New Formulations of the Optimal Power Flow Problem

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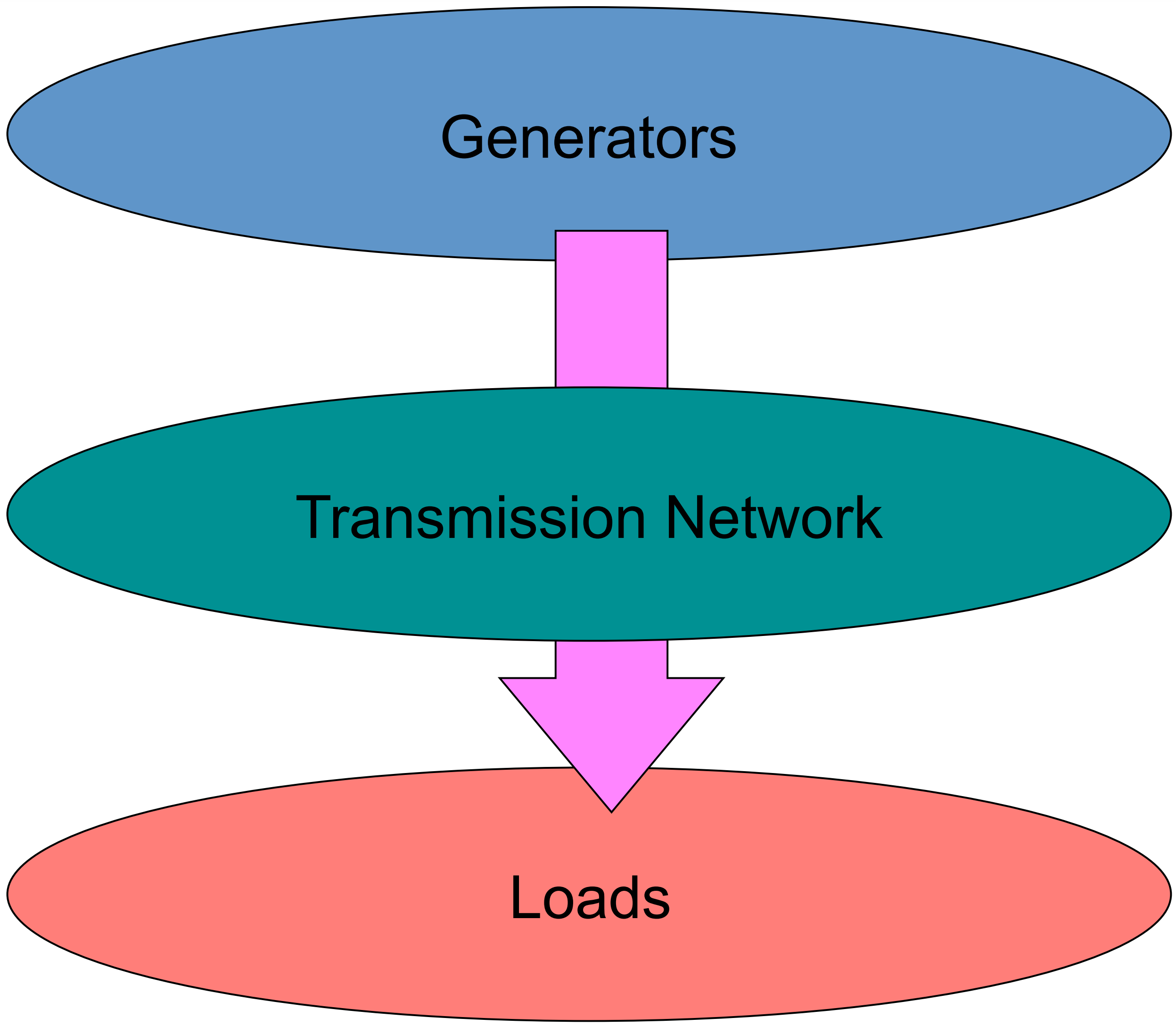
Outline

- A bit of background
- The power flow problem
- The optimal power flow problem (OPF)
- The security-constrained OPF (SCOPF)
- The worst-case problem

What is a power system?

