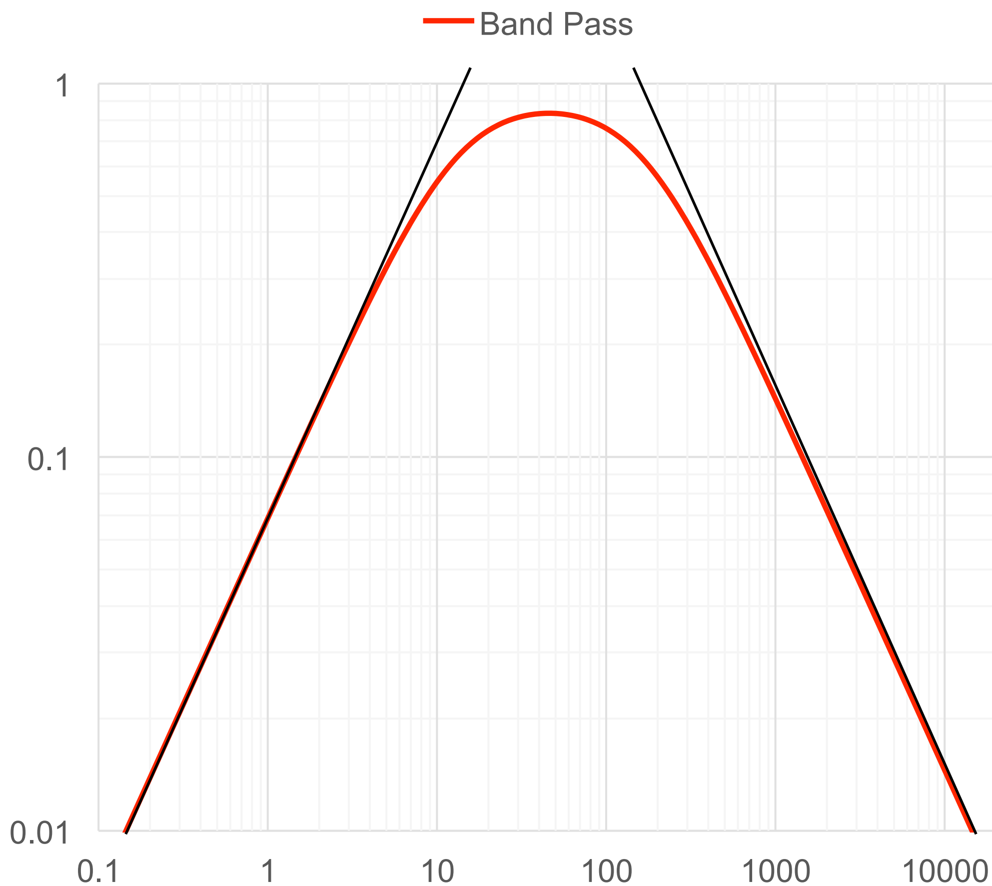




Band Pass – Combining the Low and High Pass







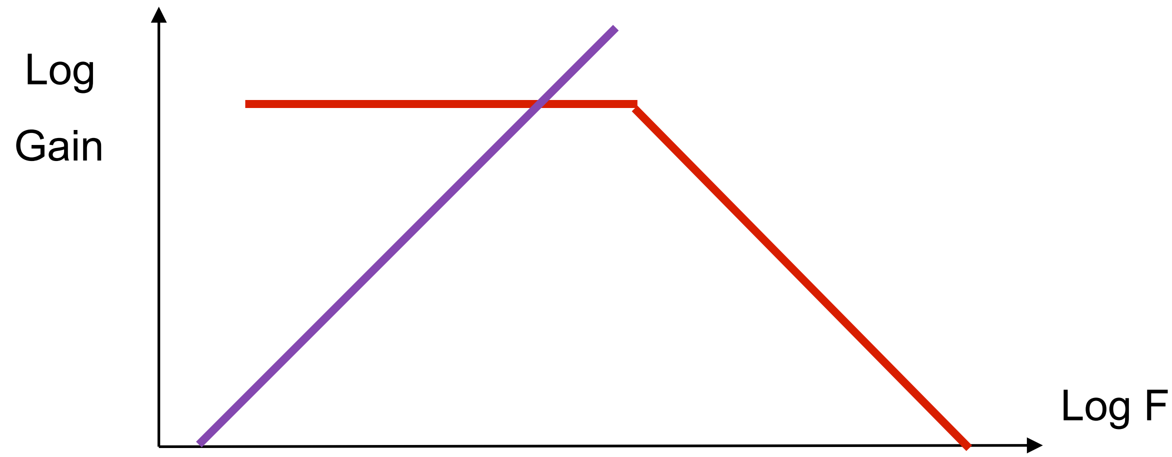
One More Trick With Log-Log Plots

- Remember:

  log(A*B) = log(A) + log(B), and   log(A/B) = lob(A) – log(B)

