Parameterization and Order Reduction of Geological Models for History Matching

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CME-334

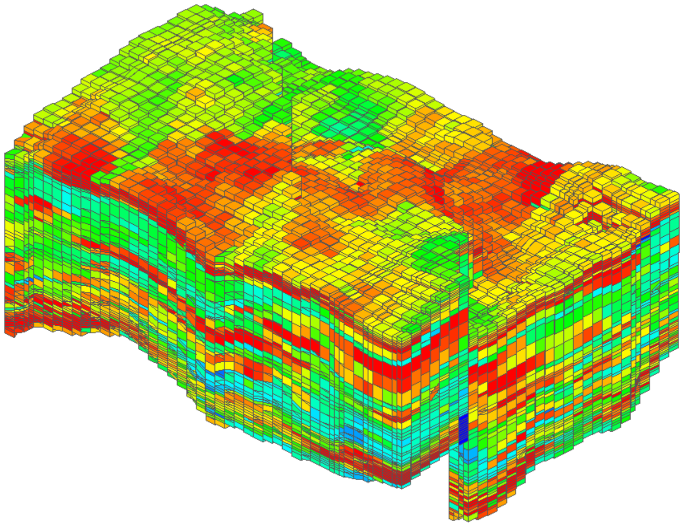
Outline

- Research motivation
- Karhunen-Loève (K-L) Expansion or Principal Component Analysis (PCA)
- Limitations of existing methods
- Optimization-based PCA (O-PCA)
- Application of O-PCA to history matching using AD-GPRS
- Conclusions and future work

History Matching Problem

$$\min_{y}\left\{S = \sum_{n=0}^{N-1}\left(d_{obs}^{n} - d_{sim}^{n}(y)\right)^2 + regularization\ term\right\}$$

$$y = \left[k_1, k_2, \ldots., k_{N_c}\right]^T$$



- Idea is to find a geological model y such that prediction matches production and honors prior information about geological model (regularization)

Challenges in History Matching

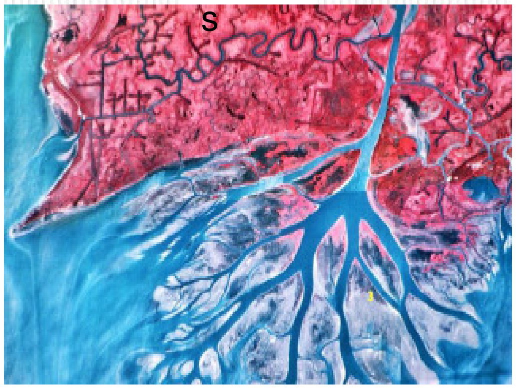
- Real models can have millions of gridblocks
- Updating permeabilities for all blocks independently is expensive and may not maintain geology
- Useful to have algorithms that represent reservoir model as $y(\xi)$ and maintain geology
- Now write history matching problem as:

$$\min_{\xi}\left\{S=\sum_{n=0}^{N-1}\left(d_{obs}^{n}-d_{sim}^{n}(\xi)\right)^{2}\right\}$$

$y(\xi)$ implicitly honors regularization

Types of Geological Systems

Deltas

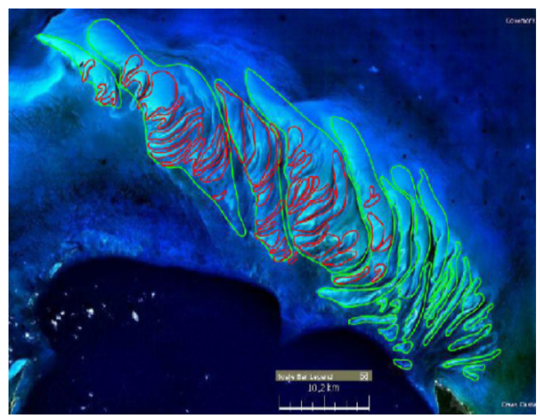


Tidal and bars

(Caers, 2011)

Meandering rivers

Carbonate reefs



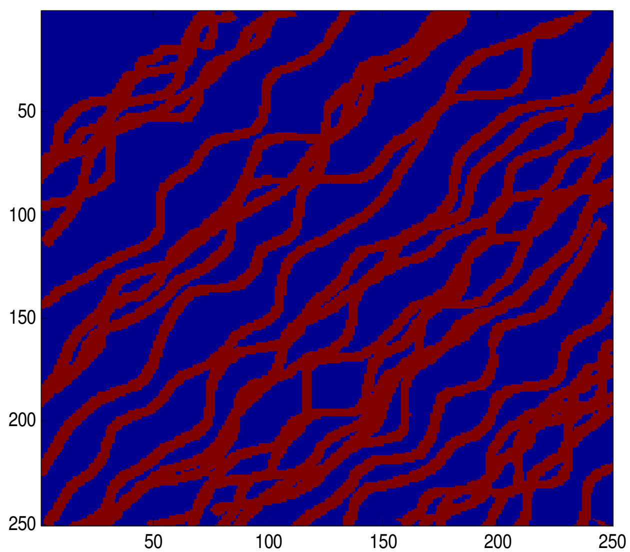
- Goal is to determine parameterization $y(\xi)$ for such systems

Previous Work

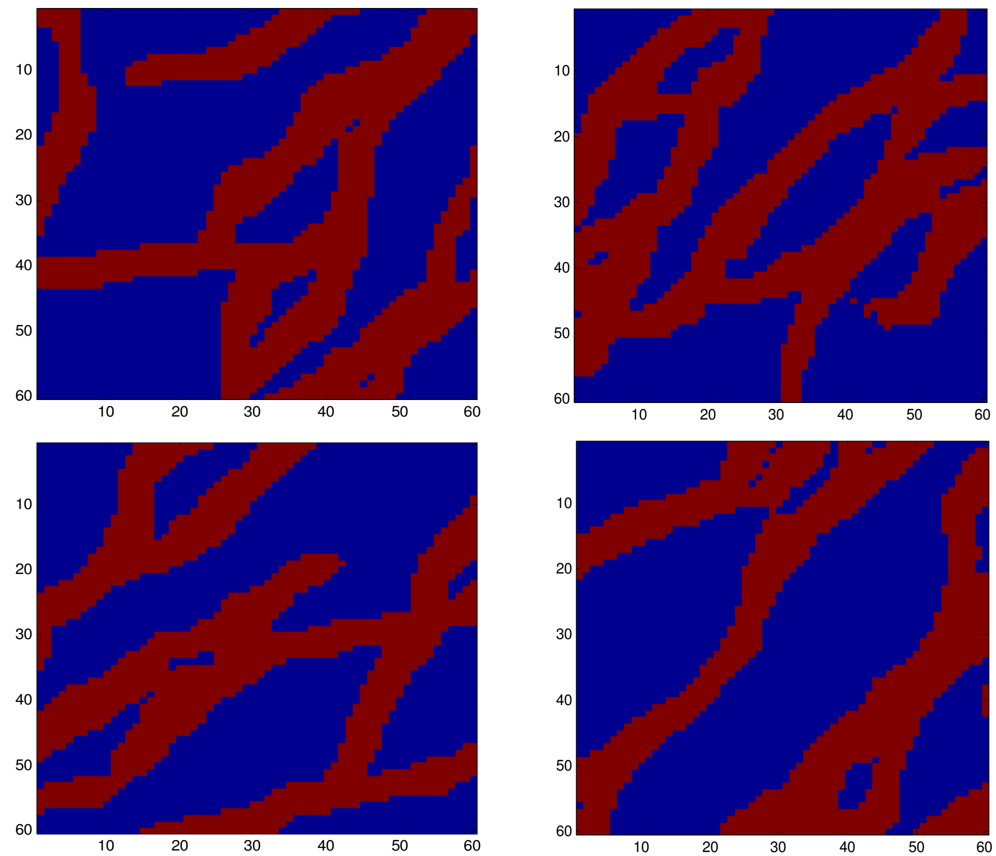
- PCA: Oliver (1996), Sarma et al. (2006)
  - Linear representation: $\boldsymbol{y} = \boldsymbol{\Phi}\boldsymbol{\xi} + \bar{\boldsymbol{y}}$
- KPCA: Sarma et al. (2008), Ma and Zabaras (2011): perform PCA in high-dimensional feature space
  - KPCA can generate more “channelized” realizations but
  - Pre-image problem is strongly nonlinear, nonconvex
  - Resulting histograms may not honor geology

Constructing PCA Representation (1)

- Run Gaussian or training-image based geostatistical algorithms (SGeMS, GSLIB) to create a set of ***N*** realizations



Training image



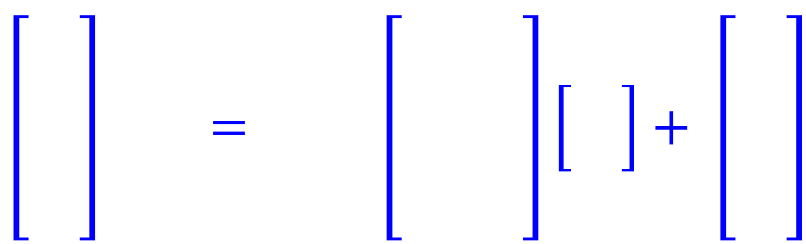
SGeMS realizations

Constructing PCA Representation (2)

- Construct: $\boldsymbol{X} = [\boldsymbol{y}_1, \boldsymbol{y}_2, \dots., \boldsymbol{y}_N]$
- Conceptually, define covariance matrix as: $\boldsymbol{C} = \frac{1}{N}\boldsymbol{X}\boldsymbol{X}^T$
- Eigen-decomposition of $\boldsymbol{C}$ to give: $\boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^T = \boldsymbol{C}$ (in practice use SVD of X)
- K-L (PCA) representation

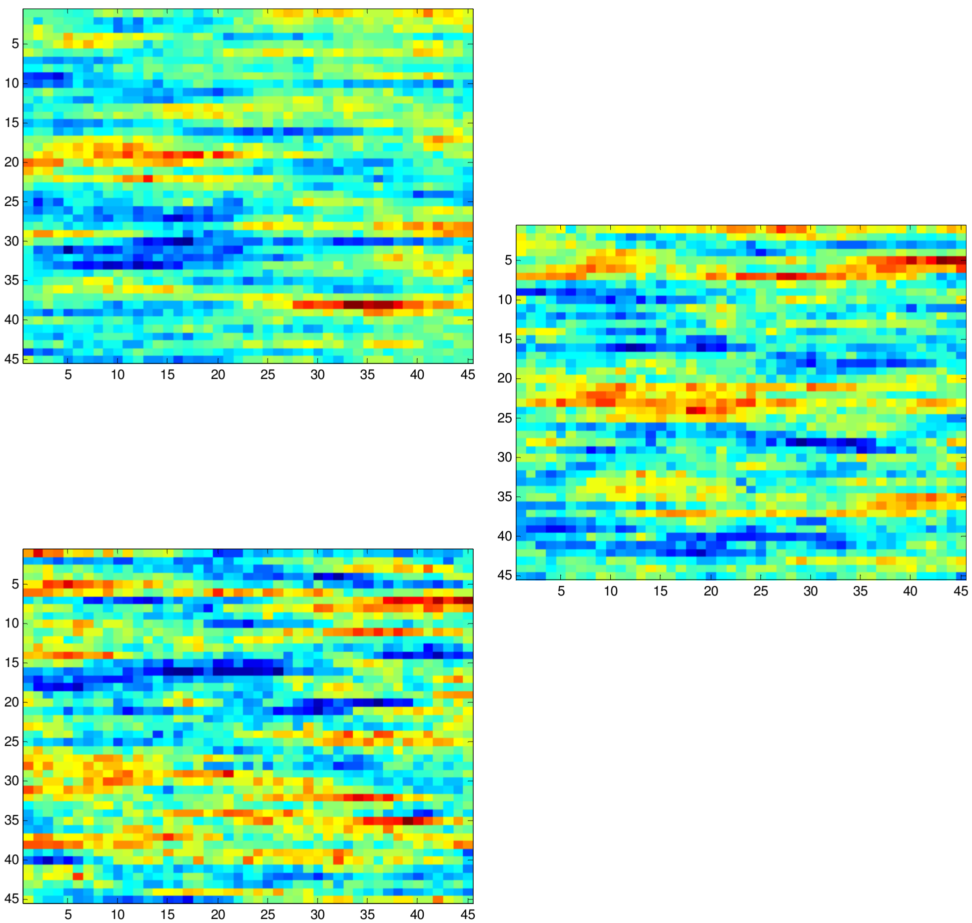
$$\boldsymbol{y}_{new} = \boldsymbol{U}_r \boldsymbol{\Lambda}_r^{1/2} \boldsymbol{\xi} + \bar{\boldsymbol{y}}$$

