E40M

Op Amps
Reading


Reader, Chapter 8

• Noninverting Amp
• Inverting Amp
  – http://www.electronics-tutorials.ws/opamp/opamp_2.html
• Summing Amp
How to Measure Small Voltages?

• Arduino input has full-scale around 5V
  – It produces a 10 bit answer (1024)
  – This means a LSB (least significant bit) is 5mV

• Need to make the signal bigger before input to Arduino
  – So, we will use an amplifier

• Many ways to build amplifiers
  – One often uses a standard building block for amplifiers
    • Called an Operational Amplifier, or Op-Amp
    • A circuit with very high gain at low frequencies (< 10 kHz)
Electrical Picture

- Signal amplitude ≈ 1 mV
- Noise level will be significant
- \( \therefore \) will need to amplify and filter
- We’ll use filtering ideas from the last two lectures
OP AMPS
Op Amp

- Is a common building block
  - It is a high-gain amplifier

- Output voltage is
  \( A (V_+ - V_-) \)
  - Gain, \( A \), is 10 K to 1 M

- Output voltage can be + or –
  - Often can swing between +\( V_{dd} \) and -\( V_{dd} \) supplies
  - Huh?
Op-Amp Power Supply

• Up to now we had one supply voltage, Vdd
  – All voltages were between Vdd and Gnd
  – Generally measured relative to Gnd
    • So all voltages were positive.

• A sinewave goes positive and negative
  – And most input signals do that too

• It is convenient to have a reference where
  – The output can be positive and negative
  – Can do that by changing what we call the reference
Moving the Reference

\[ V_{dd} = 5 \text{ V} \]

\[ V_{dd} = 2.5 \text{ V} \]

The voltages are all the same, only the reference voltage has moved.
What You Will Actually Do

- Use the USB supply
  - Just change the reference voltage
Op Amp Behavior

- Relationship between output voltage and input voltage:

\[ v_o = A(v_+ - v_-) = A(v_p - v_n) \]

A is the op-amp gain (or open-loop gain), and is huge 10K-1M

- The input currents are very, very small

so \( i_p \approx 0 \) and \( i_n \approx 0 \).
Since the Output Swing is Limited

- The high gain only exists for a small range of input voltages
  - If the input difference is too large, the output “saturates”
    - Goes to the max positive or negative value possible
    - Close to supply voltages

\[ v_o = A(v_p - v_n) \]

- Linear range
- Negative saturation region
- Positive saturation region
- \( V_{cc} \)
- \(-V_{cc}\)
What Does This Do?

\[ V_{cc} = 5 \text{ V} \]

\[ v_{in} \]

\[ 2.5 \text{ V} \]

\[ v_{out} \]

\[ v_{out} \]

\[ V_{cc} = 5 \text{ V} \]
Same Circuit Different Reference

\[ V_{dd} = 2.5 \text{ V} \]

\[ v_{out} \]

\[ 2.5 \text{ V} \]
How To Get A Useful Amplifier

- The gain of the op amp is too high to make a useful amplifier
  - We need to do something to make it useful

- We will use **analog feedback** to fix this problem
  - Feedback makes the input the error between the value of the output, and the value you want the output to have.

- Let’s see how to do this
Connect $V_{\text{out}}$ to $V_{\text{in}}$-

\[ v_{\text{out}} = A(V_+ - V_-) = A(v_{\text{in}} - v_{\text{out}}) \]

\[ \therefore (A + 1)v_{\text{out}} = Av_{\text{in}} \]

\[ \therefore v_{\text{out}} = \frac{A}{A + 1}v_{\text{in}} \approx v_{\text{in}} \]
What Is Going On

• We solved the equation to find the answer
  – But how does the op-amp get this answer?

• Think about what happens when the input increases in voltage
  – From 0 V to 0.1 V
  – Initially the output can’t change
    • There is capacitance at every node
  – The op-amp thinks it needs to create a huge output voltage
    • So it drives current into the output
    • Which charges the capacitor
    • Causing the output to increase
  – This then decreases the input difference
Feedback in an Op-amp Circuit

• As the output rises
  – The input difference decreases
  – So \( A^* \Delta V_{in} \) also decreases

• The system is stable when
  – \( A^* \Delta V_{in} \) is exactly equal to \( V_{out} \)

• If \( A \) is large (\( 10^6 \)) for any \( V_{out} \)
  – Say in the range of ± 10V
  – \( \Delta V_{in} \) will be very, very small
  – Can approximate that by saying \( \Delta V_{in} \) will be driven to 0
  – Output will be set so \( V_{in+} \approx V_{in-} \)
BUT

• This is only true if you connect the output feedback
  – To the negative terminal of the amplifier

• What happens if you connect it to the positive terminal?
The Two Golden Rules for circuits with ideal op-amps*

1. \( v_p = v_n \) (Ideal op-amp model).
   - No voltage difference between op-amp input terminals

2. \( i_p = i_n = 0 \) (Ideal op-amp model).
   - No current into op-amp inputs

* when used in negative feedback amplifiers
USEFUL OP AMPS CIRCUITS
Approach To Solve All Op-amp Circuits

• First check to make sure the feedback is negative
  – If not, STOP!

