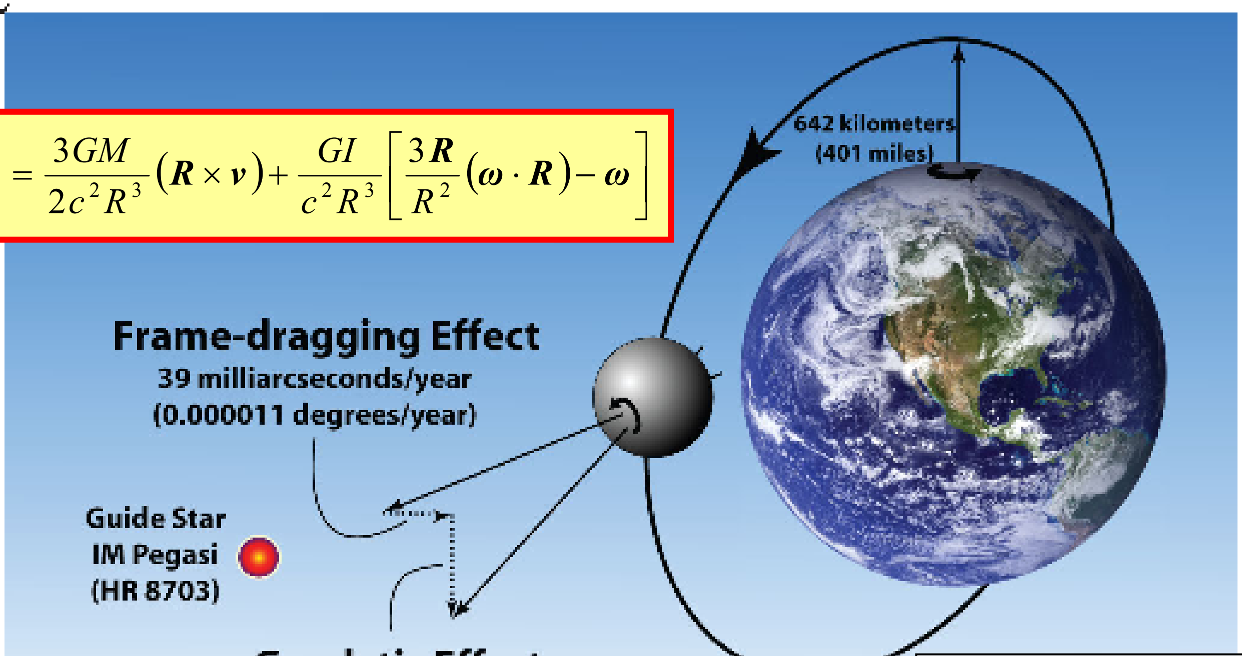
GP-B Experiment and Data Analysis Challenges

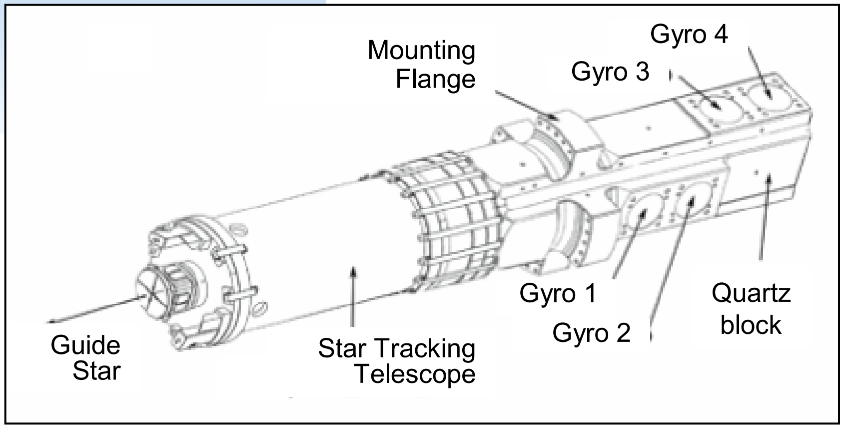
Michael Heifetz

The Relativity Mission Concept

$$\boldsymbol{\Omega} = \frac{3GM}{2c^2R^3}(\mathbf{R}\times\mathbf{v}) + \frac{GI}{c^2R^3}\left[\frac{3\mathbf{R}}{R^2}(\boldsymbol{\omega}\cdot\mathbf{R}) - \boldsymbol{\omega}\right]$$



Leonard Schiff



- **Geodetic Effect**
  - **Space-time curvature ("the missing inch")**
- **Frame-dragging Effect**
  - **Rotating matter drags space-time ("space-time as a viscous fluid")**

Brief History of Gravity Probe B

1957	**Sputnik – Dawn of the space age**
1958	**Stanford Aero-Astro Department created**
1959	**L. Schiff conceives of orbiting gyro experiment as a test of General Relativity**
1961	**L. Schiff & W. Fairbank propose gyro experiment to NASA**
1972	**1st drag-free spacecraft: TRIAD/DISCOS**
1975	**SQUID readout system developed**
1980	**Rotor machining techniques perfected**
1998	**Science instrument assembled**
2002	**Spacecraft & payload integrated**
2004	**Launch and vehicle operations**
2005	**End of data collection** **Start of Data Analysis**
2007	**Preliminary results presented at April APS meeting**

2008 -2009 **Final results**

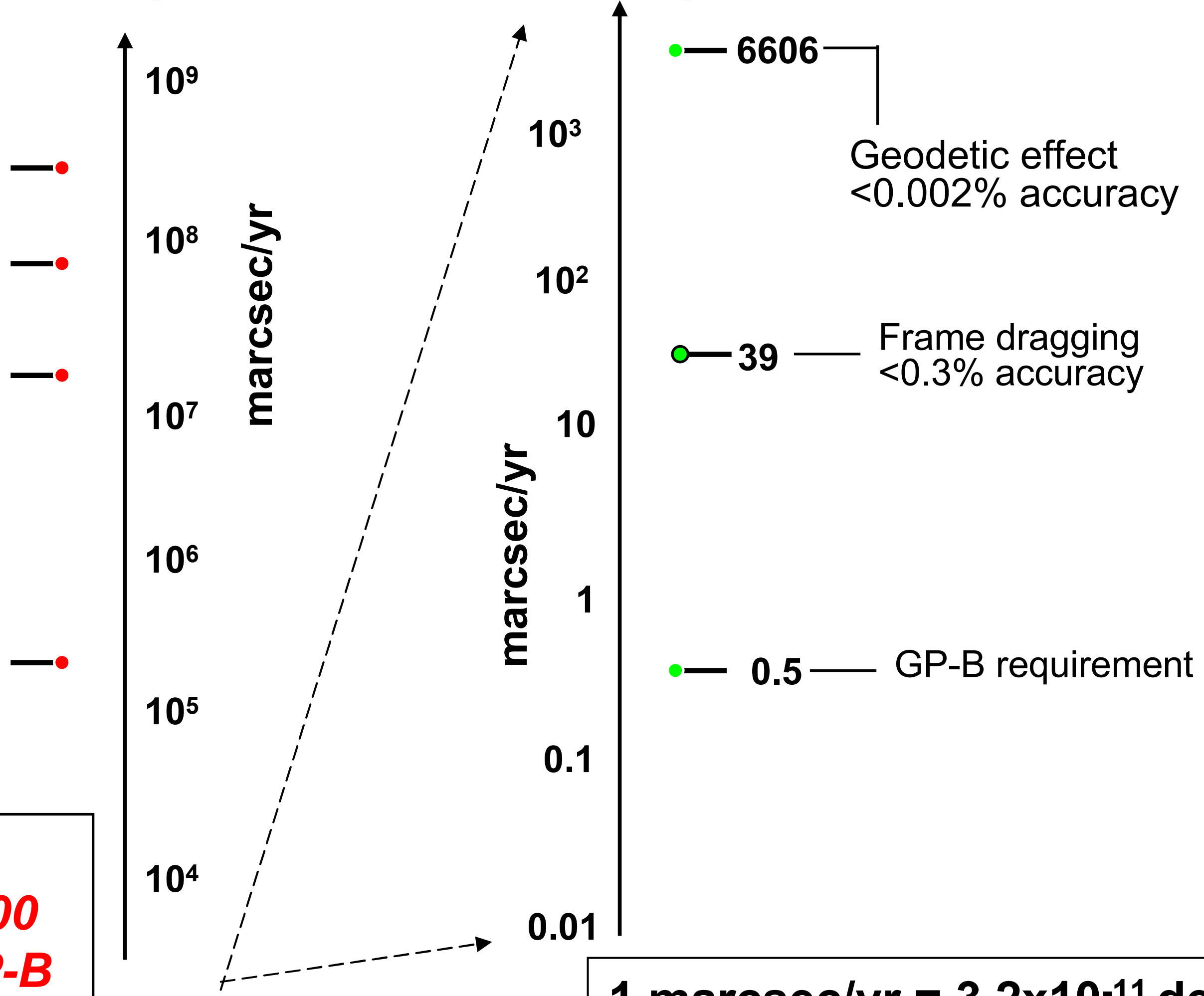
Why a Space-Based Experiment?

