

Raptor Problem

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MISS LENHART COULDN'T BE
HERE TODAY, SO SHE ASKED
ME TO SUBSTITUTE.



I'VE PUT OUT YOUR
TESTS. PLEASE GET
STARTED.

MR. MUNROE, MISS LENHART
NEVER TAUGHT US THIS.



THAT'S BECAUSE MISS LENHART DOESN'T
UNDERSTAND HOW IMPORTANT CERTAIN
KINDS OF MATH ARE.

BUT THIS JUST LOOKS --

THIS MATERIAL IS MORE VITAL THAN
ANYTHING YOU'VE EVER LEARNED

BUT --

NO BUTS.

Source: <http://xkcd.com/135/>

The problem

2. You are at the center of a 20m equilateral triangle with a raptor at each corner. The top raptor has a wounded leg and is limited to a top speed of 10 m/s.



The raptors will run toward you. At what angle should you run to maximize the time you stay alive?

— THIS IS A MATTER OF LIFE AND DEATH.

The model

choose angle to run θ

to maximize survival time T

subject to raptor motion $\frac{d\mathbf{r}_i}{dt}(t) = v_i \frac{\mathbf{p}(t) - \mathbf{r}_i(t)}{\|\mathbf{p}(t) - \mathbf{r}_i(t)\|}$