- So: $$\frac{V_{out}}{V_{in}} = \frac{j*2\pi FRC}{1+j*2\pi FRC}$$

  – Can add two lines
    - One slope +1
    - One flat and then slope -1



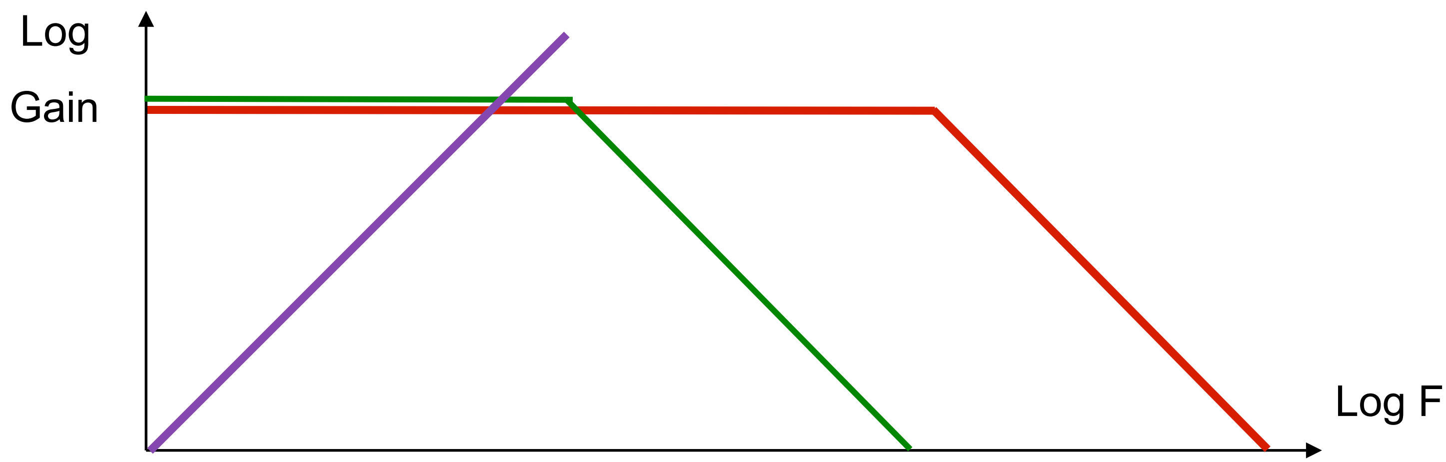
Log-Log Plot Tricks, cont'd

- Our bandpass filter earlier had a gain function of the form

$$\frac{V_{out}}{V_{in}} = \frac{j*2\pi FR_2C_2}{1+j*2\pi F(R_1C_1+R_2C_2)+\left(j*2\pi F\right)^2 R_1C_1R_2C_2}$$

