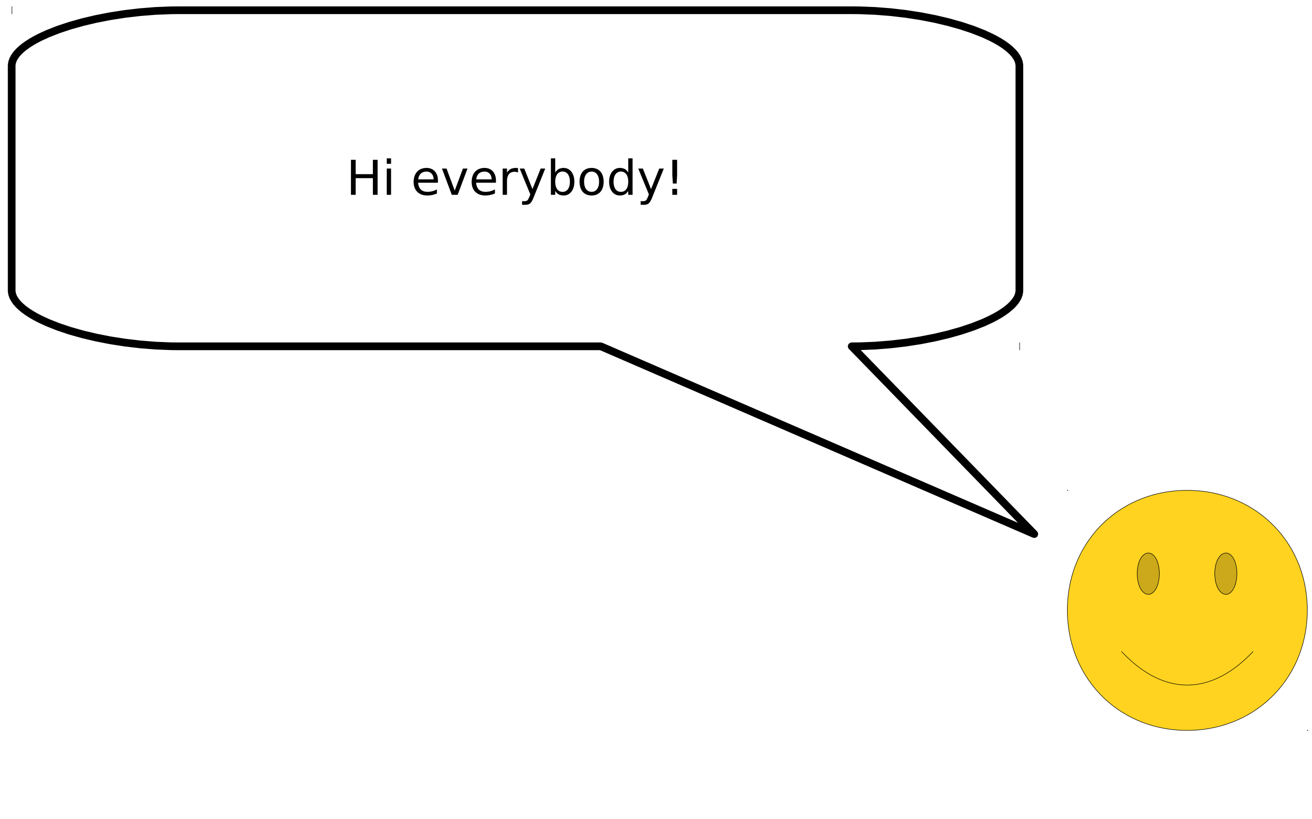
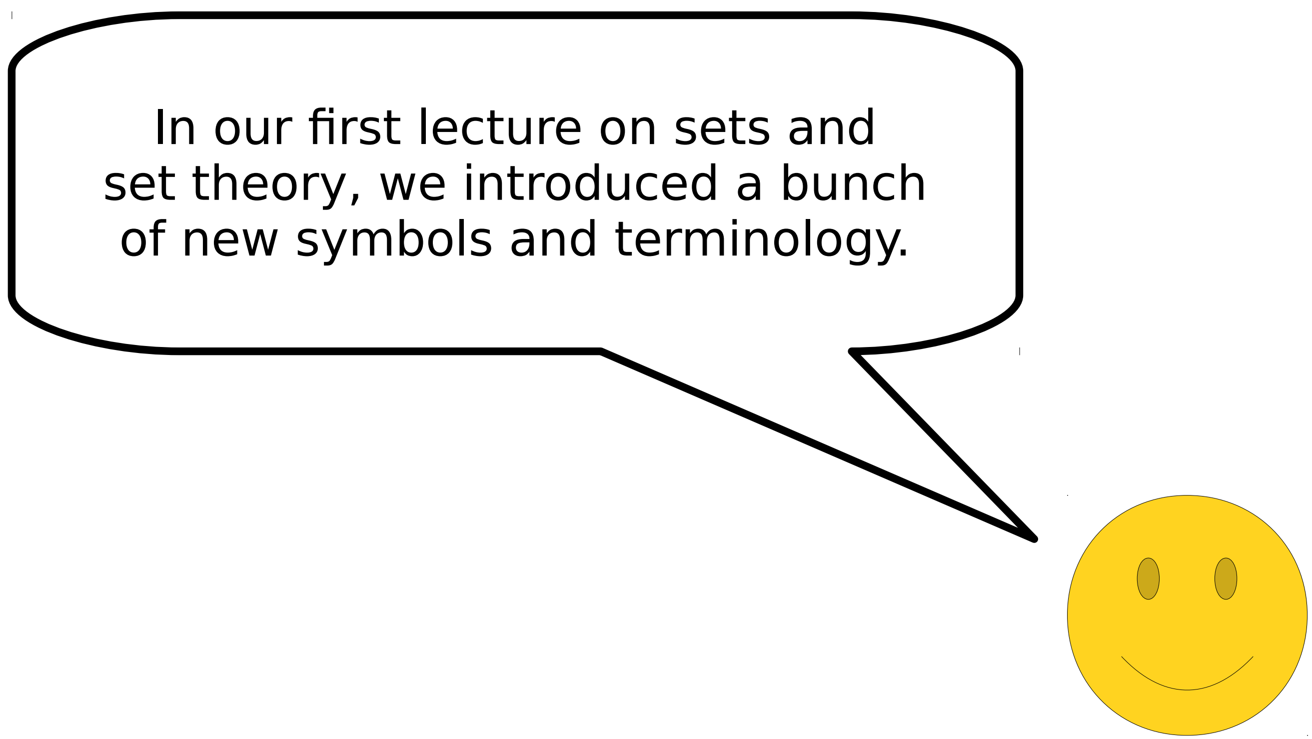