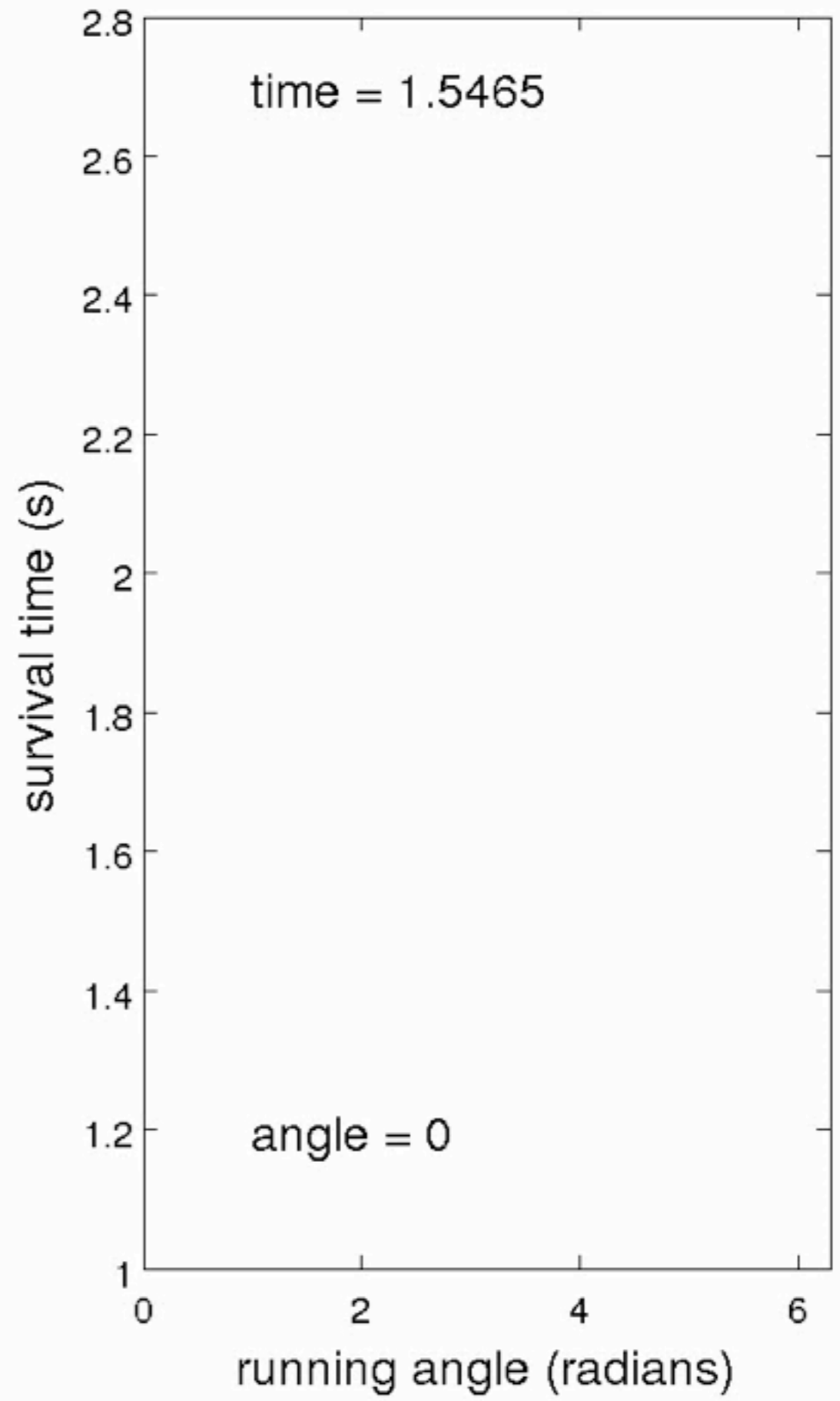
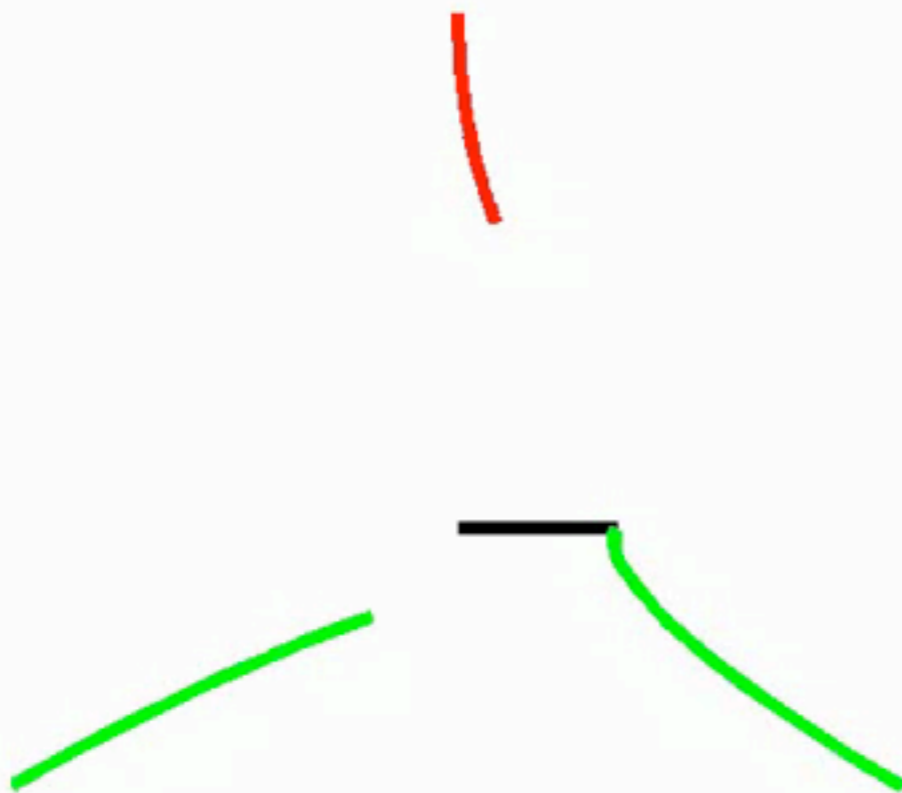
human motion $\frac{d\mathbf{p}}{dt}(t) = v_p \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix}$