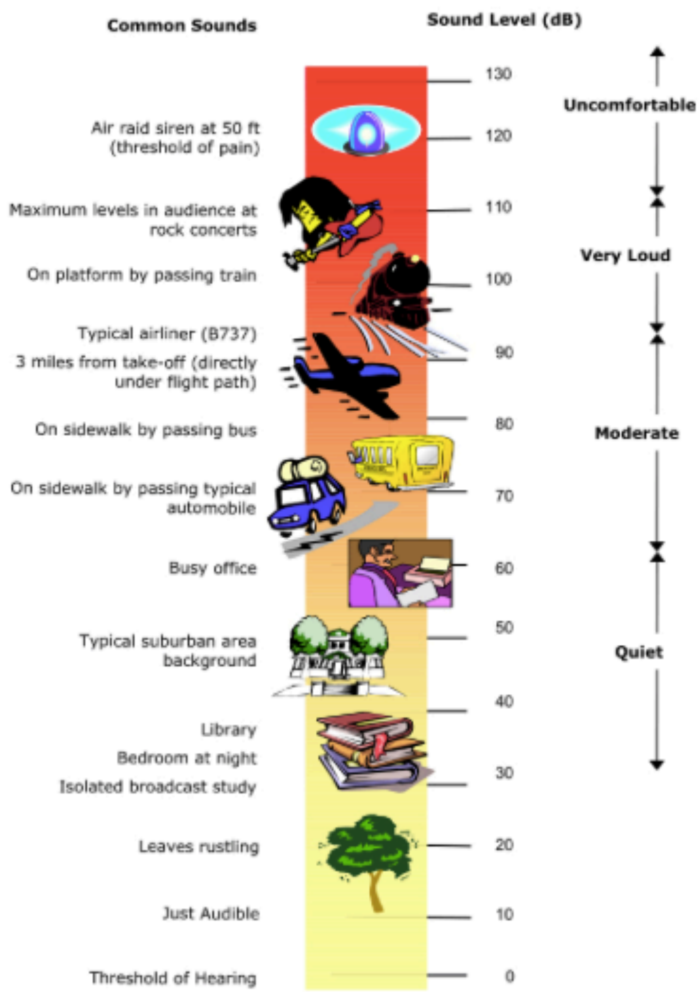
- If we can factor the polynomial, then we can add lines as on the last slide

$$\frac{V_{out}}{V_{in}} = \frac{j*2\pi FR_2C_2}{\left(1+j*2\pi FR_1C_1\right)*\left(1+j*2\pi FR_2C_2\right)}$$



dB

Use of Logarithmic Scales to Represent Wide Ranges



Source: Handbook of Environmental Acoustics, James P. Cowan, 1994

dB	power ratio	amplitude ratio
100	10 000 000 000	100 000
90	1 000 000 000	31 623
80	100 000 000	10 000
70	10 000 000	3 162
60	1 000 000	1 000
50	100 000	316 .2
40	10 000	100
30	1 000	31 .62
20	100	10
10	10	3 .162
6	3 .981	1 .995 (~2)
3	1 .995 (~2)	1 .413
1	1 .259	1 .122
0	1	1
−1	0 .794	0 .891
−3	0 .501 (~1/2)	0 .708
−6	0 .251	0 .501 (~1/2)
−10	0 .1	0 .316 2
−20	0 .01	0 .1
−30	0 .001	0 .031 62
−40	0 .000 1	0 .01
−50	0 .000 01	0 .003 162
−60	0 .000 001	0 .001
−70	0 .000 000 1	0 .000 316 2
−80	0 .000 000 01	0 .000 1
−90	0 .000 000 001	0 .000 031 62
−100	0 .000 000 000 1	0 .000 01

dB

- In many places you will see the symbol **dB**
  – This is decibel
  – It is a logarithmic measure of power gain
    - dB = 10 * log ($Power_{out}/Power_{in}$)

- It is logarithmic so
  – 10dB is a 10x change in power
  – 20dB is a 100x change in power
  – 3dB is a 2x change in power

- Since power is proportional to V^2
  – A 10x change in voltage is a 100x change in power
    - This is a 20dB change
  – 6dB is a 2x change in voltage

