Problem set 1, CME325 winter 2008,

Hand in no later than Wednesday Febuari 6

1. Consider the periodic IVP for the backward heat eqution ($u_t=-u_{xx}$) with initial data u(x,0)=sin(x). What is the solution? Show that the problem is ill-posed (not well-posed). Why does this make it difficult to solve numerically?
2. Derive an energy estimate for

$$u_t + (2 + \sin(x))u_x = F(x,t), 0 \le x \le 1, t \ge 0,$$

$$u(0,t)=g(t),$$

$$u(x,0) = f(x)$$

   Is the problem wellposed? You may assume that a solution can be constructed for all reasonable data. Is it time-stable for F=0, g=0?
3. Consider the semi-discretization

$$\frac{du_j}{dt} = Qu_j + F_j, j = 1,..N-1$$

$$Qu_j = -(2+\sin(x_j))\frac{u_{j+1}-u_{j-1}}{2h}, j=1,..N-2$$

$$Qu_{N-1} = -(2 + \sin(x_{N-1}))\frac{u_{N-1}-u_{N-2}}{h}$$

   Here $x_j=jh$, $h=1/N$, and $u_j(t)\approx u(x_j,t)$.
   Show stability.
   Is the approximation strictly stable?