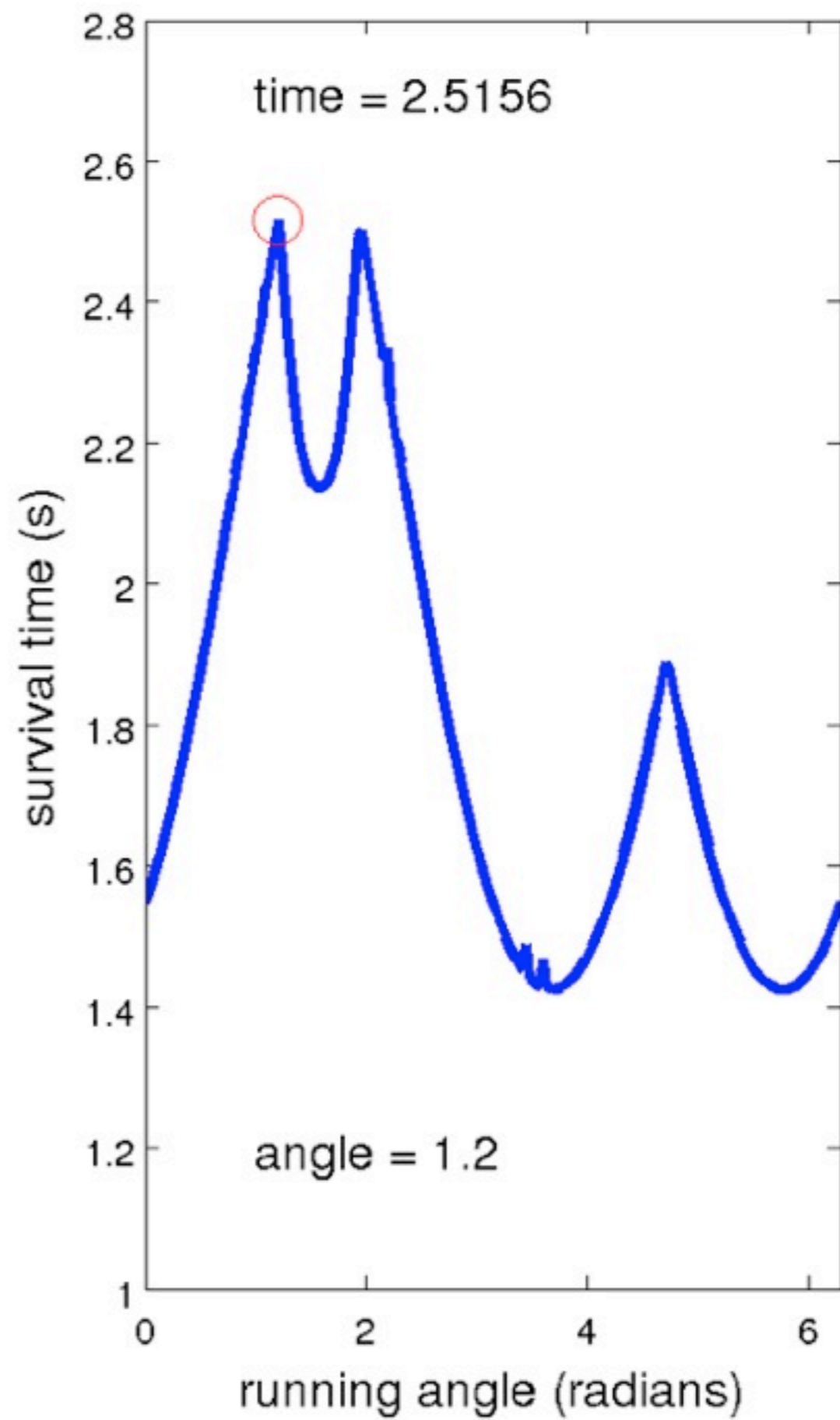
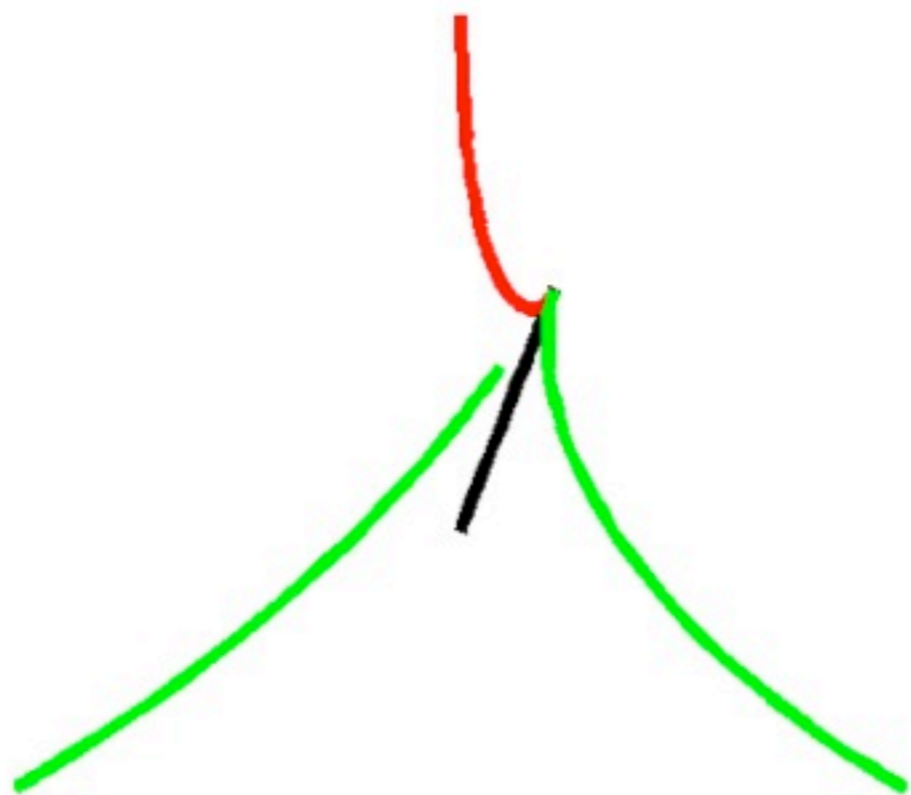
getting caught $\|\mathbf{p}(T) - \mathbf{r}_i(T)\| \leq \delta$

