Alternative methods for solving power flow problems

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Outline

1. **introduction**
2. **available methods**
3. **contributions and proposed methods**
4. **implementation**
5. **experiments**
6. **conclusions**

1. Introduction

1.1. **background**

1.2. **formulation**

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  - real and reactive power flows through every branch

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- fast and reliable solution methods needed

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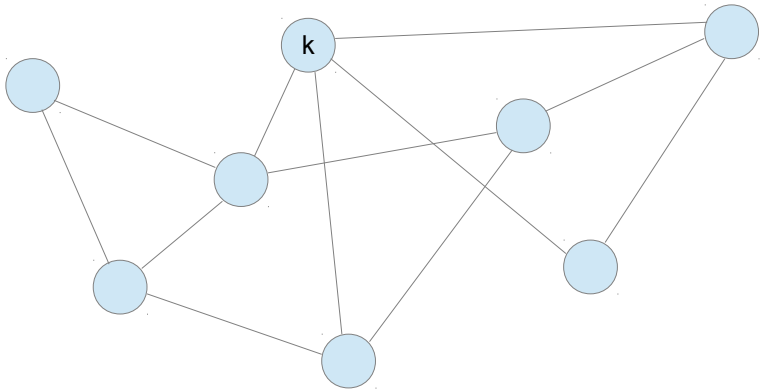

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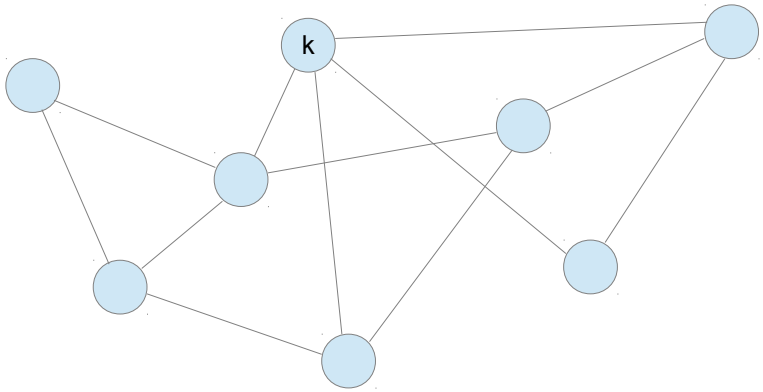

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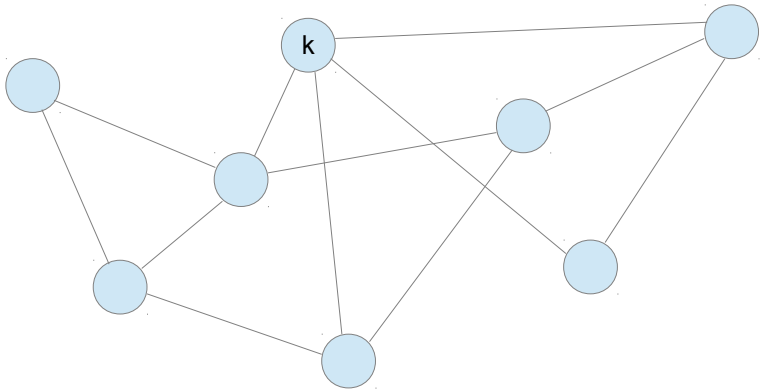

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gives power flow equations [5][6]

$$\Delta P_k = P_k^g - P_k^l - \sum_{m \in [n]} v_k v_m (G_{km} \cos(\theta_k - \theta_m) + B_{km} \sin(\theta_k - \theta_m)) = 0$$

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- bus $i \in \mathcal{U}$: unregulated
  - active and reactive power given (PQ)

2. Available methods

2.1. **overview**

2.2. **Newton-Raphson**

2.3. **Newton-Krylov**

2.4. **optimal multiplier**

2.5. **continuous Newton**

2.6. **sequential conic programming**

2.7. **holomorphic embedding**

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- Newton-Raphson (NR) method [5][47]
  - more robust, better convergence properties
  - good when combined with sparse techniques
  - most widely used method

- improving NR
  - speed: fast decoupled [5], DC power flow, simplified XB and BX [47]
  - scalability: Newton-Krylov [13][14][15][20][26][32][35][45]
  - robustness: integration [34] and optimal multiplier [9][11][27][41][42]

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- other methods
  - genetic, neural networks and fuzzy algorithms
    * not competitive with NR-based method [44]
  - sequential conic programming [28][29]
  - holomorphic embedding [43]

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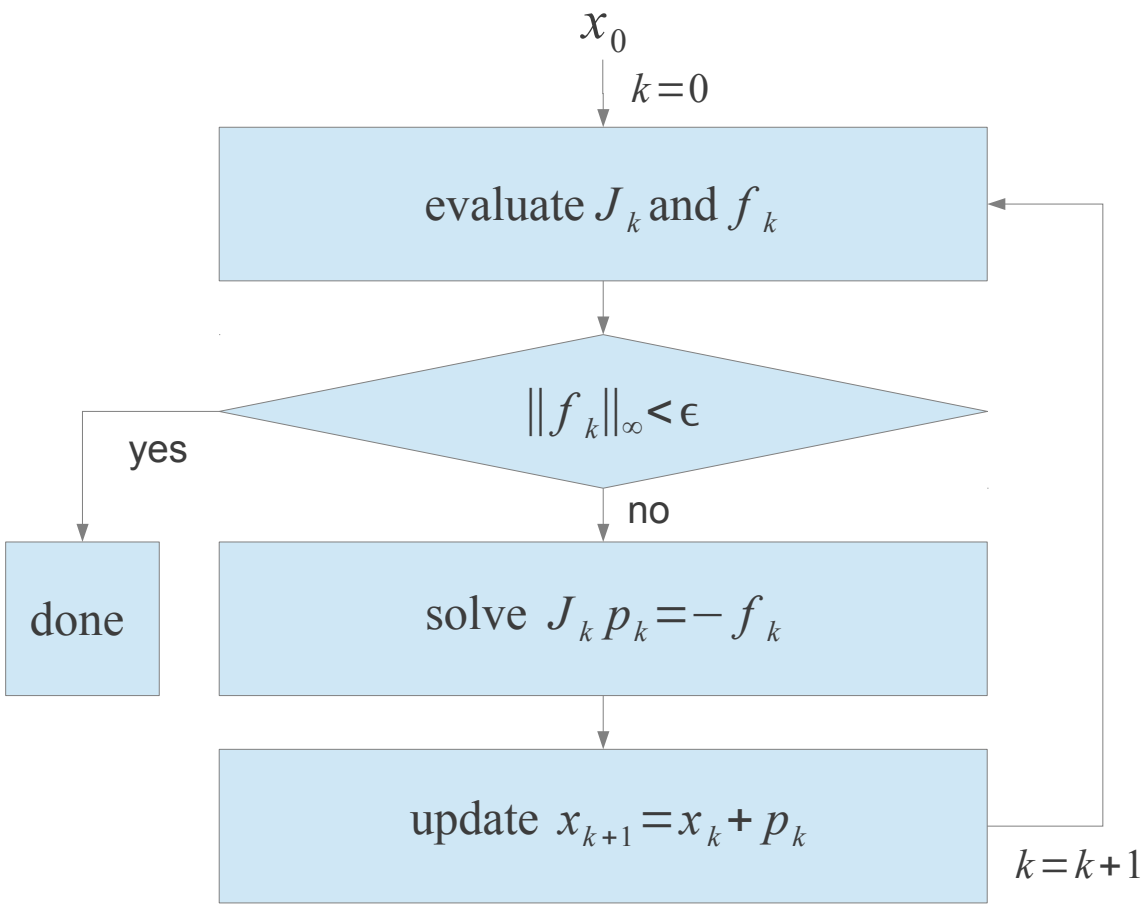
properties: [36]

- quadratic rate of convergence
- only locally convergent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  - If violation
    - $Q_i^g \rightarrow$ fixed
    - $\mathcal{R} \rightarrow \mathcal{U}$
    - $v_i \rightarrow$ free
  - more complex ones for $\mathcal{R} \leftarrow \mathcal{U}$ [46]

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preconditioners

- polynomial [14][32]
- LU-based [20][26][35][45]

J_k changes little $\rightarrow$ same preconditioner, multiple iterations [15][20][26]

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extend to polar [9][11][27][41]

- polar sometimes preferable [42]
- can only minimize approx. of $\|f(x_k + \alpha_k p_k)\|_2^2$, no guarantees

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Runge-Kutta [34]

- more robust than NR
- less iterations than the optimal multiplier method (UCTE network)
- no speed comparison

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for meshed networks [29] (same author)

- extra constraints: zero angle changes around every loop
  - destroy convexity
- linearize and solve sequence of conic programs (expensive)

relatively large number of iterations [29]

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very strong claims in [43]:

- algorithm finds solution iff "solution" exists

few experimental results

3. Contributions and proposed methods

3.1. **line search**

3.2. **starting point**

3.3. **formulation and bus type switching**

3.4. **real networks**

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measure progress with $h = \frac{1}{2} f^T f$ (merit function)