Plotting Gain vs. Frequency

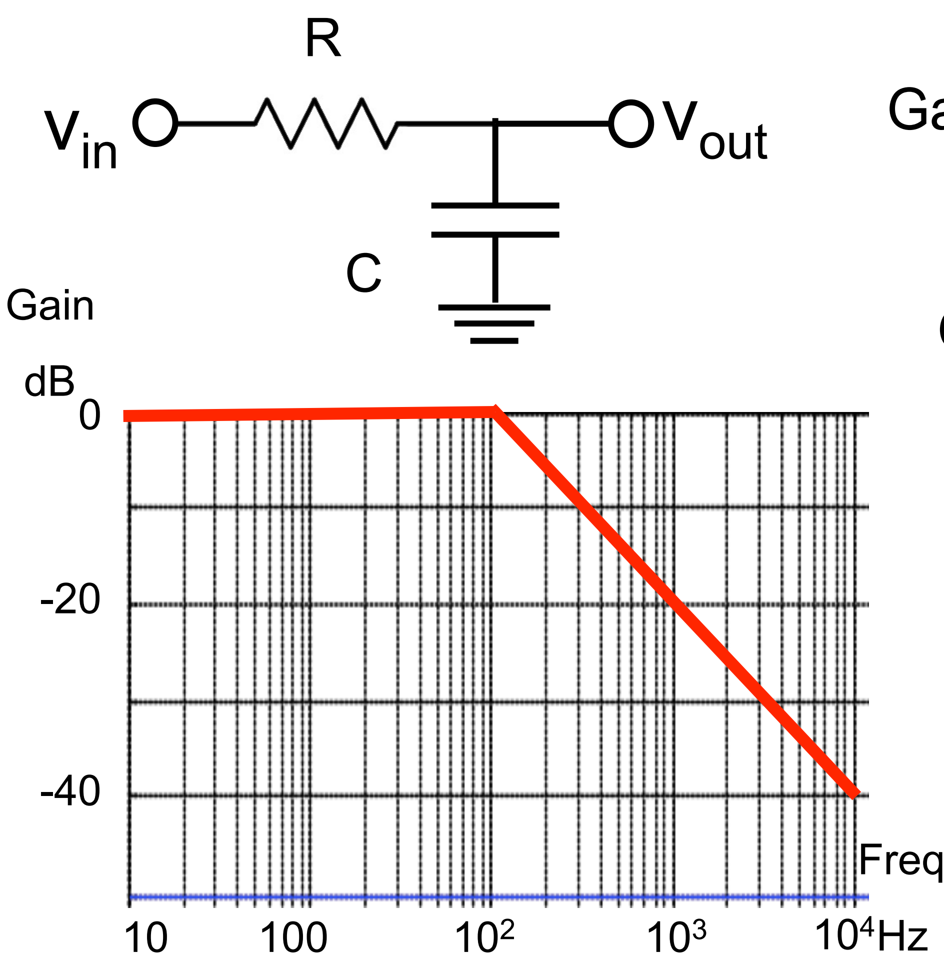
- Want to plot log(gain) vs. log(frequency)

- dB is already log of the gain
  - So the plots look semilog
    - Log of frequency in the x direction
    - dB in the y direction
  - But this is the log-log plot that we want

- Please remember that dB measures power
  - 10x in voltage = 20dB

Plotting dB vs. Frequency

- Consider the simple low pass filter we looked at earlier



$$\text{Gain} = \frac{V_{out}}{V_{in}} = \frac{1}{1 + j * 2\pi FRC} = \frac{1}{1 + jF/F_c}$$

$$\text{Gain}_{dB} = 20\log_{10}\left(\frac{1}{1 + jF/F_c}\right)$$

$$= 20\log_{10}(1) - 20\log_{10}(1 + jF/F_c)$$

$$\cong 0 - 20\log_{10}(F/F_c)$$

(assuming F is large and

neglecting the phase)