Single parameter optimization

- Sample over range of **angles**
- Choose the **angle** that gives largest **survival time**
- Simulation uses Matlab's ode45

Simulate!





Can we do better?

The new model

choose direction to run $\mathbf{v}_p(t)$

to minimize “likelihood” of being eaten

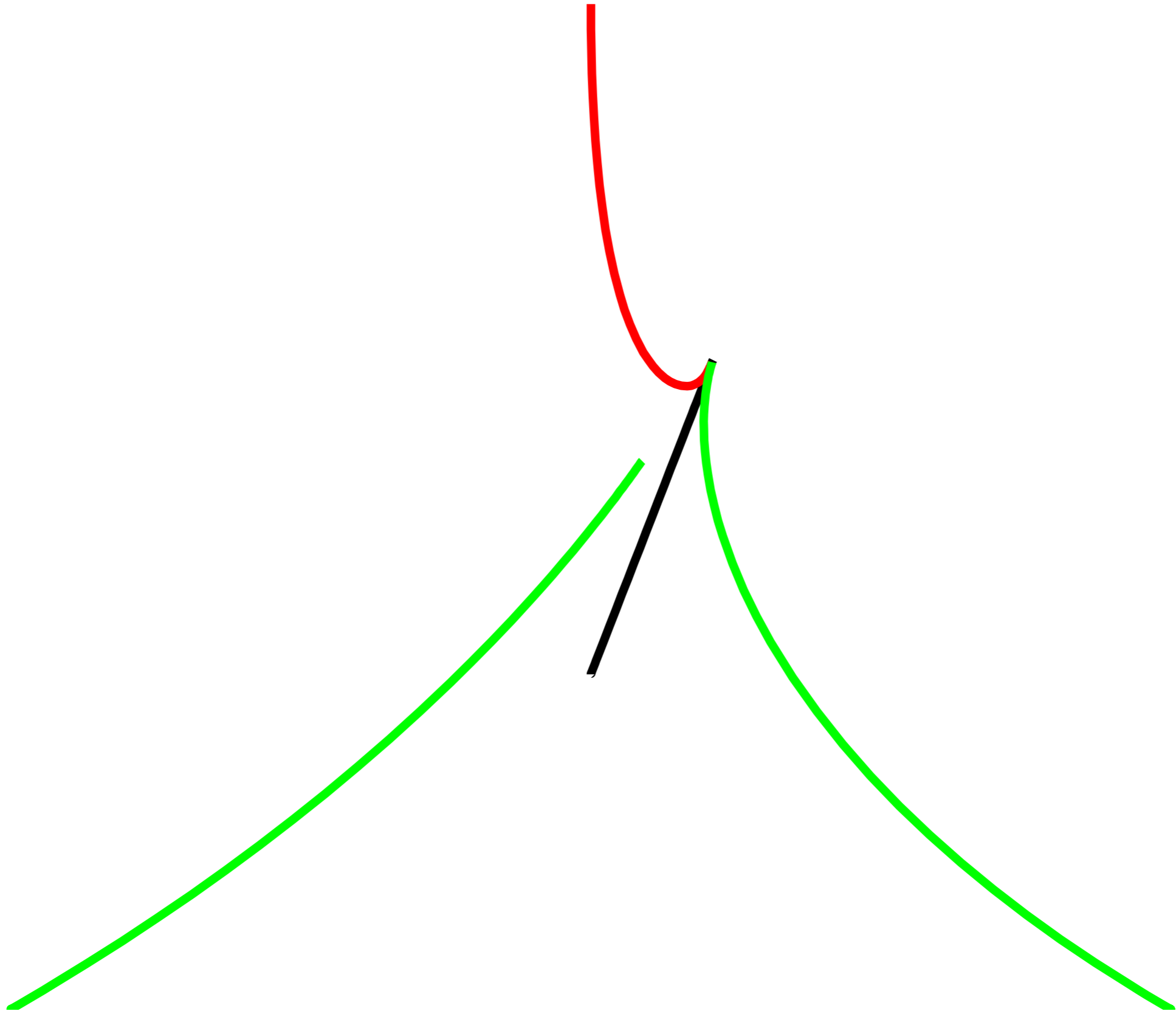
$$\int_0^T \sum_{i=1}^3 \frac{1}{\|\mathbf{p}(t) - \mathbf{r}_i(t)\|^2} dt$$