use controlled update $x_{k+1} = x_k + \alpha_k p_k$

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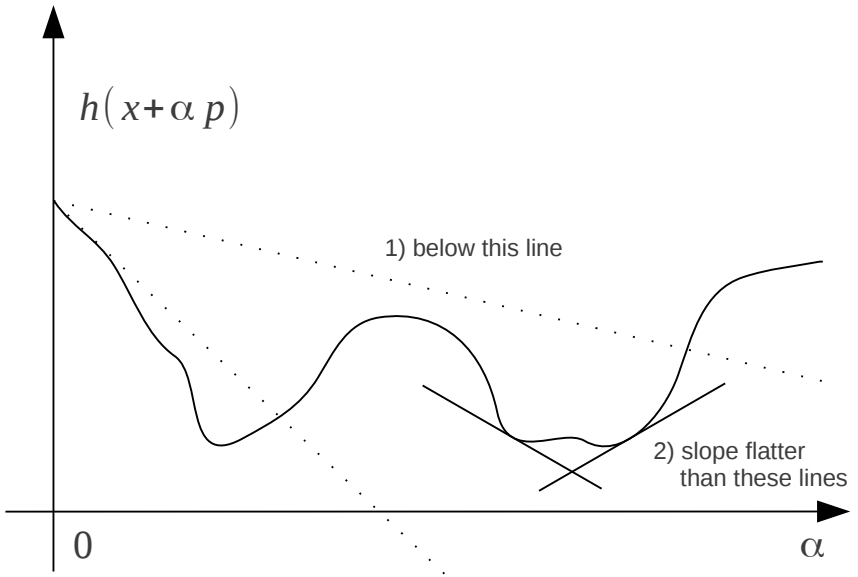
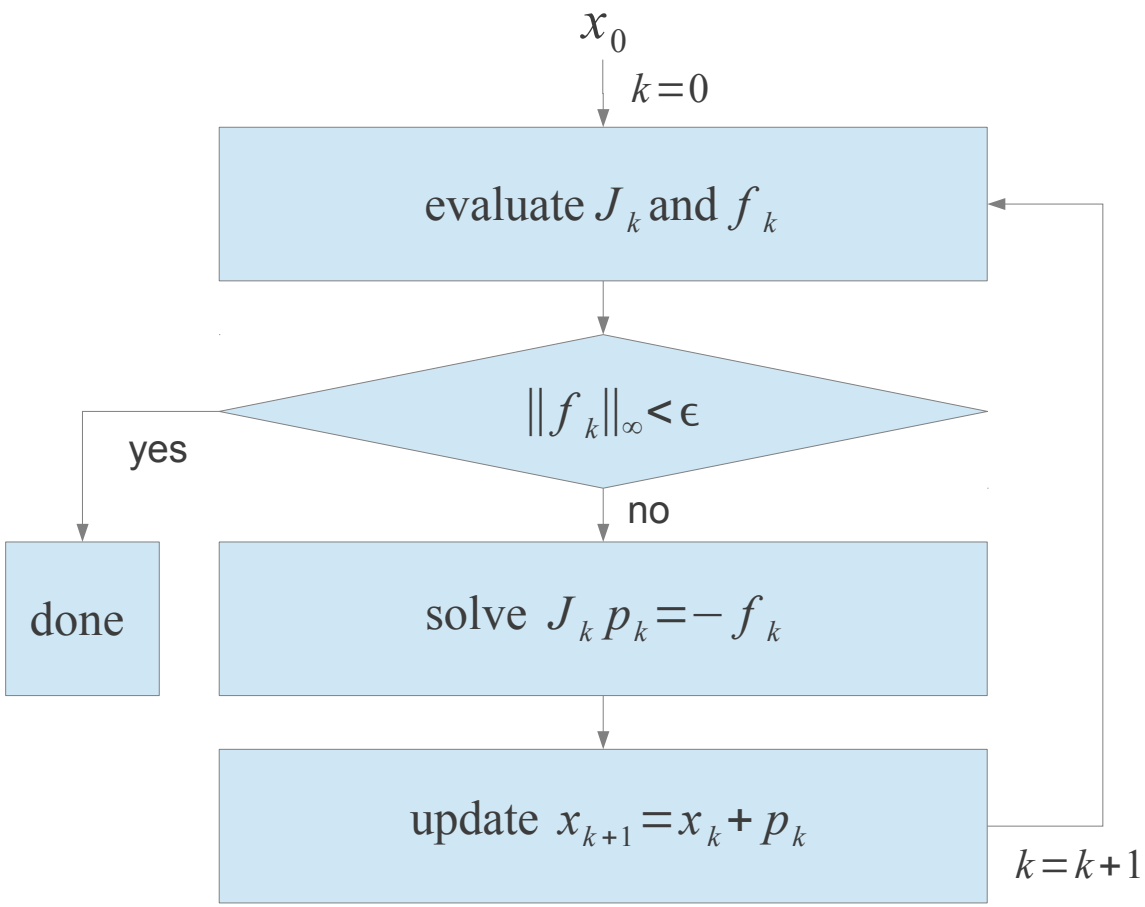
- α_k satisfies strong Wolfe conditions [38]

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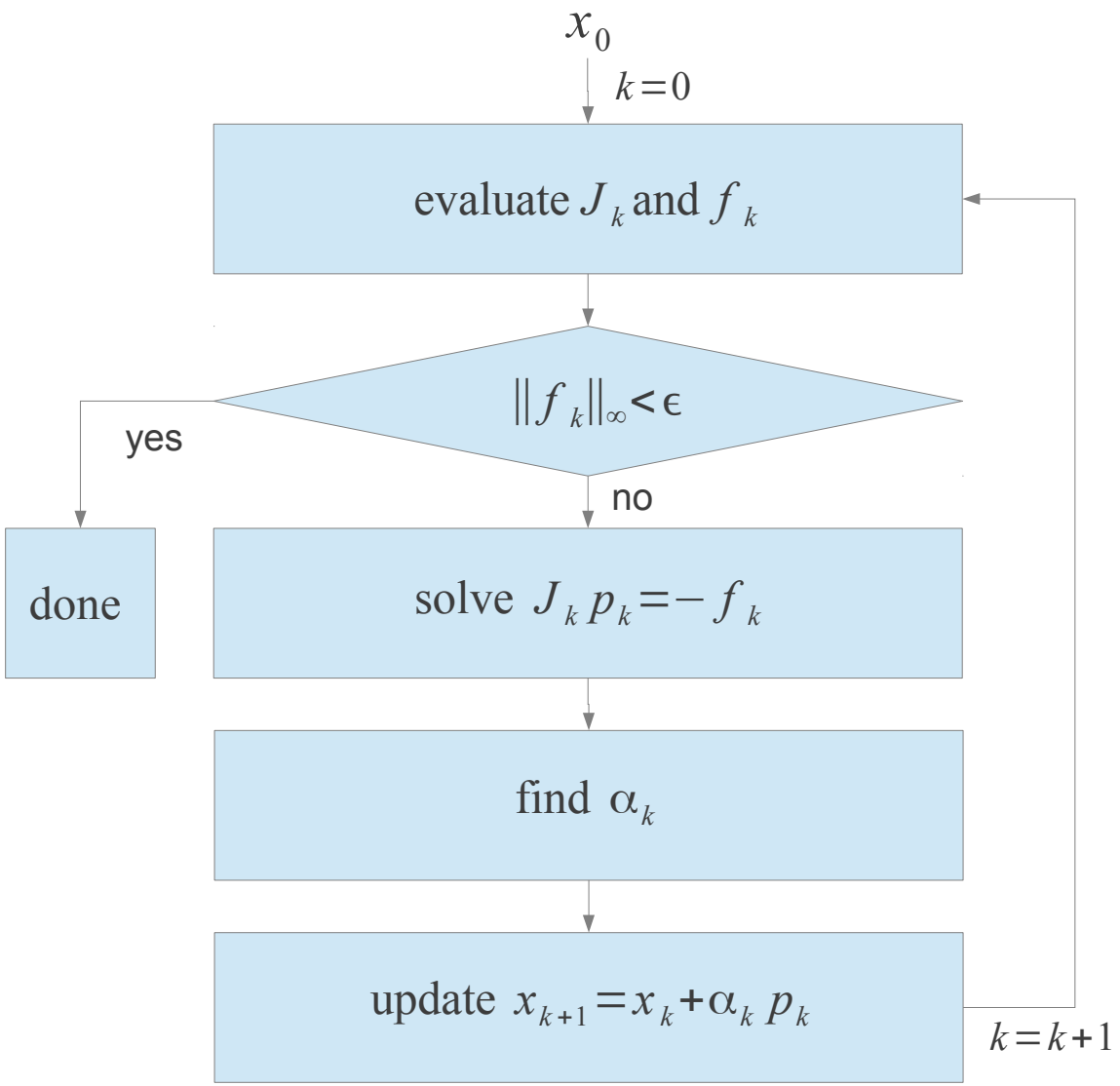
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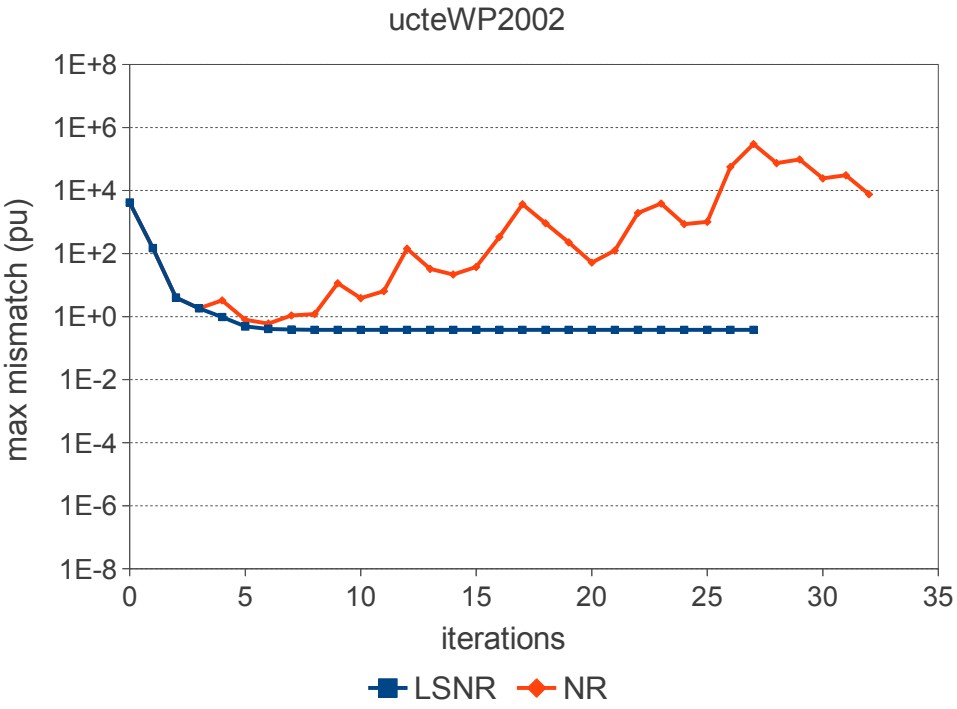
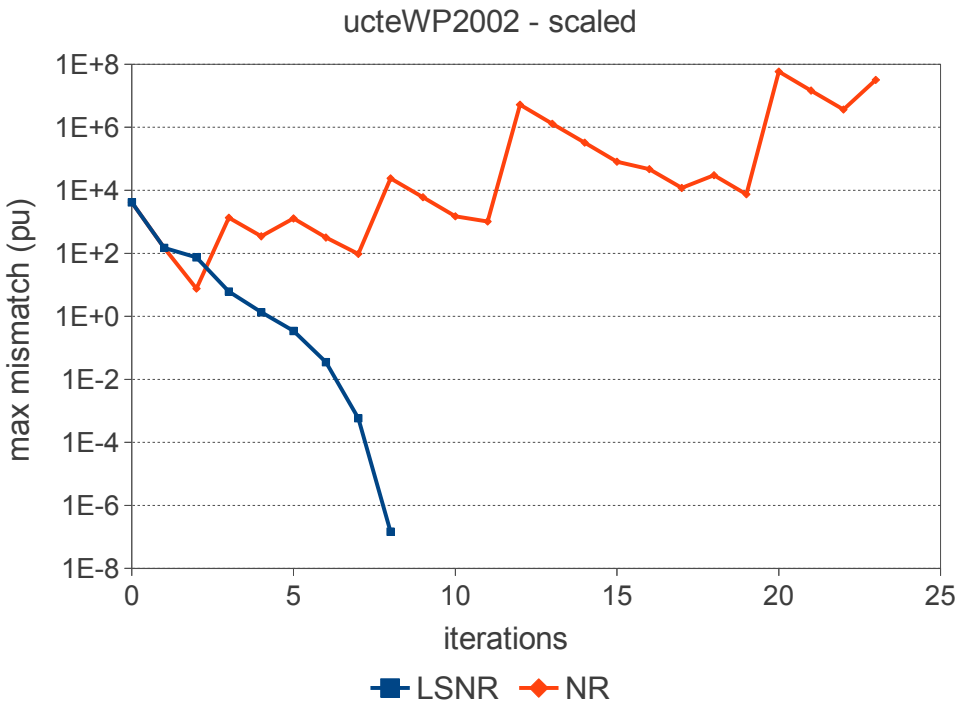
benefits





scaling: reduce net injection by factor of 1.07 approx.