Best mechanical gyros on Earth
(10^{-2} deg/hr)

Spacecraft gyros
(3x10^{-3} deg/hr)

Best laser gyros
(1x10^{-3} deg/hr)

Electrostatically suspended gyroscope (ESG) on Earth with torque modeling
(10^{-5} deg/hr)



Geodetic effect
<0.002% accuracy

Frame dragging
<0.3% accuracy

GP-B requirement

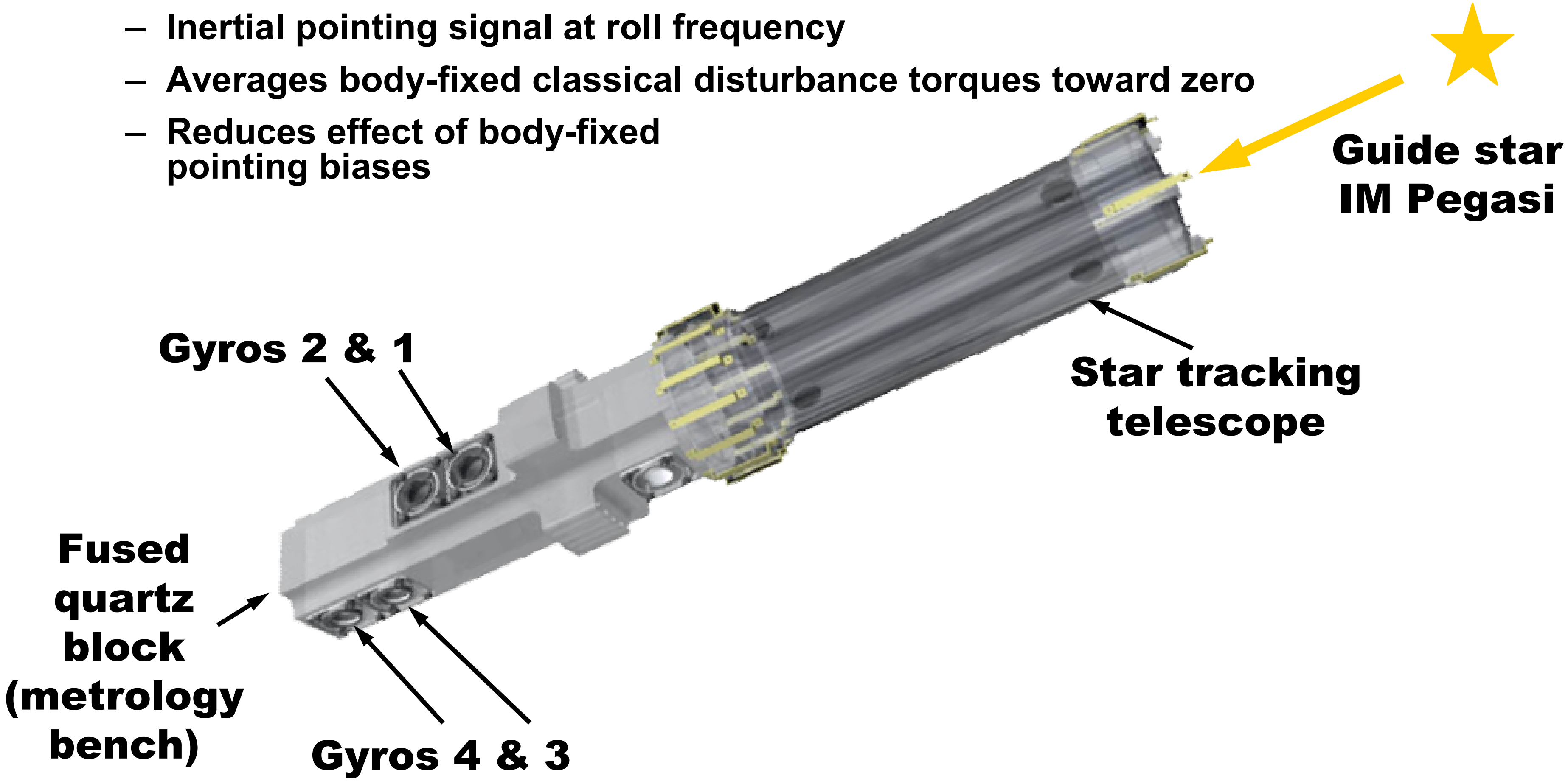
Best terrestrial gyroscopes 1,000,000 times worse than GP-B

1 marcsec/yr = 3.2x10^{-11} deg/hr

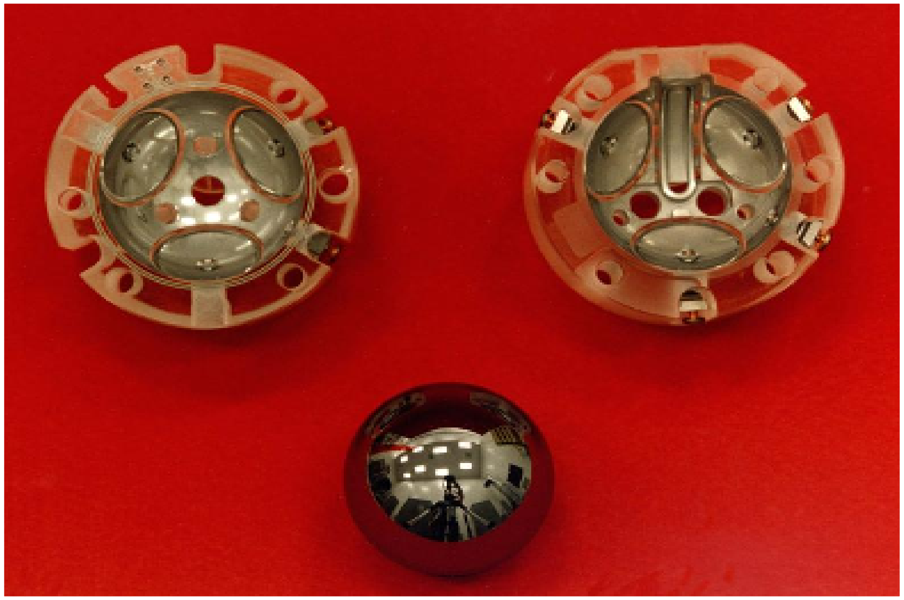
Operation in 1g environment degrades mechanical gyro performance
Laser gyroscopes and other technologies fidelity too low for GP-B

GP-B Instrument Concept

- Operates at ~ 2 K with liquid He
- Rolls about line of sight to Guide Star
  - Inertial pointing signal at roll frequency
  - Averages body-fixed classical disturbance torques toward zero
  - Reduces effect of body-fixed pointing biases



The GP-B Science Gyroscope



- ★ **Material: Fused quartz, homogeneous to a few parts in 10^7**
- ★ **Overcoated with Niobium.**
- ★ **Diameter: 38 mm.**
- ★ **Electrostatically suspended.**
- ★ **Spherical to 10 nm – minimizes suspension torques.**
- ★ **Mass unbalance: <10 nm – minimizes forcing torques.**
- ★ **All four units operational on orbit.**

> ***"Everything should be made as simple as possible, but not simpler."***
> **– A. Einstein**

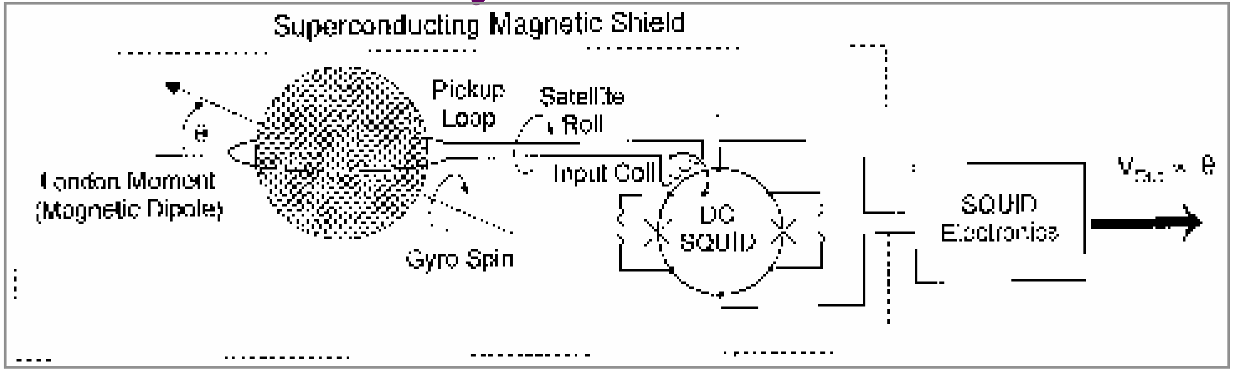
Demonstrated performance:

- Spin speed: 60 – 80 Hz.
- <1 μHz/hr spin-down.

If a GP-B rotor was scaled to the size of the Earth, the largest peak-to-valley elevation change would be only 2 meters!

Readout and Gyro Scale Factor

How to measure the spin direction of a perfect spinning sphere?

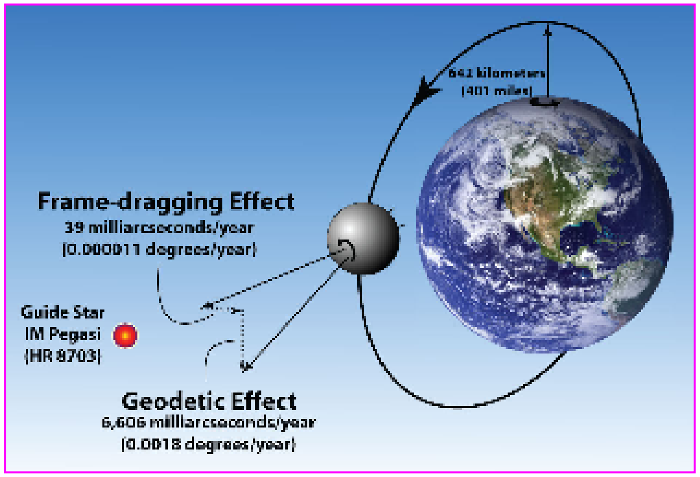


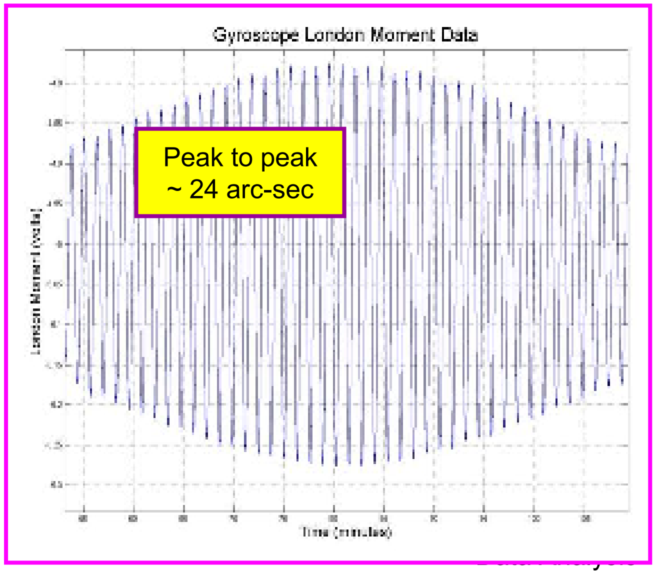
Scale Factor Calibration using guide star light aberration

