Problem set 3, CME325 winter 2008,
Hand in no later than Friday Febuari 22

1. Consider the 2π –periodic system

$$\begin{pmatrix} u \\ v \end{pmatrix}_t + \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix}_x = 0, u(x,0) = e^{-8(x-\pi)^2}, v(x,0) = 0$$

   Use the standard 4th order central scheme in space and the 4th order Runge-Kutta in time. Run until t=12, using N gridpoints so that you clearly see dispersive errors at t=12. (You choose N) Make a plot. Change to the 6th order central scheme and run again until t=12 using the N points in space and the same time stepping. Add solution to the plot.

2. Derive formulas for wave speeds (as a function of $\xi=\omega h$) for the two methods above applied to the scalar advection equation and plot together with the exact wave speed.

3. Change the grid by introducing

$$x'(x) = \begin{cases} \frac{\pi}{1+\pi} x \text{ if } 0 \le x < 2 \\ \frac{\pi}{1+\pi}(x-2) \text{ if } 2 \le x < 4 \\ \frac{\pi}{1+\pi}(x+2) \text{ if } 4 \le x < 2\pi \end{cases}$$

   What gridpoints in the old coordinate x does 100 equidistant gridpoints in x' coorespond to? Formulate the transformed problem, and solve using the 4th and the 6th order methods with N gridpoints (the same number as above) until t=12. Plot the solutions as a function of x (the original coordinate). Compare with the solutions on the uniform grid. Measure the order of accuracy for both 6th order and 4th order. Use one of the measures involving space accuracy from hw2. Also, plot the solution at suitable times, and comment on numerical artefacts.

4. Bonus question: Make a smoother grid transformation and measure the accuracy.