Spacetime Optimization for Predictive Simulation of Motion in OpenSim

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November 21, 2013

OpenSim [1]

- "OpenSim is a powerful and freely available tool for modeling and simulation of movement" developed by the National Center for Simulation in Rehabilitation Research (NCSRR) under Principal Investigator Scott Delp.
- "Musculoskeletal modeling and dynamic simulation have recently emerged as powerful tools to uncover the biomechanical causes of movement abnormalities and to design improved treatments."

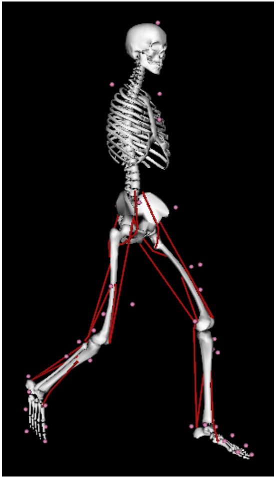
[1] http://opensim.stanford.edu/

Predictive Simulation of Motion

- Predictive simulations using OpenSim are currently under development.
- *Goal*: to generate realistic motion trajectories for a human/character that achieve a particular task using numerical optimization and biomechanics modeling tools in OpenSim.
- "Realistic" motion in this context means that it must obey the laws of physics and be consistent with our understanding of biomechanics.

Predictive Simulation of Motion

- *Motivation*:
  this could provide a tool to simulate motions for new/modified tasks without new data.
- Possible applications include
  - Loaded activity
  - Performance-enhancing or assistive devices
  - Medical interventions (e.g. physical therapy, surgery)



Forward Dynamics Approach for Predictive Simulation

- Parameterize motion using a set of initial conditions and motion controllers (design variables).
- Objective function evaluation requires forward dynamics simulation (forward integration) to compute motion trajectories and torques.
  - Slow - computation for a single simulation is approximately real-time.
  - No gradients available and noise in objective from numerical integration.
- Gradient-free optimization (Covariance Matrix Adaptation)

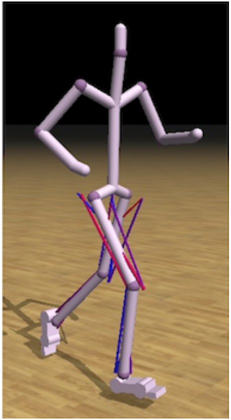


FIGURE: Related work[2] on simulation of human motion.

[2] J. M. Wang, S. R. Hamner, S. L. Delp, and V. Koltun. *Optimizing Locomotion Controllers Using Biologically-Based Actuators and Objectives.* ACM Transactions on Graphics 31(4), 2012.

Inverse Dynamics Approach

- *Idea:* reformulate optimization problem to avoid using forward dynamics.
  - Inverse dynamics computes net forces and torques on a model at a specified time given its motion and the applied forces.
  - Inverse dynamics faster to compute.
- Formulation of problem that uses inverse dynamics must include a direct representation of the motion trajectory and applied forces.
  - *New approach:* model problem of motion simulation as a trajectory optimization problem using *spacetime method*.

Spacetime Method for Optimization

- Introduced by Witkin and Kass (1988) [3]
- *Basic idea*: solve for motion trajectory and unknown forces over the entire time interval, rather than relying on forward dynamics to generate motion.
- Formulated as an optimization problem, this takes the form:

$$\begin{array}{ll} \underset{(\textit{trajectory } q(t),\ \textit{forces } F(t))}{\text{minimize}} & \textit{performance metric} \\ \text{subject to} & \textit{physical laws,} \\ & \textit{pose/task constraints} \end{array}$$

[3] A. Witkin and M. Kass, *Spacetime Constraints.* Proc. SIGGRAPH, 1988, pp. 159-168.