FYI – Hendrik Bode

Hendrik Wade Bode

Hendrik Wade Bode

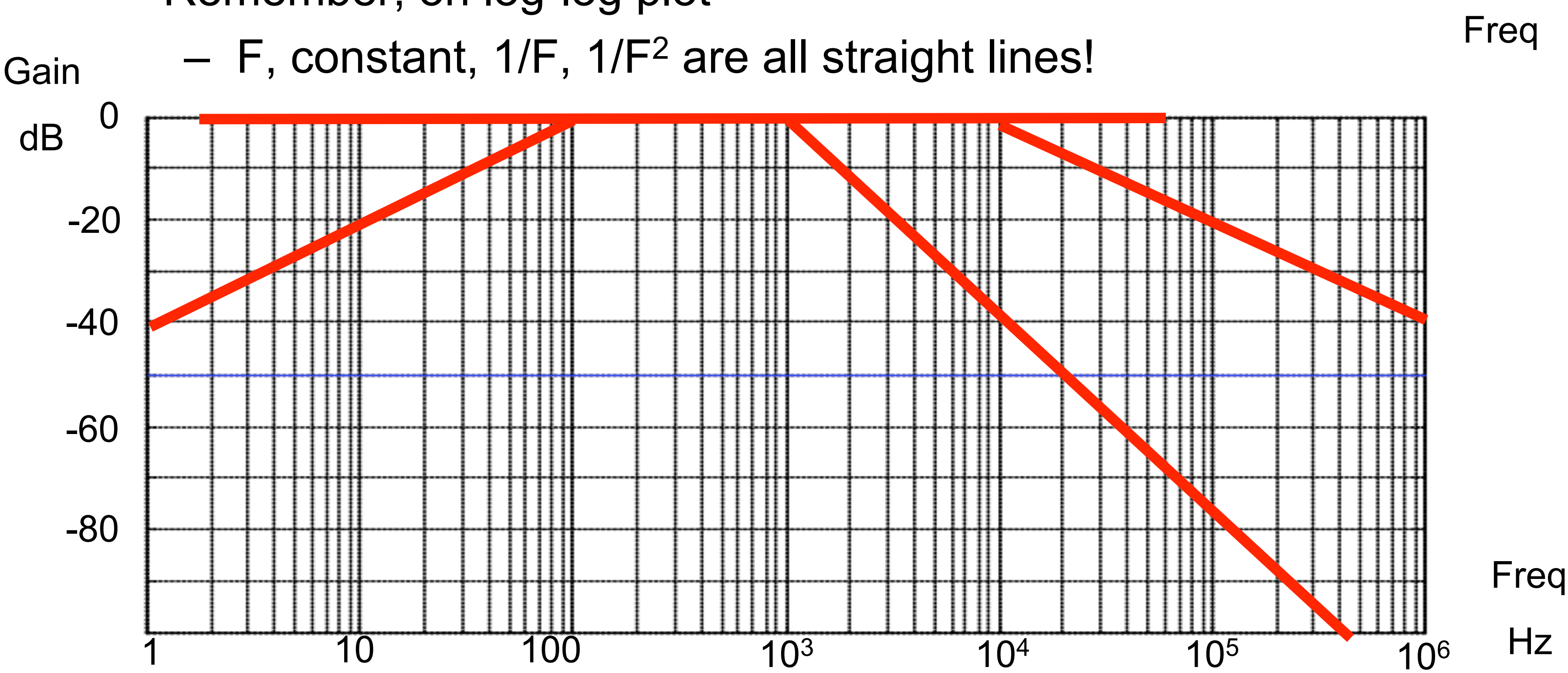
Born	December 24, 1905 Madison, Wisconsin
Died	June 21, 1982 (aged 76) Cambridge, Massachusetts
Residence	Cambridge, Massachusetts
Nationality	American
Fields	Control Systems, Physics, Mathematics, Telecommunications
Institutions	Ohio State University Bell Laboratories Harvard University
Alma mater	Ohio State University Columbia University
Known for	Bode plot, Control theory, Telecommunications
Notable awards	Richard E. Bellman Control Heritage Award (1979) Rufus Oldenburger Medal (1975) President's Certificate of Merit Edison Medal (1969) Ernest Orlando Lawrence Award (1960)

- Bode (1905 –1982) spent most of his career at Bell Labs.
- He worked on control system theory and electronic filters and during WW II he worked on using radar information to direct antiaircraft guns to try to intercept enemy aircraft and missiles like the German V2 missile.
- But today he's best remembered for inventing Bode plots used to describe the frequency behavior of linear systems.

(Wikipedia)

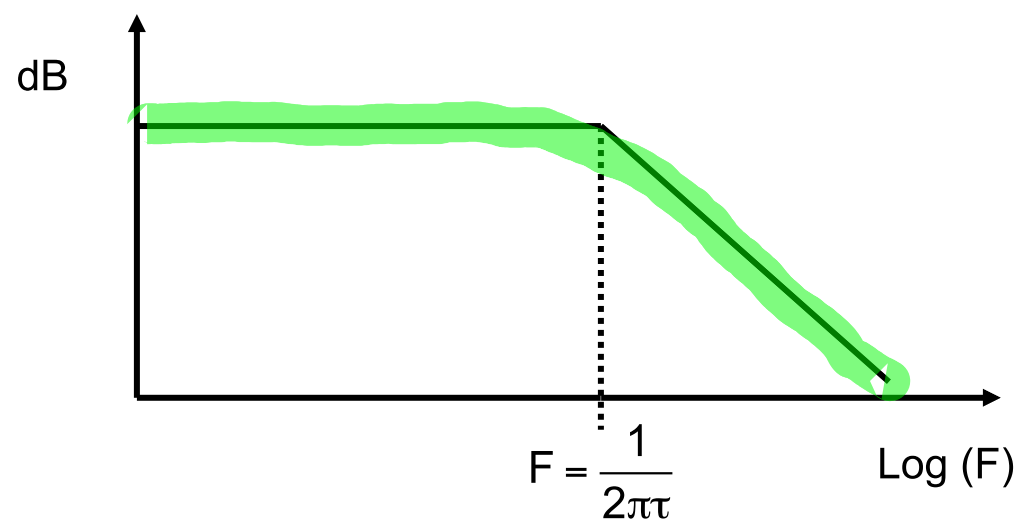
Plotting the Output

- We use a Bode Plot to plot the transfer function of a circuit
  - Remember that this is the log of gain vs. log of frequency
- Remember, on log-log plot
  - F, constant, 1/F, $1/F^2$ are all straight lines!