• Find the output voltage that makes the input difference 0
  – Assume $V_+ = V_-$
  – Find $V_{out}$ such that KCL holds

• We’ll do some examples
Non-inverting Amplifier

- $i_p = 0$ so $v_p = v_s$
- $V_+ = V_-$ so $v_n = v_p = v_s$

\[ i_1 = i_2 \quad \text{so} \quad \frac{v_o - v_s}{R_1} = \frac{v_s}{R_2} \]

\[ \therefore \quad \frac{v_o}{R_1} = v_s \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \]

\[ \therefore \quad v_o = v_s \left( \frac{R_1 + R_2}{R_2} \right) \]
Inverting Amplifier

At node $v_n$

$$\frac{v_n - v_s}{R_s} + \frac{v_n - v_o}{R_f} + i_n = 0$$

But $v_n = v_p = 0$ and $i_n = 0$, so

$$-\frac{v_s}{R_s} - \frac{v_o}{R_f} = 0 \text{ or } v_o = -v_s \frac{R_f}{R_s}$$

$$G = \frac{v_o}{v_s} = -\left(\frac{R_f}{R_s}\right).$$
Current-to-Voltage Converter

KCL at the $v_n$ node:

$$i_1 = i_s = i_2 = -\frac{v_o}{R_f} \quad \text{so} \quad v_o = -i_s R_f$$

- $i_p = i_n = 0$
- $v_n = v_p = 0$
- So $i_R = 0$ as well
OP AMP FILTERS
Adding Capacitors

• Suppose we add a capacitor in the feedback
• We can treat this exactly as we did the earlier circuits by using impedances.
• Our earlier analysis showed

\[ V_o = -V_s \frac{R_f}{R_s} \]

\[ Z_s = R_s \]

\[ Z_f = \frac{1}{\frac{1}{R_f} + j \cdot 2\pi F C_f} \]

\[ \therefore V_o = -V_s \frac{Z_f}{Z_s} = -\frac{\frac{1}{R_f} + j \cdot 2\pi F C_f}{R_s} = -V_s \frac{R_f}{R_s} \left( \frac{1}{1 + j \cdot 2\pi F R_f C_f} \right) \]
Sketching the Bode Plot

\[ \frac{V_o}{V_s} = -\frac{R_f}{R_s} \left( \frac{1}{1 + j \cdot 2\pi FR_fC_f} \right) \]

\[ R_s = 1 \text{ k}\Omega, \quad R_f = 100 \text{ k}\Omega, \quad C_f = 160 \text{ nF} \]

\[ F_c = \frac{1}{(2\pi R_fC_f)} = 10 \text{ Hz} \]
Learning Objectives

• Understand how living things use electricity

• Understand what an op amp is:
  – The inputs take no current
  – The output is $10^6$ times larger than the difference in input voltages

• The two Golden Rules of op amps in negative feedback
  – Input currents are 0; $V_{in-} = V_{in+}$

• Be able to use feedback to control the gain of the op amp
  – For inverting and non-inverting amplifiers

• Understand op amp filters and differential amplifiers
More Examples
Summing Amplifier

Output voltage is a scaled sum of the input voltages:

\[ i_1 + i_2 = i_3 \text{ so } \frac{v_1}{R_1} + \frac{v_2}{R_2} = -\frac{v_o}{R} \]

KCL at the summing point (or summing node):

- \( i_p = i_n = 0 \)
- \( v_n = v_p = 0 \)

Output voltage is a scaled sum of the input voltages:

\[ v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2\right) \]
A Subtracting (Difference) Amplifier?

• Take an inverting amplifier and put a 2\textsuperscript{nd} voltage on the other input?

\[ i_1 + i_2 = 0 \quad \text{so} \quad \frac{v_n - v_1}{R_s} + \frac{v_n - v_o}{R_f} = 0 \]

\[ v_n = v_2 \quad \text{so} \quad \frac{v_2 - v_1}{R_s} = \frac{v_o - v_2}{R_f} \]

\[ \therefore \frac{v_o}{R_f} = \frac{v_2 - v_1}{R_s} + \frac{v_2}{R_f} \]

\[ \therefore v_o = -v_1 \frac{R_f}{R_s} + v_2 \frac{R_f + R_s}{R_s} \]

• Not quite what we wanted. We’d like \( v_o \propto (v_1 - v_2) \).
Differential Amplifier 1.0

\[ \frac{v_1 - v_n}{R_1} = \frac{v_n - v_o}{R_2} \]

\[ \frac{v_1 - v_2}{R_3 + R_4} = \frac{v_2}{R_3 + R_4} - \frac{R_4}{R_1} \]

\[ v_o = \frac{-v_1 + R_4}{R_1} \left( \frac{R_4}{R_3 + R_4} + \frac{R_1}{R_2} \frac{R_4}{R_3 + R_4} \right) \]

But if \( R_3 = R_1 \) and \( R_4 = R_2 \)

\[ v_o = (v_2 - v_1) \frac{R_2}{R_1} \]