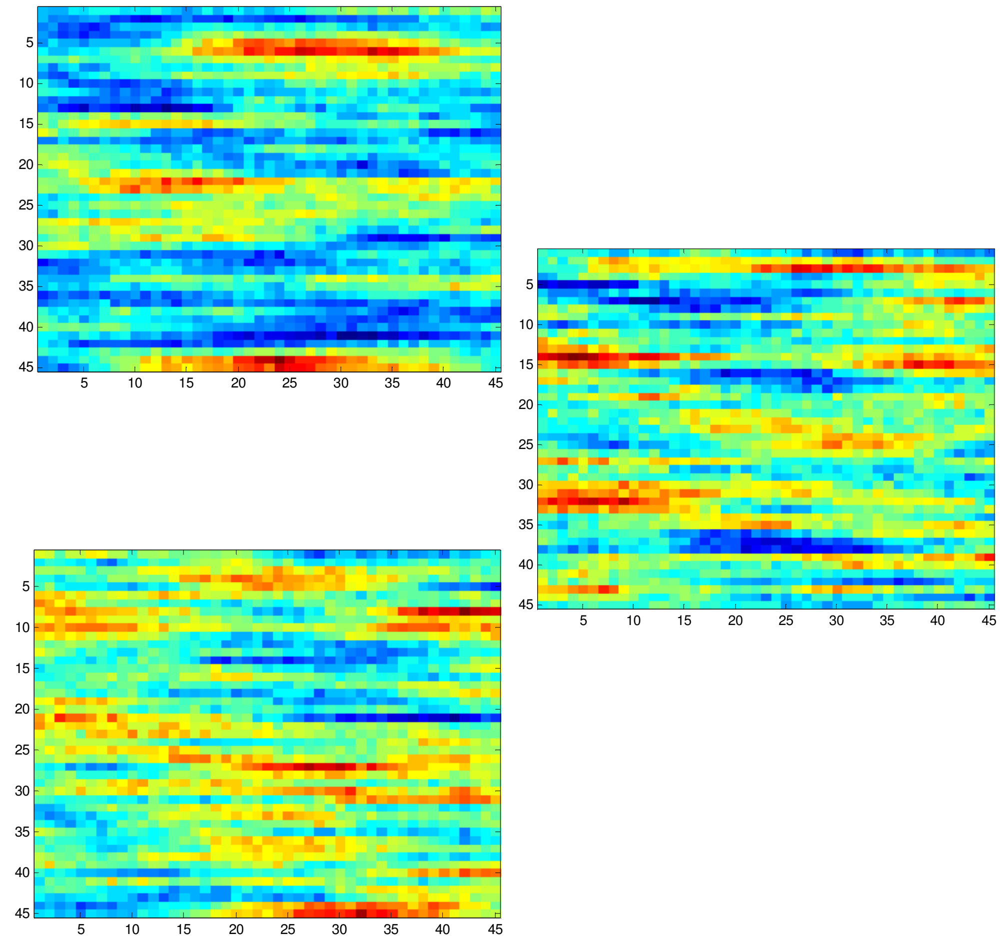
$$= \boldsymbol{\Phi}_r \, \boldsymbol{\xi} + \bar{\boldsymbol{y}} \quad (r << N)$$



$$\boldsymbol{y}_{new} \quad = \quad \boldsymbol{\Phi}_r \quad \boldsymbol{\xi} \; + \; \bar{\boldsymbol{y}}$$

Application of PCA: Gaussian Fields (1)

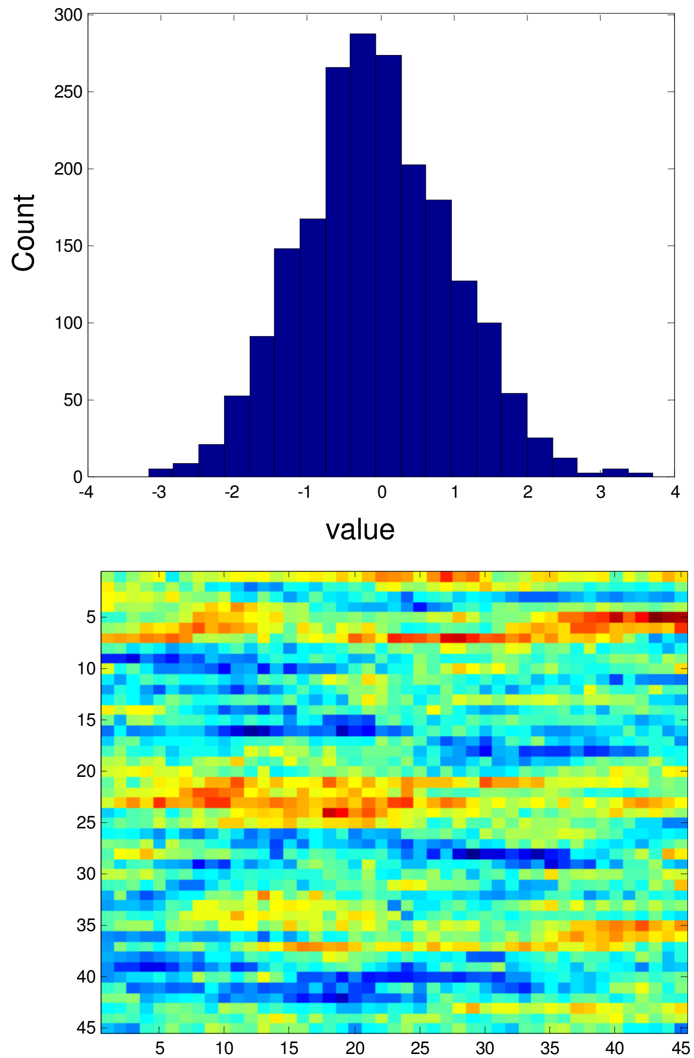




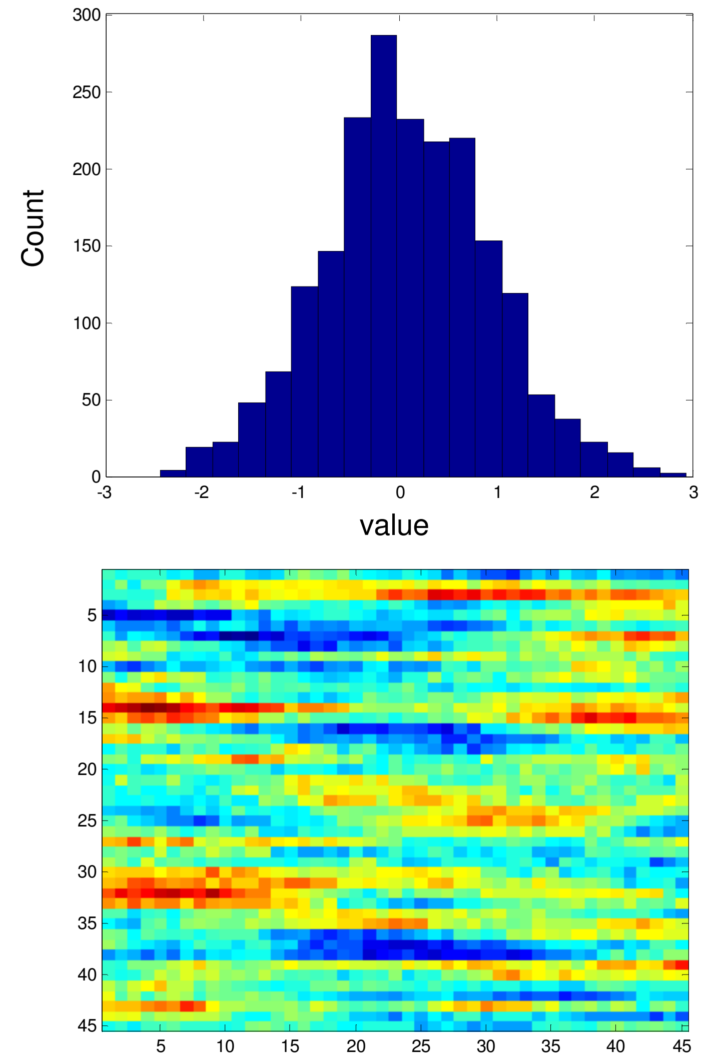
SGeMS realizations

PCA realizations: $y_{new} = \Phi_r \, \xi + \bar{y}$

Application of PCA: Gaussian Fields (2)

