E40M
Bode Plots, dB

1. HW #6 posted, due next Tues
2. If struggling with some concept, Roger or I are happy to meet 1:1 email us
Reading

• Reader
  – 7.1-7.2 – Bode Plots

or lots of stuff on internet
EKG Lab

• Concepts
  – Amplifiers
  – Impedance
  – Noise
  – Safety
  – Filters

• Components
  – Capacitors
  – Inductors
  – Instrumentation and Operational Amplifiers

In this project we will build an electrocardiogram (ECG or EKG). This is a noninvasive device that measures the electrical activity of the heart using electrodes placed on the skin.
Capacitors

\[ Z_c = \frac{1}{j\omega C} \Rightarrow \frac{1}{2\pi f C} \]

Ignore phase

For large: \( Z_c \to 0 \)

For small: \( Z_c \to \infty \) (open)
Analyzing RC Circuits Using Impedance – Review (Low Pass Filter)

\[ \frac{V_{out}}{V_{in}} = \frac{1}{j \cdot 2\pi FC} \cdot \frac{1}{R + j \cdot 2\pi FC} = \frac{1}{1 + j \cdot 2\pi FRC} \]
Analyzing RC Circuits Using Impedance – Review (High Pass Filter)

\[ \frac{V_{out}}{V_{in}} = \frac{R}{R + \frac{1}{j \cdot 2\pi FC}} = \frac{j \cdot 2\pi FRC}{1 + j \cdot 2\pi FRC} \]

RC = 11ms; \(2\pi RC\) is about 70ms

Gain vs. Freq

\[ \text{Gain} = \text{Vout} \]

M. Horowitz, J. Plummer, R. Howe
Analyzing RC Circuits Using Impedance – Review
(Band Pass Filter)

\[ V_{out} = \frac{V_{out}}{V_{in}} = \frac{j2\pi FR_4C_3}{\left(1 + j2\pi F(R_4C_3 + R_1C_2 + R_1C_3)\right)^2 R_1C_2R_4C_3} \]

- We’ll use a filter that operates like this in the ECG lab project.
BODE PLOTS
Our Plots Are Not Very Good

- Most of the plot is for the “high frequency”
  - Your ear is very interested in each octave (2x) in freq
  - If you plot the full audio spectrum (20-20kHz)
    - 50% of the plot will be from 10-20kHz
      - And that is only one octave of ten!
    - You won’t be able to see the first five octaves!
More Plot Issues

- The plots usually are proportional to:
  - Constant, or F or F^{-1} or F^2 or F^{-2}
  - It would be great if these were easy to see on a plot
There is an Easy Way to Fix Both Issues

- Use a log-log plot
  - That is plot the log(Gain) vs log(F)
  - Usually labeled with Gain and F
    - But the spacing between numbers is their log
- Any power of F is a straight line \( \log(F^n) = n \times \log(F) \)
  
  So the slope of the line is the power of F
Example – Low Pass Filter

\[ V_{out} = \frac{1}{\frac{j*2\pi FC}{V_{in}}} = \frac{1}{1 + j*2\pi FR} = \frac{1}{1 + j\frac{F}{F_c}} \]

2\pi RC is about 7ms; \( F_c = 140\text{Hz} \)
Low Pass

\[ R = 11 \text{K}\Omega \]
\[ V_{\text{in}} \quad \text{O} \quad R + \frac{1}{j \cdot 2\pi FC} \quad \text{O} \quad V_{\text{out}} \]

\[ \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{1}{j \cdot 2\pi FC} \]

\[ = \frac{1}{1 + j \cdot 2\pi FC} \]

\[ = \frac{1}{1 + jF / F_c} \]

\[ 2\pi RC \text{ is about 7ms; } F_c = 140\text{Hz} \]
High-Pass

\[ \frac{V_{out}}{V_{in}} = \frac{R}{R + \frac{1}{j*2\pi FC}} = \frac{j*2\pi FRC}{1+j*2\pi FRC} \]

