2009 CME304/MS&E315 Class Project

Linesearch routine. The first step is to design, implement, and test a linesearch routine. Your routine may use just the function's value or both its value and its derivative. In either case, the first of two termination criteria is

$$F(x_k) - F(x_k + \alpha p_k) \geq -\mu\alpha g_k^T p_k.$$

If the function's derivative is used, then the second criterion is

$$|g(x_k + \alpha p_k)^T p_k| \leq -\eta g_k^T p_k;$$

if only the function value is used,

$$\frac{|F(x_k + \alpha p_k) - F(x_k + \nu p_k)|}{\alpha - \nu} \leq -\eta g_k^T p_k,$$

where $0 \leq \nu < \alpha$. To assure that an acceptable point always exists, choose $0 < \mu \leq 1/2$, $0 \leq \eta < 1$, and $\mu \leq \eta$.

Multivariate optimization routine. You will design a routine to solve the bound-constrained optimization problem

$$\min_x F(x)$$
$$\text{subject to } l \leq x \leq u.$$

You may use any algorithm you wish.

The minimal surface problem. The problem is to find an approximation to the function s having minimal surface area on the domain Ω that obeys $s|_{\partial\Omega} = h$ on the boundary $\partial\Omega$.

Please show results for the following specification:

- Ω is $[0,1] \times [0,1]$: the unit square with southwest corner $(0,0)$.
- The boundary function is

$$h(x,y) = \begin{cases} -1 + 2x^2 & \text{on the south edge,} \\ \cos(\pi y) & \text{on the east edge,} \\ -8\left(x - \frac{1}{2}\right)^3 & \text{on the north edge,} \\ -1 + 2y & \text{on the west edge.} \end{cases}.$$

Next, consider a domain $\Omega_1 \subset \Omega$ on which a function g is defined. Find an approximation to the function s having minimal surface area that obeys $s \geq g$ on Ω_1.

Please show results for the following specification:

- Ω_1 is $[1/4, 3/4] \times [1/4, 3/4]$.
- $g = 0$.

Additionally, show the difference between the solutions for the unconstrained and constrained problems.

You are welcome to create more interesting domains, h and g functions, and other extensions.

Report. Your report should describe the following:

- each algorithm;
- how you verified your software is working properly;
- how you verified your solution is correct;
- clear, neat, and informative output;
- extensions you explored.

You should not include your source code unless an excerpt improves the clarity of your report. Please include a bibliography.

Comments. You may program in any language you wish.

Though the linesearch routine will be used in the multivariate optimization routine, it may be best to test it on 1D problems first. When solving a 1D problem, it may be helpful to choose η to be very small so that the routine terminates at a minimizer. However, when using the linesearch routine in your multivariate optimization routine, it is best to use loose tolerances. Generally, one desires that the unit step be accepted as the solution is approached.

When developing software, it is helpful to develop and then test small pieces at a time. Similarly, when solving a complicated problem, it is best to solve and verify the solutions to simpler ones first.

You may need to do some background reading to learn how to discretize continuous 2D problems. Many approaches exist; you are free to create your own.

You may not collaborate with anyone.