Raptor Problem

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Miss Lenhart couldn't be here today, so she asked me to substitute.

Mr. Munroe

I've put out your tests. Please get started.

Mr. Munroe, Miss Lenhart never taught us this.

That's because Miss Lenhart doesn't understand how important certain kinds of math are.

But this just looks --

This material is more vital than anything you've ever learned.

But --

No buts.

Source: http://xkcd.com/135/
2. You are at the center of a 20m equilateral triangle with a raptor at each corner. The top raptor has a wounded leg and is limited to a top speed of 10 m/s.

(Not to scale)

The raptors will run toward you. At what angle should you run to maximize the time you stay alive?
THIS IS A MATTER OF LIFE AND DEATH.
The model

choose angle to run $\theta$

to maximize survival time $T$

subject to raptor motion

$$\frac{dr_i}{dt}(t) = v_i \frac{p(t) - r_i(t)}{||p(t) - r_i(t)||}$$

human motion

$$\frac{dp}{dt}(t) = v_p \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix}$$

goinging caught

$$||p(T) - r_i(T)|| \leq \delta$$
Single parameter optimization

• Sample over range of angles
• Choose the angle that gives largest survival time
• Simulation uses Matlab’s ode45
Simulate!
Can we do better?
The new model

choose direction to run $v_p(t)$

to minimize "likelihood" of being eaten

subject to raptor motion

human motion

\[
\begin{align*}
\int_0^T \sum_{i=1}^{3} \frac{1}{\| p(t) - r_i(t) \|^2} \, dt \\
\frac{dr_i}{dt}(t) &= v_i \frac{p(t) - r_i(t)}{\| p(t) - r_i(t) \|} \\
\frac{dp}{dt}(t) &= v_p(t)
\end{align*}
\]
The new model

choose direction to run \( v_p[j] \) for \( j = \{1, \ldots, N\} \)

to minimize “likelihood” of being eaten

subject to

raptor motion

human motion

\[
\begin{align*}
\sum_{j=1}^{N} \sum_{i=1}^{3} \frac{1}{\|p[j] - r_i[j]\|^2} dt \\
r_i[j+1] &= r_i[j] + hv_i \frac{p[j] - r_i[j]}{\|p[j] - r_i[j]\|} \\
\end{align*}
\]
Solve!
time = 2.65 sec