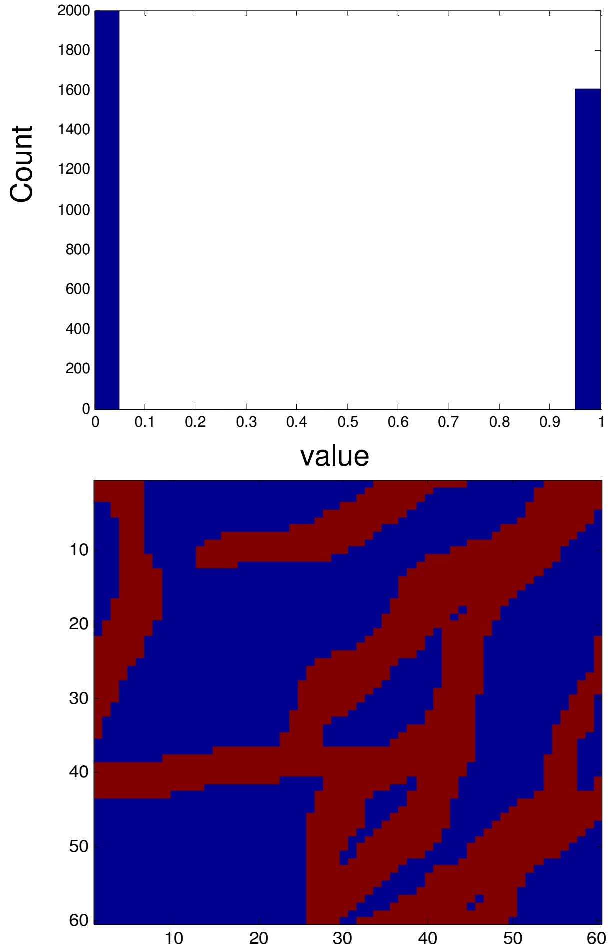


SGeMS realization

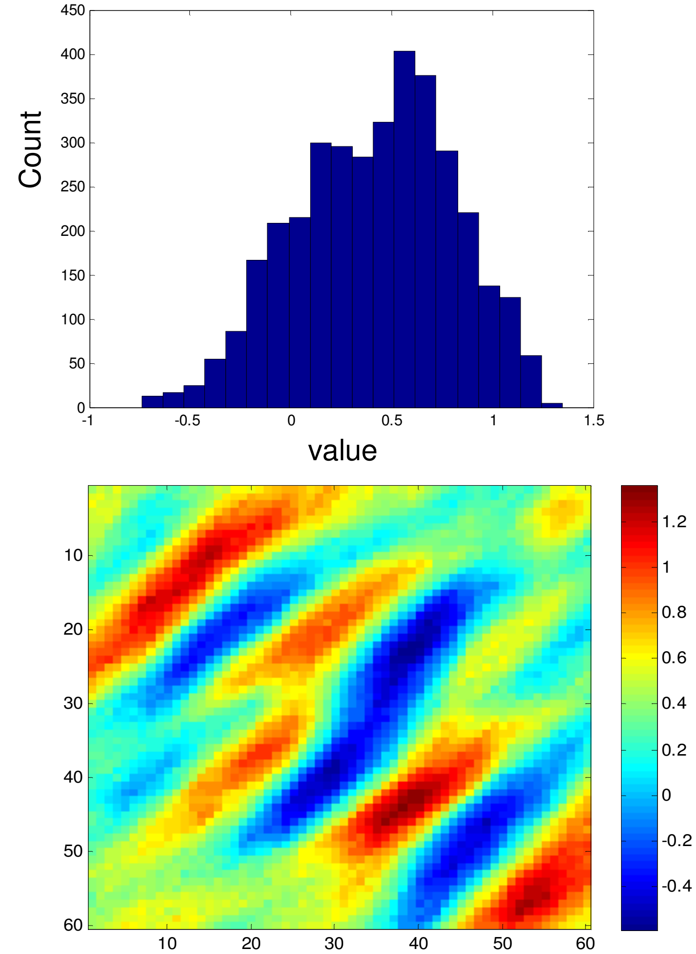


PCA realization: $y_{new} = \Phi_r \, \xi + \bar{y}$

Application of PCA: non-Gaussian Fields



SGeMS realization



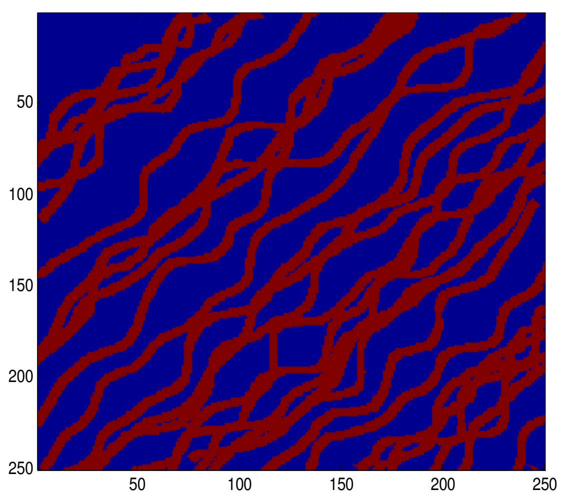
PCA realization

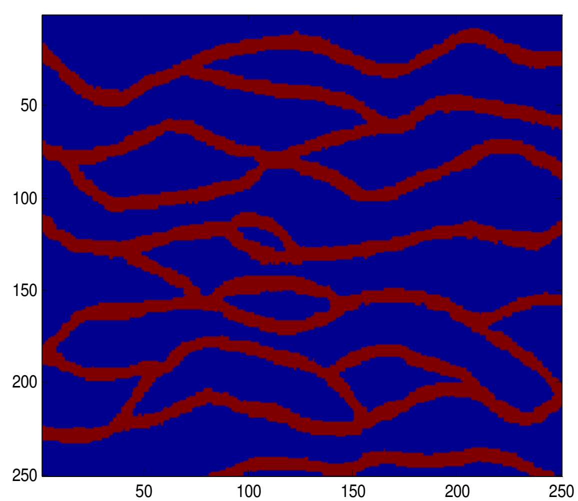
- Direct use of PCA leads to inconsistency for non-Gaussian models

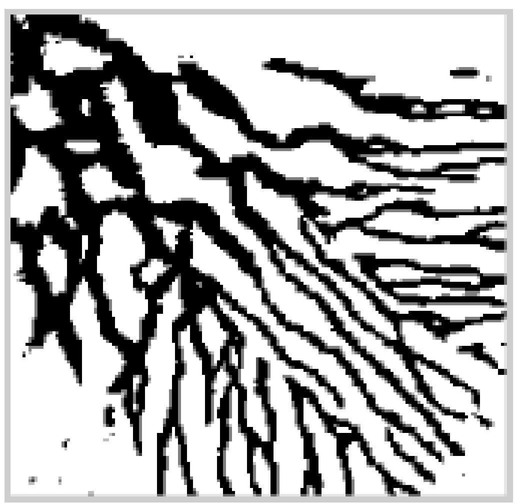
Modified PCA Procedure

- PCA is simple and linear but gives Gaussian-looking models and histograms

- Goal: modify PCA procedure to better represent complex geology







Optimization-based PCA (O-PCA)

- Standard PCA: $y_{new} = U_r \Lambda_r^{1/2} \xi + \bar{y}$

- Formulate PCA as an optimization problem:

$$y_{new} = \underset{z}{\mathbf{argmin}} \left\{ \left\| U_r \Lambda_r^{1/2} \xi + \bar{y} - z \right\|_2^2 \right\}$$

- Apply ***regularization + bound constraints*** $z \in [0, 1]$:

$$y_{new} = \underset{z}{\mathbf{argmin}} \left\{ \left\| U_r \Lambda_r^{1/2} \xi + \bar{y} - z \right\|_2^2 + \lambda z^T (1 - z) \right\}$$

- Separable quadratic optimization problem (***convex, unique, analytical*** solution for y_{new} and $dy/d\xi$)!

O-PCA Characteristics

- Define $\boldsymbol{a} = \boldsymbol{U}_r \boldsymbol{\Lambda}_r^{1/2} \boldsymbol{\xi} + \bar{\boldsymbol{y}}$. Now minimize:

$$\boldsymbol{f}(\boldsymbol{z}) = \|\boldsymbol{a} - \boldsymbol{z}\|_2^2 + \boldsymbol{\lambda} \boldsymbol{z}^T (\mathbf{1} - \boldsymbol{z})\} \,;$$

$$= (1 - \lambda) z^T \mathrm{z} - 2\{\boldsymbol{a} - \lambda/2\} \boldsymbol{z} + \textbf{constant}$$

$$(\boldsymbol{z} \in [\mathbf{0}, \mathbf{1}])$$

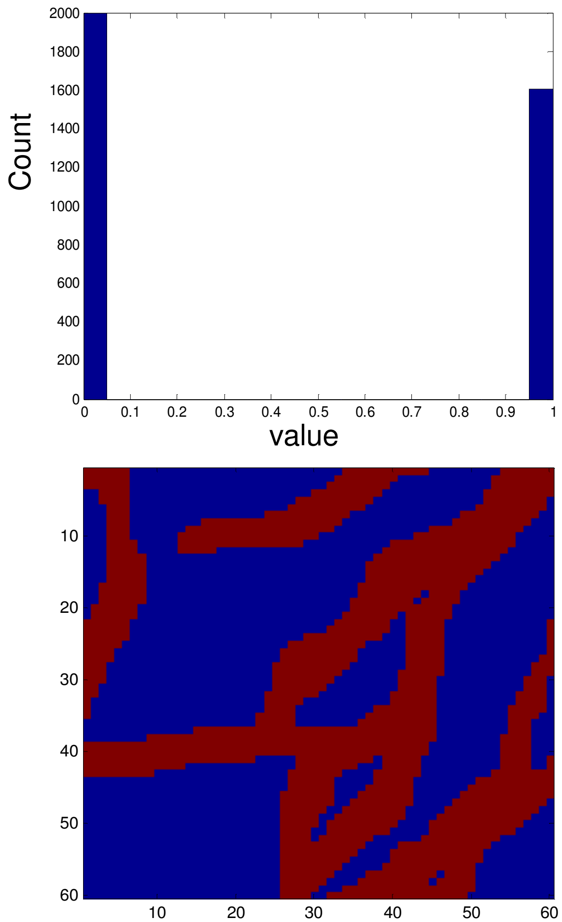
- Term depending on $\boldsymbol{z}$:

$$\boldsymbol{g}(\boldsymbol{z}) = \sum_{j=1}^{r} (\mathbf{1} - \lambda) z_j^2 - 2(\boldsymbol{a}_j - \lambda/2) \boldsymbol{z}_j$$

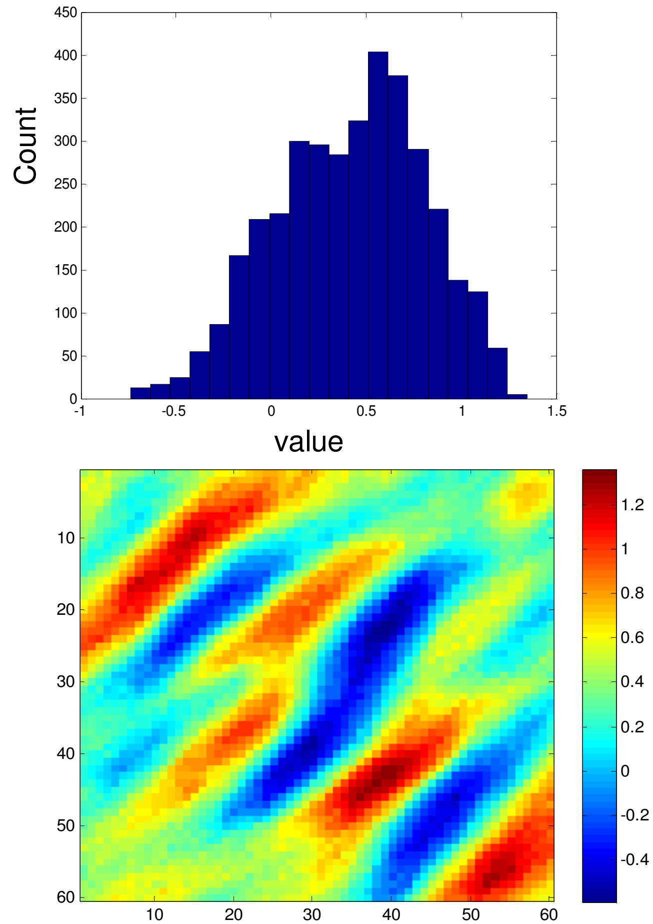
$$\boldsymbol{z}_j \in [\mathbf{0}, \mathbf{1}]$$

- Separable quadratic optimization problem (**convex, unique, analytical solution for $\boldsymbol{y}_{new}$ and $\boldsymbol{dy/d\xi}$**)!