➡ **allows conversion of measurement (volts) to angle (arcsec)**

Aberration: Vehicle motion causes star's apparent position to vary

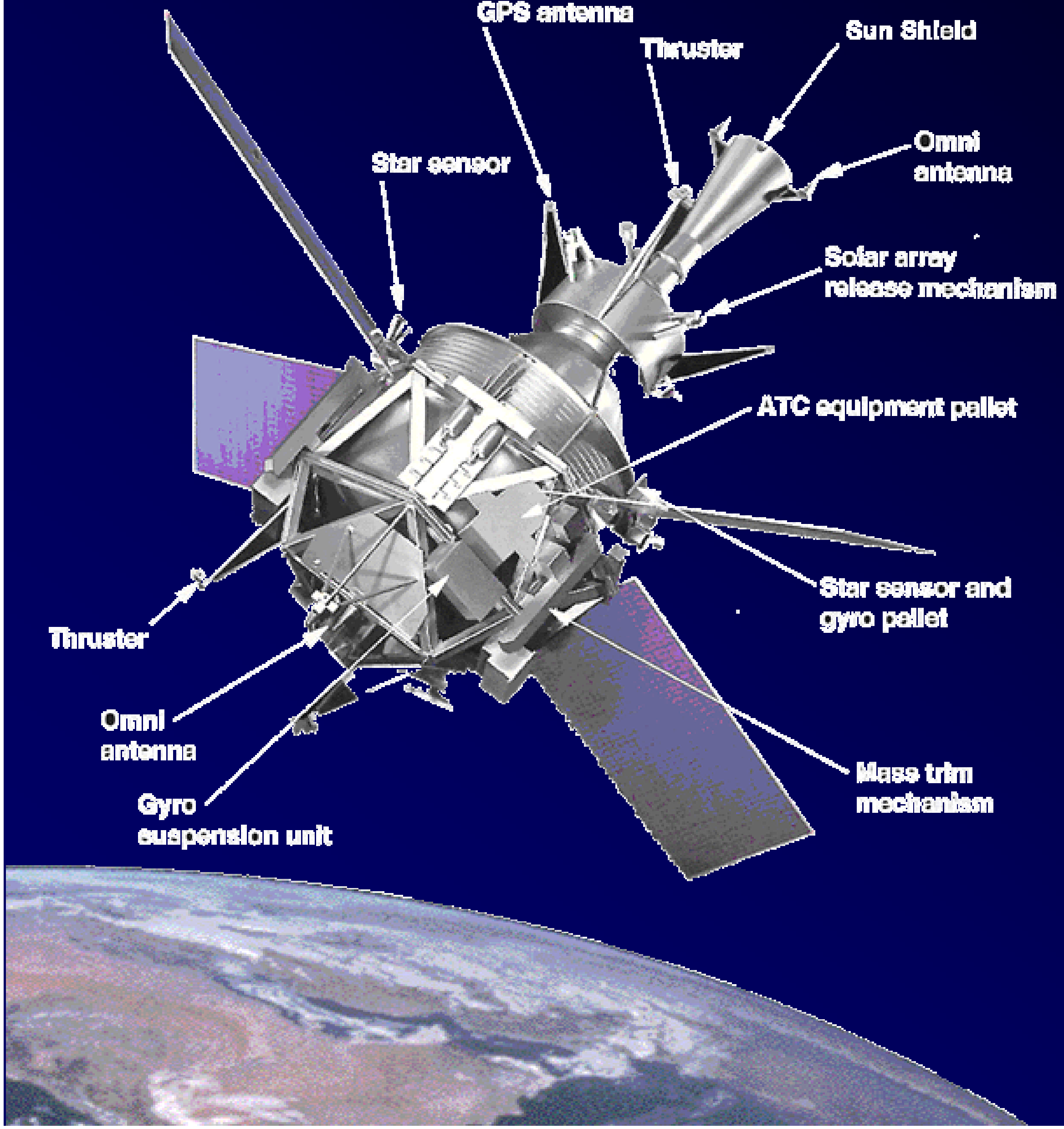
S/V around Earth -- 5.1856 arc-s @ 97.5-min period





Peak to peak
~ 24 arc-sec

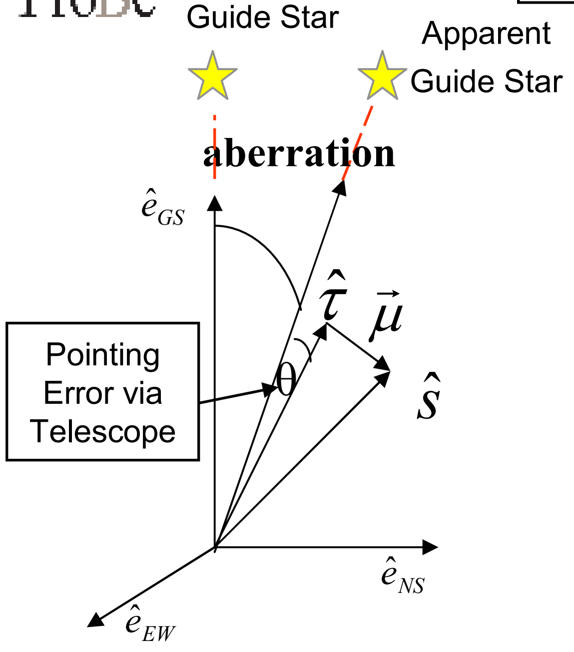
GP-B Spacecraft



- ★ Redundant spacecraft processors, transponders.
- ★ 16 Helium gas thrusters, 0-10 mN ea, for fine 6 DOF control.
- ★ Roll star sensors for fine pointing.
- ★ Magnetometers for coarse attitude determination.
- ★ Tertiary sun sensors for very coarse attitude determination.
- ★ Magnetic torque rods for coarse orientation control.
- ★ Mass trim to tune moments of inertia.
- ★ Dual transponders for TDRSS and ground station communications.
- ★ Stanford-modified GPS receiver for precise orbit information.
- ★ 70 A-Hr batteries, solar arrays operating perfectly.

6.4 m
3240 kg

‘Simple’ GP-B Data Analysis



SQUID Readout Data

Telescope Data, Orbital and Annual Aberrations

Roll Phase Data

$$Z_{SQUID}(t) = C_g[(\tau_{NS} - s_{NS})\cos(\Phi_r + \delta\varphi) + (\tau_{EW} - s_{EW})\sin(\Phi_r + \delta\varphi)] + bias + noise$$

Scale Factor

$\hat{s}$ - **gyro spin axis orientation**

$\hat{\tau}$ - **vehicle roll axis orientation**

$\vec{\mu}$ - **gyroscope misalignment**

$C_g, \delta\varphi$ - calibrated based on orbital and annual aberration

Surprise A: C_g variations

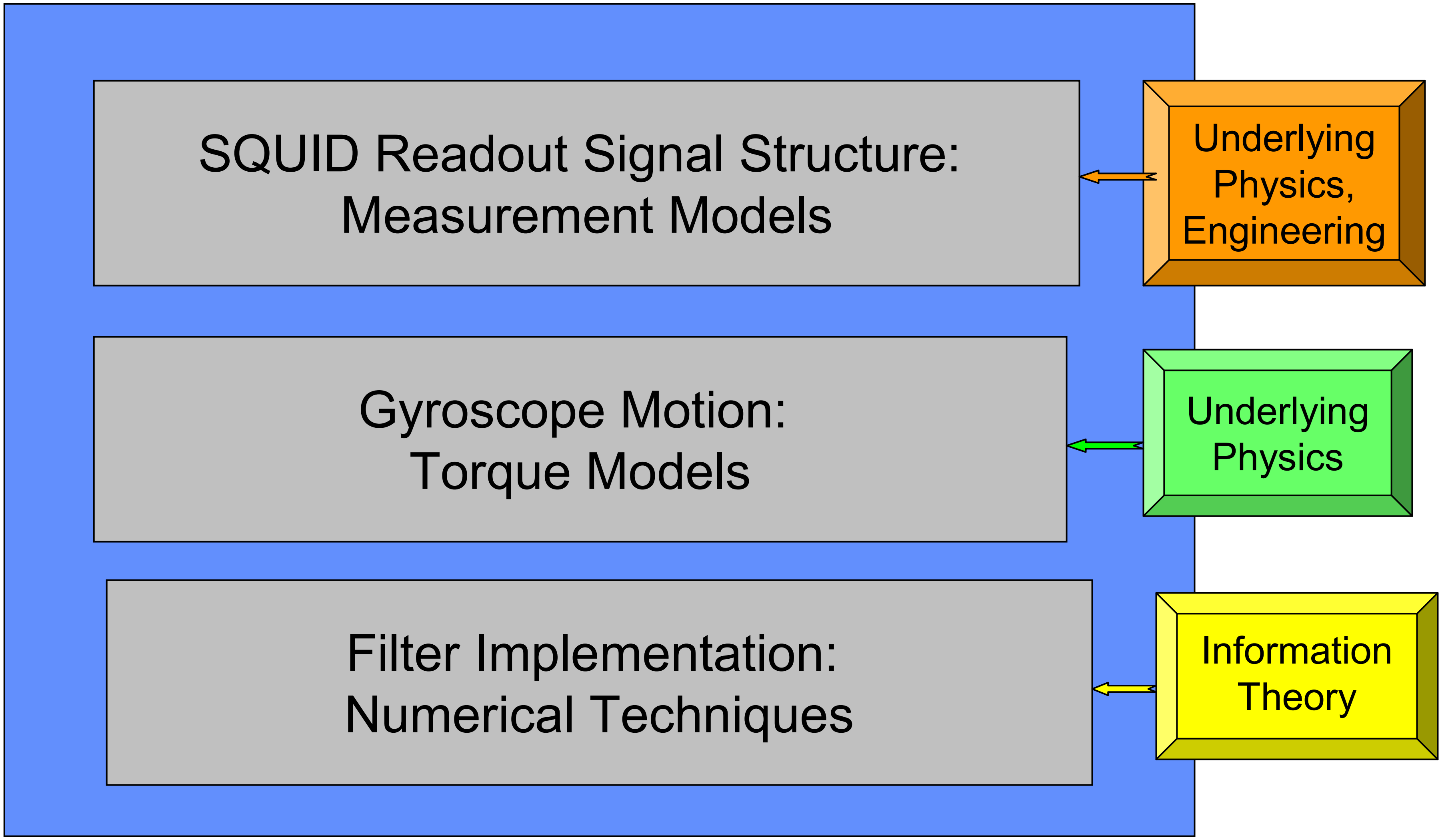
Gyro orientation trajectory $s_{NS}(t)$ and $s_{EW}(t)$ - straight lines

