Finite Automata
Part Two
Recap from Last Time
Old MacDonald Had a Symbol, ♫ Σ-eye-ε-ey∈, Oh! ♫

• You may have noticed that we have several letter-E-ish symbols in CS103, which can get confusing!

• Here’s a quick guide to remembering which is which:
  • In automata theory, Σ refers to an \textit{alphabet}.
  • In automata theory, ε is the \textit{empty string}, which is length 0.
  • In set theory, use ∈ to say “is an \textit{element of}.”
  • In set theory, use ⊆ to say “is a \textit{subset of}.”
DFAs

- A **DFA** is a
  - **D**eterministic
  - **F**inite
  - **A**utomaton
- DFAs are the simplest type of automaton that we will see in this course.
Recognizing Languages with DFAs

$L = \{ w \in \{a, b\}^* \mid w \text{ contains } aa \text{ as a substring} \}$
DFAs

- A DFA is defined relative to some alphabet \( \Sigma \).
- For each state in the DFA, there must be exactly one transition defined for each symbol in \( \Sigma \).
  - This is the “deterministic” part of DFA.
- There is a unique start state.
- There are zero or more accepting states.
New Stuff!
Which table best represents the transitions for the DFA shown below?

(A)  
\[
\begin{array}{c|cc}
q_0 & q_1 & q_0 \\
q_1 & q_3 & q_2 \\
q_2 & q_3 & q_0 \\
q_3 & q_3 & q_3 \\
\end{array}
\]

(B)  
\[
\begin{array}{c|cc}
q_0 & q_0 & q_1 \\
q_1 & q_2 & q_3 \\
q_2 & q_0 & q_3 \\
q_3 & q_3 & q_3 \\
\end{array}
\]

(C)  
\[
\begin{array}{c|ccc}
q_0 & q_1 & q_0 & / \\
q_1 & q_3 & q_2 & / \\
q_2 & q_3 & q_0 & / \\
q_3 & / & / & q_3 \\
\end{array}
\]

Answer at PollEv.com/cs103 or text CS103 to 22333 once to join, then A, B, C, or D (none of the above).
Tabular DFAs
Tabular DFAs

\[
\begin{array}{c|cc}
\text{state} & \text{0} & \text{1} \\
\hline
q_0 & q_1 & q_0 \\
q_1 & q_3 & q_2 \\
q_2 & q_3 & q_0 \\
q_3 & q_3 & q_3 \\
\end{array}
\]
Tabular DFAs

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>*q_0</td>
<td>q_1</td>
<td>q_0</td>
</tr>
<tr>
<td>q_1</td>
<td>q_3</td>
<td>q_2</td>
</tr>
<tr>
<td>q_2</td>
<td>q_3</td>
<td>q_0</td>
</tr>
<tr>
<td>*q_3</td>
<td>q_3</td>
<td>q_3</td>
</tr>
</tbody>
</table>
These stars indicate accepting states.

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>*q_0</td>
<td>q_1</td>
<td>q_0</td>
</tr>
<tr>
<td>q_1</td>
<td>q_3</td>
<td>q_2</td>
</tr>
<tr>
<td>q_2</td>
<td>q_3</td>
<td>q_0</td>
</tr>
<tr>
<td>*q_3</td>
<td>q_3</td>
<td>q_3</td>
</tr>
</tbody>
</table>
Since this is the first row, it's the start state.

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>*q₀</td>
<td>q₁</td>
<td>q₀</td>
</tr>
<tr>
<td>q₁</td>
<td>q₃</td>
<td>q₂</td>
</tr>
<tr>
<td>q₂</td>
<td>q₃</td>
<td>q₀</td>
</tr>
<tr>
<td>*q₃</td>
<td>q₃</td>
<td>q₃</td>
</tr>
</tbody>
</table>
The Regular Languages
A language $L$ is called a \textit{regular language} if there exists a DFA $D$ such that $\mathcal{L}(D) = L$.

If $L$ is a language and $\mathcal{L}(D) = L$, we say that $D$ \textit{recognizes} the language $L$. 