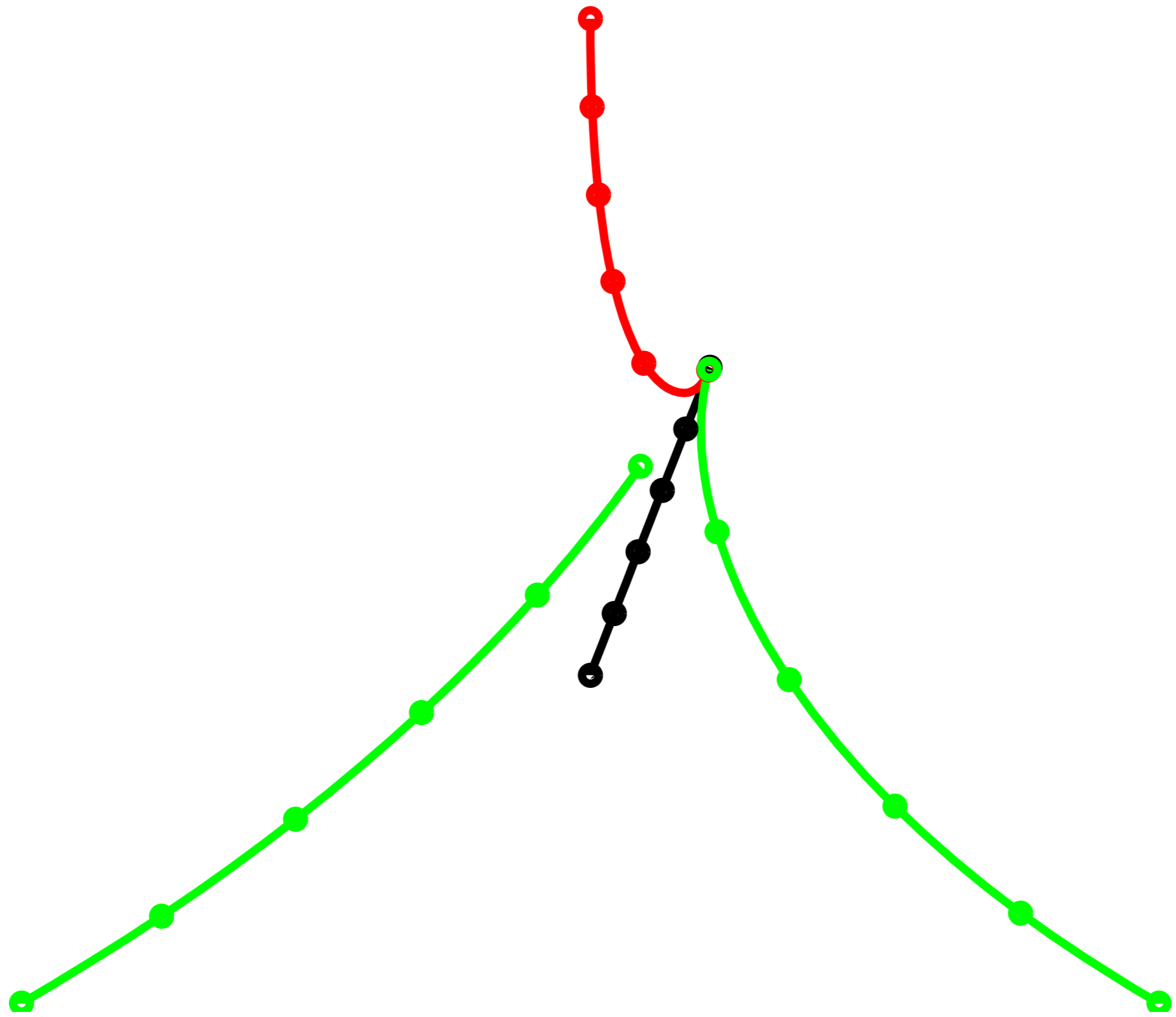
subject to raptor motion

$$\frac{d\mathbf{r}_i}{dt}(t) = v_i \frac{\mathbf{p}(t) - \mathbf{r}_i(t)}{\|\mathbf{p}(t) - \mathbf{r}_i(t)\|}$$