Surprise B: Patch Effect Torque

Relativity: slopes of $s_{NS}(t)$ (Geodetic) and $s_{EW}(t)$ (Frame- dragging)

(significantly more complex problem)

Three Cornerstones of Estimation (Filtering) Method



SQUID Readout Signal Structure:
Measurement Models

Underlying
Physics,
Engineering

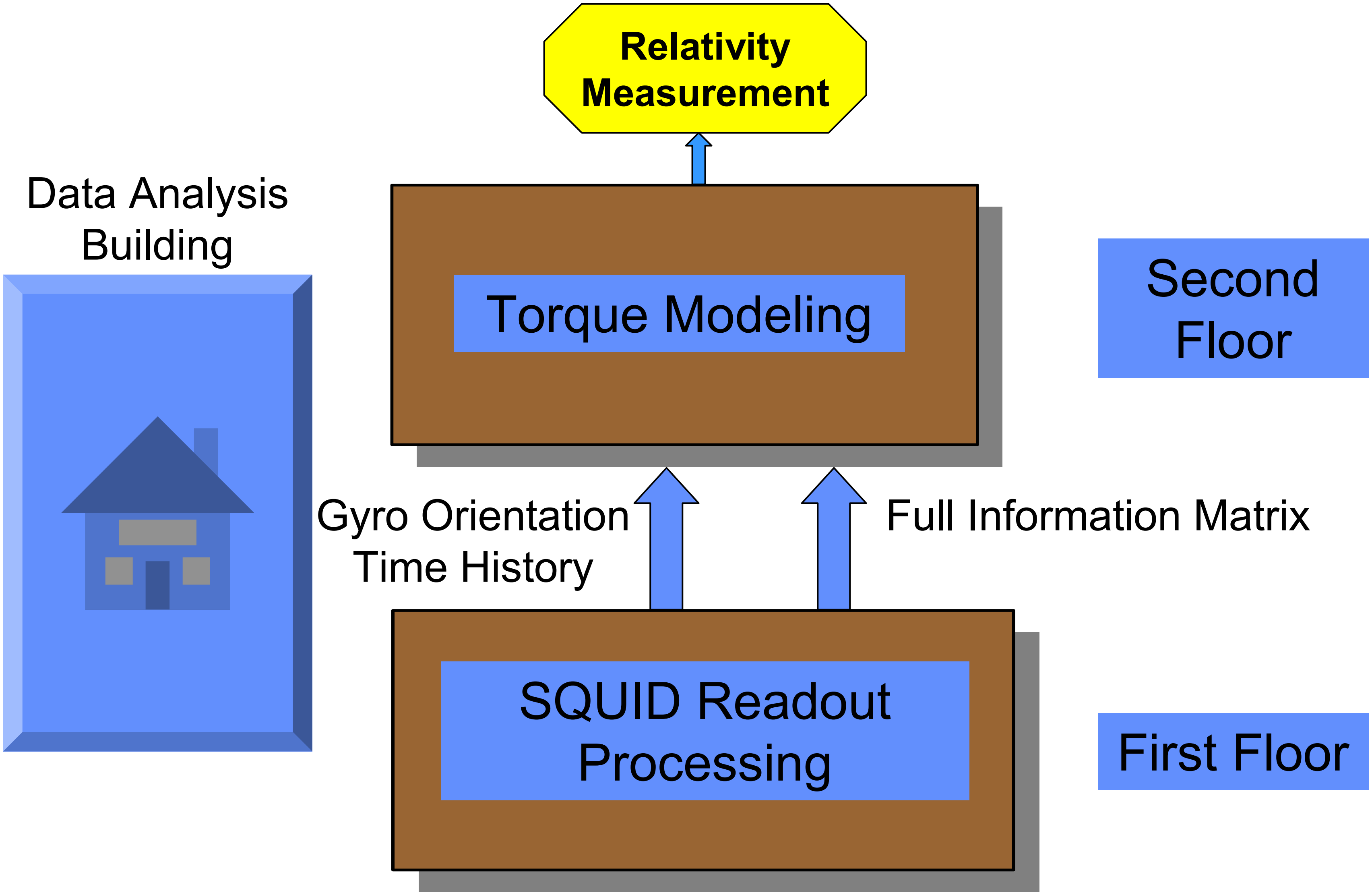
Gyroscope Motion:
Torque Models

Underlying
Physics

Filter Implementation:
Numerical Techniques

Information
Theory

Data Analysis Structure: ‘Two-Floor’ Processing



Relativity Measurement

Data Analysis Building

Torque Modeling

Second Floor

Gyro Orientation Time History

Full Information Matrix

SQUID Readout Processing

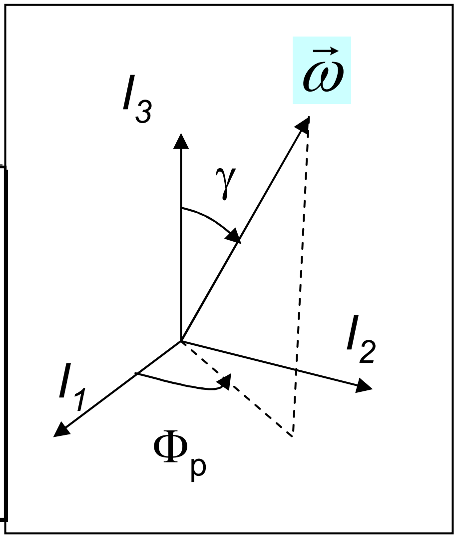
First Floor

Readout Scale Factor: γC_g Model

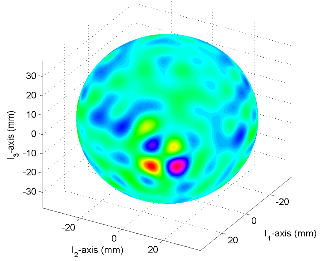
Harmonic expansion in polhode phase with coefficients that depend on polhode angle

$$C_g(t) = C_{g0}\left\{1 + \sum_{n=0}^{N}\left[a_n(\gamma)\cos(n\,\Phi_p(t)) + b_n(\gamma)\sin(n\,\Phi_p(t))\right]\right\},$$

$$a_n = \varepsilon^n \sum_{k=0}^{K} a_{nk}\,\varepsilon^{2k},\quad b_n = \varepsilon^n \sum_{k=0}^{K} b_{nk}\,\varepsilon^{2k},\qquad \varepsilon(t) = \tan(\gamma/2).$$



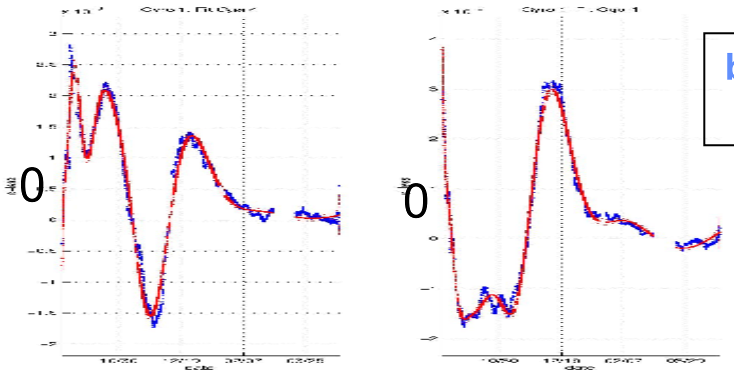
Gyro principle axes of inertia and instant spin axis position



Trapped Flux Mapping (TFM)