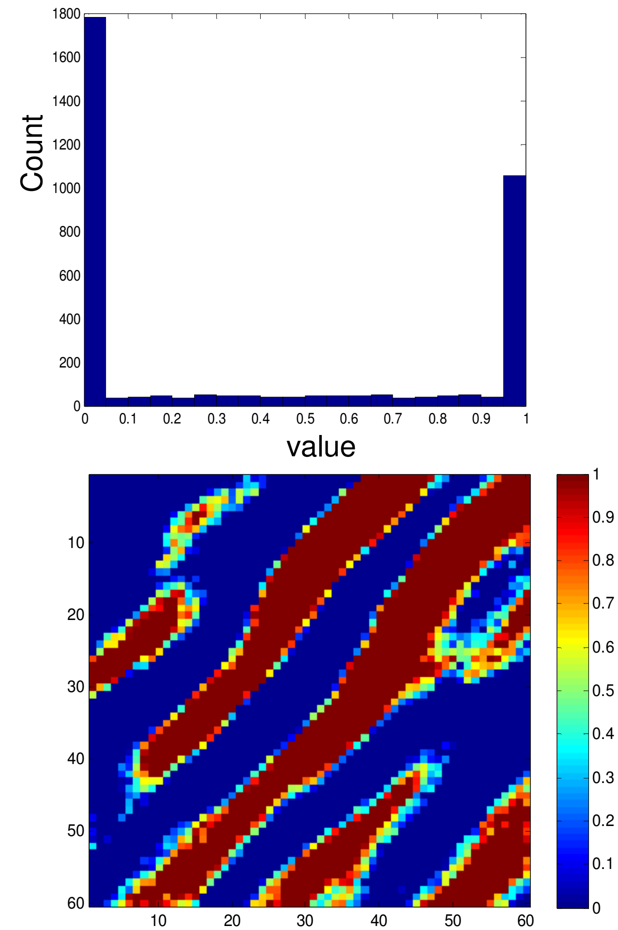
O-PCA Realization



SGeMS

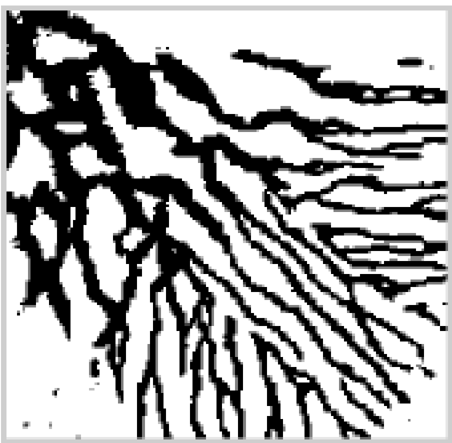


Standard PCA



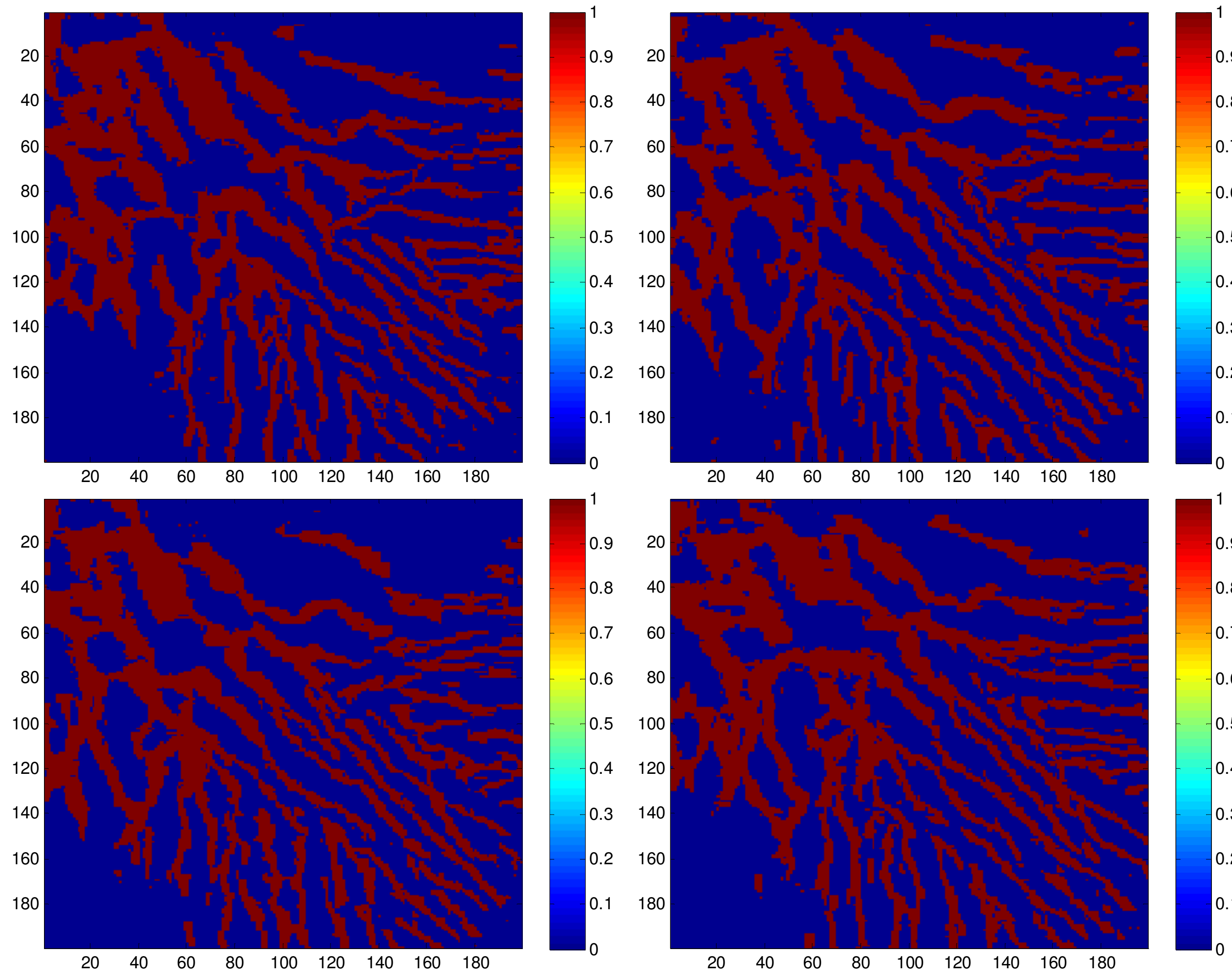
O-PCA

O-PCA for a non-Stationary Model



Training image

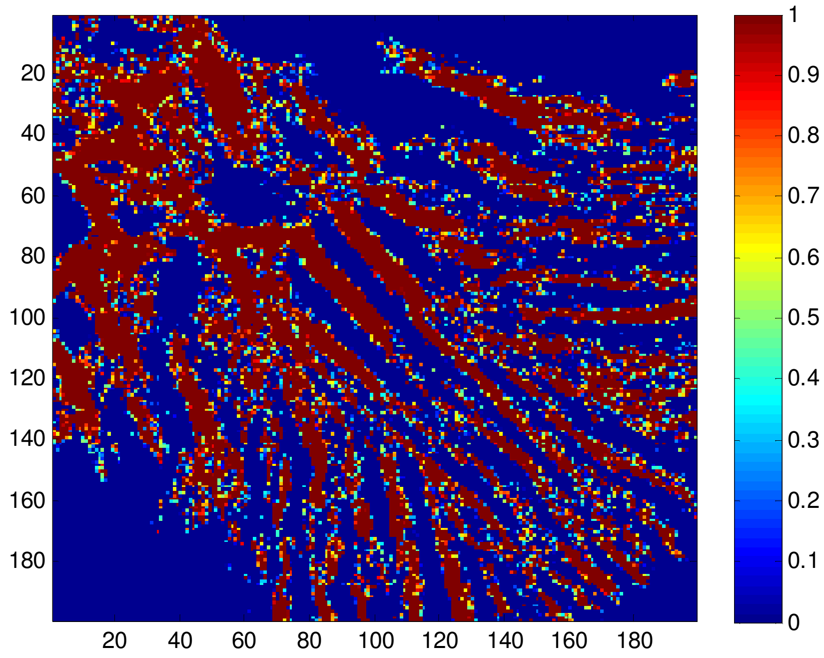
(Honarkhah and Caers, 2011)



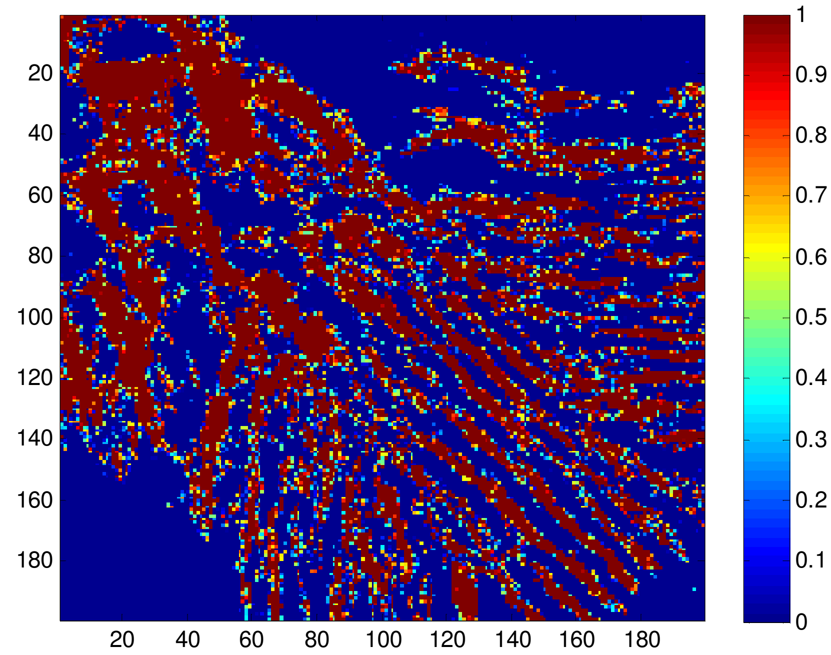
SGEMS realizations

O-PCA Results for a non-Stationary Model (1)

- 40,000 gridblock SGeMS realizations
- Represented using only 70 components of vector ξ

