Another TM Design

• We just designed a TM for this language over $\Sigma = \{0, 1\}$:
  
  $$L = \{ w \in \Sigma^* \mid w \text{ has the same number of } 0\text{s and } 1\text{s } \}$$

• Let's do a quick review of how it worked.
A Leap of Faith
A Leap of Faith
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A Leap of Faith

... 0 0 1 1 1 0 ...

...
A Leap of Faith
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A Leap of Faith
A Leap of Faith

How do we know that this blank isn't one of the infinitely many blanks after our input string?
The Solution
The Solution

... × 0 0 1 1 1 1 1 0 ...

The Solution
The Solution
The Solution
The Solution

... \times 0 \times 0 \times 1 \times 1 \times 1 \times 0 \times ...

The Solution

... × 0 0 × 1 1 1 1 0 ...

...
The Solution

| ... | × 0 0 × 1 1 1 0 | ... |
The Solution

... × 0 0 × 1 1 1 0 ...
The Solution

\[ \ldots \times 0 0 \times 1 1 1 0 \ldots \]
The Solution

| ... | × | × | 0 | × | 1 | 1 | 1 | 0 | ... |
The Solution

... × × 0 × 1 1 1 1 0 ...

...
The Solution

... × × 0 × 1 1 1 1 0 ...

...
The Solution

| ... | × | × | 0 | × | × | 1 | 1 | 0 | ... |
The Solution

| ... | × | × | 0 | × | × | 1 | 1 | 0 | ... |
The Solution

... × × 0 × × 1 1 0 ...
The Solution
The Solution
The Solution

... × × 0 × × 1 1 0 ...
A Different Idea
A Different Strategy

\[\ldots 0 0 0 1 1 1 1 1 0 \ldots\]
A Different Strategy

Could we sort the characters of this string?
A Different Strategy

Observation 1: A string of 0s and 1s is sorted if it matches the regex 0*1*.
A Different Strategy

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### A Different Strategy

**Observation 1:** A string of 0s and 1s is sorted if it matches the regex $0^*1^*$. 

| ... | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | ... |

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A Different Strategy

Observation 1: A string of 0s and 1s is sorted if it matches the regex $0^*1^*$.
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... 0 0 0 1 1 1 1 1 0 ...
A Different Strategy

Observation 1: A string of 0s and 1s is sorted if it matches the regex 0*1*.
A Different Strategy

Observation 2: A string of 0s and 1s is not sorted if it contains 10 as a substring.
A Different Strategy

Observation 2: A string of 0s and 1s is not sorted if it contains 10 as a substring.
A Different Strategy

Observation 2: A string of 0s and 1s is not sorted if it contains 10 as a substring.
A Different Strategy

Observation 2: A string of 0s and 1s is not sorted if it contains 10 as a substring.

... 0 0 0 1 1 1 1 0 1 ...
A Different Strategy

Idea: Repeatedly find a copy of 10 and replace it with 01.
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Let's Build It!