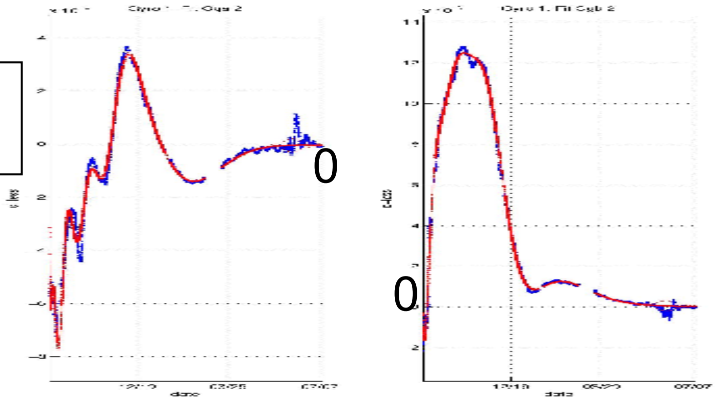
Φ_p - Polhode phase

γ - Polhode angle

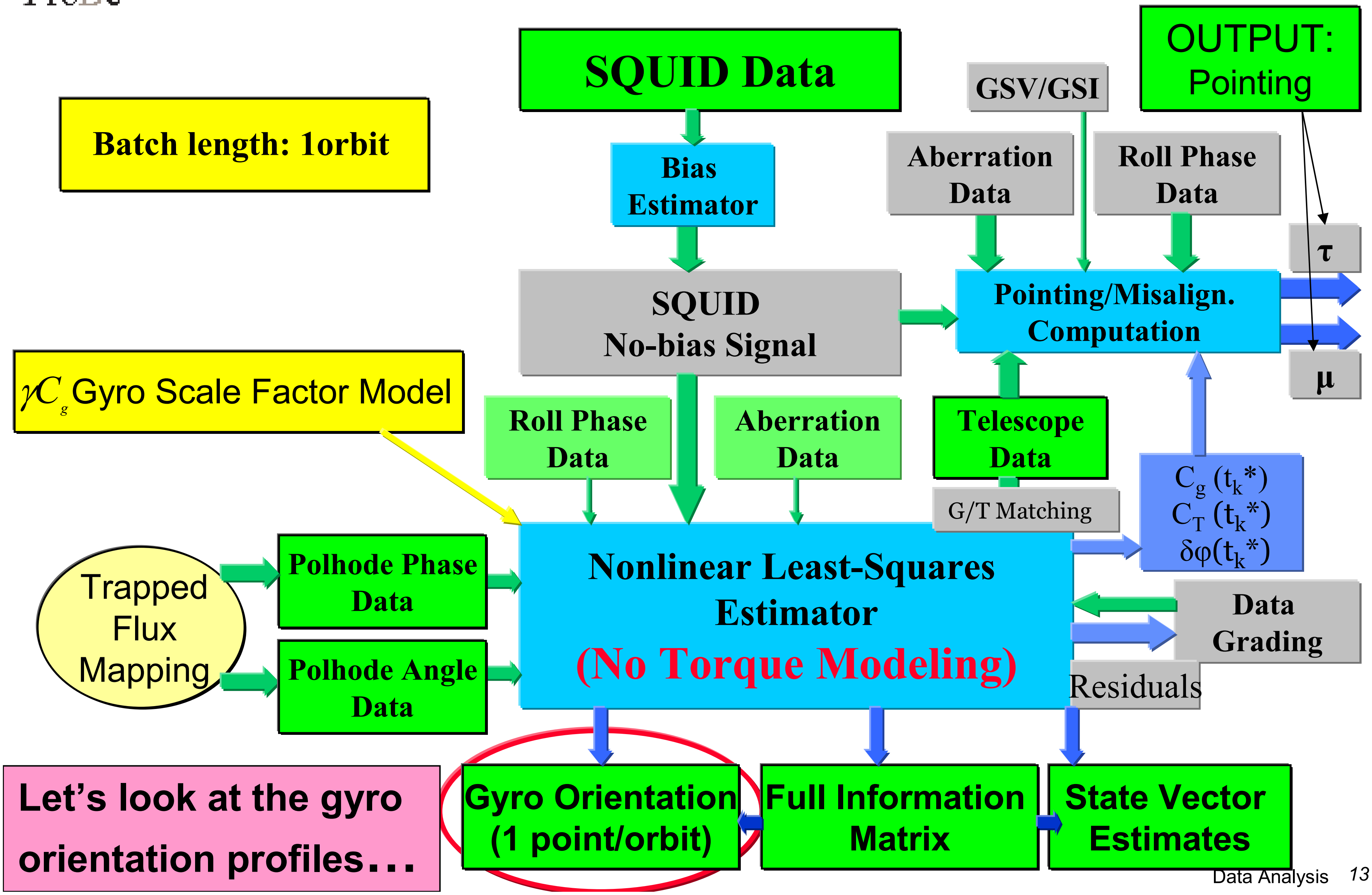


blue – $a_n(t)$ and $b_n(t)$

red - fit to $\varepsilon(t)$



First Floor: SQUID Readout Data Processing



Batch length: 1orbit

γC_g Gyro Scale Factor Model

Trapped Flux Mapping

SQUID Data

Bias Estimator

SQUID No-bias Signal

GSV/GSI

Aberration Data

Roll Phase Data

Pointing/Misalign. Computation

OUTPUT: Pointing

τ

μ

Roll Phase Data

Aberration Data

Telescope Data

G/T Matching

C_g (t_k^*)
C_T (t_k^*)
$\delta\varphi(t_k^*)$

Polhode Phase Data

Polhode Angle Data

Nonlinear Least-Squares Estimator
(No Torque Modeling)

Data Grading

Residuals

Gyro Orientation (1 point/orbit)

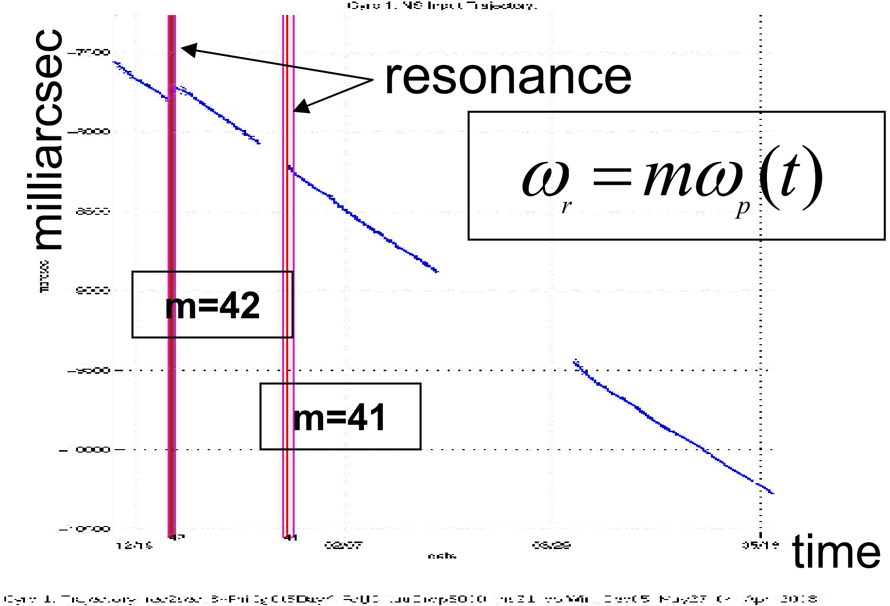
Full Information Matrix

State Vector Estimates

Let's look at the gyro orientation profiles…

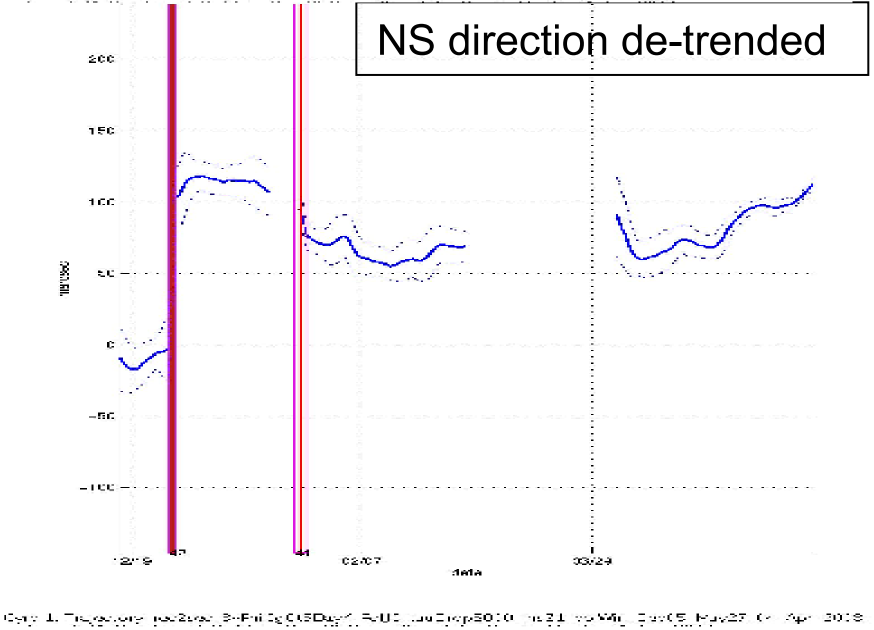
Inertial Orientation Time-history: Gyro 1

