Problem set 4, CME325 winter 2008,

Hand in no later than Friday Febuari 29

1. Consider solving
   $u_{xx}=sin(x)+\delta(x-\pi/4),\ 0\le x\le 1,\ u(0)=1, u(1)=0$
   on an equidistant grid by adding the appropriate corrections for the singularity. Use the standard second order central approximtion for the second derivative. An equivalent formulation without the singular forcing term is $u_{xx}=sin(x),\ 0\le x\le \pi/4,\ u(0)=1,$
   $v_{xx}=sin(x),\ 0\le x\le \pi/4, v(1)=0,$
   $-u_x(\pi/4)+v_x(\pi/4)=1.$
   Write a matlab program and solve on different grids. Plot the solution and check the convergence rate.
2. Consider solving the inital value problem for the heat equation in one space dimension, with an explicit finite volume method. Assume the grid consists of gridcells of length h except one of length αh. The method is
   $$u_i^{n+1} = u_i^n - \frac{k}{h_i}(F^n_{i+1/2} - F^n_{i-1/2}),$$
   $$F^n_{i+1/2} = 2\frac{u_i^n - u_{i+1}^n}{h_i + h_{i+1}}$$
   Away from the small grid cell we recover the standard formula and expect a time step limit $k\le h^2/2$. The validity of this bound when one cell is smaller can be checked by normal mode analysis. Derive the determinant in the Godunov-Ryabenkii condition in the case $k=h^2/2$.
   Hint1: use the condition $||u^n||_h$ bounded.
   Hint2: there are 2 solutions of $(z-1)\kappa=0.5(\kappa-1)^2$, $\kappa_{1,2}(z)$. Precisely one has absolut value <1 for all z with $|z|>1$.
3. Bonus question: Is the Godunov-Ryabenkii condition satisfied?