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method: Flat Network Homotopy (FNH) (phase shifters only)

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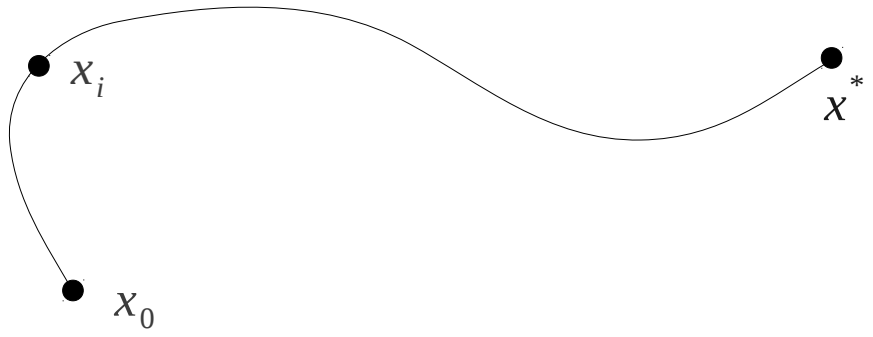
approx. solve $f(x,t_i) = 0$ for $i \in \mathbb{N}$, $t_1 = 0$, $t_i \uparrow 1$ as $i \to \infty$



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starting points:

- subproblem $i = 1$ (flat network), use flat start
- subproblem $i > 1$, modify x_{i-1}

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methods: Vanishing Regulation (VR), Penalty-Based Power Flow (PBPF)

analogy ...

Q violations

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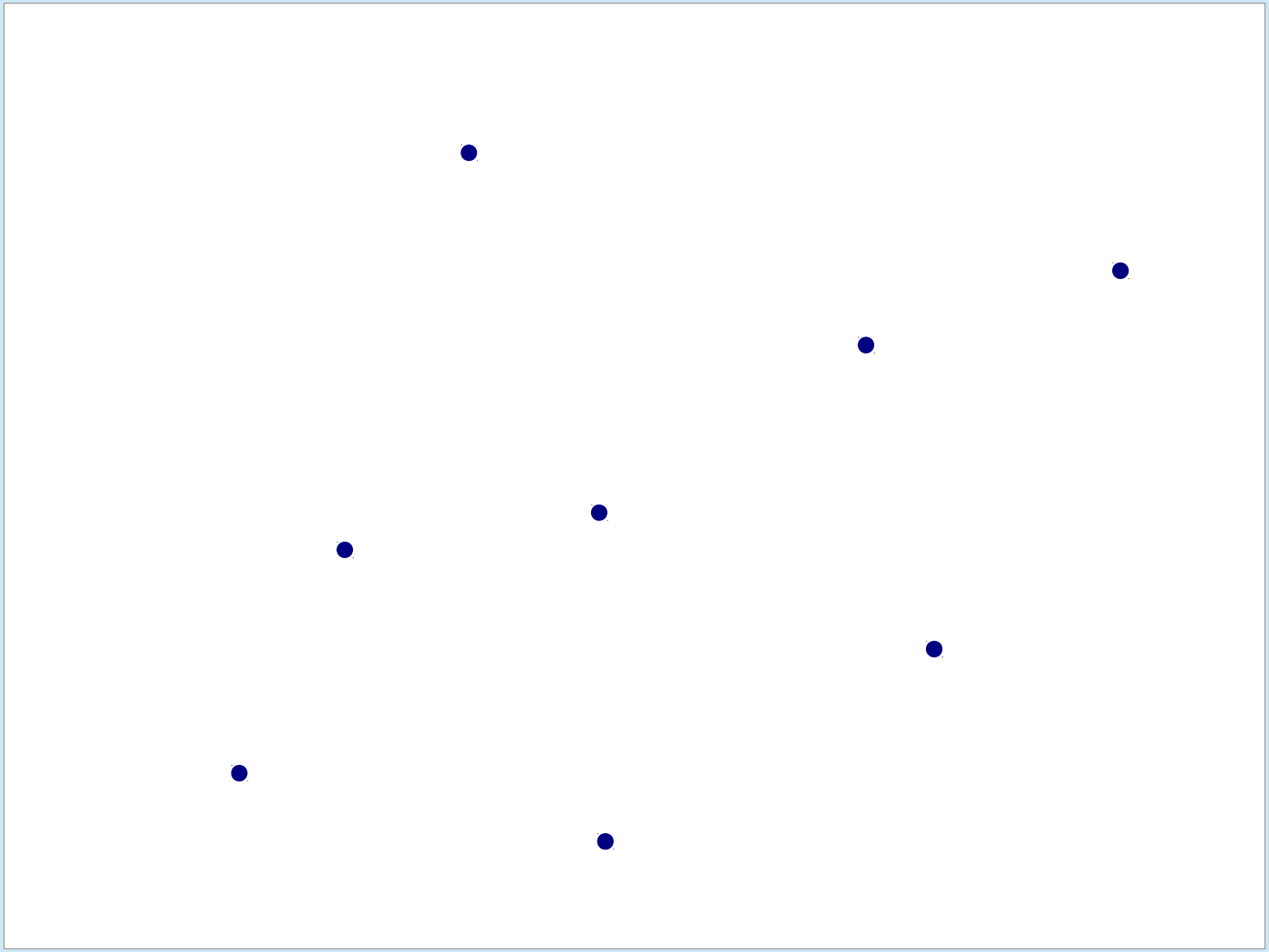
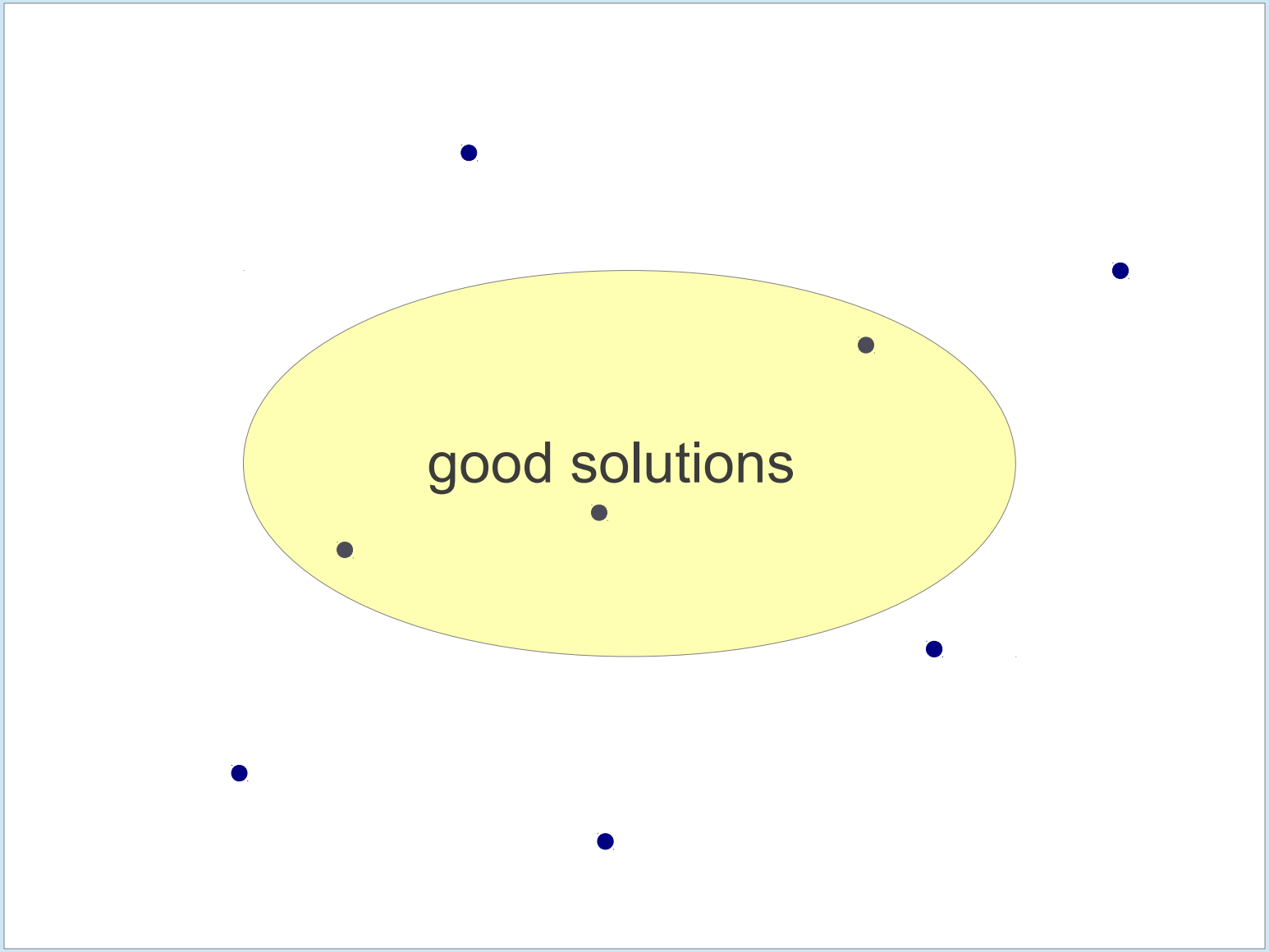
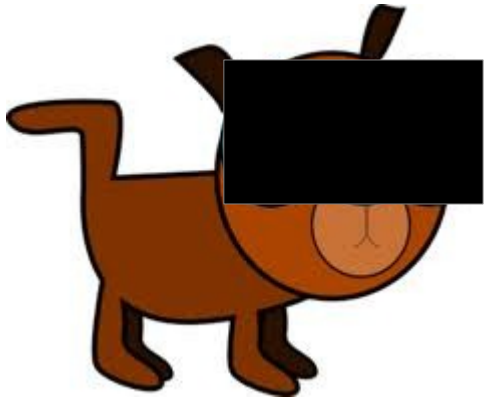


