CME350Q – Spring 2022
The ABCs of TQC: An introduction to the mathematics of Topological Quantum Computing

Guillermo (Willie) Aboumrad will be teaching CME350Q. Please don't hesitate to reach him at willieab@stanford.edu if you have any questions.

Course description

Computation is a mechanical process. Computers process information by manipulating physical systems encoding bits, and quantum computers manipulate encodings in quantum mechanical systems. This process is extremely delicate and error-prone, so we must develop fault-tolerant computation protocols to make quantum computers useful. Quantum error-correcting codes provide a means of developing fault-tolerance at the software level. This course will explore Topological Quantum Computing (TQC) as a means of achieving fault-tolerance at the hardware level instead, by encoding information in topological phases of matter that are intrinsically protected from local deformations and interactions. TQC promises scalable quantum computing and it lies at the crossroads of cutting-edge research in Physics, Engineering, and Mathematics. This course will introduce the mathematical machinery modeling TQC. The main players are Anyons, Braids, and Categories: braiding anyons, which are certain quasiparticles existing only in two-dimensional systems, results in unitary state transformations implementing logical gates on encoded qubits. The mathematical theory of anyons, which are neither bosons nor fermions, as simple objects in unitary modular tensor categories is quite interesting, and this course will develop it from the ground up.

Prerequisites

We will develop algebraic models for relevant topological objects from scratch, so no background in topology is required. We will, however, assume some familiarity with linear and abstract algebra. Familiarity with basic quantum mechanics is recommended. We will review quantum computing basics, including the circuit model.

Grading basis

The course is graded on a CR/No Credit basis. To obtain credit, you must solve at least **one** exercise from each problem set. There will be **two** problem sets: the first will be published at the beginning of Week 5, and the second at the beginning of Week 7. Solutions are due at the end of the quarter, on June 3. Instead of submitting written solutions via email, you may choose to meet with Willie at some point in the quarter to discuss your chosen problems and their solutions.

Office hours

We will have office hours by appointment only. Please reach out to Willie at willieab@stanford.edu to set up a time. We can also talk after lecture.

Schedule

We aim to cover the following topics.

1. Introduction. Logistics, course set-up, and overview. Computation is a mechanical process. Circuit model and QC basics. Fault-tolerance at the hardware level. Topological phases of matter. Anyons, braids, and categories. Non-abelian particle statistics. Braiding anyons implements logical gates that transform encoded information. The promise of TQC: scalable quantum computing.

2. (Interlude.) Algebraic theory of braids, knots, ribbons, and tangles. Reidemeister moves. Knot polynomial invariants. Skein relations. Jones, HOMFLY, Kauffman polynomials.

3. (Continued.) Categorification. RIBBON and TANGLE. Monoidal, braided, and ribbon categories. Diagrammatical calculus. Representations of RIBBON and TANGLE.

4. Anyon systems. Topological approach, via punctured surfaces. Algebraic approach, via modular functors and unitary modular tensor categories. Anyonic quantum computing with graphical interpretation.

5. Anyon models as unitary modular tensor categories. F-, R-, S-, and T-matrices. Pentagon and hexagon equations. Fusion spaces and the standard basis. Unitary braid group representations. The Yang-Baxter equation.

6. Example. Universal quantum computation with Fibonacci anyons.

7. $SU(2)_k$ Chern-Simons theory. Small rank anyon system examples. Anyon systems as a semi-simplification of the finite-dimensional representation categories of quasi-triangular Hopf algebras.

8. Bi-algebras, Hopf algebras, braiding, rigidity, and ribbon structures. Categorical properties via Tannaka-Krein duality.

9. Jones polynomial via $SU(2)_2$ representation theory. Generalizations to quantum groups. Numerical invariants via SAGE.