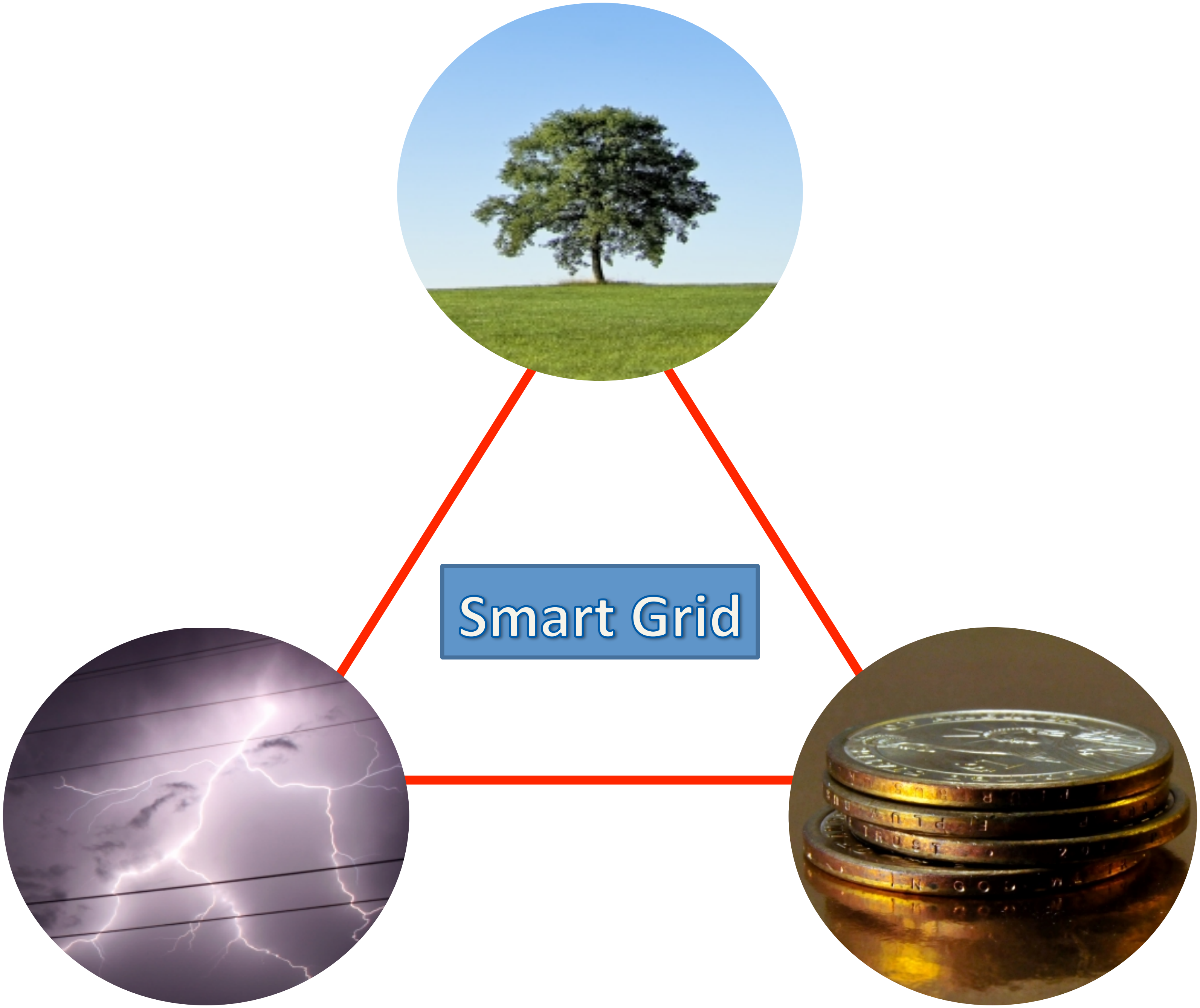


What is running a power system about?

What is running a power system about?

Balancing the greed and the fear

What is running a power system about?


Smart Grid

Structure of the optimization problems

- Objective function
  - Minimization of operating cost (mostly fuel)
  - Minimization of deviation from current conditions
- Equality constraints
  - Physical flows in the network (power flow)
- Inequality constraints
  - Safety margin to provide stability, reliability
- Renewable energy sources
  - Tend to be taken as given so far

The Power Flow Problem

State variables

- Voltage at every node (a.k.a. "bus") of the network
- Because we are dealing with ac, voltages are represented by phasors, i.e. complex numbers in polar representation:
  - Voltage magnitude at each bus: V_k
  - Voltage angle at each bus: θ_k