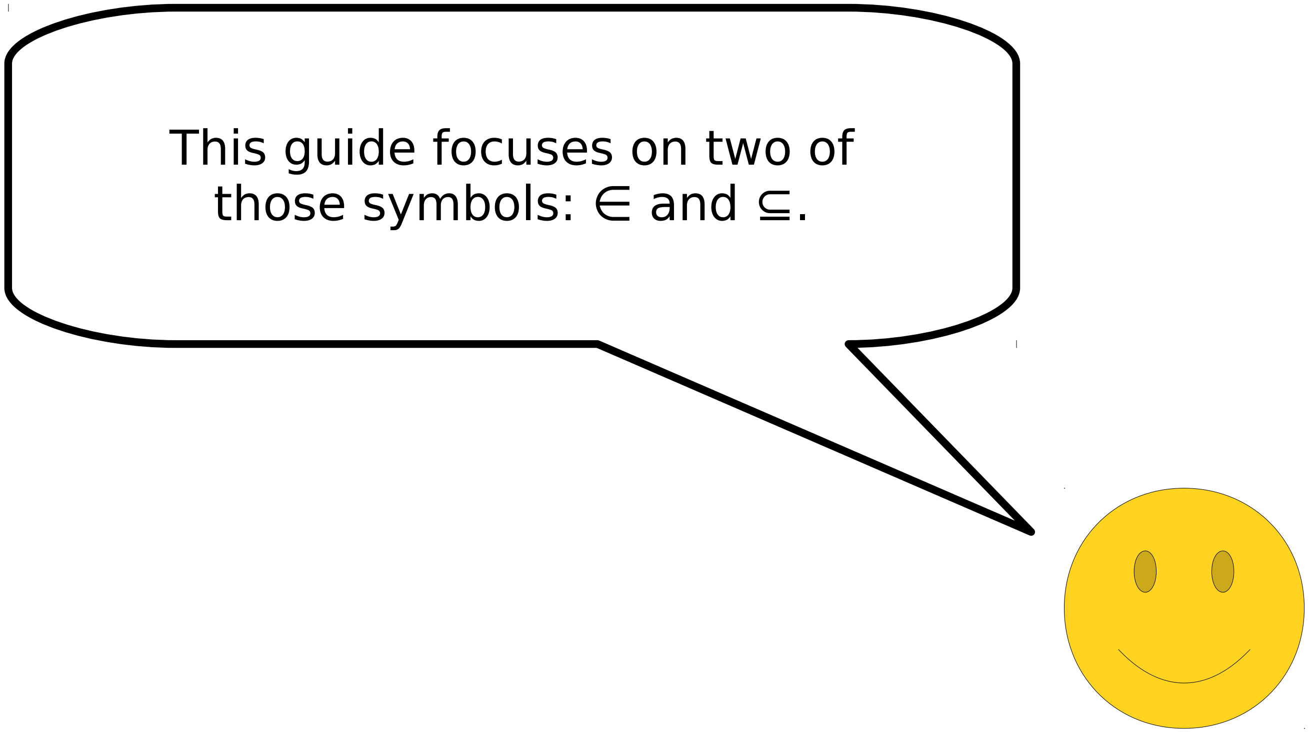