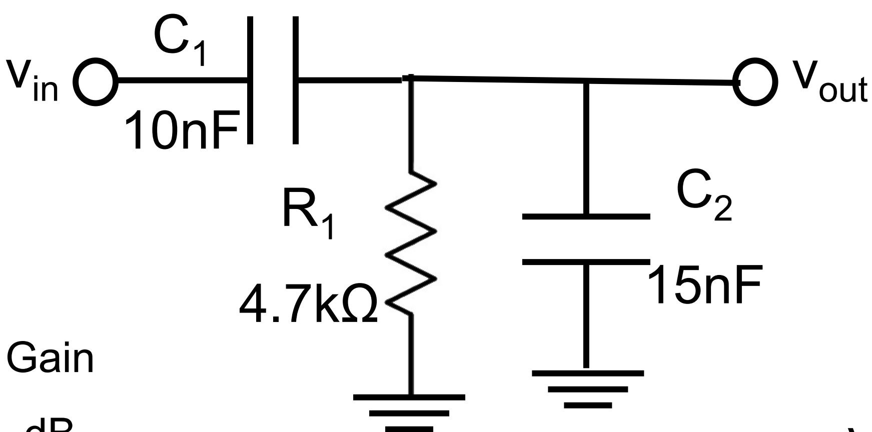
Circuit Bode Plots

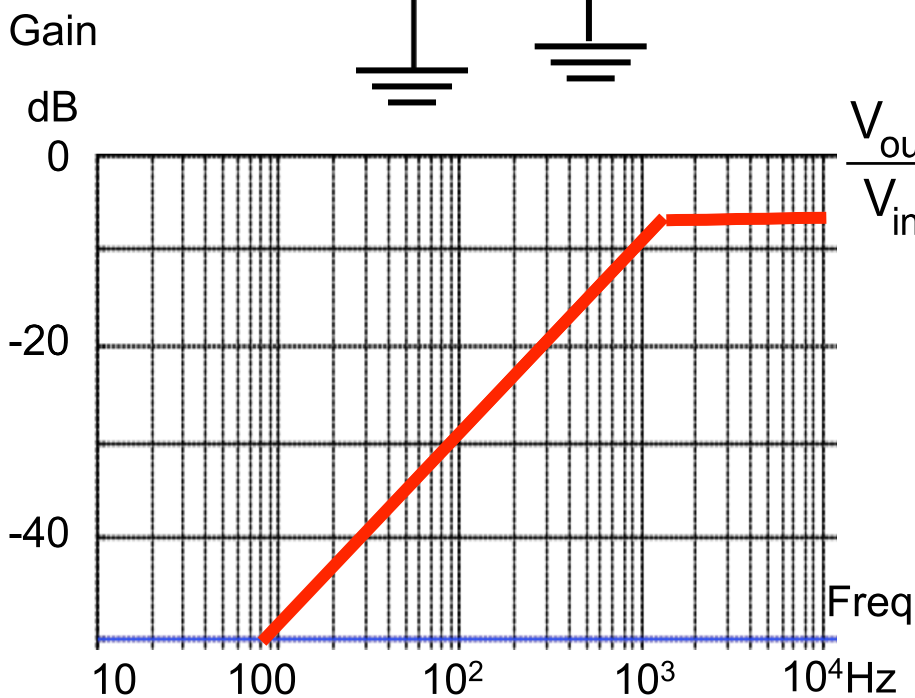
- These are generally easy to draw
  - Know the slopes at different frequency ranges
    - Plot those straight lines
  - These lines will intercept at the F where the terms are equal i.e. $F = 1/2\pi\tau \;\; = 1/2\pi RC$



- Actual curve will be close to the straight lines

Example





$$\frac{V_{out}}{V_{in}} = \frac{Z_{R1} \,||\, Z_{C2}}{Z_{C1} + Z_{R1} \,||\, Z_{C2}}$$

