Week 3 Tutorial

*Mathematical Logic*
Take-Home Exam Revision Resources

- Feedback for the first round of Week 2 Take-Home Exam has been released – review on Gradescope ASAP
- Specific by-problem resources linked on course website
- Videos on Canvas for Problems 2.i and 2.ii – under Course Videos where you find the lecture videos
- Ask for help in office hours or on the Q&A forum
- Special Take-Home Exam office hours Thursday 4/23 after PS2 deadline
- If you feel you would benefit from additional 1-on-1 support beyond these resources, please email the staff mailing list cs103-spr1920-staff@lists.stanford.edu
Other Announcements

• Updates to office hours – will be posted to office hours page on course website once finalized

• Please tag your pages on Gradescope

• CS103 Week 3 Pulse Check: https://forms.gle/FgDH2NzHWjParh9D6
Part 1: *Logic Recap and Warmup*
represents $Loves(x, y)$
1. Determine the minimum number of arrows that must be added to make the following statements true.

You can’t remove existing arrows. If a formula is already true, the answer will be “0”. Tell us what the arrows are and briefly explain why it’s the smallest number that need to be added.

a) Loves(A, A) → Loves(E, E)
b) Loves(A, B) → Loves(C, D)
c) Loves(A, C) → Loves(C, F)
d) Loves(A, D) → Loves(D, E)
e) Loves(A, D) ↔ Loves(B, C)
f) Loves(A, C) ↔ Loves(E, C)

*Fill in answer on Gradescope!*
2. Repeat the previous exercise with the following statements:

a) $\forall x. \exists y. (Loves(x, y))$
b) $\forall x. \exists y. (x \neq y \land Loves(x, y))$
c) $\exists x. \forall y. (Loves(x, y))$
d) $\exists x. \forall y. (x \neq y \rightarrow Loves(x, y))$

*Fill in answer on Gradescope!*
Part 2: Negating Statements
∀p. (Person(p) →
   ∃q. ((Person(q) ∧ p ≠ q) ∧ Loves(p, q)
   )
)
\neg \forall p. (\text{Person}(p) \rightarrow \\
\exists q. ((\text{Person}(q) \land p \neq q) \land \\
\text{Loves}(p, q)) \}
)\)
\[ \neg \forall p. (\text{Person}(p) \rightarrow \exists q. ((\text{Person}(q) \land p \neq q) \land \text{Loves}(p, q)) \) \]
\[\neg \forall p. (\text{Person}(p) \rightarrow \exists q. ((\text{Person}(q) \land p \neq q) \land \text{Loves}(p, q)) \land \neg \forall x. A \quad \downarrow \]
\[
\exists x. \neg A
\]
\[ \neg \forall p. \ ( Person(p) \rightarrow \\
\exists q. \ (( Person(q) \land p \neq q) \land \\
\text{Loves}(p, q) \) \) \]
\neg \forall p. (Person(p) \rightarrow \\
\exists q. ((Person(q) \land p \neq q) \land \\
Loves(p, q)) \\
)
\[\exists p. \neg (Person(p) \rightarrow \\
\exists q. ((Person(q) \land p \neq q) \land \\
Loves(p, q))\]

\[
\begin{array}{c}
\neg \forall x. A \\
\hline \\
\exists x. \neg A
\end{array}
\]
$\exists p. \neg (\text{Person}(p) \rightarrow \exists q. ((\text{Person}(q) \land p \neq q) \land \text{Loves}(p, q))$
3. Finish the negation of this first-order logic formula. Your final formula should not have any negations in it except for direct negations of predicates.

Here is the formula so far in LaTeX:

$$\exists p. \neg (\text{Person}(p) \rightarrow \\
\exists q. ((\text{Person}(q) \land p \neq q) \land \\
\text{Loves}(p, q))$$

Fill in answer on Gradescope!
\[\exists p. \neg (Person(p) \to \exists q. ((Person(q) \land p \neq q) \land Loves(p, q)) \land \neg (A \to B) \land A \land \neg B)\]
\[ \exists p. \neg (\text{Person}(p) \rightarrow \exists q. ((\text{Person}(q) \land p \neq q) \land \text{Loves}(p, q)) \land \neg(A \rightarrow B) \land A \land \neg B \]
\exists p. \neg (\text{Person}(p) \rightarrow \\
\exists q. ((\text{Person}(q) \land p \neq q) \land \\
\text{Loves}(p, q) \\
) \\
) \\