NS Direction

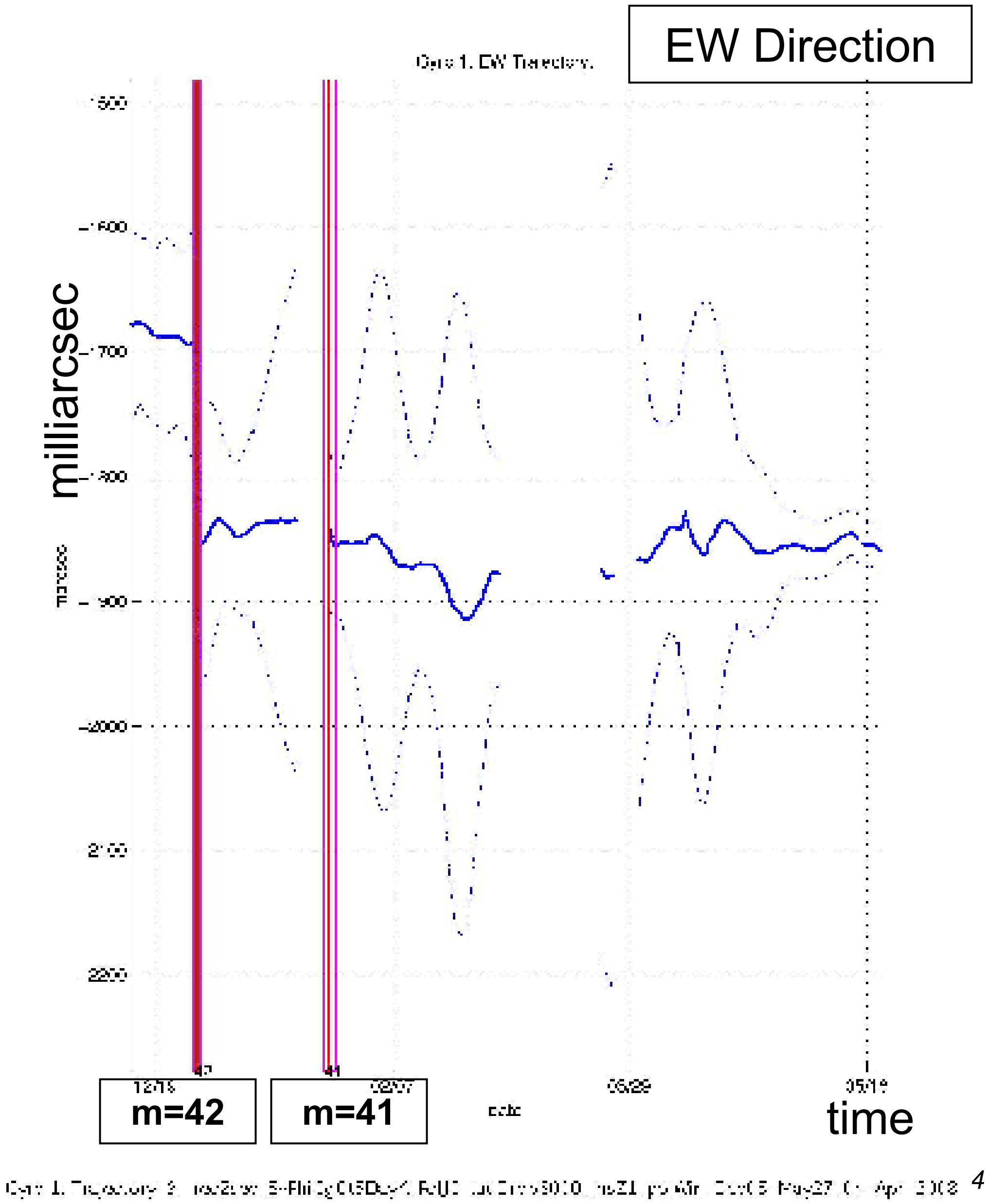


$$\omega_r = m\omega_p(t)$$

NS direction de-trended



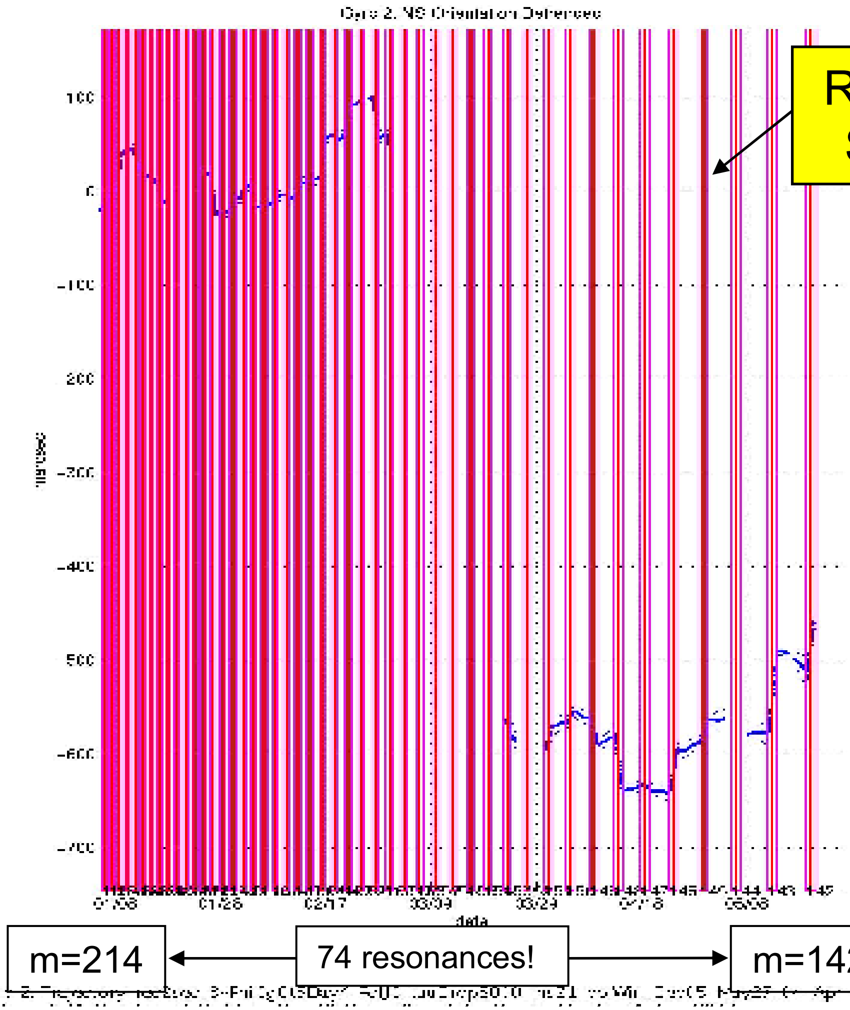
EW Direction

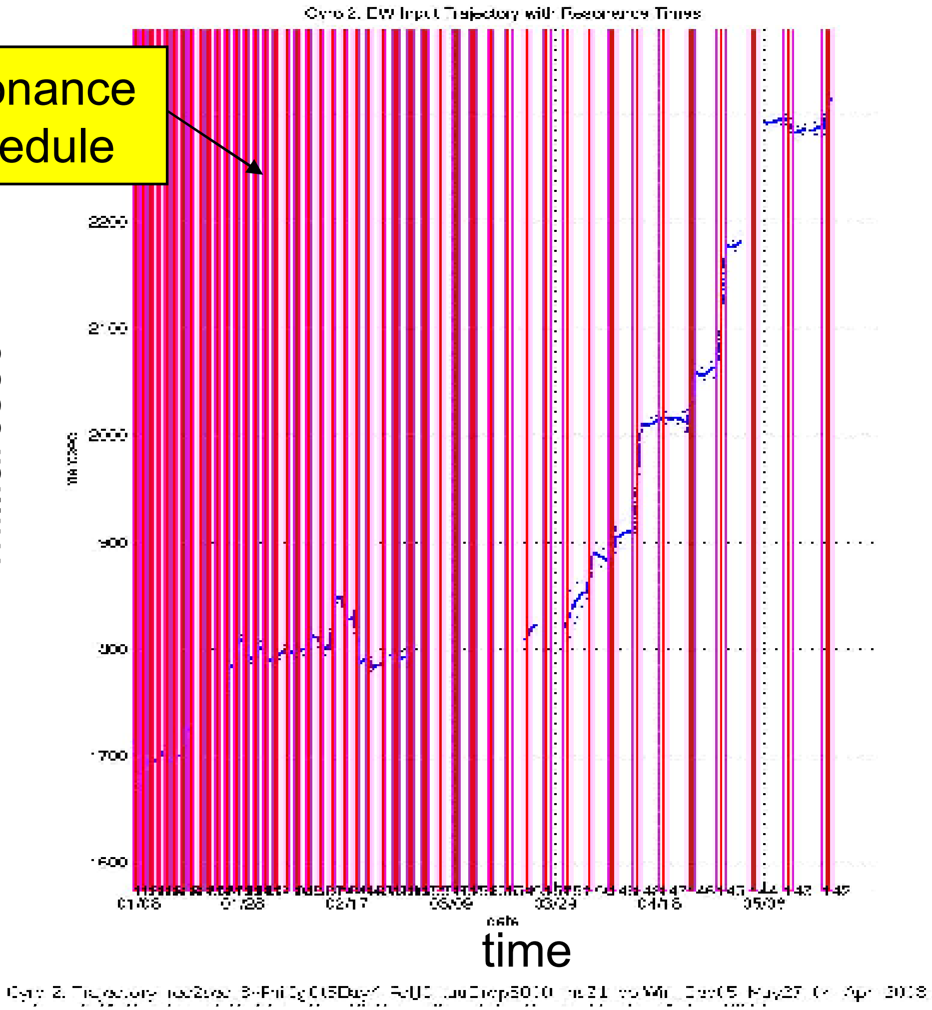


Inertial Orientation Time-history: Gyro 2

NS direction de-trended

EW Direction





Resonance Schedule

milliarcsec

time

m=214

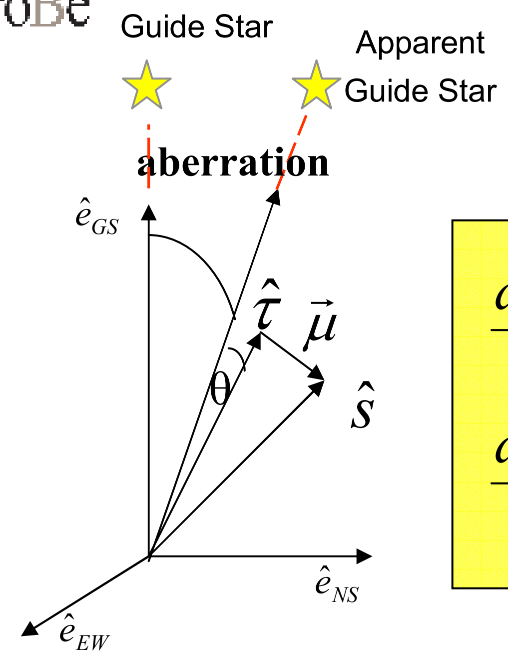
74 resonances!

m=142

Resonances:

$$\omega_{roll} = m\omega_p(t)$$

Torque Modeling



$\hat{s}$ - **gyro spin axis orientation**

$\hat{\tau}$ - **vehicle roll axis orientation**

$\vec{\mu}=\hat{\tau}-\hat{s}$ - **gyroscope misalignment**

$$\vec{R}_{MisTorq}(t) \perp \vec{\mu}$$

relativity **Misalignment torque** **Roll-Resonance torque**

$$\frac{ds_{NS}}{dt} = r_{NS} + k(t)(\tau_{EW} - s_{EW}) + [c^{-}(t)\cos(\theta+\Phi_r) - c^{+}(t)\sin(\theta+\Phi_r)]$$

$$\frac{ds_{EW}}{dt} = r_{EW} - k(t)(\tau_{NS} - s_{NS}) + [c^{-}(t)\sin(\theta+\Phi_r) + c^{+}(t)\cos(\theta+\Phi_r)]$$

