Problem set 2, CME325 winter 2008,

Hand in no later than Wednesday Febuari 13

1. Consider the 2π –periodic system

$$\begin{pmatrix} u \\ v \end{pmatrix}_t + \begin{pmatrix} 0 & 1+0.2\sin(x) \\ 1 & 0 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}_x = 0$$

$$u(x,0) = e^{-8(x-\pi)^2} \sin(5x)$$

$$v(x,0) = 0$$

Formulate a fully discrete problem using the standard 4th order central scheme in space and the 4th order Runge-Kutta in time. Determine a stability condition for the time step k in terms of the space step h using frozen coefficient technique.

2. Write a program that computes the solution until $t=8$. Is the stability condition valid? Make a plot that demonstrate the temporal evolution of u and v in a stable case.

3. Measure the rate of convergence at $t=8$ by considering pointwise differences and differences in norm, when
   (a) k is fixed and h goes to zero
   (b) h is fixed and k goes to zero
   (c) both k and h go to zero with k/h fixed

4. In real computations k/h is usully kept fixed close to the stability limit. Is the error dominated by the spatial or the temporal error?