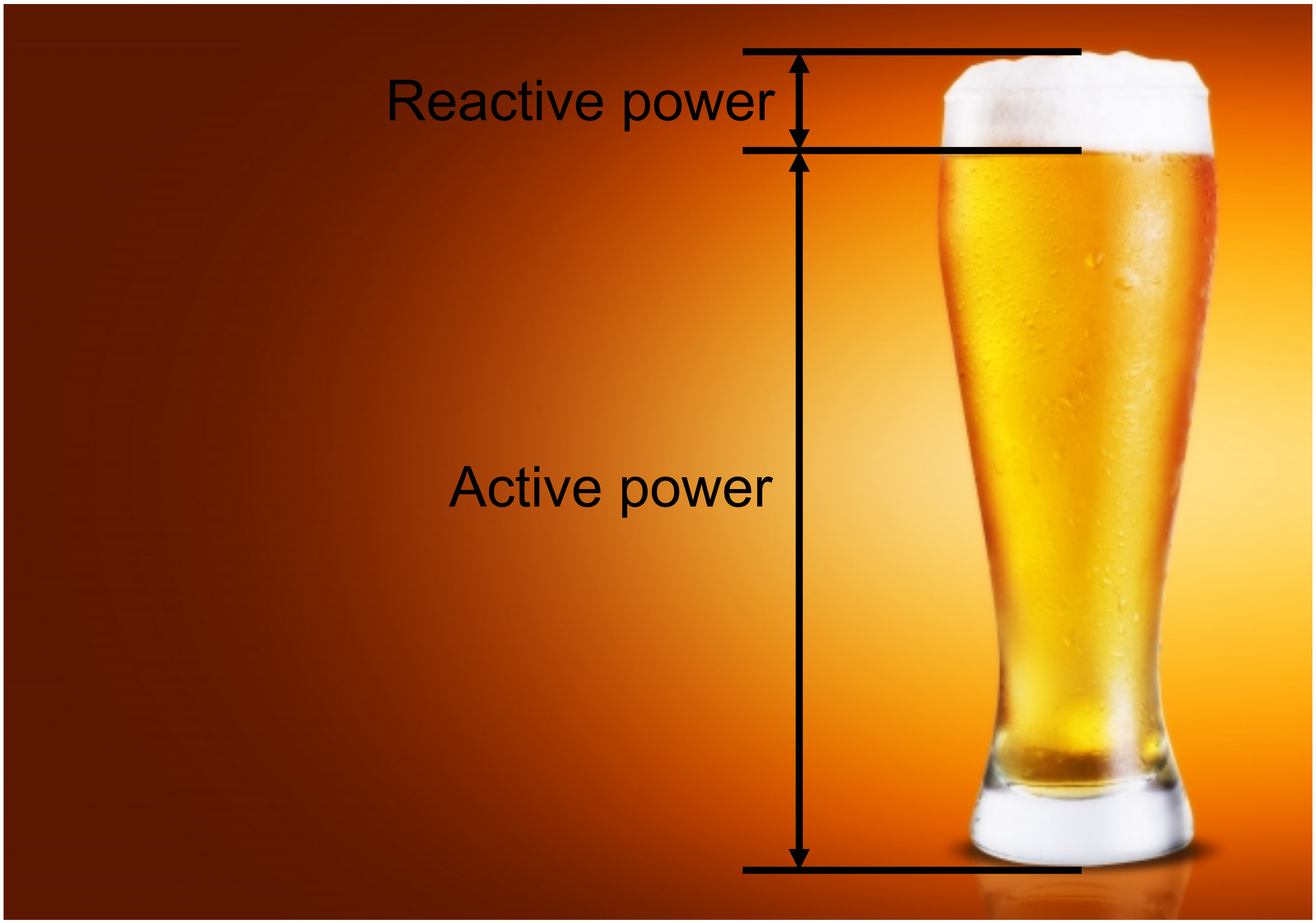
Other variables

- Active and reactive power consumed at each bus: P_k^L, Q_k^L
  - a.k.a. the load at each bus

- Active and reactive power produced by renewable generators: P_k^W, Q_k^W

- Assumed known in deterministic problems
- In practice, they are stochastic variables

What is reactive power?

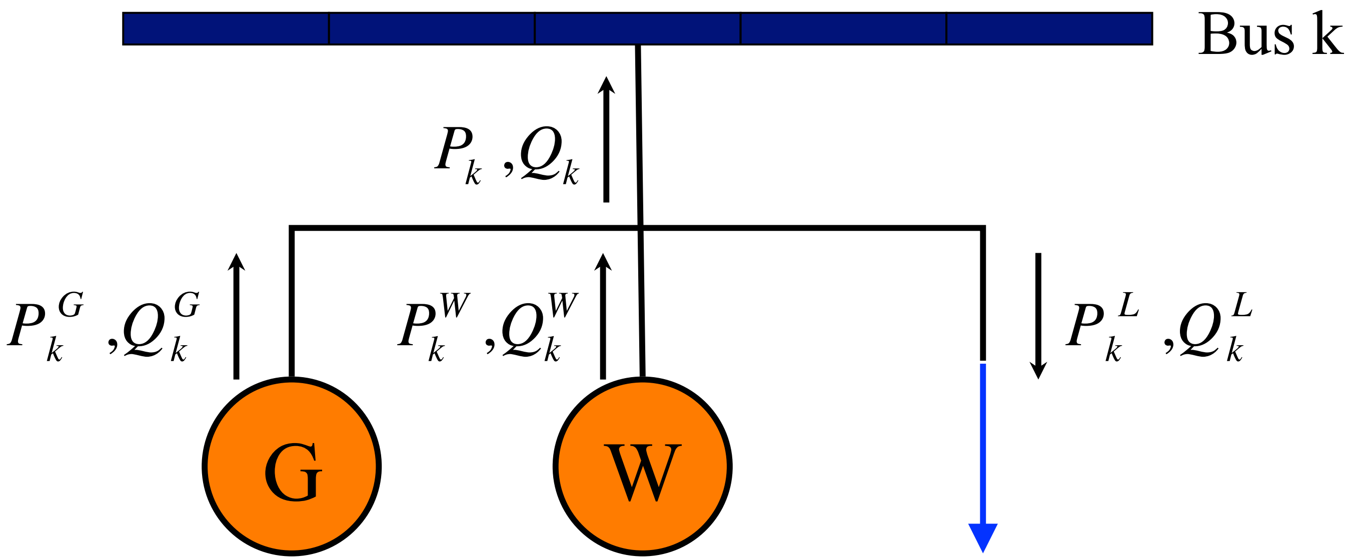


What is reactive power?