$k(t)$, $c^{+}(t)$, $c^{-}(t)$ **are modulated by harmonics of polhode frequency – roll/polhode resonance:**

$$\boxed{\omega_{roll} = m\omega_p(t)}$$

Torque Coefficients: Polhode Variation

Roll-resonance torque coefficients c^+, c^-:

$$c^{\pm}(t) = c_{10}^{\pm}(\gamma_0) + \sum_{m=1}^{M_c} (c_{1m}^{\pm} \cos m\Phi_p + c_{2m}^{\pm} \sin m\Phi_p)$$

$$c_{10}^{\pm} = \sum_{n=0}^{N_c} c_{10n}^{\pm} \varepsilon_0^n, \quad \begin{bmatrix} c_{1m}^{\pm} \\ c_{2m}^{\pm} \end{bmatrix}_{m=1,2,..M_c} = \varepsilon_0 \sum_{n=0}^{N_c} \begin{bmatrix} c_{1mn}^{\pm} \\ c_{2mn}^{\pm} \end{bmatrix} \varepsilon_0^n, \quad \varepsilon_0 = \tan\left(\frac{\gamma_0(t)}{2}\right)$$

Trapped Flux Mapping

$\Phi_p(t)$ - polhode phase

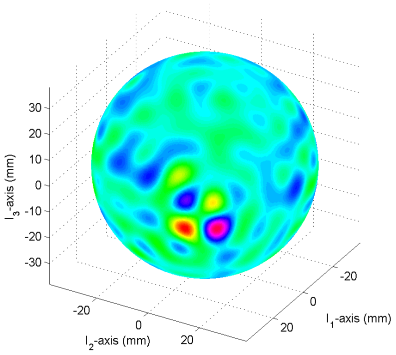
$\gamma_0(t)$ - polhode angle

Misalignment torque coefficient k:

$$k(t) = \sum_{m=0}^{M_k} (k_{1m} \cos m\Phi_p^0 + k_{2m} \sin m\Phi_p^0)$$

k_{1m} and k_{2m} have the same structure as $c_{1m}^{\pm}$ and $c_{2m}^{\pm}$

The same polhode structure as in Readout Scale Factor Model (1st Floor)



2nd Floor Roll-Resonance Torque Dynamic Estimator

Orientations Profiles

Roll Phase

Misalignment

Polhode Phase/Angle

State vector: $x = \left[s_{NS}(t), s_{EW}(t), r_{NS}, r_{EW}, \{k\}, \{c\}\right]$

Propagation Model:

$$x(t_{k+1}) = F(t_{k+1}, t_k)\, x(t_k)$$

Measurement Model:

$$z(t_{k+1}) = Hx(t_k) + \nu_k$$

Full 1st Floor Information is not yet used

Estimator (separate for each segment)

Output:

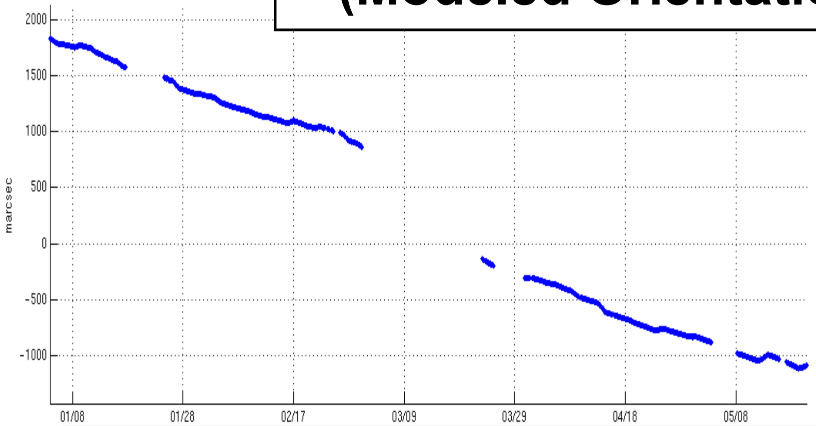
- **Torque related variables:**
  - torque coefficients
  - modeled torque contributions
- **Reconstructed "relativistic" trajectory (Orientation profile minus torque contributions)**

Combine reconstructed trajectories for all segments

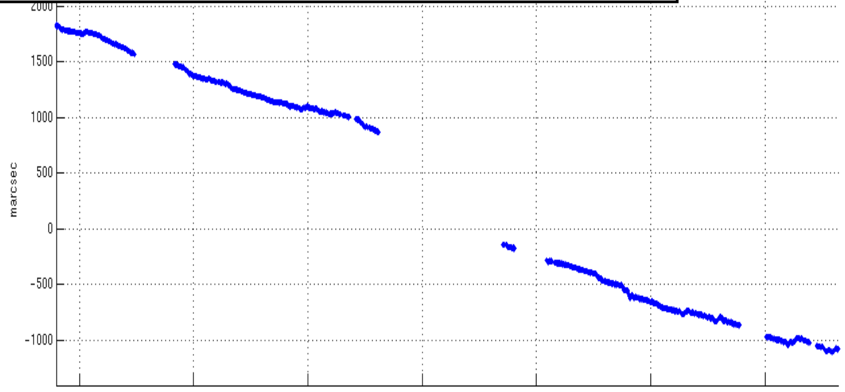
Fit to a straight line

Relativity: Slope estimate

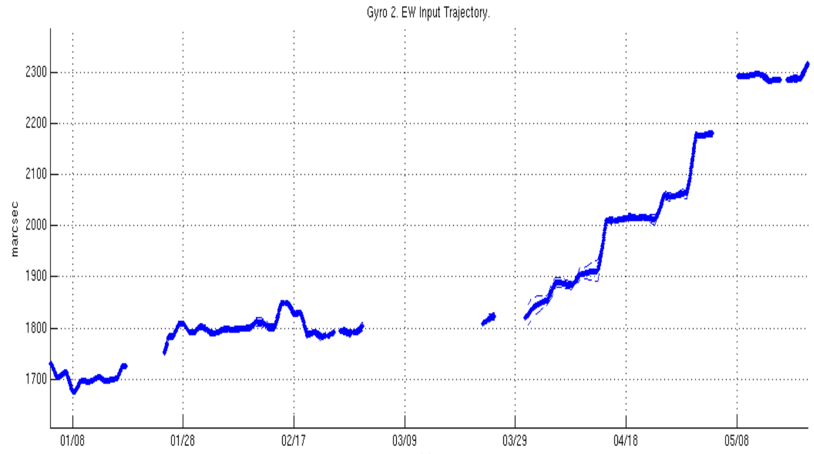
Gyro 2: Estimation Results
(Modeled Orientation vs Measured Orientation)