Simple Example: spacetime particle [4]

- Consider a particle of mass m with a jetpack that generates a time-varying force $f(t)$.
- *Problem*: find the trajectory, from initial position $x(0) = a$ to target position $x(T) = b$, that minimizes fuel consumption.
  - Unknowns: trajectory $x(t)$, and jet force $f(t)$, $0 \leq t \leq T$.
  - Physical law: $m\ddot{x}(t) - f(t) - mg = 0$, $\forall t$.
  - Total fuel consumption: $F = \int_{t=0}^{T} |f(t)|^2 dt$

[4] from Witkin and Kass (1988)

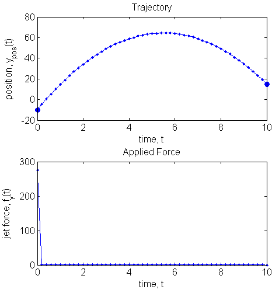
Simple Example: spacetime particle (2)

- Spacetime optimization formulation:

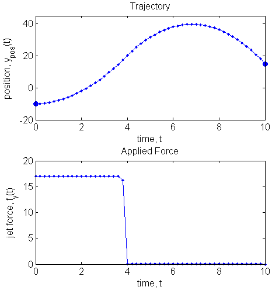
$$
\begin{aligned}
\underset{x_i,\, \dot{x}_i,\, f_i,\, \forall i=0,\ldots,n}{\text{minimize}} \quad & \Delta t \sum_i |f_i|^2 \\
\text{subject to} \quad & m\left(\frac{\dot{x}_i - \dot{x}_{i-1}}{\Delta t}\right) - f_{i-1} - mg = 0, \quad i = 1, \ldots, n \\
& \dot{x}_i = \frac{x_{i+1} - x_i}{\Delta t}, \quad i = 1, \ldots, n \\
& \dot{x}_0 = 0 \\
& x_0 = a, \quad x_n = b
\end{aligned}
$$

- This a convex optimization problem, and therefore the global minimum can be computed efficiently (e.g. using cvx).

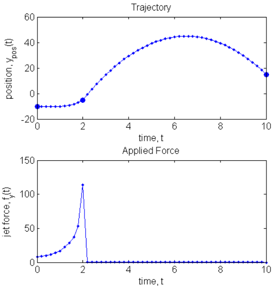
Simple Example: spacetime particle (3)



(A) original formulation; obj value: 54.78



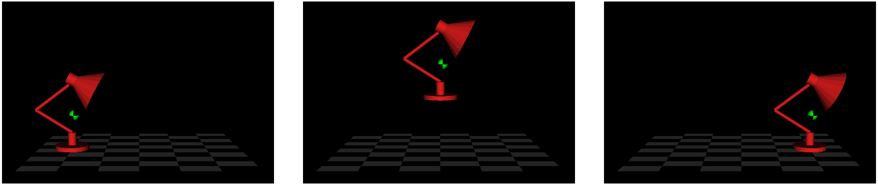
(B) rate limit $|f_i| \leq f_{max}$; obj value: 67.84



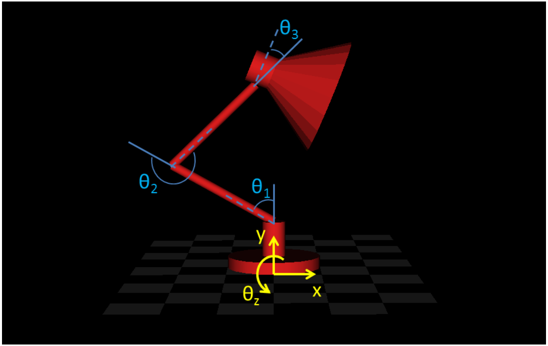
(C) intermediate target point; obj value: 64.78

Jumping Luxo Lamp Problem

- *Task*: use spacetime method to generate a physically realistic trajectory for a jumping Luxo lamp in OpenSim.
- Test problem, used in Witkin and Kass (1988) paper, is restricted to 2 dimensions and uses simple character with only 6 degrees of freedom.