power flow solutions


good solutions

Algorithm



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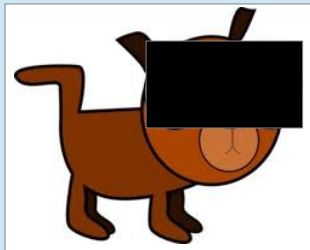
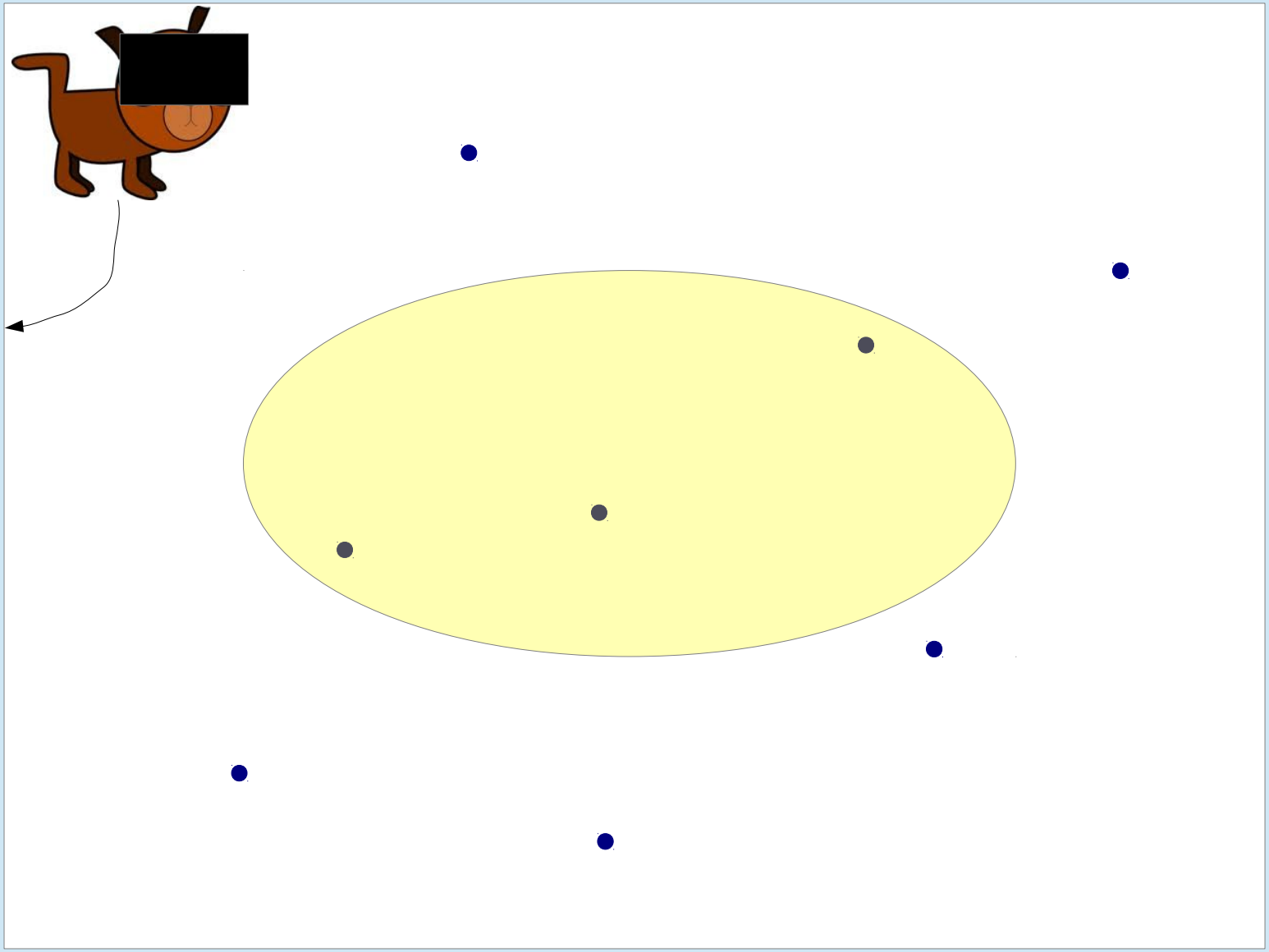
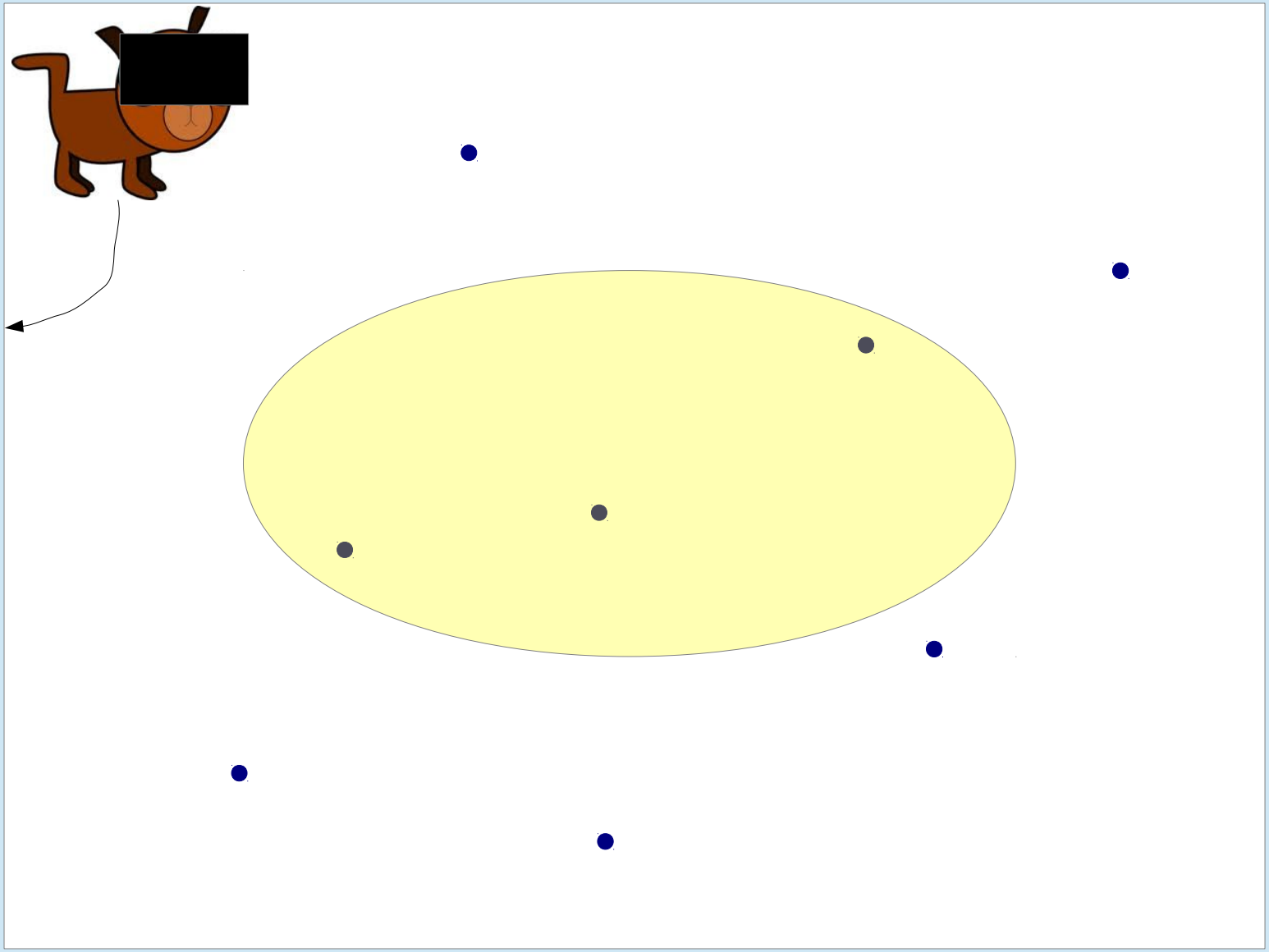
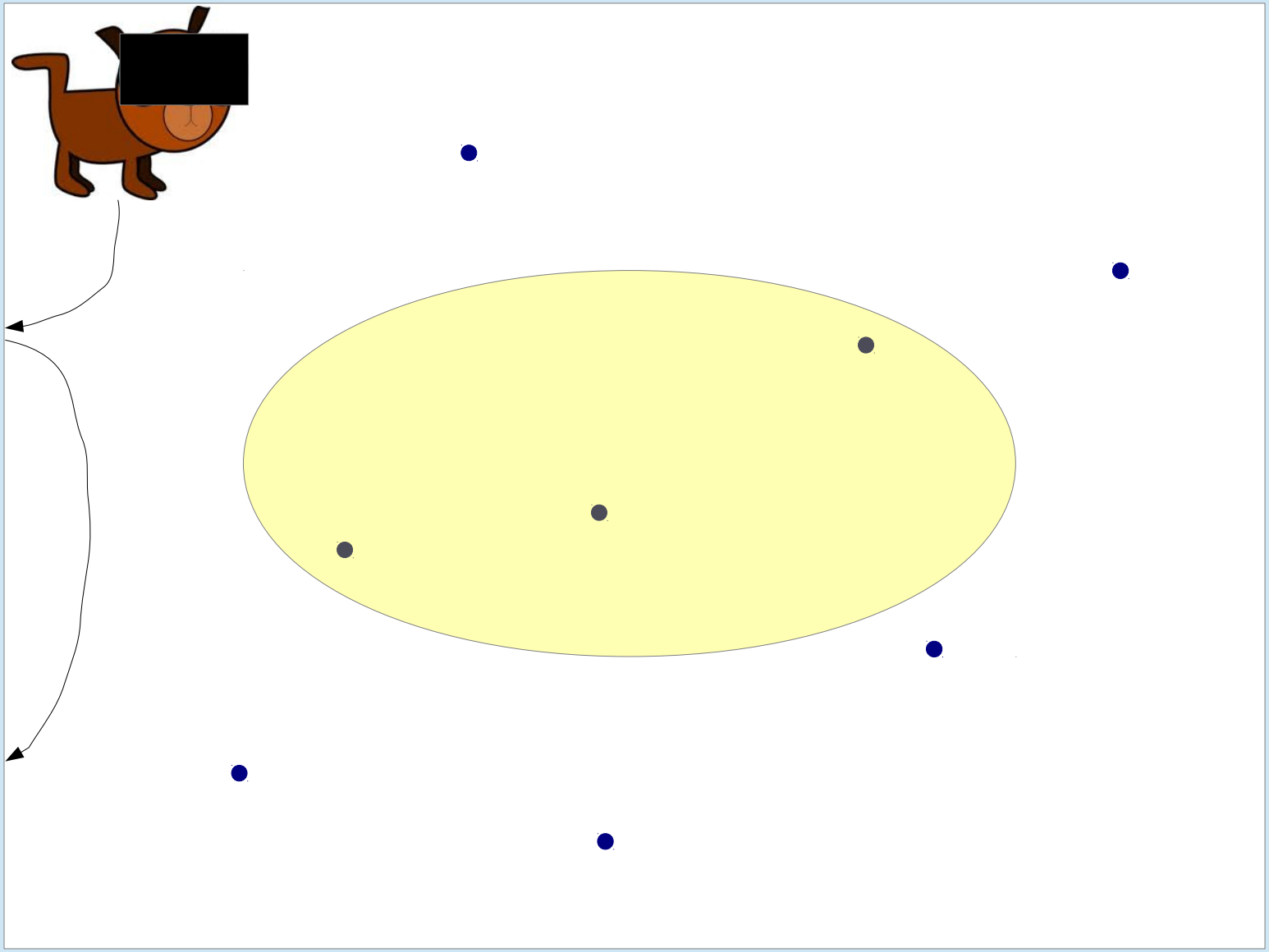





SWITCH!