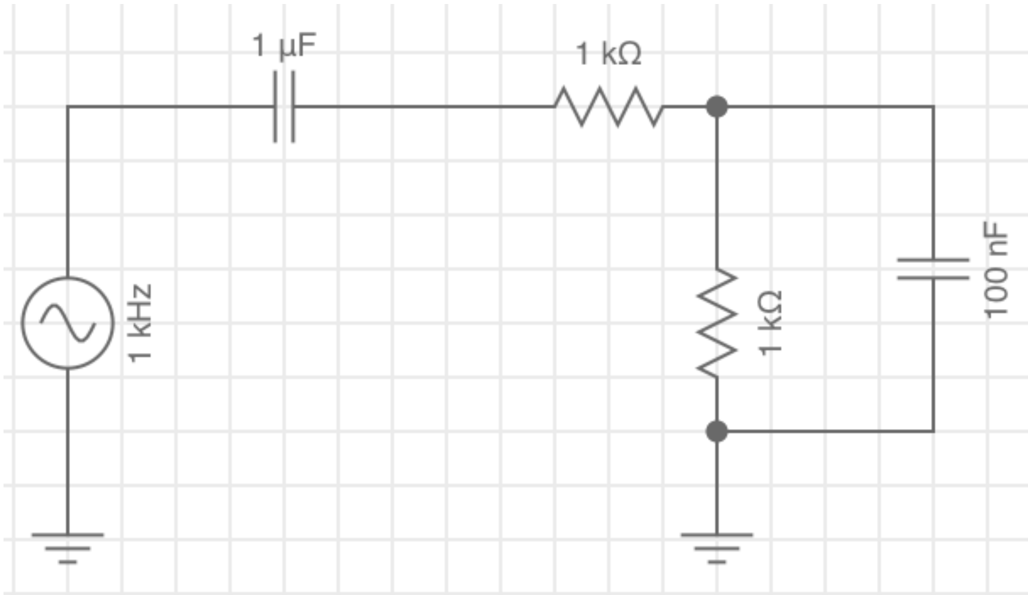
$$Z_{R1} \,||\, Z_{C2} = \frac{Z_{R1} * Z_{C2}}{Z_{R1} + Z_{C2}} = \frac{R_1}{1 + 2\pi F R_1 C_2}$$

$$\frac{V_{out}}{V_{in}} = \frac{\dfrac{R_1}{1 + 2\pi F R_1 C_2}}{\dfrac{1}{2\pi F C_1} + \dfrac{R_1}{1 + 2\pi F R_1 C_2}} = \frac{2\pi F R_1 C_1}{1 + 2\pi F R_1 (C_1 + C_2)}$$

$$F_c = \frac{1}{2\pi R_1 (C_1 + C_2)} = 1.35\text{kHz}$$

$$\text{Gain}_{dB} \,@\,\text{HighF} = 20\log\left(\frac{C_1}{C_1 + C_2}\right) = -7.96\text{dB}$$

EveryCircuit – Frequency Resonse

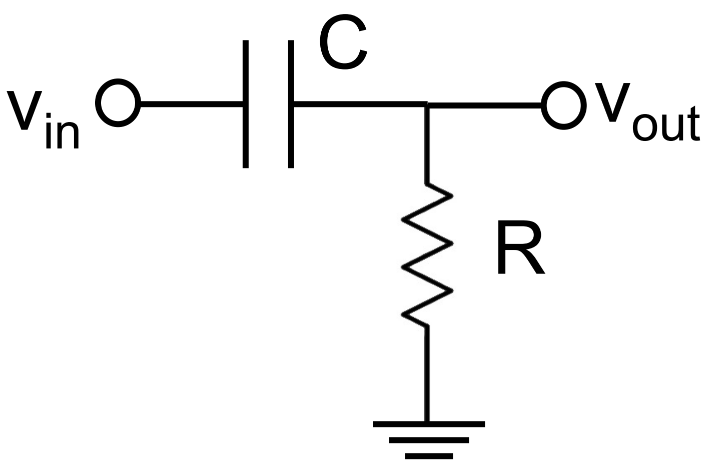


- EveryCircuit can be used to calculate and plot Bode plots for circuits. We'll demo this in class.
- This is very useful for checking answers to HW problems or for developing understanding about circuit frequency response.