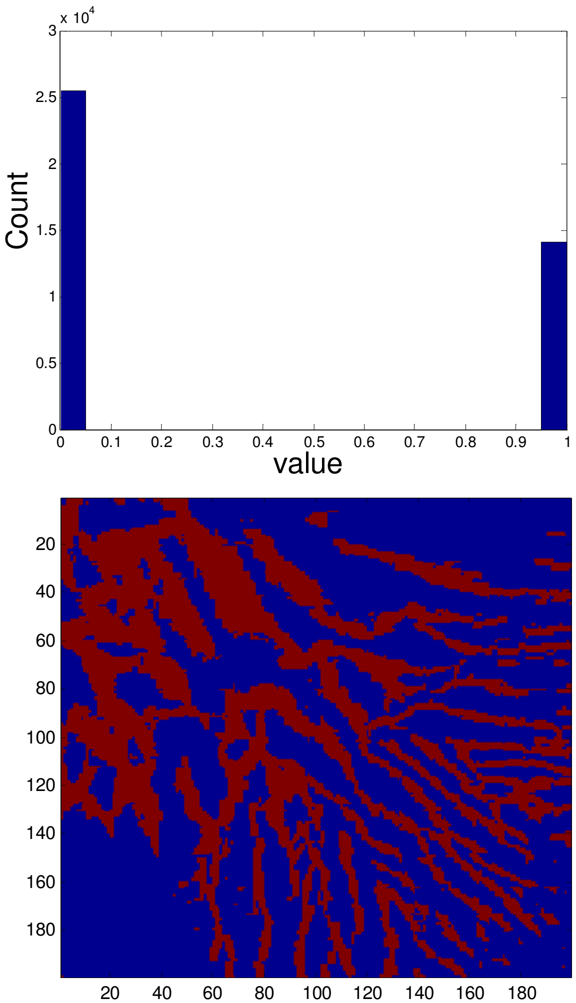


One realization

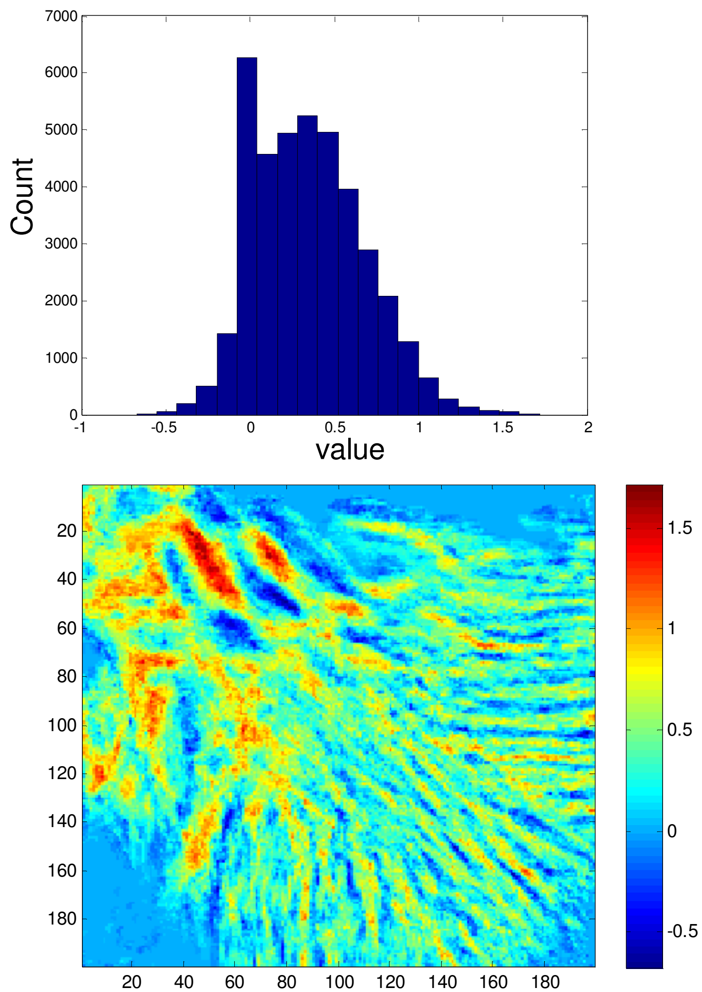


Another realization

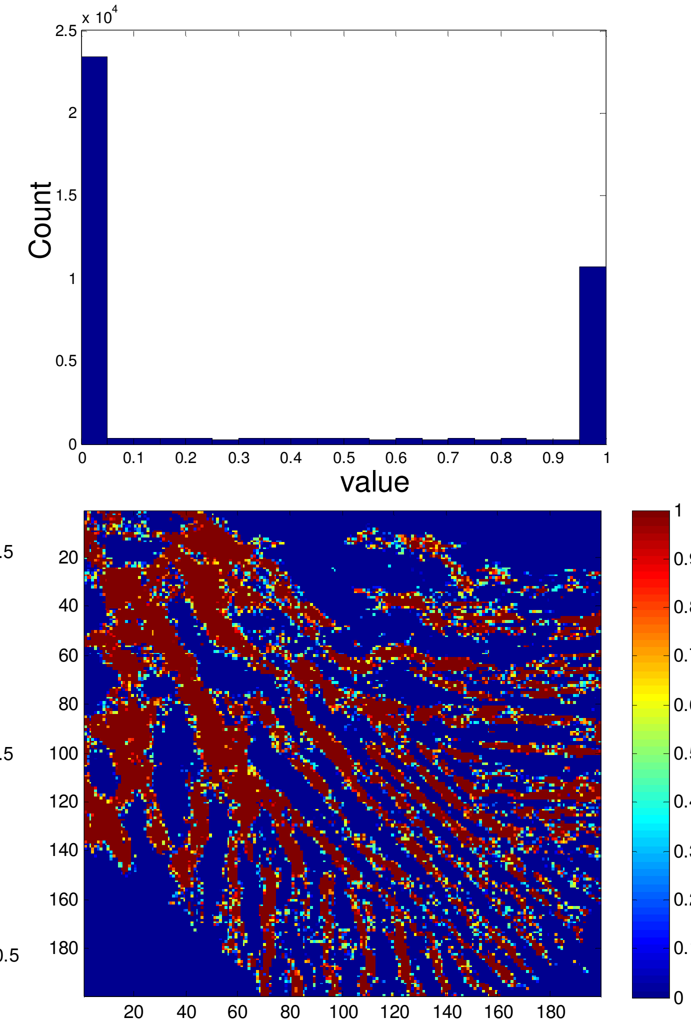
O-PCA Results for a non-Stationary Model (2)



SGeMS

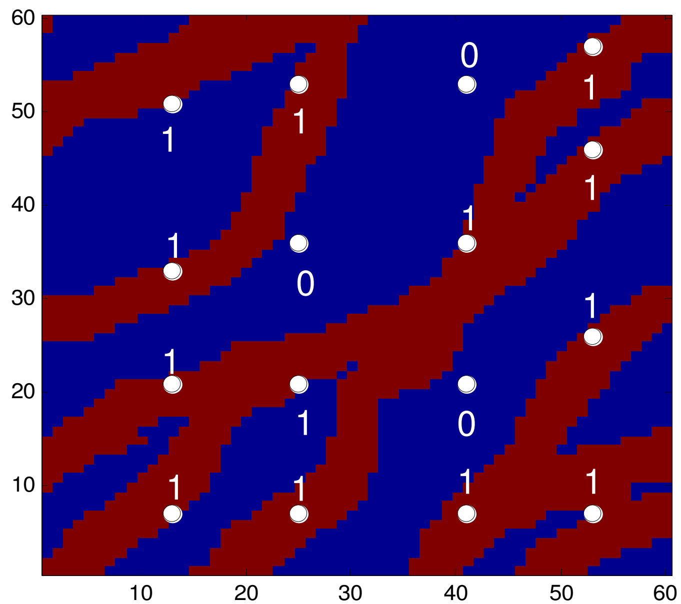


Standard PCA

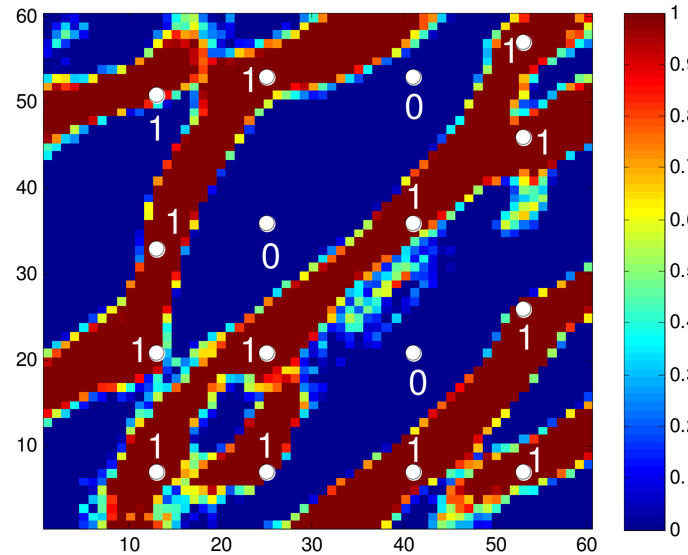


O-PCA

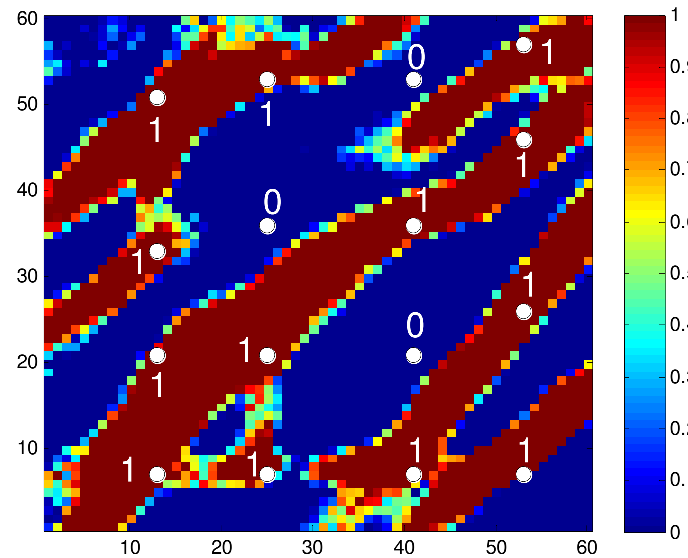
Honoring Hard Data with O-PCA



Hard Data



O-PCA



- Hard data is honored by O-PCA

O-PCA Analytical Gradient

- Need $dS/d\xi$ for history matching

$k = a * exp(b * y)$

- Construct using: $\frac{dS}{d\xi} = \frac{dS}{dk}\frac{dk}{dy}\frac{dy}{d\xi}$

from simulator

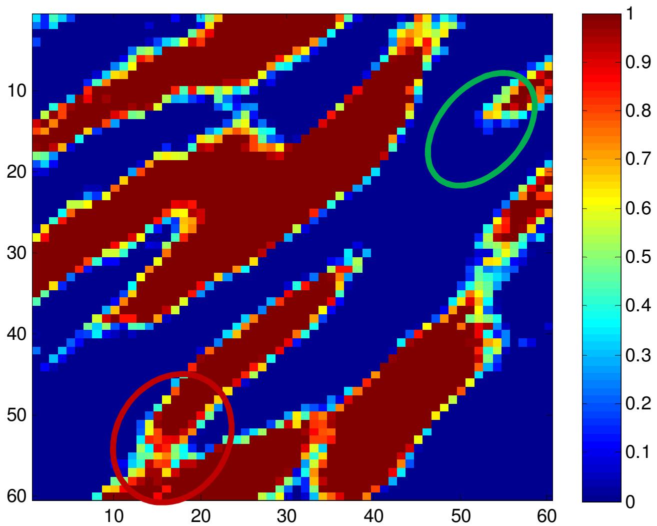
from O-PCA

- O-PCA gives $dy/d\xi$ **analytically**:

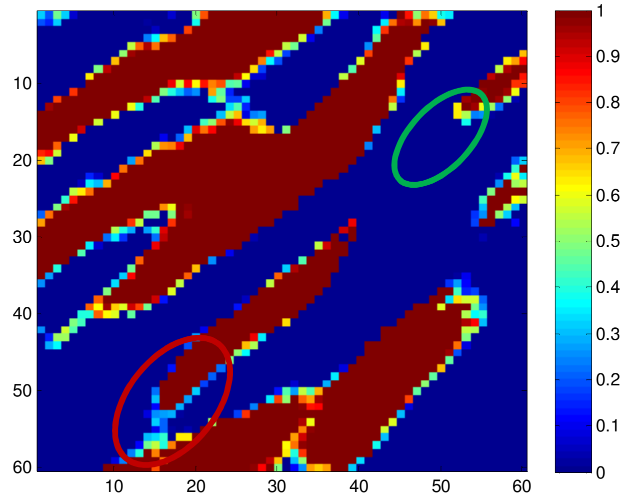
$$\frac{\partial y_i}{\partial \xi_j} = \frac{2y_i(1-y_i)\frac{\partial a_i}{\partial y_j}}{\mu_i + 2y_i + \eta_i - 2\lambda y_i - \mu_i y_i + 2y_i^2(\lambda - 1)}$$

μ_i, η_i = Lagrange multipliers

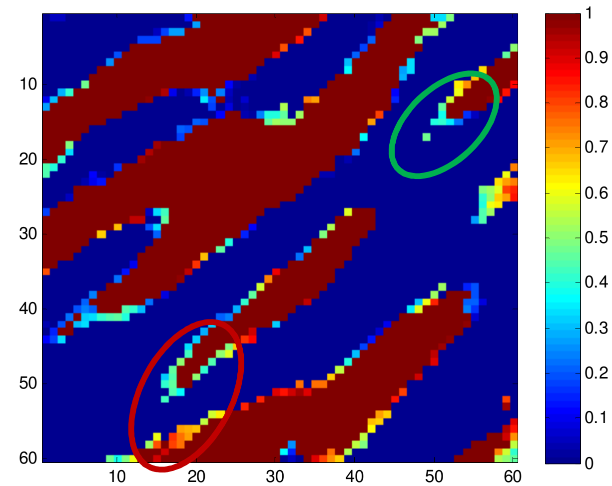
Differentiability of O-PCA Representation