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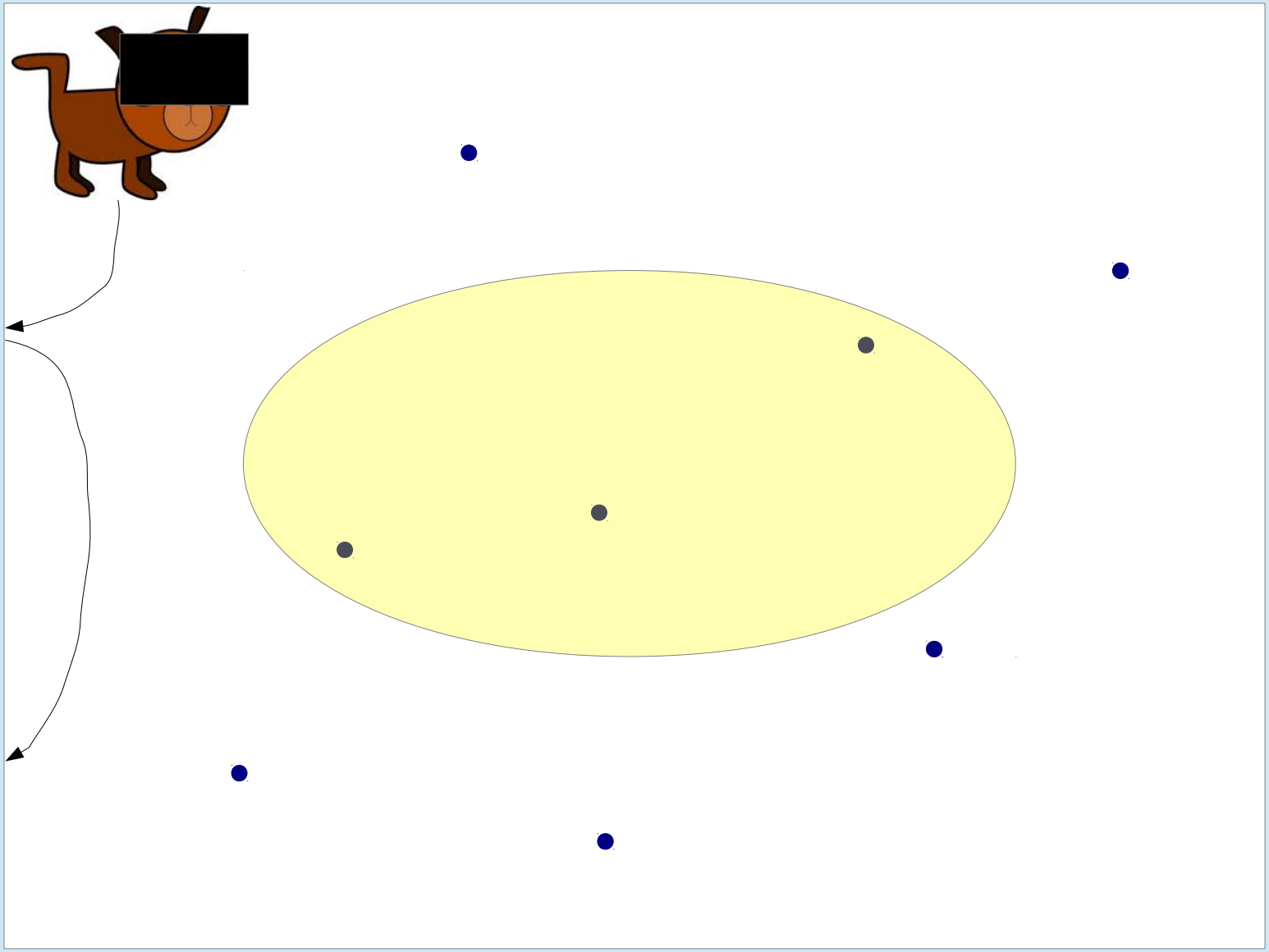

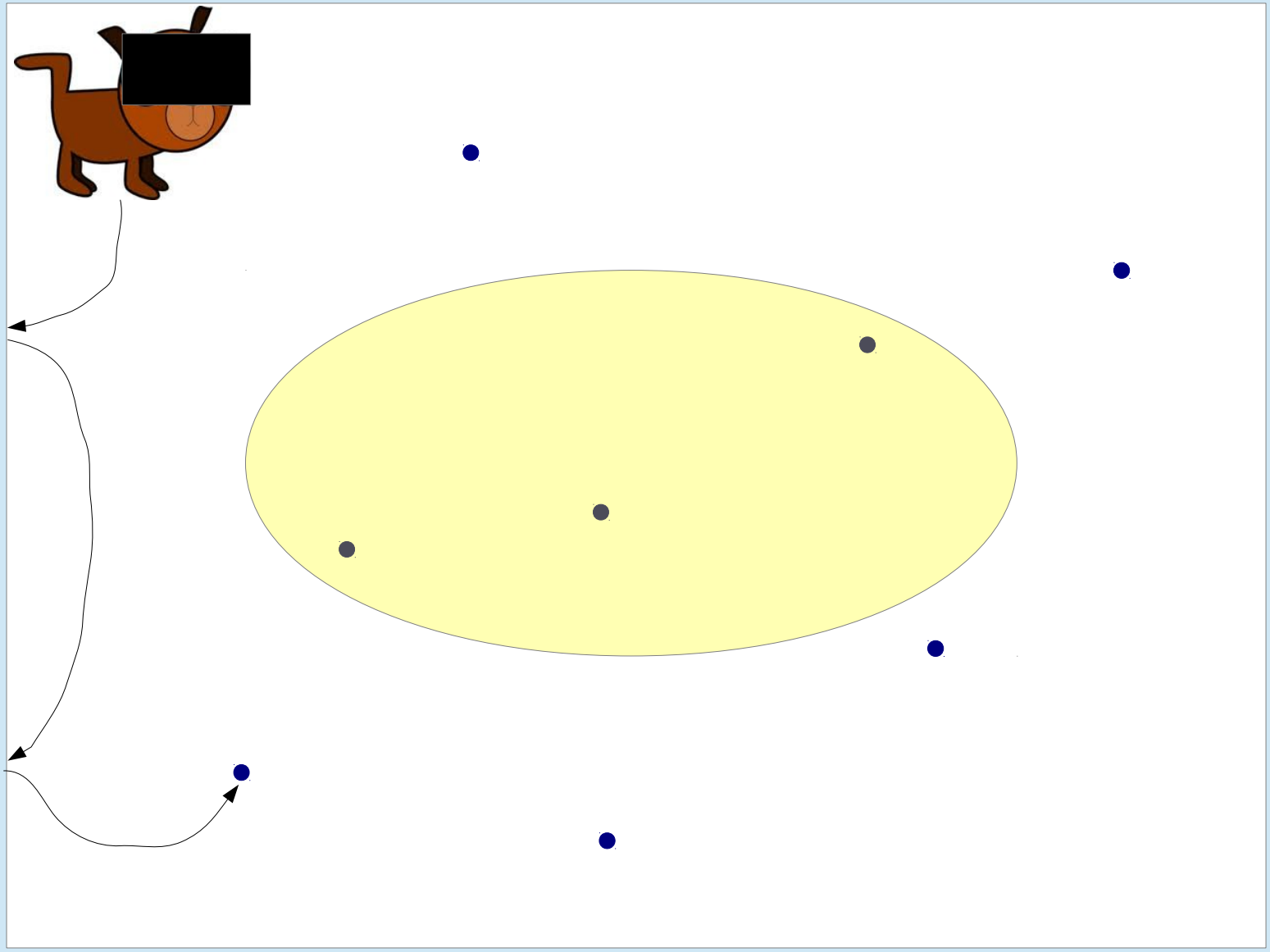

Algorithm

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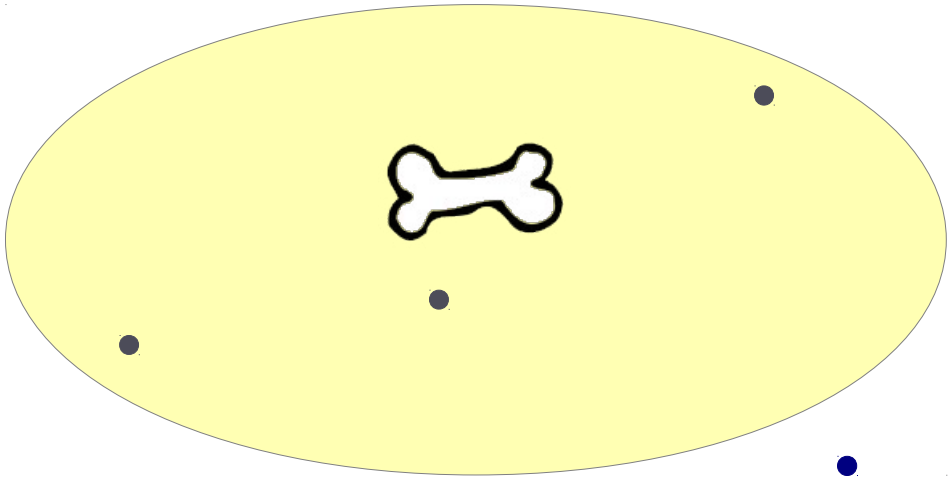