\(2\pi RC\) is about 70ms; \(F = 14\text{Hz}\)
Band Pass – Combining the Low and High Pass

\[ \begin{align*}
R_1 &= 11 \text{K} \\
C_2 &= 0.1 \mu \text{F} \\
C_3 &= 0.1 \mu \text{F} \\
R_4 &= 110 \text{K}
\end{align*} \]
One More Trick With Log-Log Plots

- Remember:
  \[ \log(A \cdot B) = \log(A) + \log(B), \quad \text{and} \quad \log(A/B) = \log(A) - \log(B) \]

- So:
  \[ \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{j \cdot 2\pi F_{RC}}{1 + j \cdot 2\pi F_{RC}} \]

- Can add two lines
  - One slope +1
  - One flat and then slope -1

\[ \log \frac{V_{\text{out}}}{V_{\text{in}}} = \log(2\pi F_{RC}) \quad \sim \quad -\log(1 + 2\pi F_{RC}) \]
Log-Log Plot Tricks, cont’d

• Our bandpass filter earlier had a gain function of the form

\[
\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{j \cdot 2\pi F R_2 C_2}{1 + j \cdot 2\pi F (R_1 C_1 + R_2 C_2) + (j \cdot 2\pi F)^2 R_1 C_1 R_2 C_2}
\]

• If we can factor the polynomial, then we can add lines as on the last slide

\[
\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{j \cdot 2\pi F R_2 C_2}{(1 + j \cdot 2\pi F R_1 C_1)(1 + j \cdot 2\pi F R_2 C_2)}
\]
Use of Logarithmic Scales to Represent Wide Ranges

\[ AB = 10 \log \left( \frac{P_2}{P_1} \right) \]

- **Common Sounds**
  - Air raid siren at 50 ft (threshold of pain)
  - Maximum levels in audience at rock concerts
  - On platform by passing train
  - Typical airliner (B737)
  - 3 miles from take-off (directly under flight path)
  - On sidewalk by passing bus
  - On sidewalk by passing typical automobile
  - Busy office
  - Typical suburban area background
  - Library
  - Bedroom at night
  - Isolated broadcast study
  - Leaves rustling
  - Just Audible
  - Threshold of Hearing

*Source: Handbook of Environmental Acoustics, James P. Cowan, 1994*
dB

- In many places you will see the symbol \( \text{dB} \)
  - This is decibel
  - It is a logarithmic measure of power gain
    - \( \text{dB} = 10 \times \log \left( \frac{\text{Power}_{\text{out}}}{\text{Power}_{\text{in}}} \right) \)

- It is logarithmic so
  - 10dB is a 10x change in power
  - 20dB is a 100x change in power
  - 3dB is a 2x change in power

- Since power is proportional to \( V^2 \)
  - A 10x change in voltage is a 100x change in power
    - This is a 20dB change
  - 6dB is a 2x change in voltage
Plotting Gain vs. Frequency

- Want to plot log(gain) vs. log(frequency)

- dB is already log of the gain
  - So the plots look semilog
    - Log of frequency in the x direction
    - dB in the y direction
  - But this is the log-log plot that we want

- Please remember that dB measures power
  - 10x in voltage = 20dB
Plotting dB vs. Frequency

Consider the simple low pass filter we looked at earlier:

\[
\text{Gain} = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{1}{1 + j \cdot 2\pi F R C} = \frac{1}{1 + j F / F_c}
\]

Gain (dB) = \[20 \log_{10} \left( \frac{1}{1 + j F / F_c} \right)\]

\[= 20 \log_{10} (1) - 20 \log_{10} \left(1 + j F / F_c\right)\]

\[\approx 0 - 20 \log_{10} \left( \frac{F}{F_c} \right)\]

(assuming \(F\) is large and neglecting the phase)
FYI – Hendrik Bode

Bode (1905 –1982) spent most of his career at Bell Labs.

He worked on control system theory and electronic filters and during WW II he worked on using radar information to direct antiaircraft guns to try to intercept enemy aircraft and missiles like the German V2 missile.