Luxo Lamp Model



Jumping Luxo Lamp Model

- generalized coordinates $q(t)$
  - lamp base and joint coordinates are denoted $q_b(t)$ and $q_j(t)$.

$$\begin{aligned} q(t) &= \left[q_b(t) \,\middle|\, q_j(t) \right]^T \\ &= \left[x(t), \;\; y(t), \;\; \theta_z(t) \;\; \middle| \;\; \theta_1(t), \;\; \theta_2(t), \;\; \theta_3(t) \right]^T \end{aligned}$$

- contact forces $F(t;\mathcal{T})$, which are non-zero for $t \in \mathcal{T}$.
- residual forces $\tau(t)$
  - generalized forces not accounted for by inertial and applied forces.

$$\tau(t, q, \dot{q}, \ddot{q}) = M(q)\ddot{q} + C(q, \dot{q}) - (G(q) + F(t;\mathcal{T}))$$

where M is the mass matrix, C are the inertial forces (coriolis and gyroscopic forces), G is the gravitational force.

Jumping Luxo Lamp Formulation

- Spacetime optimization formulation:

$$\begin{aligned} \underset{q(t),\, F(t)}{\text{minimize}} \quad & \int_{t=0}^{T} f\left(q(t),\, \tau(t);\, \rho\right) dt \\ & \left(= \int_{t=0}^{T} \|\tau_j(t) - k\left(q_j(t) - q_j^{rest}(t)\right)\|_2^2 dt\right) \\ \text{subject to} \quad & \tau_b(t) = 0, \quad 0 \le t \le T \\ & q_b(t) = p(t), \quad t \in \mathcal{T} \end{aligned}$$

  for performance metric $f(\cdot)$, with lamp base positions $p(t)$, time interval(s) $\mathcal{T}$, and lamp model parameters ρ given.

- This problem is *non-convex* and more difficult to solve than the spacetime particle problem.

Challenges: Model formulation

- Parameterization of the trajectory
  - Must represent continuous functions $q(t)$ (also $\dot{q}(t)$ and $\ddot{q}(t)$) with a finite set of parameters.
  - Initially used piece-wise linear approximation of $q(t)$ and approximate derivatives using finite differences.
  - Currently using cubic Bezier splines: require fewer control points to represent smooth motion, but are not well-suited for representing discontinuities (e.g. contact).
- Modeling contact forces
  - Currently solving for the magnitude/direction of a single contact force centered at lamp base, but the position and times of contact must be specified.
  - Contact forces are still represented by a piece-wise linear function due to sharp discontinuity.

Challenges: Model formulation (2)

- Constraints and objective should be realistic (i.e. represent physical laws and constraints).
  - Physical laws are modeled as (nonlinear) constraints - must be satisfied with equality.
  - Modeling of hard versus soft constraints: e.g. enforcing target joint angles less critical than avoiding ground penetration.
  - Balancing multiple objectives: e.g. limit work/effort expenditure, achieve desired task, and avoid falling over/tripping.

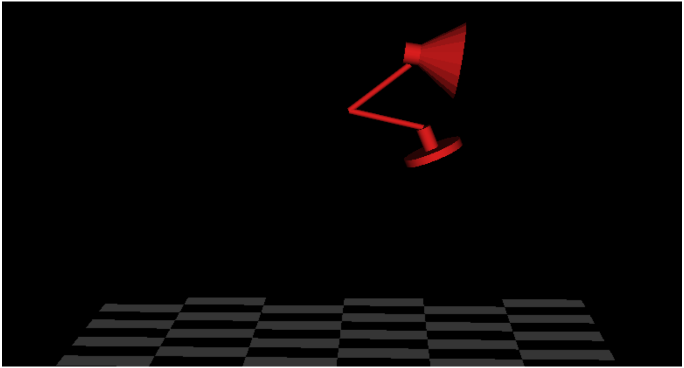
Challenges: Computing the Solution

- The problem is non-convex, and finding the global minimum of a non-convex problem is hard!
- Choice of optimization algorithm.
  - Available in OpenSim: L-BFGS, L-BFGS-B, IPOPT, and CFSQP
  - Mostly using L-BFGS with quadtratic penalty terms to handle constraints - other algorithms may be better at handling constraints.
  - Also used IPOPT, but so far this seems prohibitively slow (computing numerical Jacobian by finite differences).
- Choosing an initial point for the algorithm.
  - Choice of initial point may determine which local minima the algorithm identifies as the solution.
  - Difficult to find a feasible initial point.