- Reactive power represents power that oscillates between the sources and the reactive components (inductors, capacitors)
- It does not do any real work
- Because transmission lines are inductive, the flow of reactive power is closely linked to the magnitudes of the voltages
- Controlling the reactive power is thus important
- Complex power: $S = P + jQ$

Photo credit: FreeDigitalPhotos.net

Injections

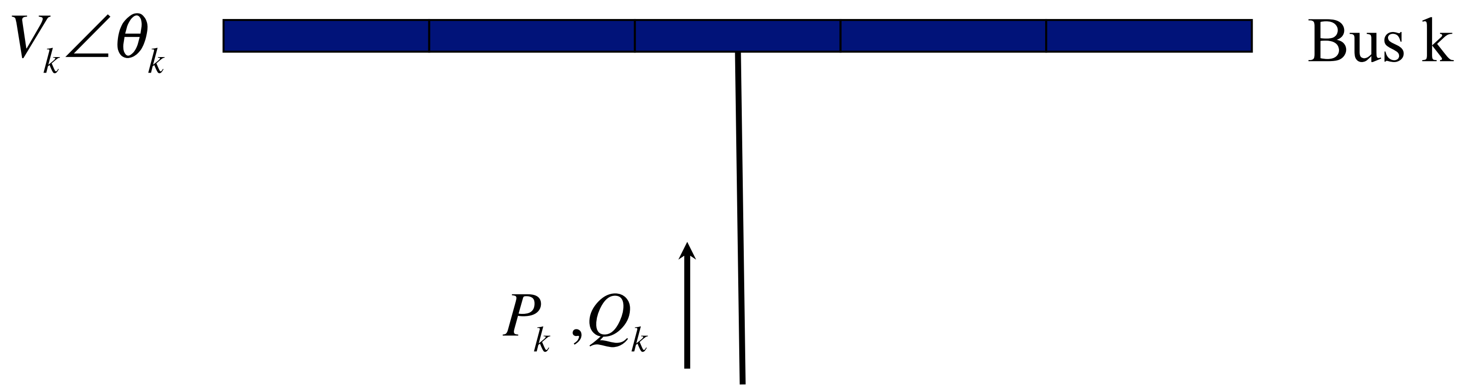


$$P_k = P_k^G + P_k^W - P_k^L$$

$$Q_k = Q_k^G + Q_k^W - Q_k^L$$

There is usually only one *P* and *Q* component at each bus

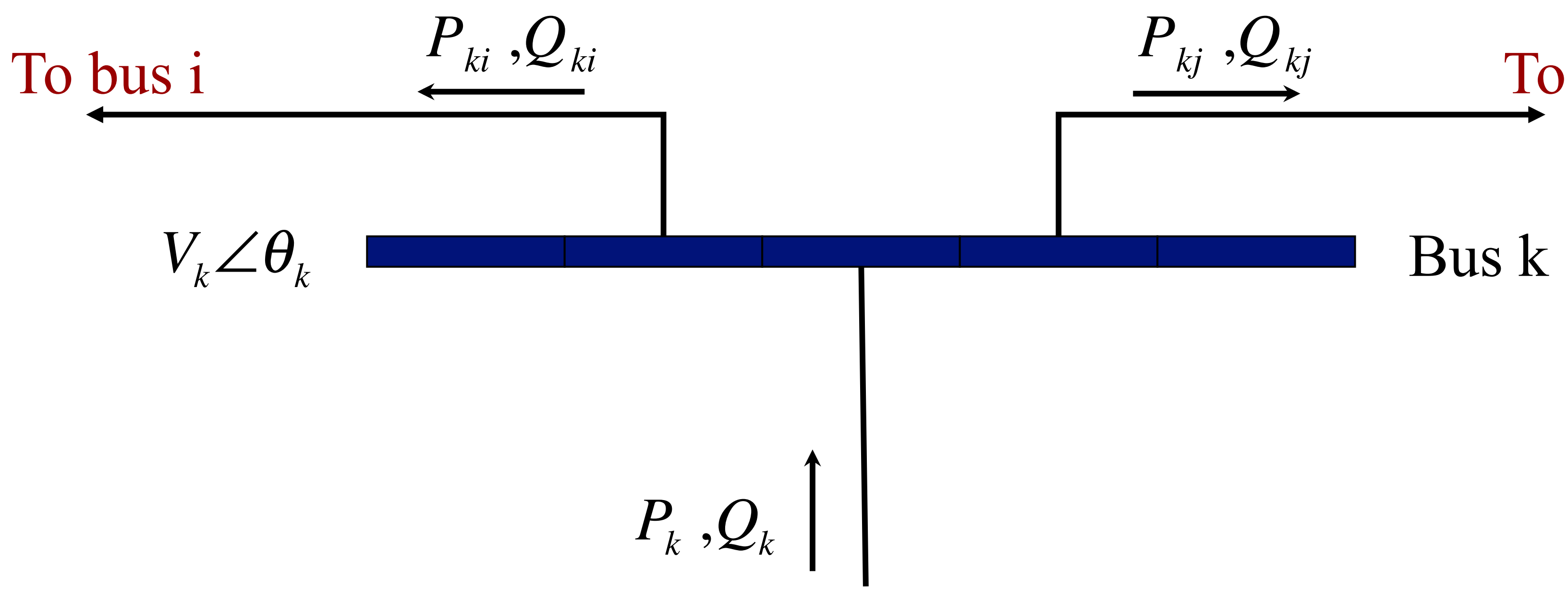
Injections



Two of these four variables are specified at each bus:

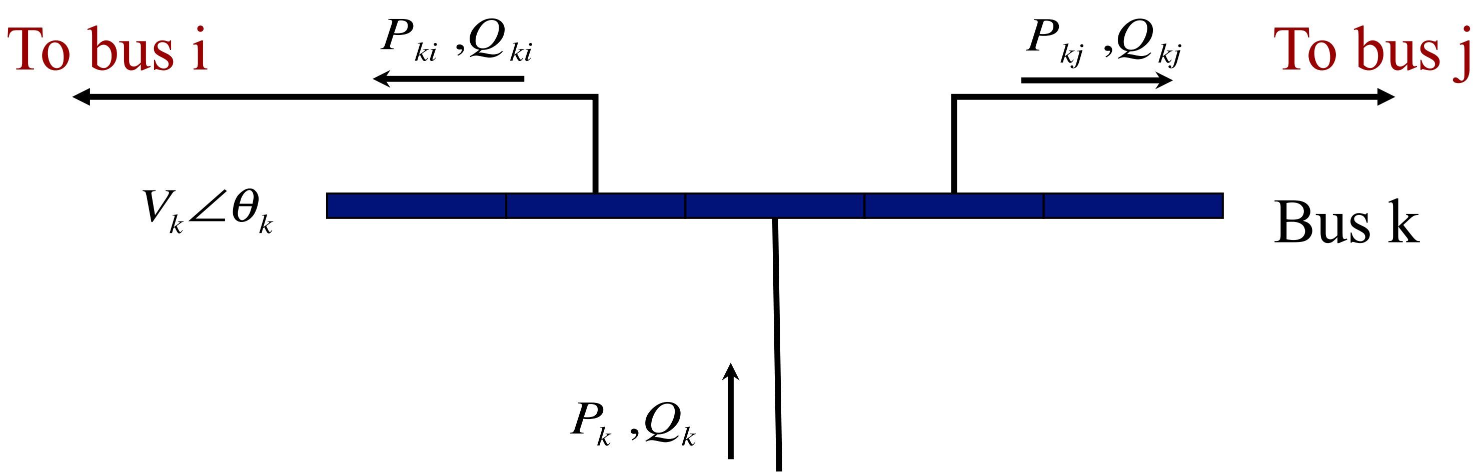
- Load bus: $P_k \ ,Q_k$
- Generator bus: $P_k \ ,V_k$
- Reference bus: V_k ,θ_k

Line flows



The line flows depend on the bus voltage magnitude and angle as well as the network parameters G_{ki}, B_{ki} (real and imaginary part of the network admittance matrix)

Power flow equations



Write active and reactive power balance at each bus:

$$P_k = \sum_{i=1}^{N} V_k V_i [G_{ki} \cos\theta_{ki} + B_{ki} \sin\theta_{ki}]$$

$$k = 1, \cdots N$$

$$Q_k = \sum_{i=1}^{N} V_k V_i [G_{ki} \sin\theta_{ki} - B_{ki} \cos\theta_{ki}]$$

with: $\theta_{ki} = \theta_k - \theta_i$, N : number of nodes in the network

The power flow problem

Given the injections and the generator voltages, Solve the power flow equations to find the voltage magnitude and angle at each bus and hence the flow in each branch

$$P_k = \sum_{i=1}^{N} V_k V_i [G_{ki} \cos\theta_{ki} + B_{ki} \sin\theta_{ki}]$$

$$Q_k = \sum_{i=1}^{N} V_k V_i [G_{ki} \sin\theta_{ki} - B_{ki} \cos\theta_{ki}]$$

$$k = 1, \cdots N$$

Typical values of N:
GB transmission network: N~1,500
Continental European network (UCTE): N~13,000

However, the equations are highly sparse!