But today he’s best remembered for inventing Bode plots used to describe the frequency behavior of linear systems.

(Wikipedia)
Plotting the Output

- We use a Bode Plot to plot the transfer function of a circuit
  - Remember that this is the log of gain vs. log of frequency

- Remember, on log-log plot
  - $F$, constant, $1/F$, $1/F^2$ are all straight lines!
Circuit Bode Plots

- These are generally easy to draw
  - Know the slopes at different frequency ranges
    - Plot those straight lines
  - These lines will intercept at the F where the terms are equal
    i.e. $F = \frac{1}{2\pi \tau} = \frac{1}{2\pi RC}$

- Actual curve will be close to the straight lines
Example

\[ \frac{V_{out}}{V_{in}} = \frac{\frac{Z_{R1} \parallel Z_{C2}}{Z_{C1} + Z_{R1} \parallel Z_{C2}}}{\frac{R_1}{1 + 2\pi F R_1 C_2}} \]

\[ Z_{R1} \parallel Z_{C2} = \frac{Z_{R1} \cdot Z_{C2}}{Z_{R1} + Z_{C2}} = \frac{R_1}{1 + 2\pi F R_1 C_2} \]

\[ V_{out} \quad \text{dB} \quad V_{in} \]

\[ \text{Gain}_{dB} @ \text{HighF} = 20 \log \left( \frac{C_1}{C_1 + C_2} \right) = -7.96 \text{dB} \]

\[ F_c = \frac{1}{2\pi R_1 (C_1 + C_2)} = 1.35 \text{kHz} \]
EveryCircuit – Frequency Response

- EveryCircuit can be used to calculate and plot Bode plots for circuits. We’ll demo this in class.
- This is very useful for checking answers to HW problems or for developing understanding about circuit frequency response.
Bonus Section (Not on HW, Exams)
See Class Reader For Details

WHAT DOES \((1 + j^x)\) REALLY MEAN?
An Example - High Pass RC Filter
Bonus Material – See Class Reader For Details

\[
\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{R}{R + \frac{1}{j*2\pi FC}} = \frac{j*2\pi FRC}{1+j*2\pi FRC}
\]

\[2\pi FRC = 1 \text{ or } F = \frac{1}{2\pi RC}\]
3dB?
Bonus Material – See Class Reader For Details

• Notice that when the two terms are the same, $F = 1/2\pi RC$
  – The overall gain is -3dB
  – This might seem right since -3dB is $\frac{1}{2}$, BUT
  – This is the power ratio!
    • The voltage ratio is only $1/\sqrt{2}$ !

• What is going on?

• To understand, we’ll need to think about what $(1 + j^*x)$ means
Adding Sine and Cosine Waveforms
Bonus Material – See Class Reader For Details

• 1 + j*x means the waveform is the sum of
  – A sinewave with amplitude 1
  – With a cosine wave with amplitude “x”

• The good news is that this always results in a sinusoidal waveform
  – With some phase shift between a sin and cos (+90°)
  – So we can write

\[
A \sin(2\pi Ft + \phi) = \sin(2\pi Ft) + x \cos(2\pi Ft)
\]

  – And we want to find A and \( \phi \)
The Trick
Bonus Material – See Class Reader For Details

\[ C \sin(2\pi Ft + \phi) = A \sin(2\pi Ft) + B \cos(2\pi Ft) \]

- This equation is always true so
  - At \( t = 0 \), the sine term is zero
    \[ C \sin(\phi) = B \]
  - At \( t = 1/(4F) \), the cosine term is zero, so
    \[ C \cos(\phi) = A \]
  - So the amplitude of the resulting sine wave is
    \[ \sqrt{A^2 + B^2} \]
Learning Objectives For Today

• Understand how to create Bode plots for RC circuits
  – You plot two straight lines that intercept at $F = 1/2\pi RC$

• (Bonus Section) Understand what an amplitude of $(1 + j*2\pi RCF)$ means
  – The result is a sum of a sine and cosine wave