\neg (A \rightarrow B) \\
\hline \\
A \land \neg B
$\exists p. \ (\text{Person}(p) \land \\
\neg \exists q. \ ((\text{Person}(q) \land p \neq q) \land \\
\text{Loves}(p, q) \\
) \\
)$

\[
\neg (A \rightarrow B) \\
\hline
\]
\[A \land \neg B\]
\( \exists p. \ (\text{Person}(p) \land \neg \exists q. \ ((\text{Person}(q) \land p \neq q) \land \text{Loves}(p, q)) ) \)
\[ \exists p. \, (\text{Person}(p) \land \neg \exists q. \, ((\text{Person}(q) \land p \neq q) \land \text{Loves}(p, q)) \land \\
\neg \exists x. \, A) \quad \forall x. \, \neg A \]
\[ \exists p. (\text{Person}(p) \land \\
\neg \exists q. ((\text{Person}(q) \land p \neq q) \land \\
\text{Loves}(p, q)) \land \\
\neg \exists x. A) \]
\[ \exists p. \ (\text{Person}(p) \land \neg \exists q. ((\text{Person}(q) \land p \neq q) \land \text{Loves}(p, q))) \]

[Diagram:]

\[ \neg \exists x. A \]

\[ \forall x. \neg A \]
\[ \exists p. \ (\text{Person}(p) \land \forall q. \neg((\text{Person}(q) \land p \neq q) \land \text{Loves}(p, q)) \) \]
\[ \exists p . (\text{Person}(p) \land \\
\forall q . \neg((\text{Person}(q) \land p \neq q) \land \\
\text{Loves}(p, q)) \) \)
\[\exists p. \ (\text{Person}(p) \land \forall q. \ \neg((\text{Person}(q) \land p \neq q) \land \text{Loves}(p, q)) \land \neg(A \land B) \rightarrow \neg B)\]
\[ \exists p. \ (\text{Person}(p) \land \forall q. \ \neg((\text{Person}(q) \land p \neq q) \land \text{Loves}(p, q)) \land \neg(A \land B) \land A \rightarrow \neg B] \]
∃p. (Person(p) \land
    \forall q. \neg((Person(q) \land p \neq q) \land
    Loves(p, q))
)

\neg(A \land B)

A \rightarrow \neg B
∀q. (Person(q) ∧ p ≠ q) → ¬Loves(p, q)
\( \exists p. \ ( \text{Person}(p) \land \forall q. \ ((\text{Person}(q) \land p \neq q) \rightarrow \neg \text{Loves}(p, q)) \) \)
∀p. (Person(p) →
   ∃q. ((Person(q) ∧ p ≠ q) ∧
   Loves(p, q)
   )
)

∃p. (Person(p) ∧
   ∀q. ((Person(q) ∧ p ≠ q) →
   ¬Loves(p, q)
   )
)

Part 3: *First-Order Logic Translations*
Consider this statement:

“If someone is happy, then everyone is happy.”

What is the contrapositive of this statement?

4. Find the contrapositive of the above statement. You may find it helpful to first write out the original statement in first order logic.

Fill in answer on Gradescope!
If someone is happy, then everyone is happy
someone is happy $\rightarrow$ everyone is happy
someone is happy $\rightarrow \left( \forall x. \ Happy(x) \right)$
$$(\exists x. \text{Happy}(x)) \rightarrow (\forall x. \text{Happy}(x))$$
(∃x. \textit{Happy}(x)) \rightarrow (\forall x. \textit{Happy}(x))
(\exists x. \text{Happy}(x)) \rightarrow (\forall x. \text{Happy}(x))
\neg (\forall x. \text{Happy}(x)) \rightarrow \neg (\exists x. \text{Happy}(x))
(∃x. ¬Happy(x)) → ¬(∃x. Happy(x))
(∃x. ¬Happy(x)) \rightarrow (∀x. ¬Happy(x))
(∃x. ¬Happy(x)) → (∀x. ¬Happy(x))

“If someone is not happy, then everyone is not happy.”
Consider this statement:

“If someone is happy, then everyone is happy.”

What is the negation of this statement?

5. Find the negation of the above statement. You may find it helpful to first write out the original statement in first order logic.

*Fill in answer on Gradescope!*
(∃x. Happy(x)) → (∀x. Happy(x))
(∃x. Happy(x)) → (∀x. Happy(x))

¬((∃x. Happy(x)) → (∀x. Happy(x)))
(∃x. Happy(x)) → (∀x. Happy(x))

(∃x. Happy(x)) ∧ ¬(∀x. Happy(x))
(∃x. \text{Happy}(x)) \rightarrow (\forall x. \text{Happy}(x))

(∃x. \text{Happy}(x)) \land (∃x. \neg \text{Happy}(x))
\[ (\exists x. \text{Happy}(x)) \rightarrow (\forall x. \text{Happy}(x)) \]

\[ (\exists x. \text{Happy}(x)) \land (\exists x. \neg\text{Happy}(x)) \]

“Someone is happy and someone is not happy.”