Applications of the power flow problem

- Check the state of the network
  - for an actual or postulated set of injections
  - for an actual or postulated network configuration

- Are all the line flows within limits?
- Are all the voltage magnitudes within limits?

Linear approximation

$$P_k = \sum_{i=1}^{N} V_k V_i [G_{ki} \cos\theta_{ki} + B_{ki} \sin\theta_{ki}]$$

$$Q_k = \sum_{i=1}^{N} V_k V_i [G_{ki} \sin\theta_{ki} - B_{ki} \cos\theta_{ki}]$$

$$\Rightarrow \quad P_k = \sum_{i=1}^{N} B_{ki} \theta_{ki}$$

- Ignores reactive power
- Assumes that all voltage magnitudes are nominal
- Useful when concerned with line flows only

The Optimal Power Flow Problem (OPF)

Control variables

- Control variables which have a cost:
  - Active power produced by thermal generating units: P_i^G

- Control variables that do not have a cost:
  - Magnitude of voltage at the generating units: V_i^G
  - Tap ratio of the transformers: t_{ij}

Possible objective functions

- Minimise the cost of producing power with conventional generating units:

$$\min \sum_{i=1}^{g} C_i(P_i^G)$$

- Minimise deviations of the control variables from a given operating point (e.g. the outcome of a market):

$$\min \sum_{i=1}^{g} \left(c_i^+ \Delta P_i^{G+} - c_i^- \Delta P_i^{G-} \right)$$

Equality constraints

- Power balance at each node bus, i.e. power flow equations

$$P_k = \sum_{i=1}^{N} V_k V_i [G_{ki} \cos\theta_{ki} + B_{ki} \sin\theta_{ki}]$$

$$k = 1, \cdots N$$

$$Q_k = \sum_{i=1}^{N} V_k V_i [G_{ki} \sin\theta_{ki} - B_{ki} \cos\theta_{ki}]$$

Inequality constraints

- Upper limit on the power flowing though every branch of the network
- Upper and lower limit on the voltage at every node of the network
- Upper and lower limits on the control variables
  – Active and reactive power output of the generators
  – Voltage settings of the generators
  – Position of the transformer taps and other control devices

Formulation of the OPF problem

$$\min_{\mathbf{u}_0} f_0\left(\mathbf{x}_0, \mathbf{u}_0\right)$$
$$\mathbf{g}\left(\mathbf{x}_0, \mathbf{u}_0\right) = 0$$
$$\mathbf{h}\left(\mathbf{x}_0, \mathbf{u}_0\right) \leq 0$$

$\mathbf{x}_0$: vector of dependent (or state) variables

$\mathbf{u}_0$: vector of independent (or control) variables

Nothing extraordinary, except that we are dealing with a fairly large (but sparse) non-linear problem.

The Security Constrained Optimal Power Flow Problem (SCOPF)

Bad things happen…

Sudden changes in the system

- A line is disconnected because of an insulation failure or a lightning strike
- A generator is disconnected because of a mechanical problem
- A transformer blows up

- The system must keep going despite such events
- "N-1" security criterion

N-1 Security criterion

- System with N components should be able to continue operating after any single outage
- Losing two components at about the same time is considered “not credible”
- Beloved by operators
  - Implementation is straightforward
  - Results are unambiguous
  - No need for judgment
  - Compliance is easily demonstrated

Security-constrained OPF

- How should the control variables be set to minimise the cost of running the system while ensuring that the operating constraints are satisfied in both the normal and all the contingency states?

Formulation of the SCOPF problem

$$\begin{aligned} \min_{\mathbf{u}_k} \quad & f_0\left(\mathbf{x}_0, \mathbf{u}_0\right) & \\ \text{s.t.} \quad & \mathbf{g}_k(\mathbf{x}_k, \mathbf{u}_k) = \mathbf{0} & k = 0, \ldots, N_c \\ & \mathbf{h}_k(\mathbf{x}_k, \mathbf{u}_k) \leq \mathbf{0} & k = 0, \ldots, N_c \\ & \left|\mathbf{u}_k - \mathbf{u}_0\right| \leq \Delta\mathbf{u}_k^{\max} & k = 1, \ldots, N_c \end{aligned}$$

$k = 0$: normal conditions

$k = 1, \ldots, N_c$: contingency conditions

$\Delta\mathbf{u}_k^{\max}$: vector of maximum allowed adjustments after contingency *k* has occured

Preventive or corrective SCOPF

$$
\begin{aligned}
\min_{\mathbf{u}_k} \quad & f_0\left(\mathbf{x}_0, \mathbf{u}_0\right) \\
\text{s.t.} \quad & \mathbf{g}_k(\mathbf{x}_k, \mathbf{u}_k) = \mathbf{0} \qquad k = 0,\dots,N_c \\
& \mathbf{h}_k(\mathbf{x}_k, \mathbf{u}_k) \le \mathbf{0} \qquad k = 0,\dots,N_c \\
& \left|\mathbf{u}_k - \mathbf{u}_0\right| \le \Delta\mathbf{u}_k^{\max} \qquad k = 1,\dots,N_c
\end{aligned}
$$