human motion

$$\frac{d\mathbf{p}}{dt}(t) = \mathbf{v}_p(t)$$





The new model

choose direction to run $\mathbf{v}_p[j]$ for $j = \{1, \dots, N\}$

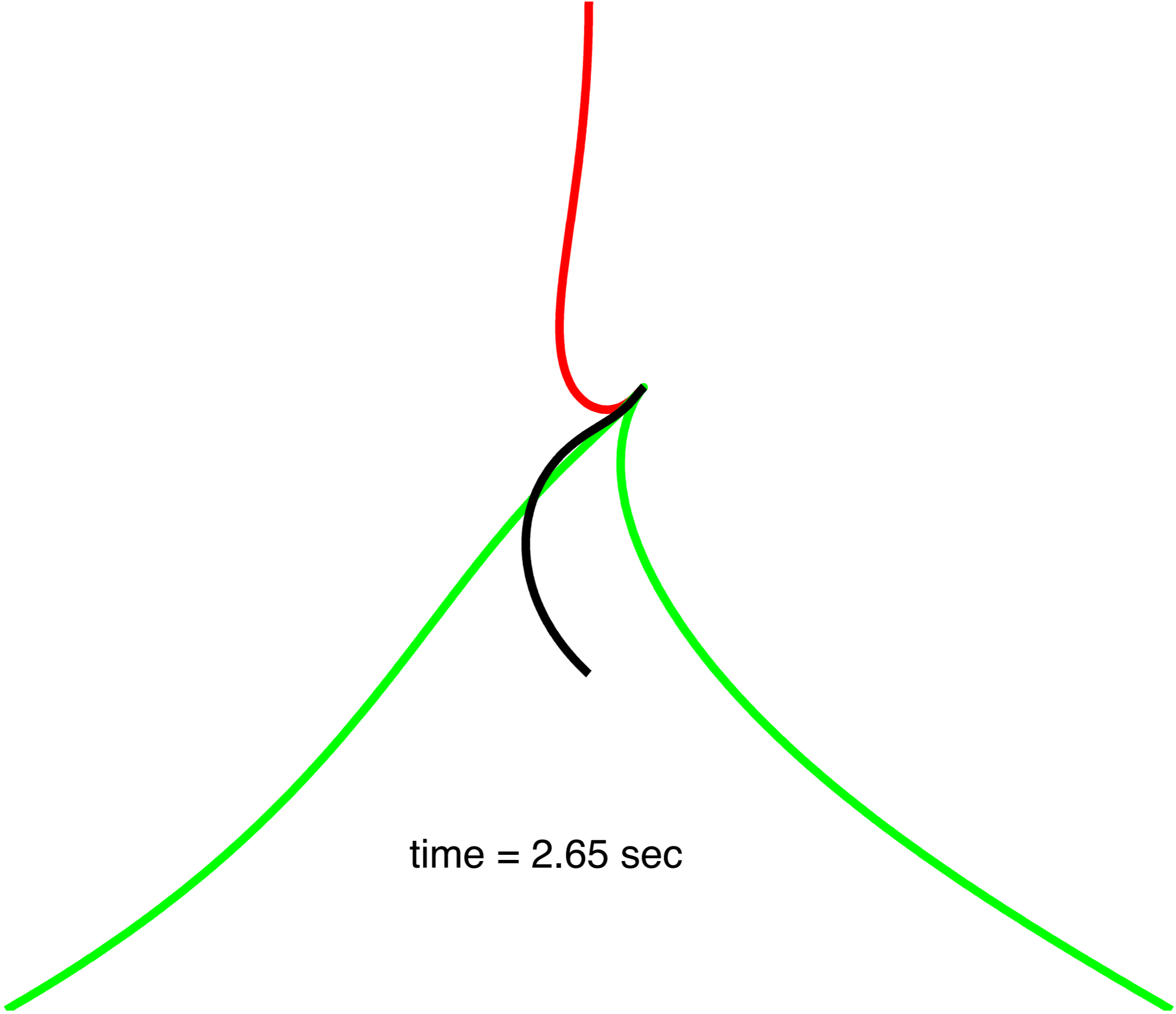
to minimize “likelihood” of being eaten $\sum_{j=1}^N \sum_{i=1}^3 \frac{1}{\|\mathbf{p}[j] - \mathbf{r}_i[j]\|^2} dt$

subject to raptor motion $\mathbf{r}_i[j + 1] = \mathbf{r}_i[j] + hv_i \frac{\mathbf{p}[j] - \mathbf{r}_i[j]}{\|\mathbf{p}[j] - \mathbf{r}_i[j]\|}$

human motion

$$\mathbf{p}[j + 1] = \mathbf{p}[j] + h\mathbf{v}_p[j]$$

Solve!



time = 2.65 sec