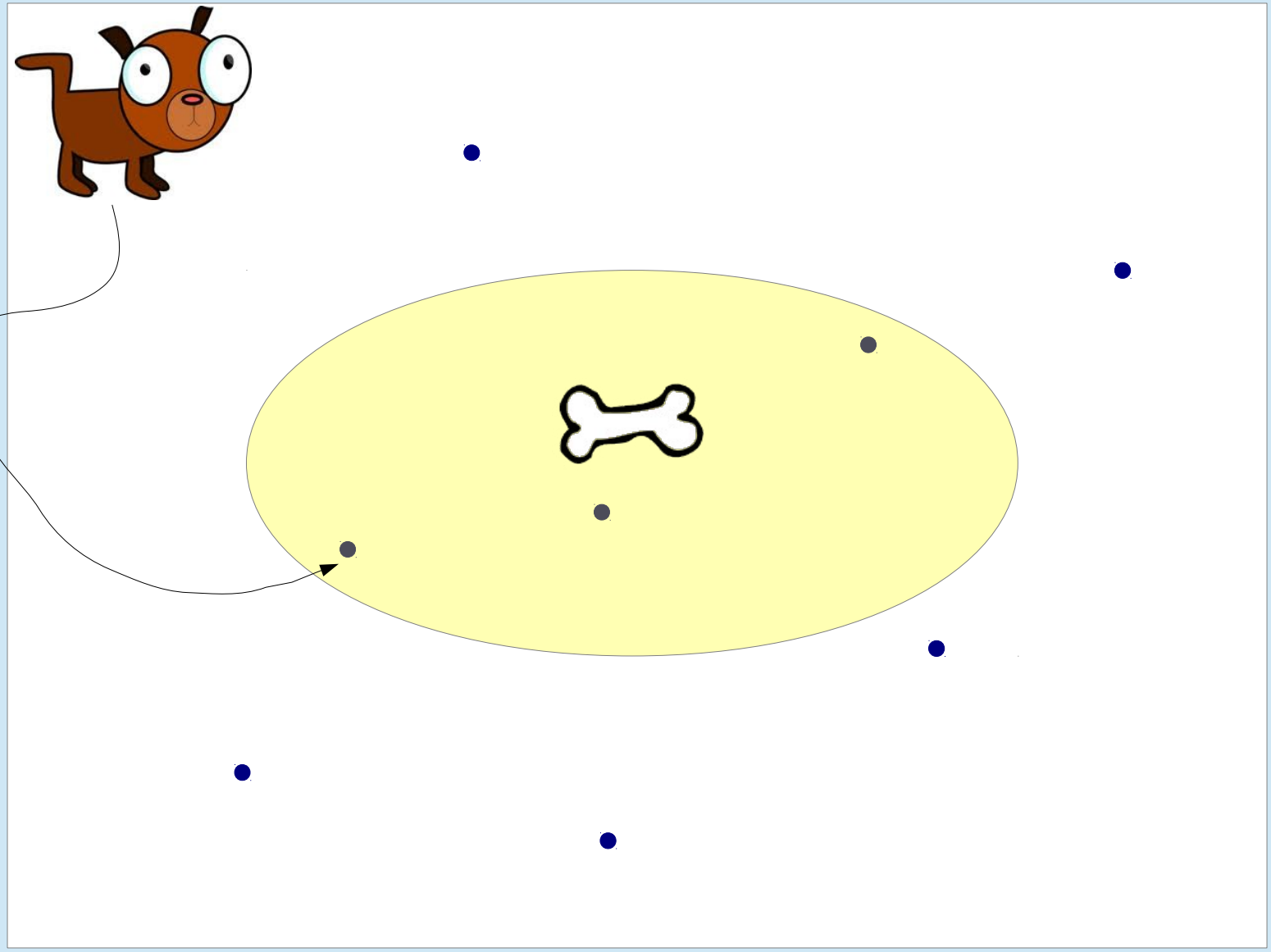

vanishing regulation (VR)

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formulate as optimization, encode preferences in objective

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & \varphi(x) = \frac{\alpha}{2} \sum_{j \in \mathcal{R}} (v_j - v_j^t)^2 + \frac{\beta}{2} \sum_{j \in \mathcal{U}} (v_j - 1)^2 \\ \text{subject to} \quad & f(x) = 0 \end{aligned}$$

$\beta \ll \alpha$ (specific values unclear), x also includes $\{v_i\}_{i \in \mathcal{R}}$

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$$\underset{x}{\text{minimize}} \quad F_i(x) = \mu_i \varphi(x) + \frac{1}{2} \| f(x) \|_2^2$$

indexed by i, generate approx. solutions $\{x_i\}_{i \in \mathbb{N}}$

solving subproblems: apply LSNR to $\nabla F_i(x) = 0$

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LSNR iteration needs to solve

$$\Big(\mu_i G_k + J_k^T J_k + \sum_j f_j(x_k)\nabla^2 f_j(x_k)\Big)p_k = -\mu_k g_k - J_k^T f_k,$$

G_k and g_k are Hessian and gradient of φ at x_k

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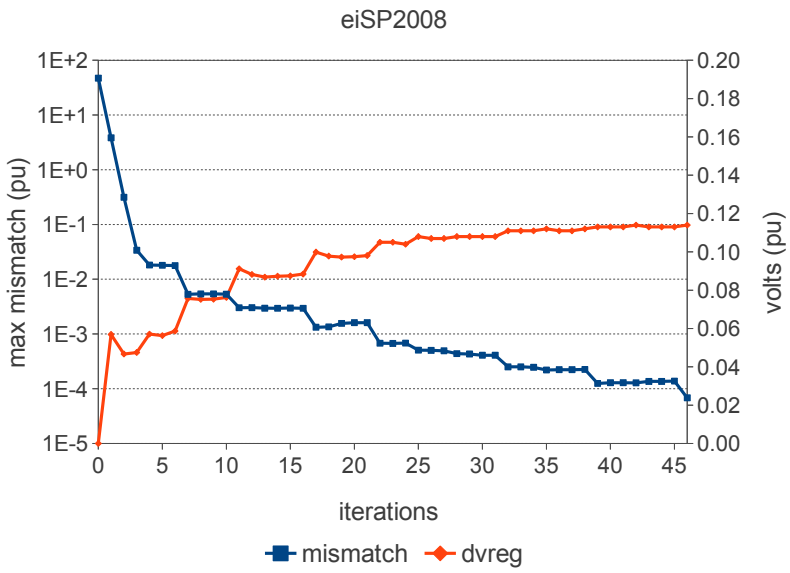
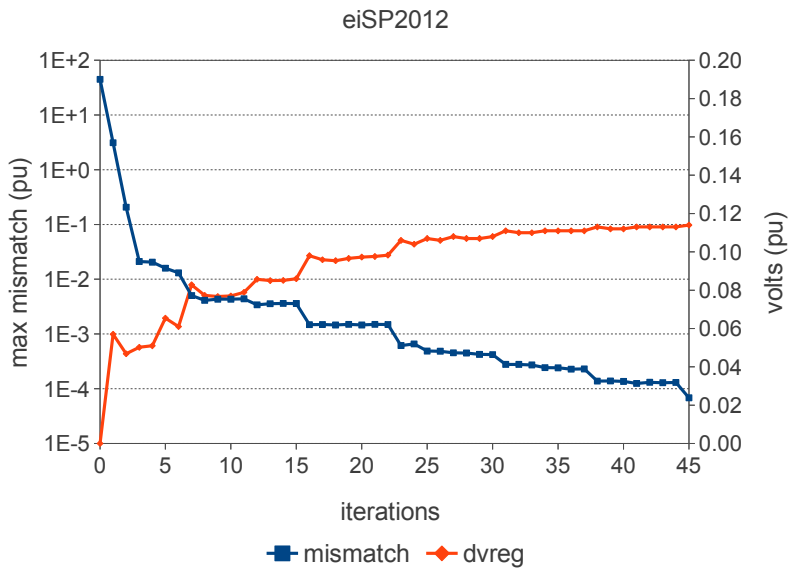
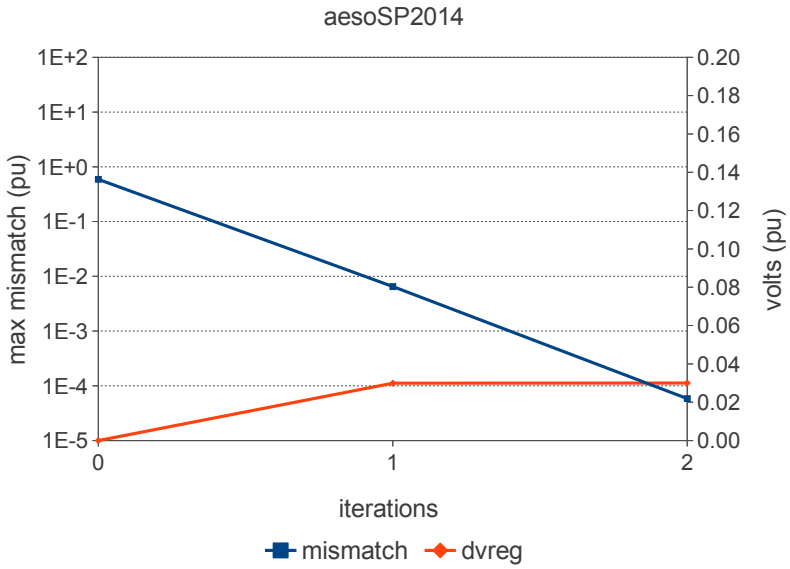
VR ignores $\sum_j f_j(x_k)\nabla^2 f_j(x_k)$ to get descent directions easily, but then

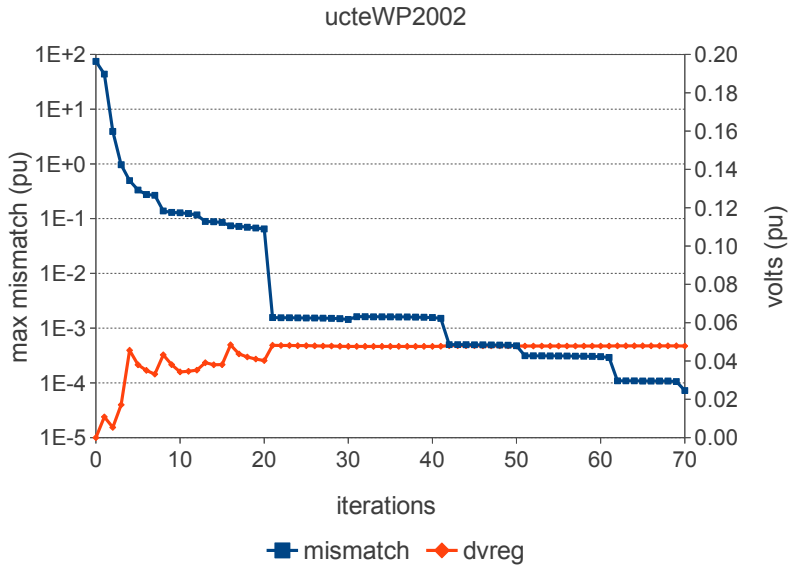
- descent directions get poorer as $\mu_i \downarrow 0$ ($J_k^T J_k$ singular)
- affects robustness of method

mismatch vs regulation trade-off









penalty-based power flow (PBPF)

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extend VR:

- consider physical limits $Q_i^{\min} \leq Q_i^g \leq Q_i^{\max}$, $i \in \mathcal{R}$

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- consider physical limits $Q_i^{\min} \leq Q_i^g \leq Q_i^{\max}$, $i \in \mathcal{R}$

improve VR:

- use better optimization algorithm
- keep second order terms in Newton system
- use better penalties to encode preferences

formulate as optimization, encode preferences & physical limits in objective

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & \varphi(x) = \varphi^u(x) + \varphi^q(x) + \varphi^r(x) \\ \text{subject to} \quad & f(x) = 0 \end{aligned}$$

x also includes $\{Q_i\}_{i \in \mathcal{R}}$

f is now mismatches at all buses except slack

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penalties:

$$\varphi^u(x) = \sum_{j \in \mathcal{U}} \varphi_j^u(v_j)$$

$$\varphi^q(x) = \sum_{j \in \mathcal{R}} \varphi_j^q(Q_j^g)$$

$$\varphi^r(x) = \sum_{j \in \mathcal{R}} \varphi_j^r(v_j)$$

for unregulated magnitudes (quartic)

$$\varphi_j^u(z) = \left(\frac{2z - v_j^{\max} - v_j^{\min}}{v_j^{\max} - v_j^{\min}} \right)^4$$