Preventive SCOPF: no corrective actions are considered

$$\Delta\mathbf{u}_k^{\max} = 0 \Rightarrow \mathbf{u}_k = \mathbf{u}_0 \forall k = 1,\ldots N_c$$

Corrective SCOPF: some corrective actions are allowed

$$\exists\ k = 1,\ldots N_c \quad \Delta\mathbf{u}_k^{\max} \neq 0$$

Size of the SCOPF problem

- SCOPF is (N_c+1) times larger than the OPF
- Pan-European transmission system model contains about 13,000 nodes, 20,000 branches and 2,000 generators
- Based on N-1 criterion, we should consider the outage of each branch and each generator as a contingency
- However:
  - Not all contingencies are critical (but which ones?)
  - Most contingencies affect only a part of the network (but what part of the network do we need to consider?)

A few additional complications…

- Some of the control variables are discrete:
  - Transformer and phase shifter taps
  - Capacitor and reactor banks
  - Starting up of generating units
- There is only time for a limited number of corrective actions after a contingency

Limitations of N-1 criterion

- Not all contingencies have the same probability
  - Long lines vs. short lines
  - Good weather vs. bad weather
- Not all contingencies have the same consequences
  - Local undervoltage vs. edge of stability limit
- N-2 conditions are not always "not credible"
  - Non-independent events
- Does not ensure a consistent level of risk
  - Risk = probability x consequences

Probabilistic security analysis

- Goal: operate the system at a given risk level
- Challenges
  – Probabilities of non-independent events
    - "Electrical" failures compounded by IT failures
  – Estimating the consequences
    - What portion of the system would be blacked out?
  – What preventive measures should be taken?
    - Vast number of possibilities

The Worst-Case Problems

Good things happen…

… but there is no free lunch!

- Wind generation and solar generation can only be predicted with limited accuracy
- When planning the operation of the system a day ahead, some of the injections are thus stochastic variables
- Power system operators do not like probabilistic approaches

Incorporating uncertainty in the OPF

$$\min \quad \overbrace{\mathbf{c}^{\mathrm{T}}\left(\mathbf{p}_0^{*}-\mathbf{p}_0^{M}\right)}^{\text{market-based generation}} + \overbrace{\mathbf{b}_0^{*T}\left(\mathbf{c}_0+\mathbf{p}_0^{nd\,*}\mathbf{c}^{\mathrm{T}}\right)}^{\text{additional generation}}$$

$$\text{s.t.} \quad \mathbf{g}_0(\mathbf{x}_0,\mathbf{u}_0,\mathbf{p}_0,\mathbf{b}_0,\mathbf{p}_0^{nd},\mathbf{s})=\mathbf{0}$$

$$\mathbf{h}_0(\mathbf{x}_0,\mathbf{u}_0,\mathbf{p}_0,\mathbf{b}_0,\mathbf{p}_0^{nd},\mathbf{s})\le\mathbf{0}$$

$$\left|\mathbf{u}_0-\mathbf{u}_0^{\text{init}}\right|\le\Delta\mathbf{u}_0^{\max}$$ ← Deviations in cost-free controls

$$\left|\mathbf{p}_0-\mathbf{p}_0^{M}\right|\le\Delta\mathbf{p}_0^{\max}$$ ← Deviations in market generation

$$\mathbf{p}_{\min}^{nd}\mathbf{b}_0^{\mathrm{T}}\le\mathbf{p}_0^{nd}\mathbf{b}_0^{\mathrm{T}}\le\mathbf{p}_{\max}^{nd}\mathbf{b}_0^{\mathrm{T}}$$ ← Deviations in extra generation