Bonus Section (Not on HW, Exams)
See Class Reader For Details

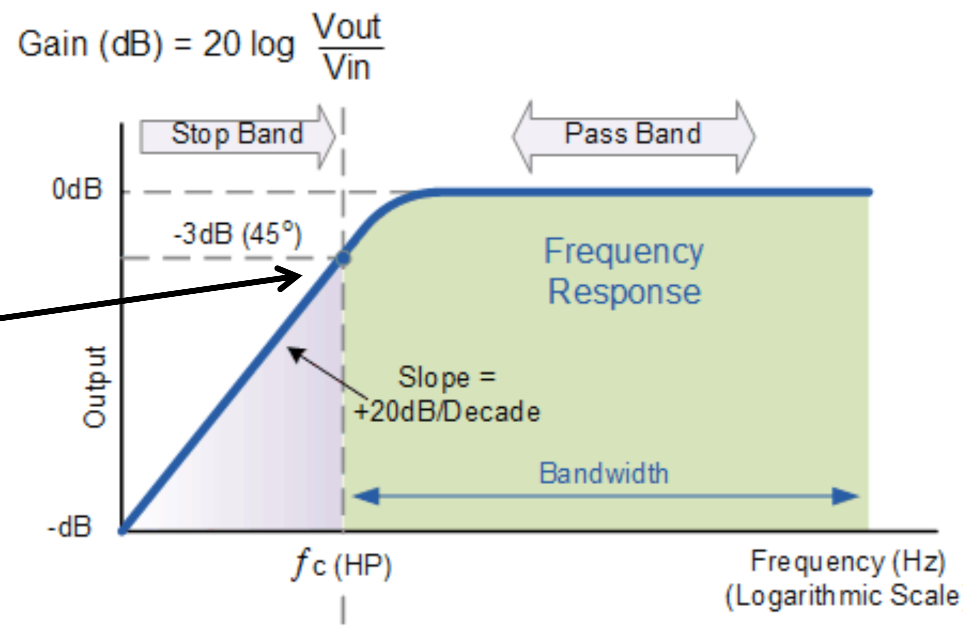
WHAT DOES (1 + j*x) REALLY MEAN?

An Example - High Pass RC Filter
Bonus Material – See Class Reader For Details



$$\frac{V_{out}}{V_{in}} = \frac{R}{R + \dfrac{1}{j * 2\pi FC}} = \frac{j * 2\pi FRC}{1 + j * 2\pi FRC}$$

$$2\pi FRC = 1 \quad \text{or} \quad F = \frac{1}{2\pi RC}$$



(Electronic Tutorials)

3dB?
Bonus Material – See Class Reader For Details

- Notice that when the two terms are the same, F = 1/2πRC
  - The overall gain is -3dB
  - This might seem right since -3dB is ½, BUT
  - This is the power ratio!
    - The voltage ratio is only $1/\sqrt{2}$!

- What is going on?

- To understand, we'll need to think about what (1 + j*x) means

Adding Sine and Cosine Waveforms
Bonus Material – See Class Reader For Details

- 1 + j*x means the waveform is the sum of
  - A sinewave with amplitude 1
  - With a cosine wave with amplitude “x”

- The good news is that this always results in a sinusoidal waveform
  - With some phase shift between a sin and cos (+90˚)
  - So we can write

$$A\sin\left(2\pi Ft+\phi\right)=\sin\left(2\pi Ft\right)+x*\cos\left(2\pi Ft\right)$$

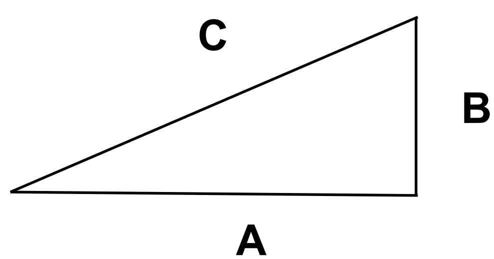
  - And we want to find A and ϕ

The Trick
Bonus Material – See Class Reader For Details

$$C * \sin(2\pi Ft + \phi) = A * \sin(2\pi Ft) + B * \cos(2\pi Ft)$$

- This equation is always true so
  - At t = 0, the sine term is zero

$$C * \sin(\phi) = B$$



  - At t = 1/(4F), the cosine term is zero, so

$$C * \cos(\phi) = A$$

  - So the amplitude of the resulting sine wave is

$$\sqrt{A^2 + B^2}$$

Learning Objectives For Today

- Understand how to create Bode plots for RC circuits
  - You plot two straight lines that intercept at $F=1/2\pi RC$
- (Bonus Section) Understand what an amplitude of $(1+ j*2\pi RCF)$ means
  - The result is a sum of a sine and cosine wave