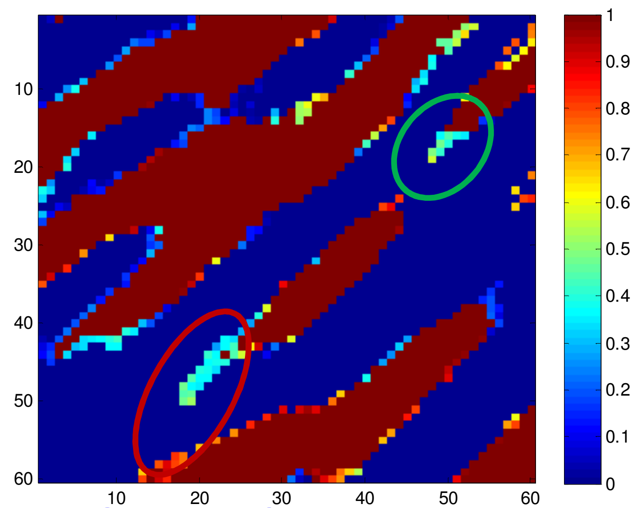
Current realization



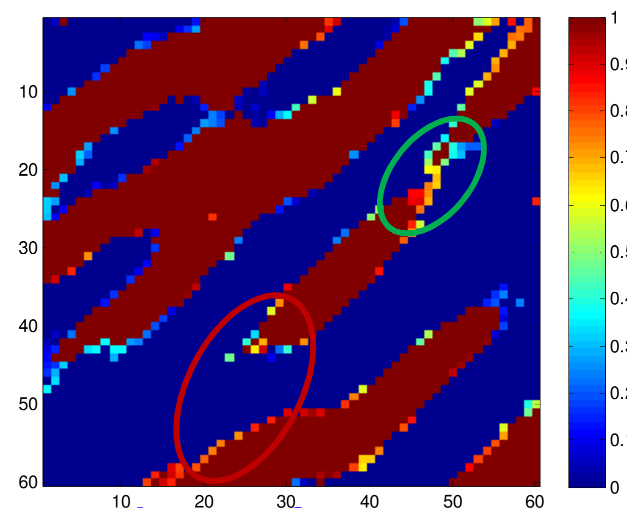
$\xi^1 = \xi^0 + \Delta\xi$



$\xi^2 = \xi^0 + 2\Delta\xi$



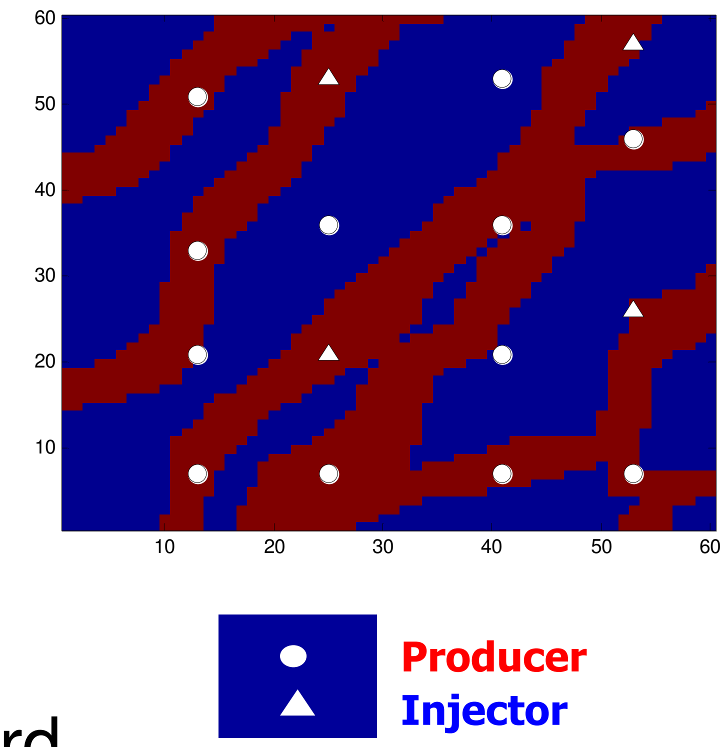
$\xi^3 = \xi^0 + 3\Delta\xi$



$\xi^4 = \xi^0 + 4\Delta\xi$

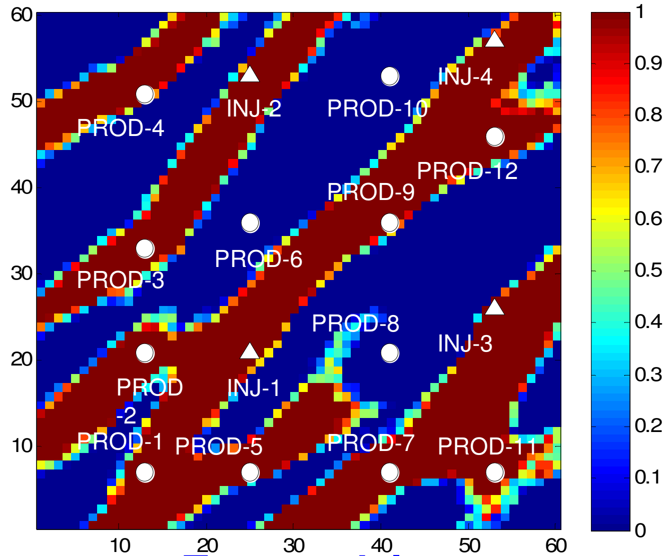
Application of O-PCA to a HM Problem

- Black-oil formulation with AD-GPRS
- $N_x = 60$, $N_Y = 60$; $l = 30$ components
- 4 water injectors at 1,000 m^3/d
- 12 producers, BHP control at 150 bar
- $k_{sand} = 2{,}000$ md, $k_{mud} = 20$ md
- Using injection pressures and production rates for history matching
- SGeMS realizations conditioned to hard data at wells
- Using off-the-shelf LBFGS optimizer (not tuned)