Challenges specific to implemenation in OpenSim

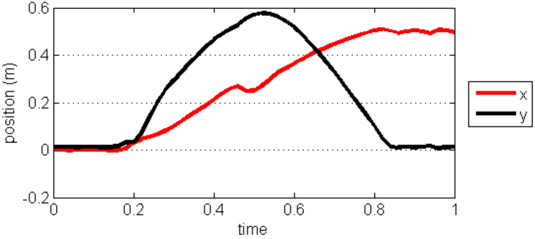
- No analytic gradients.
  - Gradients/Jacobians computed by finite differences, Hessian approximation generated by BFGS update.
  - Finite differences are less accurate than analytic gradients.
  - Finite difference computation slows down the algorithms significantly (although this is parallelizable).
  - Adding gradient calculation / automatic differentiation requires significant changes to software.
- Limitations of optimization tools
  - Solvers provide very limited diagnostic information to user.
  - Reading and modifying these codes can be challenging.

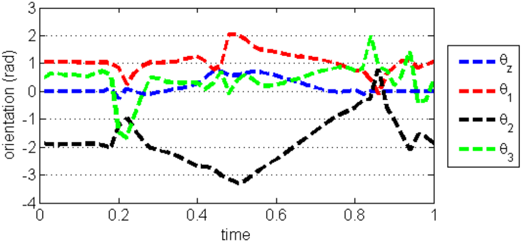
Results: Jumping Luxo Lamp

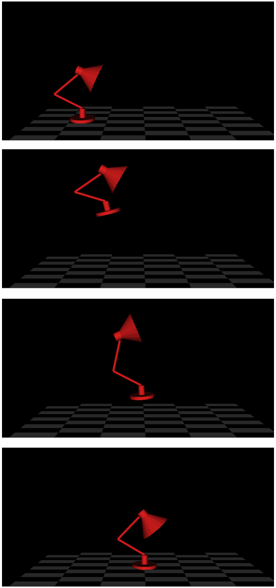


Click here to open demo in YouTube.

Results: Jumping Luxo Lamp

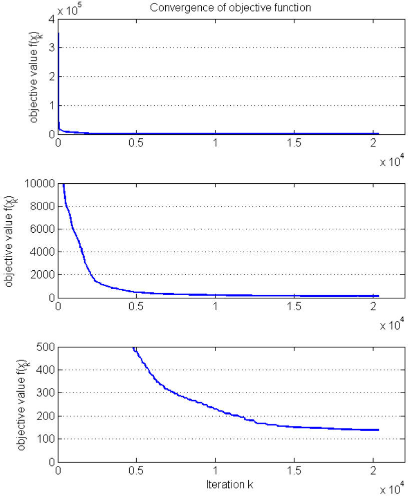






Results: Convergence

- Slow convergence: for 200-300 variables can take $\approx$ 2.0-5.5 $\times$ 10^4 iterations to converge to a local minima.
- In most cases, algorithm terminates due to failure in line-search before converging to a local minima.



References

[1] S.L. Delp, F.C. Anderson, A.S. Arnold, J.P. Loan, A. Habib, C. John, E. Guendelman, and D.G. Thelen, *OpenSim: Open-source software to create and analyze dynamic simulations of movement.* IEEE Trans. Biomedical Engineering, vol. 54, no. 11, pp. 1940-1950, 2007.

[2] J.M. Wang, S.R. Hamner, S.L. Delp, and V. Koltun. *Optimizing Locomotion Controllers Using Biologically-Based Actuators and Objectives.* ACM Transactions on Graphics 31(4), 2012.

[3] A. Witkin and M. Kass. *Spacetime Constraints.* Proc. SIGGRAPH, 1988, pp. 159-168.

[4] K. Bergen. Jumping Luxo Lamp demo (`www.youtube.com/watch?v=lWqc52FWKJg`).