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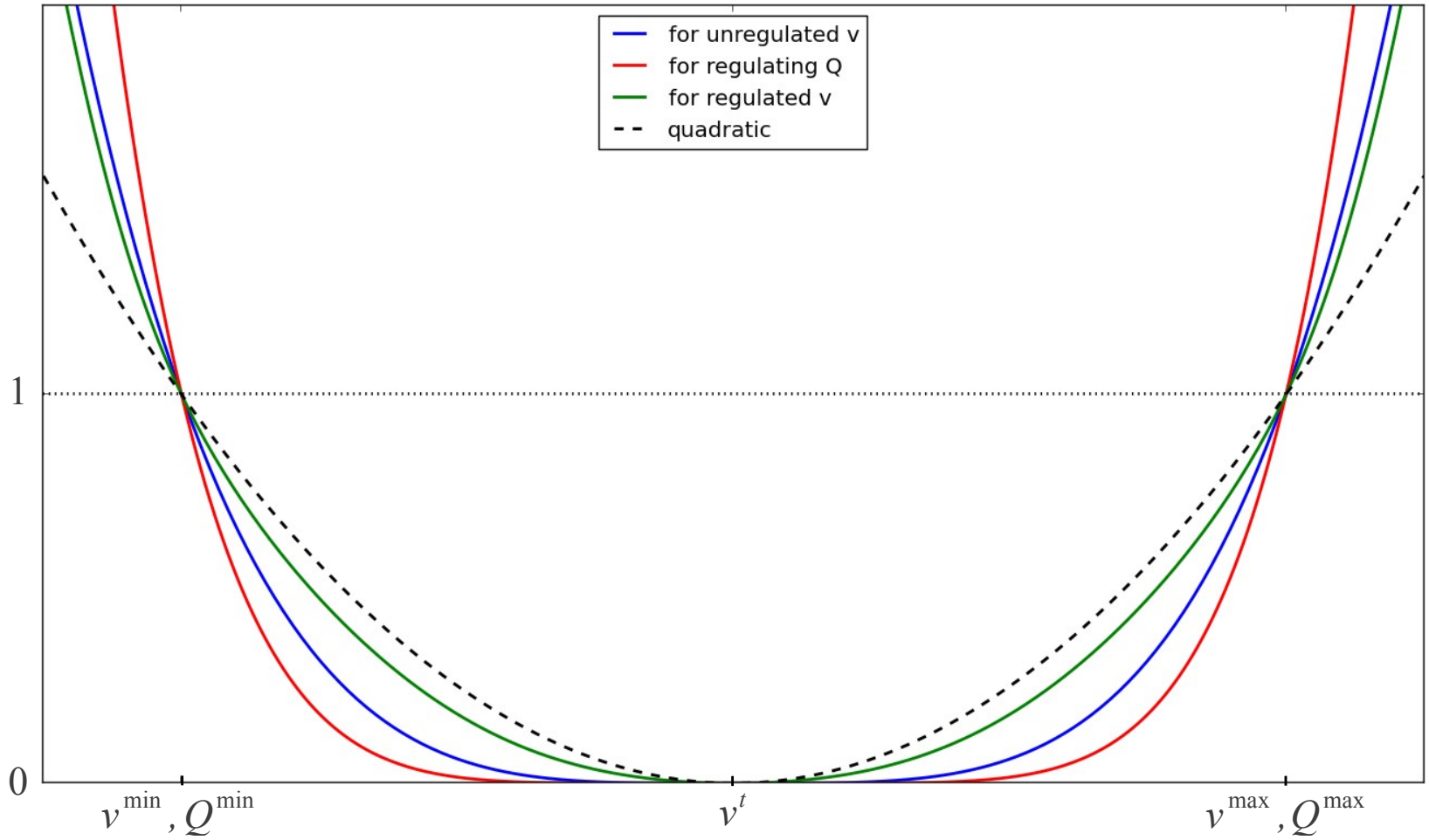
for regulated magnitudes (softmax between quartic & quadratic)

$$\varphi_j^r(z) = \frac{1}{2} \log \left(e^{2\varphi_j^u(z)} + e^{2\varphi_j^2(z)} \right) + b_j,$$


for unregulated v
for regulating Q
for regulated v
quadratic
1
0
$v^{\min}, Q^{\min}$
v^t
$v^{\max}, Q^{\max}$

apply augmented Lagrangian method [8][10][22][23]

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approx. solve sequence

$$\underset{x}{\text{minimize}} \quad L_{\mu_i}(x, \lambda_i) = \mu_i \varphi(x) - \mu_i \lambda_i^T f(x) + \frac{1}{2} \|f(x)\|_2^2,$$

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truncated LSNR iteration solves $H_k p_k = -d_k$ only approx.

- H_k is $\nabla_x^2 L_{\mu_i}(x_k, \lambda_i)$, d_k is $\nabla_x L_{\mu_i}(x_k, \lambda_i)$
- x_k is current iterate

we use | modified Conjugate Gradient | [12]

- exits with $p_k = -\Phi_k d_k$, $\Phi_k \succeq 0$, sufficient descent direction for $L_{\mu_i}(\cdot, \lambda_i)$

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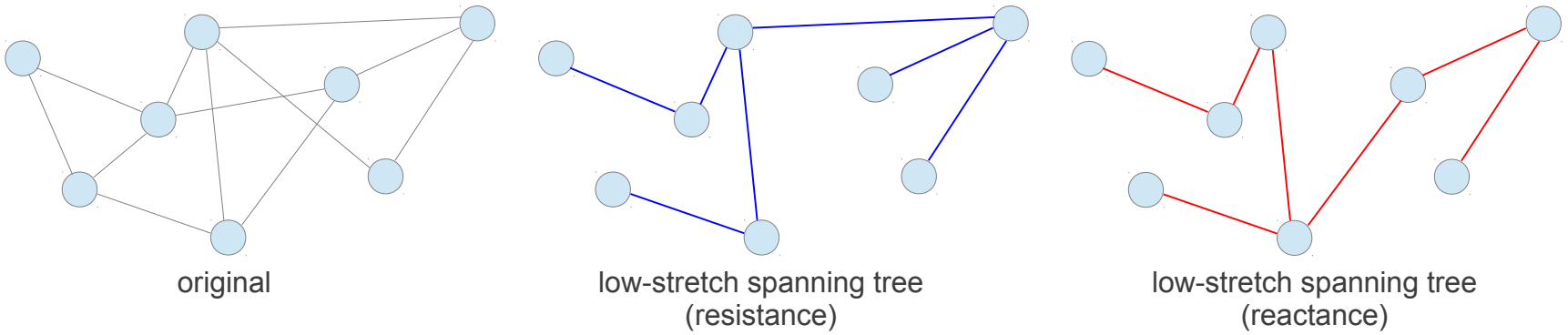
preconditioner:



merge low-stretch spanning trees [4][21], find $\tilde{H}_k$ based on resulting tree

- captures important info of H_k and sparser

factorize $\tilde{H}_k = LDL^T$, modify D if necessary (positive definite)

3.4. real networks

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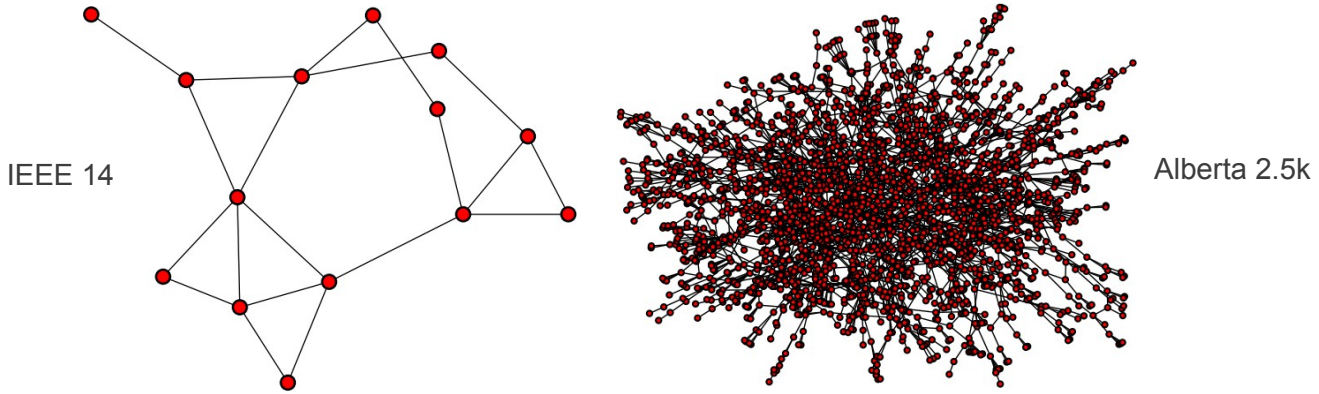

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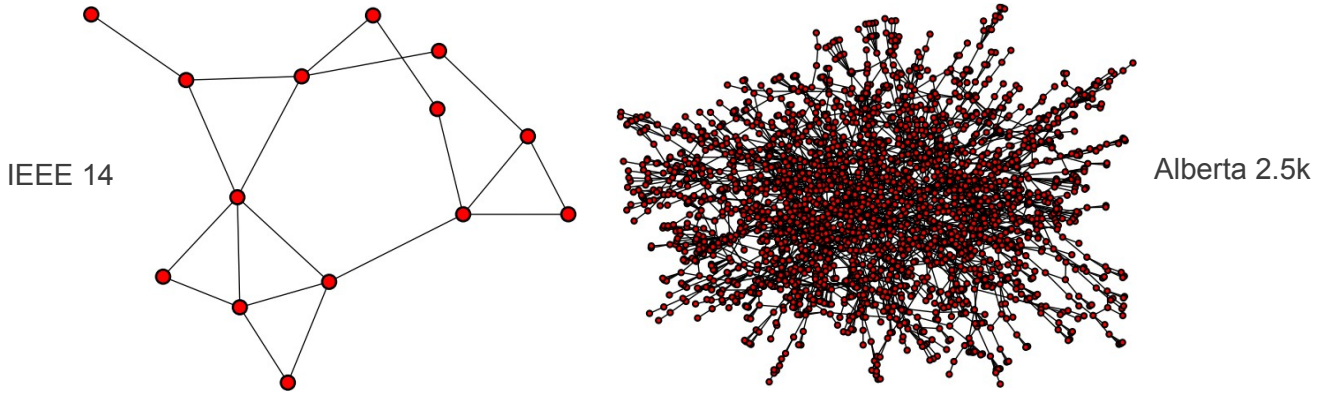

performance on toy problems gives little info

our data: real power networks

4. implementation

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algorithms

- Python [2] (fast prototyping)
- objects and basic numerical routines with NumPy [3] and SciPy [30]
- efficient sparse LU with UMFPACK [16] [17] [18] [19]
- analysis of topology and modifications with NetworkX [25]

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parsers

- also in Python
- handle IEEE common data format [24] and PSS®E raw format version 32