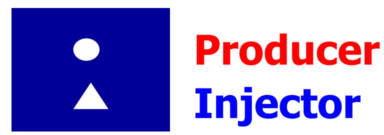


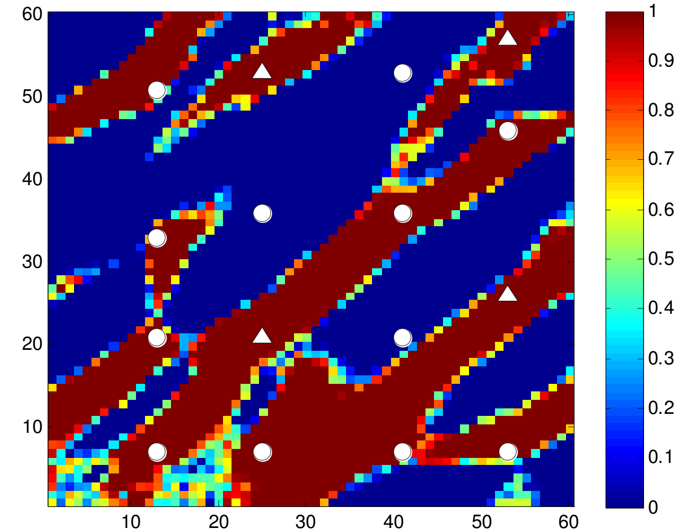
SGeMS realization

Results of O-PCA History Matching

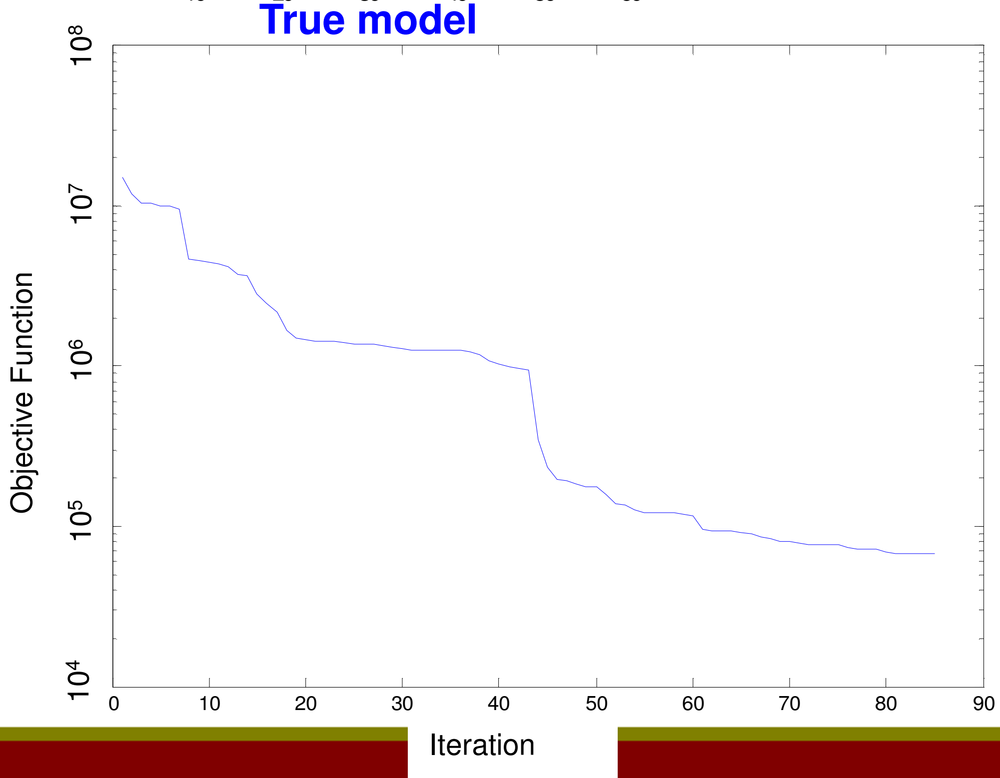


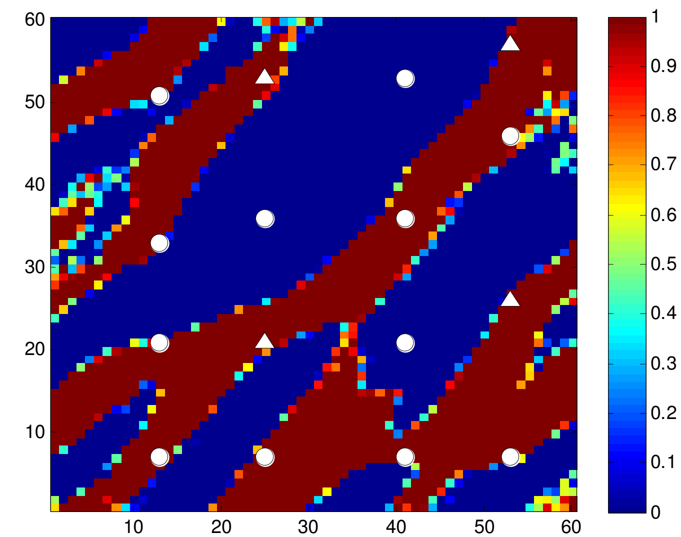
True model





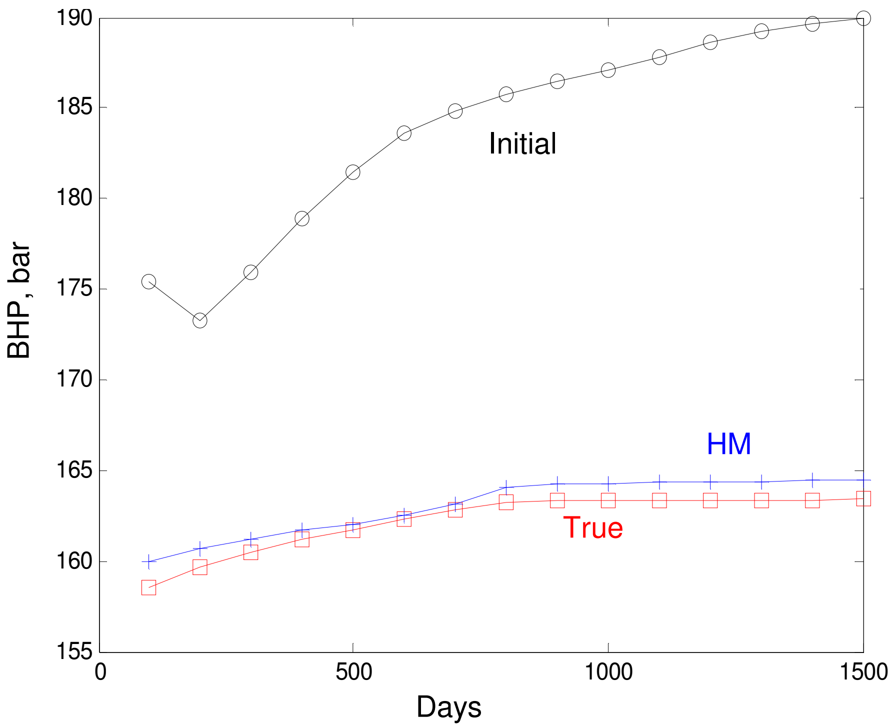
Initial guess



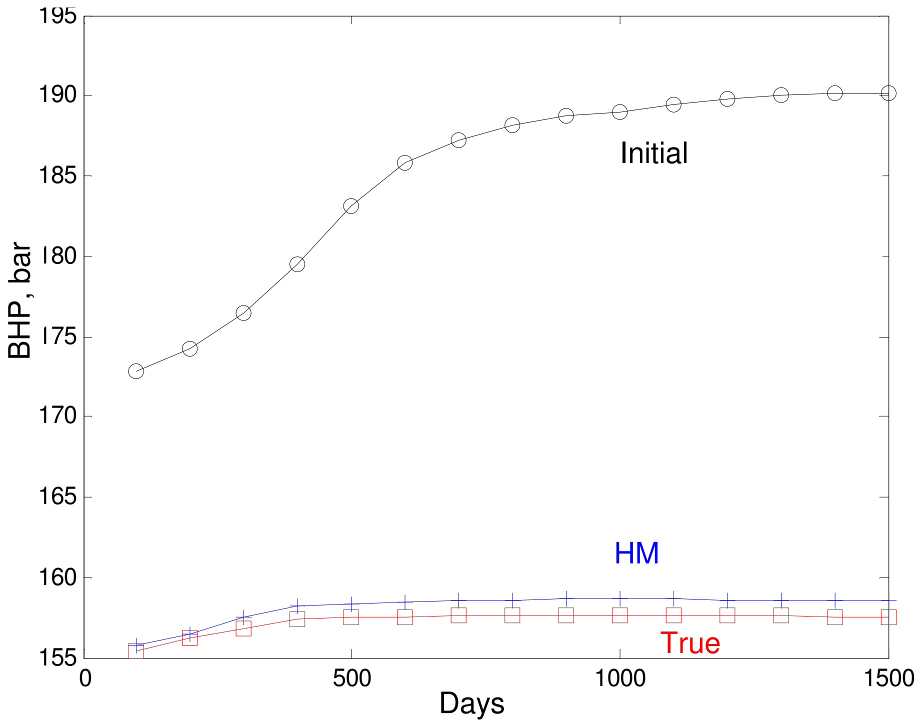


History matched

Results of O-PCA History Matching
Injection BHP's



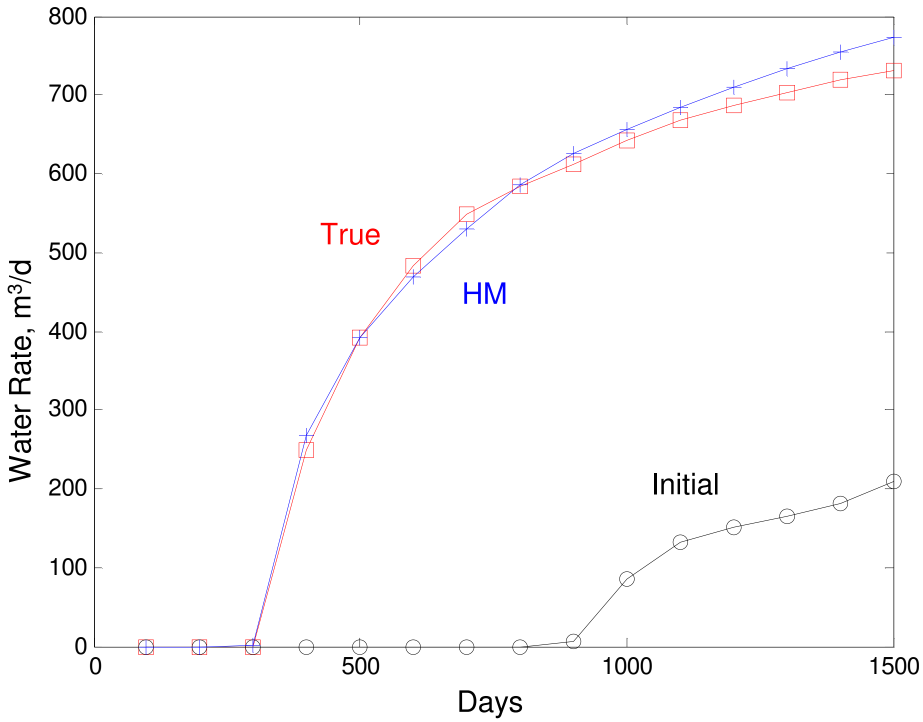
INJ-2



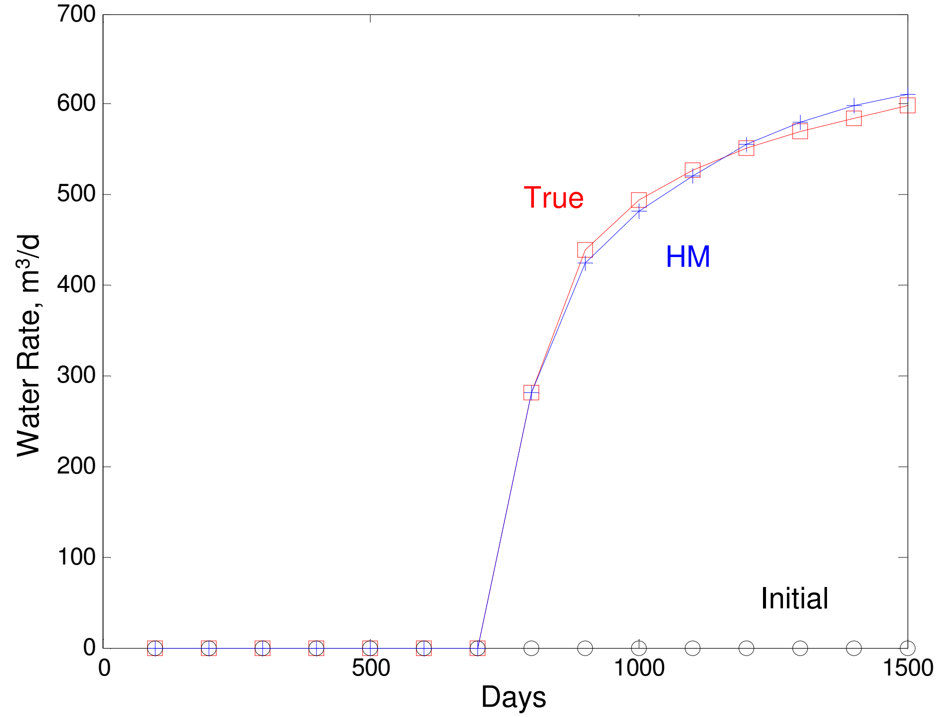
INJ-4

Results of O-PCA History Matching

Water Rates



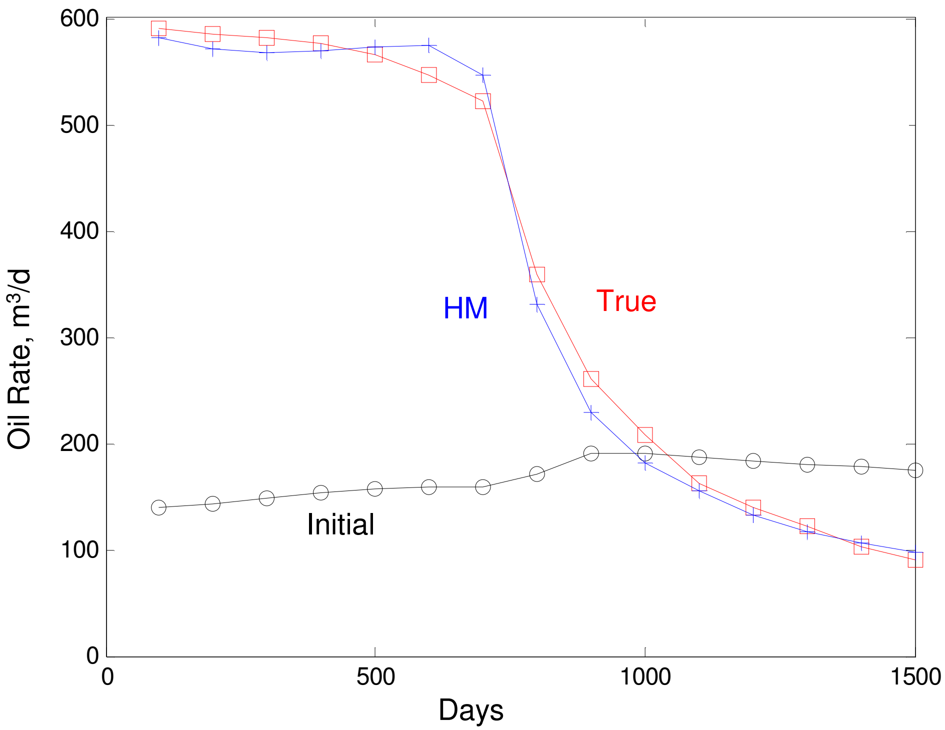
PROD-12



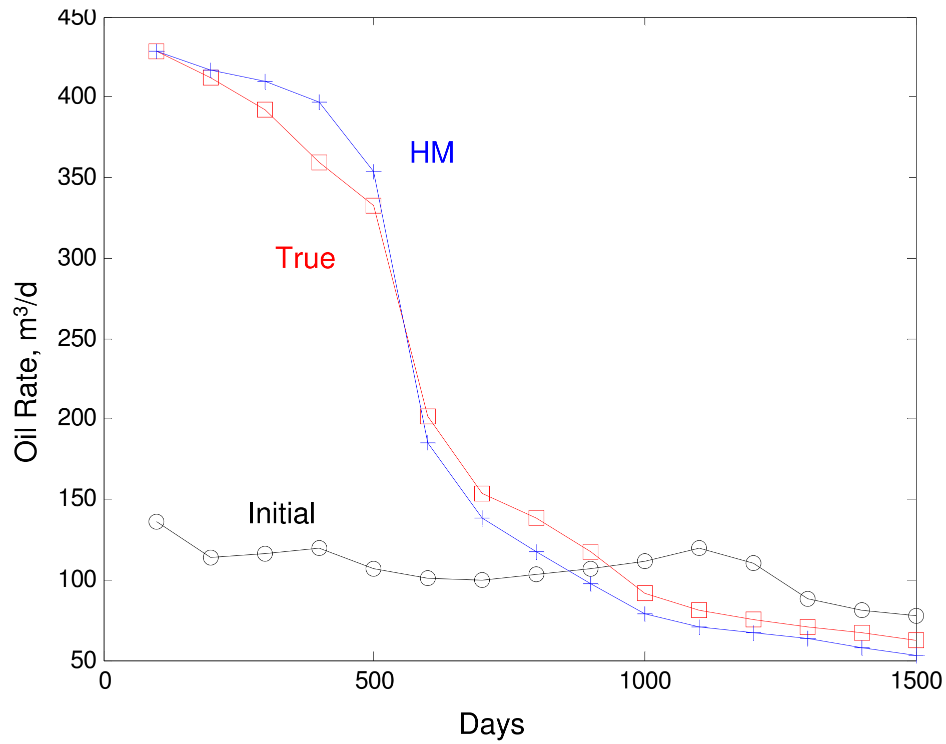
PROD-3

Results of O-PCA History Matching

Oil Rates