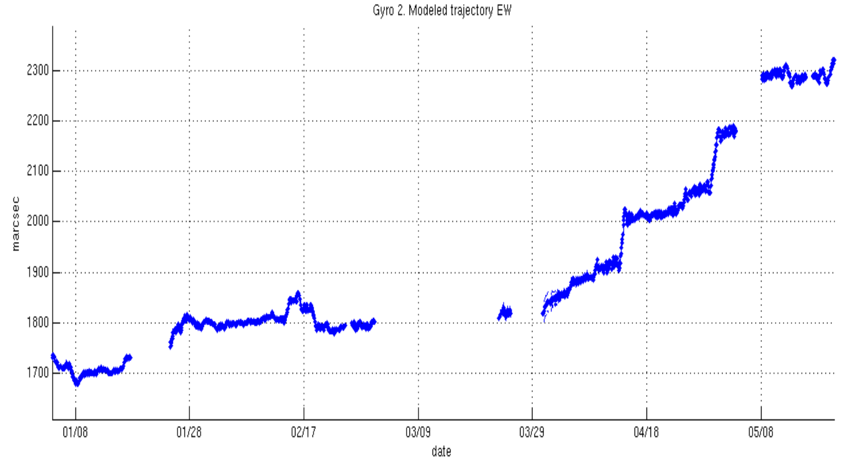


Measured **Inertial Orientation**



Modeled **Inertial Orientation**





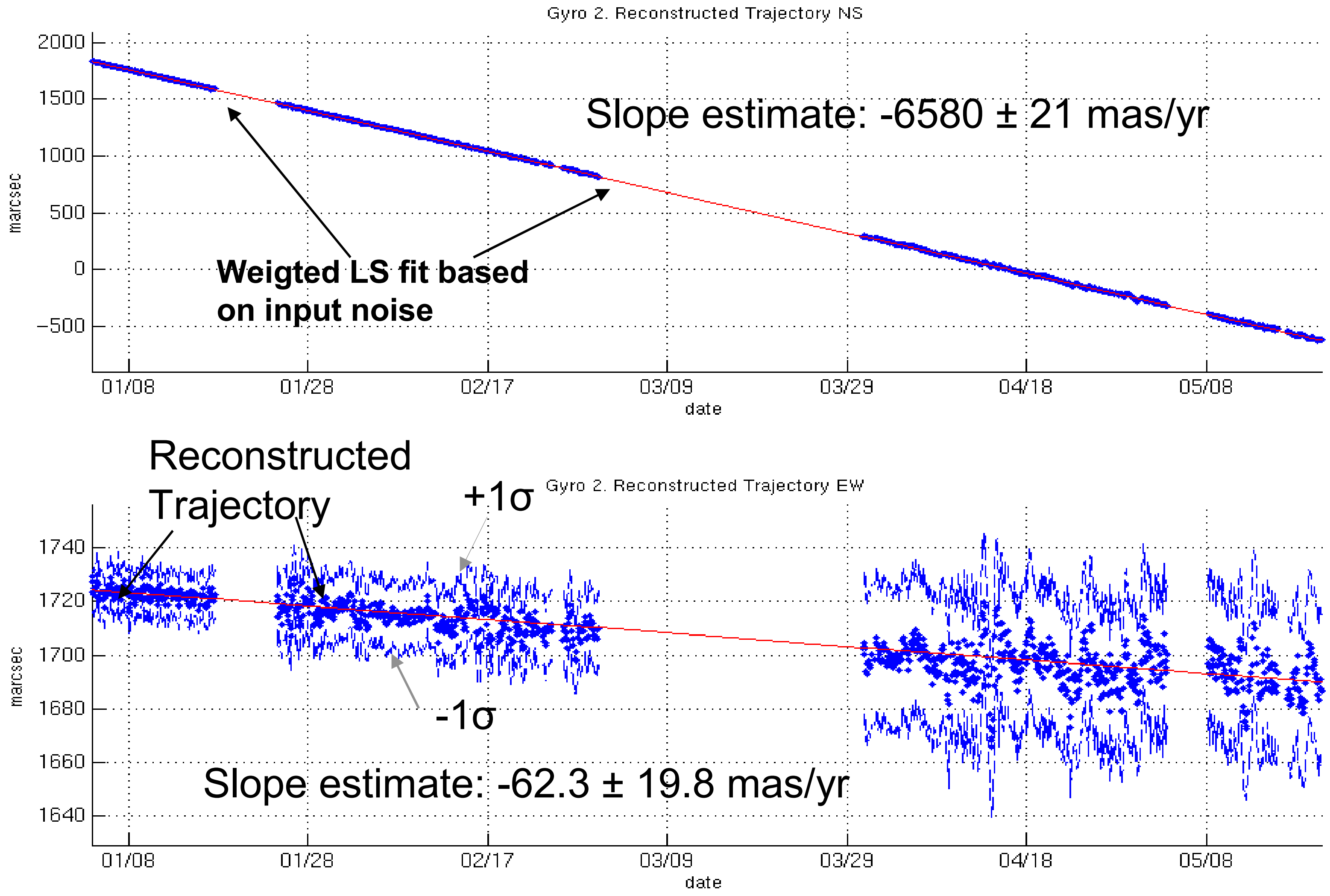
74 Resonances!

Gyro 2. Trajectory: rec2sec_B~PhiCgCt5DayK_PolJC_tauDrop300 [illegible]

Gyro 2. Modeled trajectory Seg. 9. useAb 0, useTheta 1. , estR 1.

Subtracting the torque contributions…

Gyro 2: Reconstructed “Relativistic” Trajectory



Slope estimate: -6580 ± 21 mas/yr

Weigted LS fit based on input noise

Reconstructed Trajectory

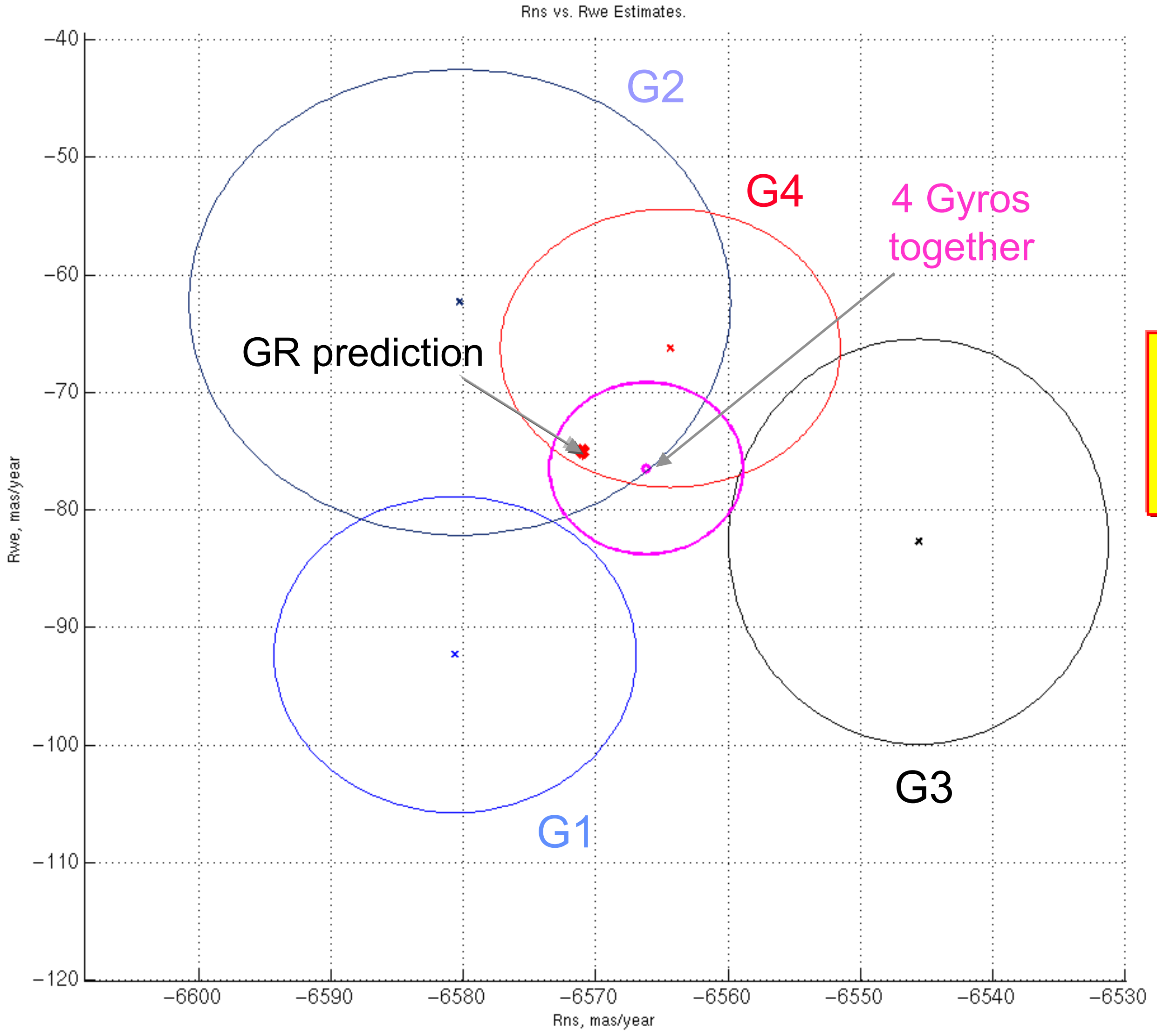
+1σ

-1σ

Slope estimate: -62.3 ± 19.8 mas/yr

Gyro 2. Reconstructed Trajectory Seg. 9. useAb 0, useTheta 1. , estR 1.

Current Relativity Estimates for Gyros 1,2,3, and 4



Four Gyros Combined:
NS -6566 ± 7 *mas/yr*
EW -76.4 ± 7.3 *mas/yr*

Where we stand now

- **Roll-Resonance Torque Modeling:**
  - reduced large part of systematic errors: previously unmodeled torque-related errors are now modeled properly
  - dramatically enhanced the agreement between the gyroscopes

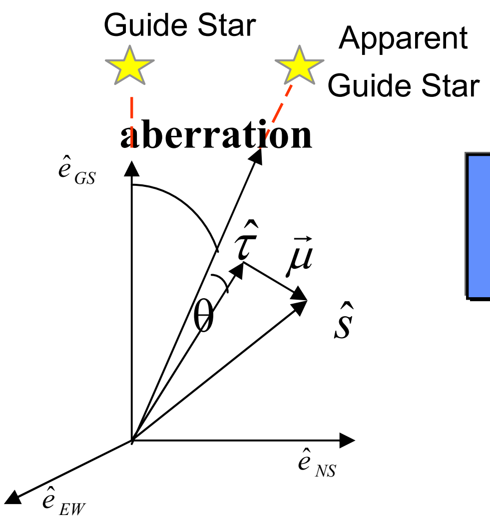
- **The same torque model works for all 4 gyros over entire mission**

- **Developed estimator is not yet perfect:**
  - Orientation and misalignment profiles not reliable enough
  - Available full information matrix is not used
  - Orientation time step (currently 1-orbit) should be made much less than 1 roll period

- **Final improvement of Algebraic Method: “2-sec Filter”**

Two-Second Filter: Nonlinear Optimization Problem

- **New Filter is formulated as a Dynamic <u>Nonlinear</u> Estimation Problem:**



For 1 Gyro

SQUID Data

Nonlinear Model

$$Z(t_k) = h_k(\vec{s}, \vec{\tau}, \{C_g\}, \{C_T\}, \ldots, t) + noise$$

$$k = 1,2,\ldots N \cong 11.8\times 10^6 \quad \textbf{(!)}$$

$$\Delta t = t_{k+1} - t_k = 2\sec$$

307 days = 4605 orbits x 97 min x 30 (2-sec data points)

- **Nonlinear Dynamic Gyro Motion Model**

$$\frac{d\vec{s}}{dt} = f(\vec{s}, \vec{\tau}, \{k\}, \{c\}, t)$$

- **Requires multiple cost-function minimum search iterations going through millions of data points**

Challenges of 2-sec Filter (2)

- **Data Analysis Concept:**
GP-B Instrument is ONE INTEGRATED SYSTEM

- **Common Features of all 4 Gyroscopes**
  - Relativistic drift rates
  - (Apparent) position of the Guide Star
    - Annual and Orbital Aberrations
    - Bending of Starlight
    - Parallax
  - S/V Attitude & Translation Control system (ATC)
    - Roll Phase
    - Pointing Error
  - Science telescope
    - Normalized Pointing Signal (Np)
    - Telescope Scale Factor (CT)
  - Quartz block
    - Roll Phase Offset

➢**10 data segments interrupted by anomalous events**

Challenges of 2-sec Filter - 3

- **Dealing with several millions of ‘measurement’ equations requires new assessment of numerical techniques and computational capabilities**

- **Analyzing gyroscopes together and the nonlinear structure of the estimation problem probably will require parallel processing (in which we have no experience)**

- **Evaluation of the analysis results, given the complexity of 2-sec filter, will probably require the development of new “truth model” simulations**