$$\mathbf{b}_0\in\{0,1\}$$ ← Decisions about extra generation

$$\mathbf{s}_{\min}\le\mathbf{s}\le\mathbf{s}_{\max}$$ ← Vector of uncertainties

Worst-case OPF bi-level formulation

$$
\begin{aligned}
\max_{\mathbf{s}} \quad & \mathbf{c}^{\mathbf{T}}\left(\mathbf{p}_0^{*}-\mathbf{p}_0^{M}\right)+\mathbf{b}_0^{*T}\left(\mathbf{c}_0+\mathbf{p}_0^{nd\,*}\mathbf{c}^{\mathbf{T}}\right) \\
\text{s.t.} \quad & \mathbf{s}_{\min} \le \mathbf{s} \le \mathbf{s}_{\max} \\
& \left(\mathbf{p}_0^{*},\mathbf{u}_0^{*},\mathbf{b}_0^{*},\mathbf{p}_0^{nd\,*}\right) = \arg \quad \min \quad \mathbf{c}^{\mathbf{T}}\left(\mathbf{p}_0-\mathbf{p}_0^{M}\right)+\mathbf{b}_0^{\mathbf{T}}\left(\mathbf{c}_0+\mathbf{p}_0^{nd}\mathbf{c}^{\mathbf{T}}\right) \\
& \qquad \text{s.t.} \quad \mathbf{g}_0(\mathbf{x}_0,\mathbf{u}_0,\mathbf{p}_0,\mathbf{b}_0,\mathbf{p}_0^{nd},\mathbf{s})=\mathbf{0} \\
& \qquad\qquad \mathbf{h}_0(\mathbf{x}_0,\mathbf{u}_0,\mathbf{p}_0,\mathbf{b}_0,\mathbf{p}_0^{nd},\mathbf{s})\le\mathbf{0} \\
& \qquad\qquad \left|\mathbf{u}_0-\mathbf{u}_0^{\text{init}}\right| \le \Delta\mathbf{u}_0^{\max} \\
& \qquad\qquad \left|\mathbf{p}_0-\mathbf{p}_0^{M}\right| \le \Delta\mathbf{p}_0^{\max} \\
& \qquad\qquad \mathbf{p}_{\min}^{nd}\mathbf{b}_0^{\mathbf{T}} \le \mathbf{p}_0^{nd}\mathbf{b}_0^{\mathbf{T}} \le \mathbf{p}_{\max}^{nd}\mathbf{b}_0^{\mathbf{T}} \\
& \qquad\qquad \mathbf{b}_0 \in \left\{0,1\right\}
\end{aligned}
$$

Worst-case SCOPF bi-level formulation

$$\max_{\mathbf{s}} \quad \mathbf{c}^{\mathbf{T}}\left(\mathbf{p}_0^* - \mathbf{p}_0^M\right) + \mathbf{b}_0^{*T}\left(\mathbf{c}_0 + \mathbf{p}_0^{nd\,*}\mathbf{c}^{\mathbf{T}}\right)$$

$$\text{s.t.} \quad \mathbf{s}_{\min} \le \mathbf{s} \le \mathbf{s}_{\max}$$

$$\left(\mathbf{p}_0^*, \mathbf{p}_k^*, \mathbf{u}_0^*, \mathbf{u}_k^*, \mathbf{b}_0^*, \mathbf{p}_0^{nd\,*}\right) = \arg \quad \min \quad \mathbf{c}^{\mathbf{T}}\left(\mathbf{p}_0 - \mathbf{p}_0^M\right) + \mathbf{b}_0^{\mathbf{T}}\left(\mathbf{c}_0 + \mathbf{p}_0^{nd}\mathbf{c}^{\mathbf{T}}\right)$$

$$\begin{aligned}
\text{s.t.} \quad & \mathbf{g}_0(\mathbf{x}_0, \mathbf{u}_0, \mathbf{p}_0, \mathbf{b}_0, \mathbf{p}_0^{nd}, \mathbf{s}) = \mathbf{0} \\
& \mathbf{h}_0(\mathbf{x}_0, \mathbf{u}_0, \mathbf{p}_0, \mathbf{b}_0, \mathbf{p}_0^{nd}, \mathbf{s}) \le \mathbf{0} \\
& \mathbf{g}_k(\mathbf{x}_k, \mathbf{u}_k, \mathbf{p}_k, \mathbf{b}_0, \mathbf{p}_0^{nd}, \mathbf{s}) = \mathbf{0} \\
& \mathbf{h}_k(\mathbf{x}_k, \mathbf{u}_k, \mathbf{p}_k, \mathbf{b}_0, \mathbf{p}_0^{nd}, \mathbf{s}) \le \mathbf{0} \\
& \left|\mathbf{p}_k - \mathbf{p}_0\right| \le \Delta\mathbf{p}_k^{\max} \\
& \left|\mathbf{u}_k - \mathbf{u}_0\right| \le \Delta\mathbf{u}_k^{\max} \\
& \mathbf{p}_{\min}^{nd}\mathbf{b}_0^{\mathbf{T}} \le \mathbf{p}_0^{nd}\mathbf{b}_0^{\mathbf{T}} \le \mathbf{p}_{\max}^{nd}\mathbf{b}_0^{\mathbf{T}} \\
& \mathbf{b}_0 \in \{0,1\}
\end{aligned}$$

Interpretation of worst-case problem

- Assume a “credible” range of uncertainty
- Try to answer the question:
  - Would there be enough resources to deal with any contingency under the worst-case uncertainty?
  - Do I need to start-up some generating units to deal with such a situation?
- Not very satisfactory but matches the power system operator’s needs and view of the world

Conclusions

- Lots of interesting optimization problems
  - Large, non-convex
  - Not always properly defined
  - Mathematical elegance does not always match the operator's expectations
- Develop "acceptable" probabilistic techniques?
- Increased availability of demand control creates more opportunities for post-contingency corrective actions