MS&E 315 and CME 304
Solar sailing between Earth and Mars

1 Overview

In this project, you will use your knowledge of nonlinear optimization to help a space agency design a cargo mission. The mission consists of making a cargo spacecraft perform interplanetary orbit transfers between Earth and Mars, and the agency is considering using a solar sail to propel the spacecraft. They need you to determine how the spacecraft should be operated to perform these orbit transfers in the shortest possible time for different solar sail technologies.

2 Mission information

2.1 Simplifications

The agency has determined that the following simplifications are acceptable for your analysis:

1. Orbits are heliocentric, circular and co-planar.
2. Gravitational effects due to celestial bodies other than the Sun are ignored.
3. The Sun and spacecraft are modeled as point masses.
4. The Sun has spherically symmetric gravitational and radiation fields.

2.2 Problem description

The problem you have to solve consists of determining how the solar sail of the spacecraft should be oriented to make the spacecraft perform each of the interplanetary orbit transfers Earth-Mars and Mars-Earth in the shortest possible time. You have to perform this analysis for some, say three, of the available solar sail technologies so that the agency can choose the most appropriate one in terms of cost and performance. Each solar sail technology is identified by a characteristic acceleration, as described in the next subsection. Figure 1 shows a diagram of the orbits of interest, where r_E and ω_E denote the radius and angular velocity, respectively, of Earth's orbit around the Sun, and r_M and ω_M denote the radius and angular velocity, respectively, of Mars's orbit around the Sun. You should note that for the spacecraft to be in an orbit around the Sun, its radial velocity must be zero and hence the spacecraft's trajectory for each of the orbit transfers must be characterized by a zero initial and final radial velocity.

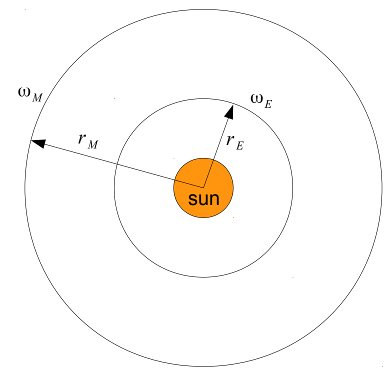

Figure 1: Orbits of interest.

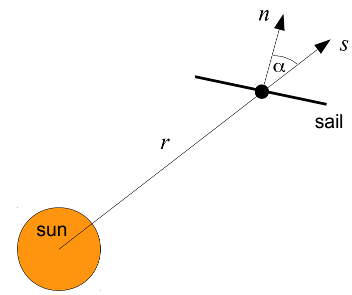

Figure 2: Solar sail model.

2.3 Spacecraft model

The spacecraft considered is propelled by a solar sail. A solar sail is a form of propulsion that is based on radiation pressure, or in other words, based on the push generated by photons reflecting off a surface. For this analysis, we assume that the solar sail is perfectly flat and reflective. Control of the spacecraft is achieved by changing the sail angle $\alpha \in [-\pi/2, \pi/2]$, which is the angle between the unit vector $\boldsymbol{s}$ pointing from the Sun to the spacecraft and the unit vector $\boldsymbol{n}$ pointing in the direction normal to the sail surface area, as shown in Figure 2.

The force exerted on the spacecraft due to photons reflecting off the sail is modeled as

$$\boldsymbol{f}_\gamma = ma\left(\frac{r_0}{r}\right)^2 \cos^2(\alpha)\boldsymbol{n},$$

where m is the spacecraft's mass, r_0 is one astronomical unit, r is the distance from the Sun to the spacecraft and a is the characteristic acceleration of the solar sail technology. Available solar sail technologies have $a \in [1, 2]$ mm/s^2.

2.4 Equations of motion

The equation that describes the spacecraft's motion subject to the Sun's gravitational pull and the force due to the solar sail is

$$m\ddot{\boldsymbol{r}} = -\frac{\mu m}{r^2}\boldsymbol{s} + \boldsymbol{f}_\gamma,$$

where $\boldsymbol{r}$ is the position vector of the spacecraft with respect to the Sun and μ is the heliocentric gravitational constant.

2.5 Constants

The constants that you need to perform the analysis are listed in the following table:

Name	Description	Value	Units
r_E	Radius of Earth's orbit around the Sun.	1.496×10^{11}	m
ω_E	Angular velocity for Earth's orbit around the Sun.	1.991×10^{-7}	rad/s
r_M	Radius of Mars's orbit around the Sun.	2.279×10^{11}	m
ω_M	Angular velocity for Mars's orbit around the Sun.	1.059×10^{-7}	rad/s
r_0	Astronomical unit distance.	1.496×10^{11}	m
μ	Heliocentric gravitational constant.	1.327×10^{20}	m^3/s^2

3 Report

You are required to write a report in LaTeX describing how you solved the interplanetary orbit transfer problems and the results you obtained. In particular, your report must include all of the following:

1. A description of how you formulated and solved the problem.
2. A description of the results you obtained for at least three different solar sail technologies, *i.e.*, for three different characteristic accelerations $a \in [1, 2]$ mm/s^2, including the minimum times and diagrams showing the spacecraft's trajectories, control (sail angle) and state (distance from the Sun, angular velocity and radial velocity) as a function of time.
3. A justification of why you are confident that your solver works correctly.
4. A bibliography including any reference consulted.

You are not required to submit code and may use any programming language you wish.

4 Hints

Here are some hints that may help you perform the required analysis successfully:

1. You can minimize time in two ways. The simplest way is to treat the total time as a parameter of the problem and solve a sequence of problems for decreasing values of this parameter. The other is to treat total time as an optimization variable and include it in the objective function.
2. You should choose appropriate units to avoid numerical issues.
3. You should identify clearly what the boundary conditions are for the problem.
4. You can construct starting points for your optimization algorithm using simple trajectories.
5. You can start with a coarse time resolution and refine it to obtain more accurate solutions.
6. You do not need to know the mass of the spacecraft. The effects of this are already taken into account in the characteristic acceleration of the solar sail.
7. You should make your solver print informative output during every iteration.
8. You should test your solver and its parts before trying it on an orbit transfer problem. For example, first test the line search and the solver using simple problems for which you know the answer.

A movie showing a sample trajectory and control of a spacecraft with solar sail of characteristic acceleration $a = 2$ mm/s^2 can be found here.