5. experiments

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power flow test cases

name	buses	branches
ieee118	118	186
ieee300	300	411
ucteSL2002	1254	1944
ucteWL2002	1254	1944
ucteWP2002	1254	1944
aesoSL2014	2495	2823
aesoSP2014	2495	2823
aesoWP2014	2529	2869
eiSP2008	46197	60177
eiSP2012	43792	57483

method comparison (warm start)

name	LSNR	PSSE	PSSE Q-lim	VR	PBPF
ieee118					
ieee300					
ucteSL2002					
ucteWL2002					
ucteWP2002					
aesoSL2014					
aesoSP2014					
aesoWP2014					
eiSP2008					
eiSP2012					

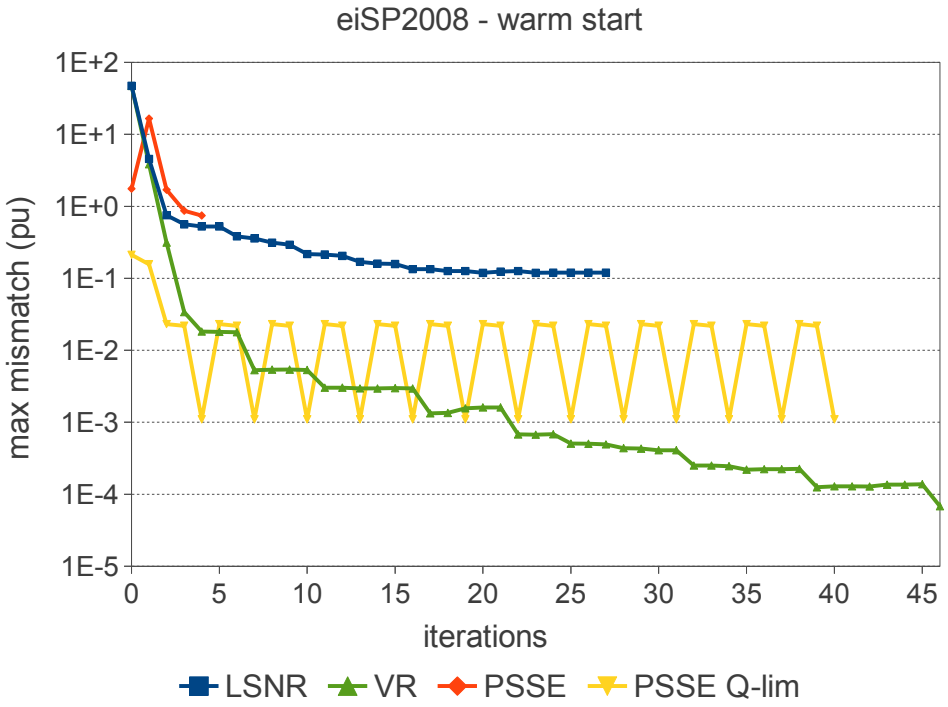
	converged
	failed
	unavailable

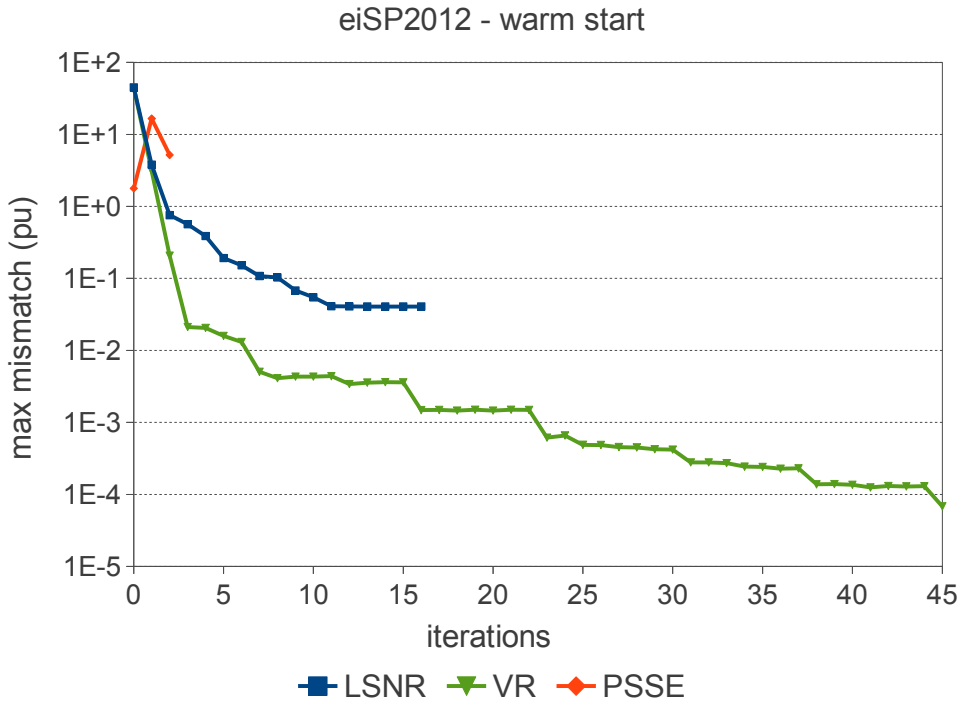
method comparison (flat start)

name	LSNR	PSSE	FNH	PSSE Q-lim	VR	PBPF
ieee118						
ieee300						
ucteSL2002						
ucteWL2002						
ucteWP2002						
aesoSL2014						
aesoSP2014						
aesoWP2014						
eiSP2008						
eiSP2012						

	converged
	failed
	unavailable

method comparison (progress on large cases)





method comparison (perturbing x_0)

aesoSL2014			
seed	NR	LSNR	VR
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			

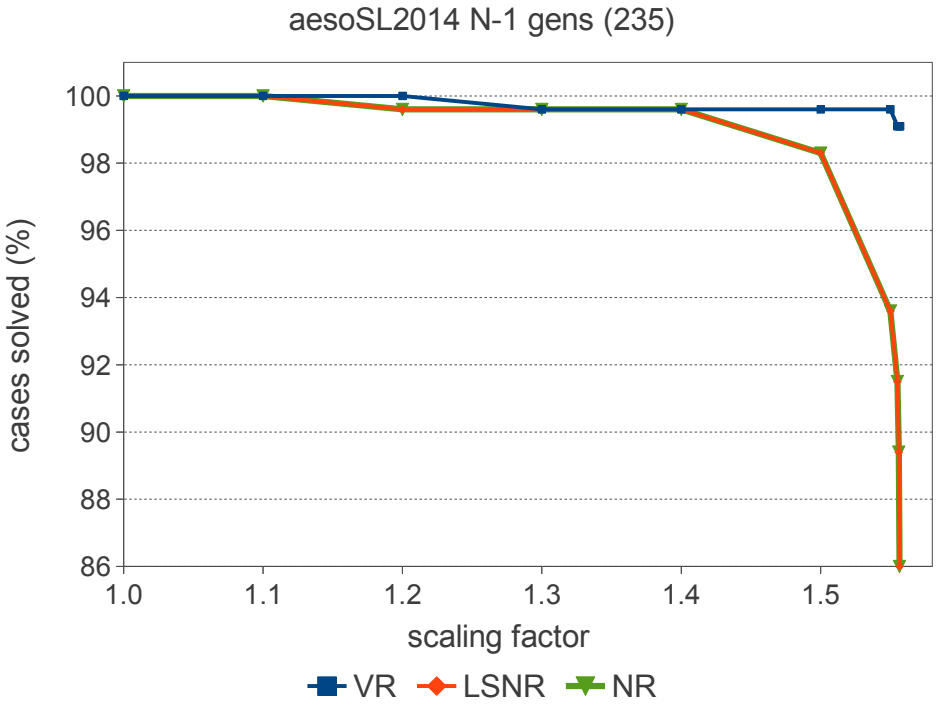
initial angle + $\mathcal{N}(0, 0.3^2)$ (degrees)

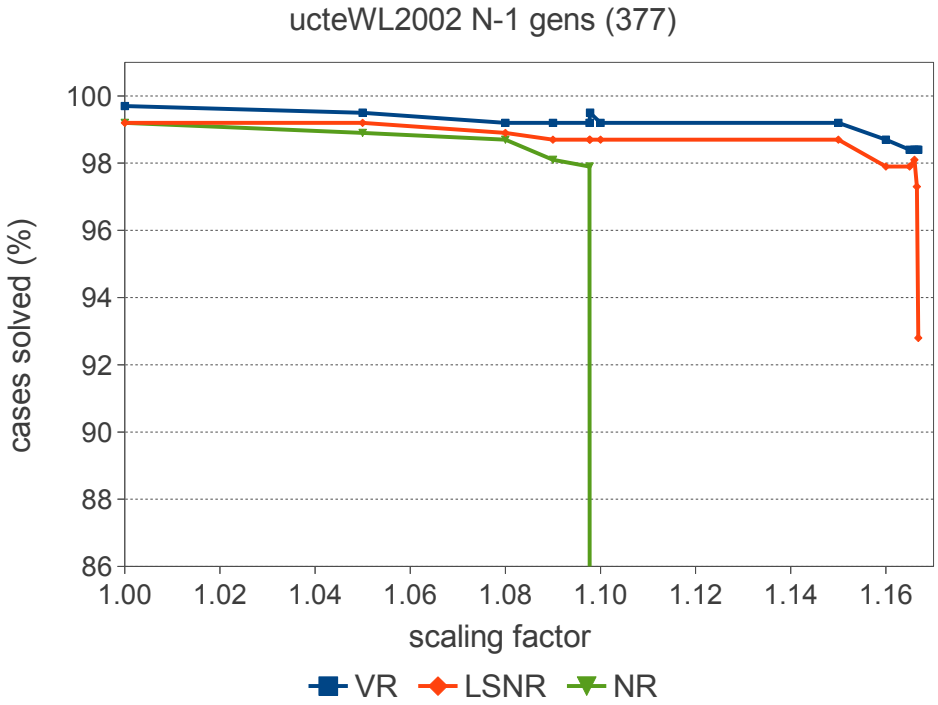
method comparison (perturbing x_0)

ucteWL2002			
seed	NR	LSNR	VR
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			

initial angle $+ \mathcal{N}(0, 0.5^2)$ (degrees)

contingency analysis (scaling net injections)





6. conclusions

line search:

- simple to implement
- not expensive
  - only a few function evaluations
    - easy to parallelize
- big value

flat network homotopy:

- phase shifters are important but not enough
- need more transformations to simplify problem
- try to make DC power flow assumptions [39] hold:
  - near flat θ profile
  - near flat v profile
  - small r and high x/r ratio

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- try to make DC power flow assumptions [39] hold:
  - near flat θ profile
  - near flat v profile
  - small r and high x/r ratio

to do

- understand θ & v profile as net injections vary
- understand θ & v profile as r becomes small and x/r large
- use homotopy with net injections, r, x and phase shifters

formulation and bus type switching

- moving away from square formulation is promising
- more robust, no more switching, preferred solutions
- VR has limitations, PBPF should solve them

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to do

- finish developing theory for PBPF
- start implementing PBPF
- parallelize function evaluations
- combine with FNH

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