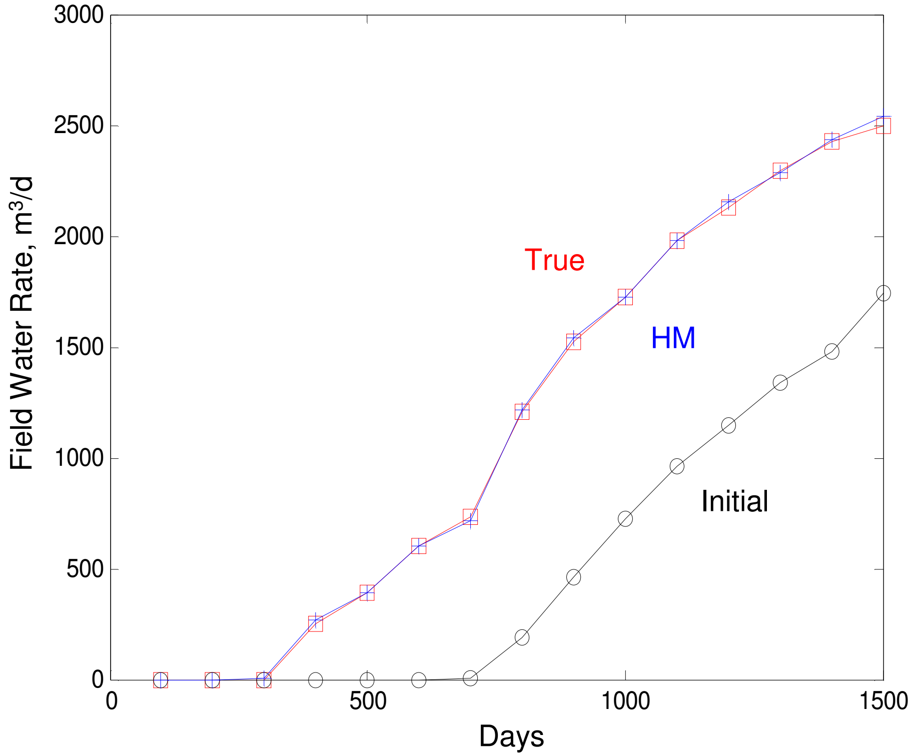
PROD-11



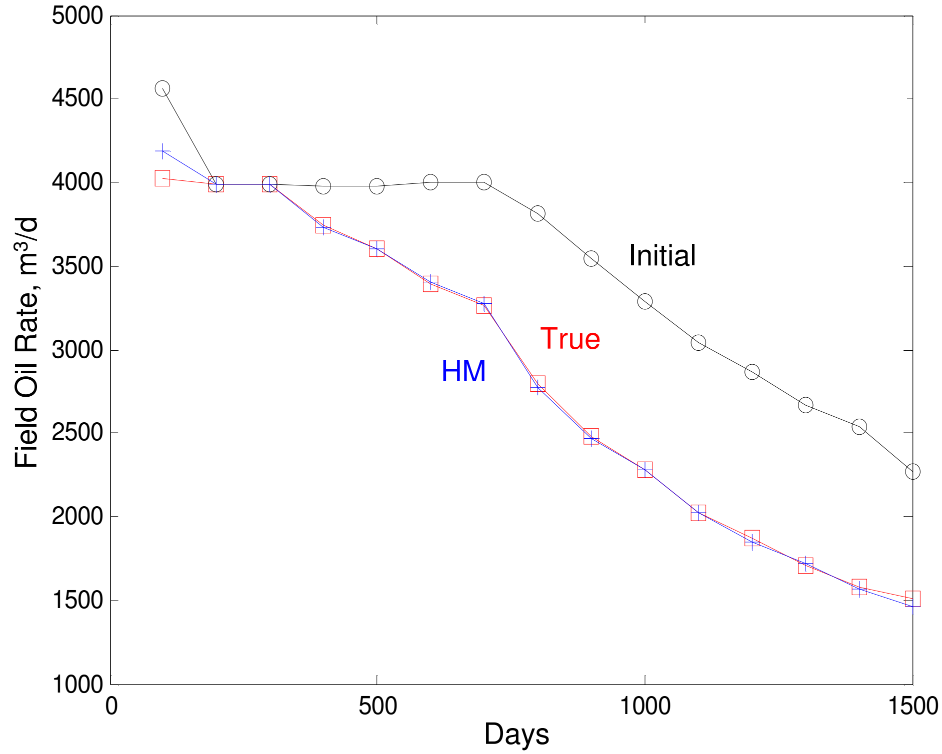
PROD-2

Results of O-PCA History Matching

Field Rates

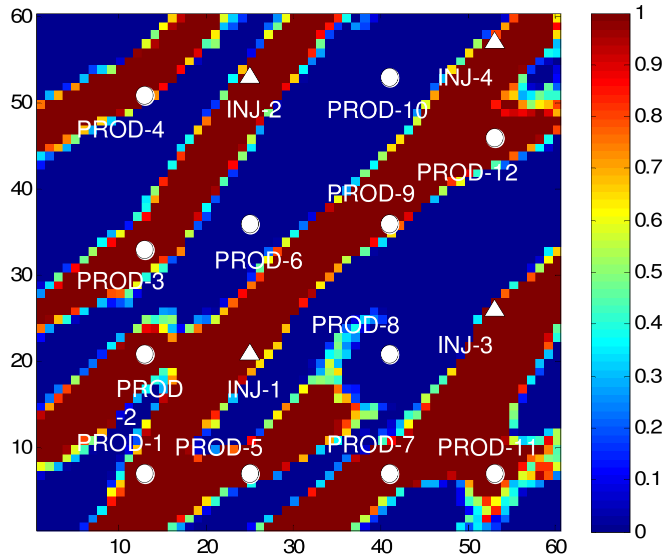


Water

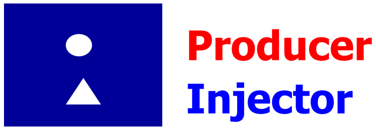


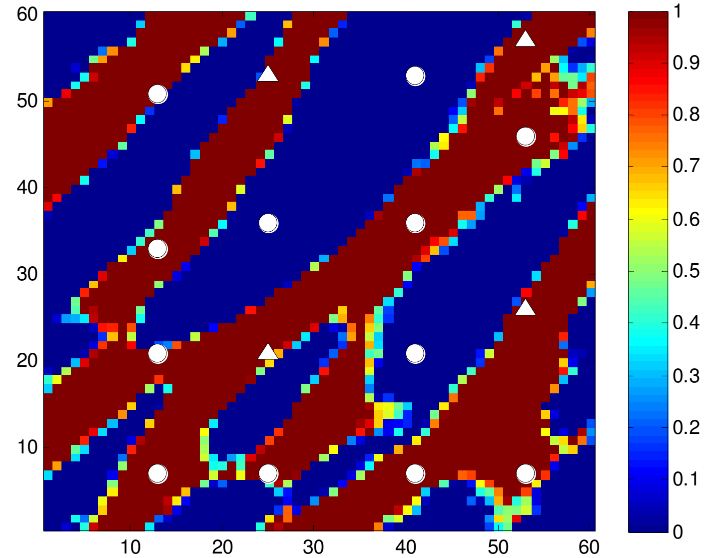
Oil

HM Results for Another Initial Guess

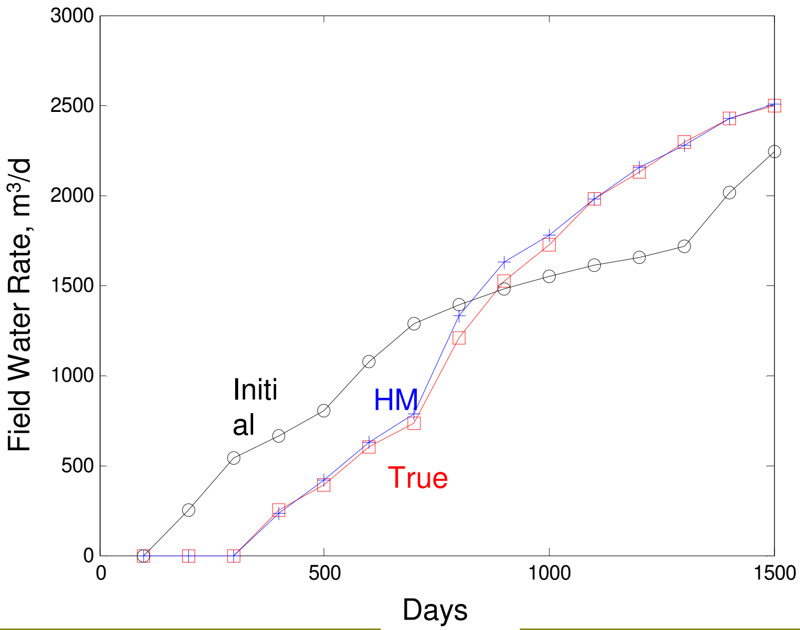


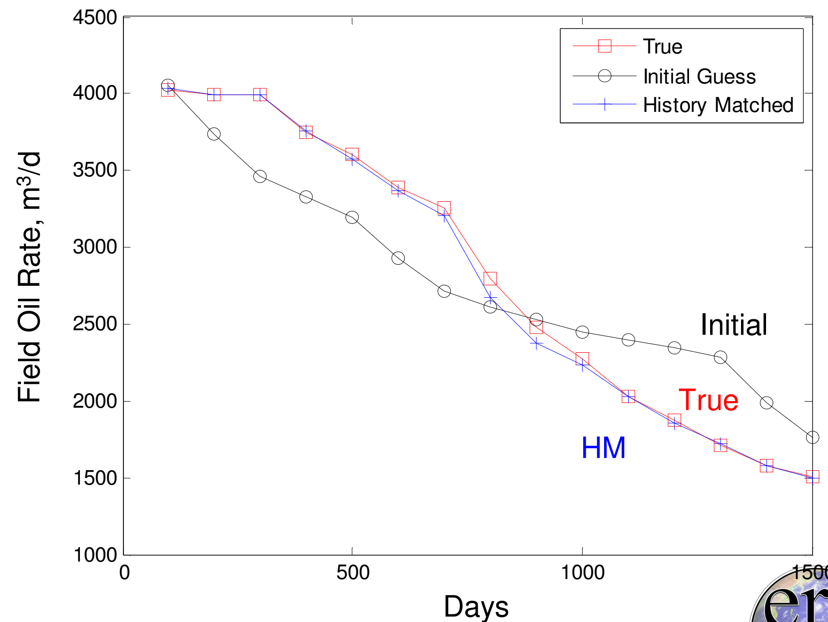
True model





History matched





Conclusions

- Introduced optimization-based PCA (O-PCA) to address limitations of standard PCA
- O-PCA representation honors hard data and is continuously differentiable
- O-PCA provides realistic realizations and can be solved analytically
- Results from history matching show O-PCA capabilities