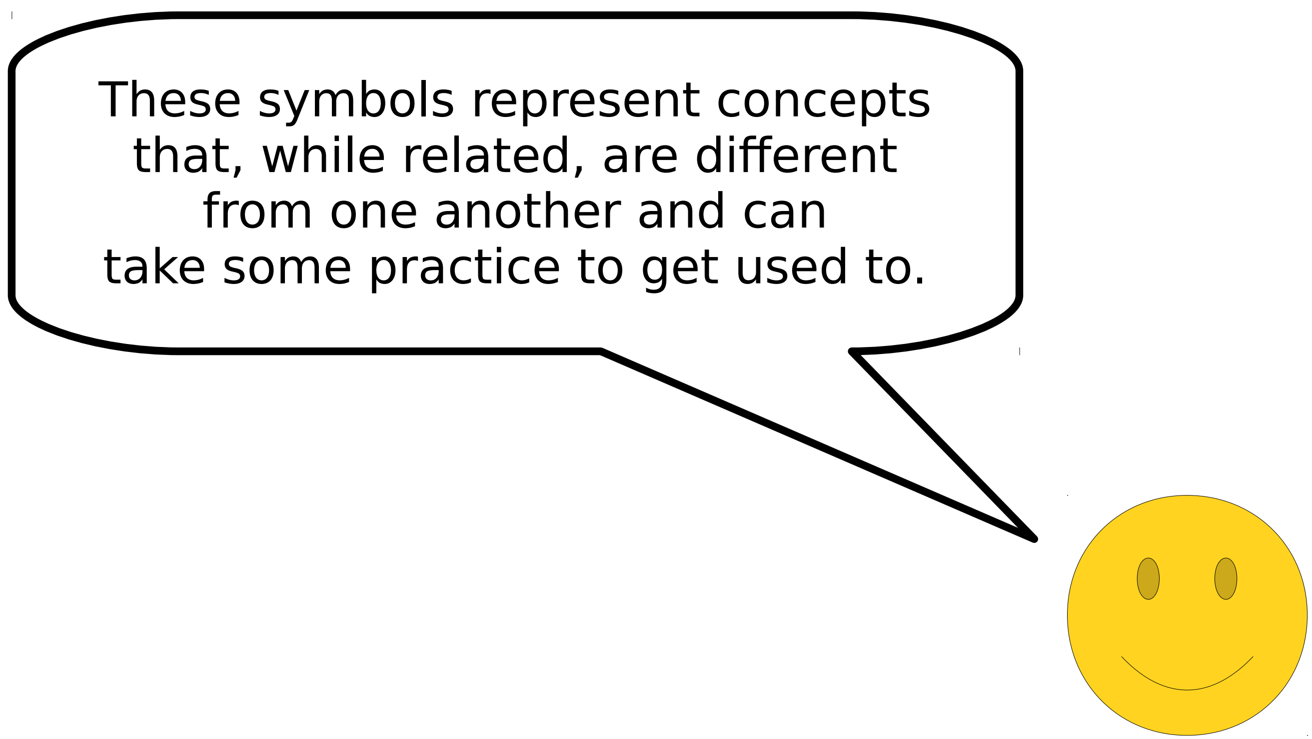
Guide to $\in$ and $\subseteq$


Hi everybody!

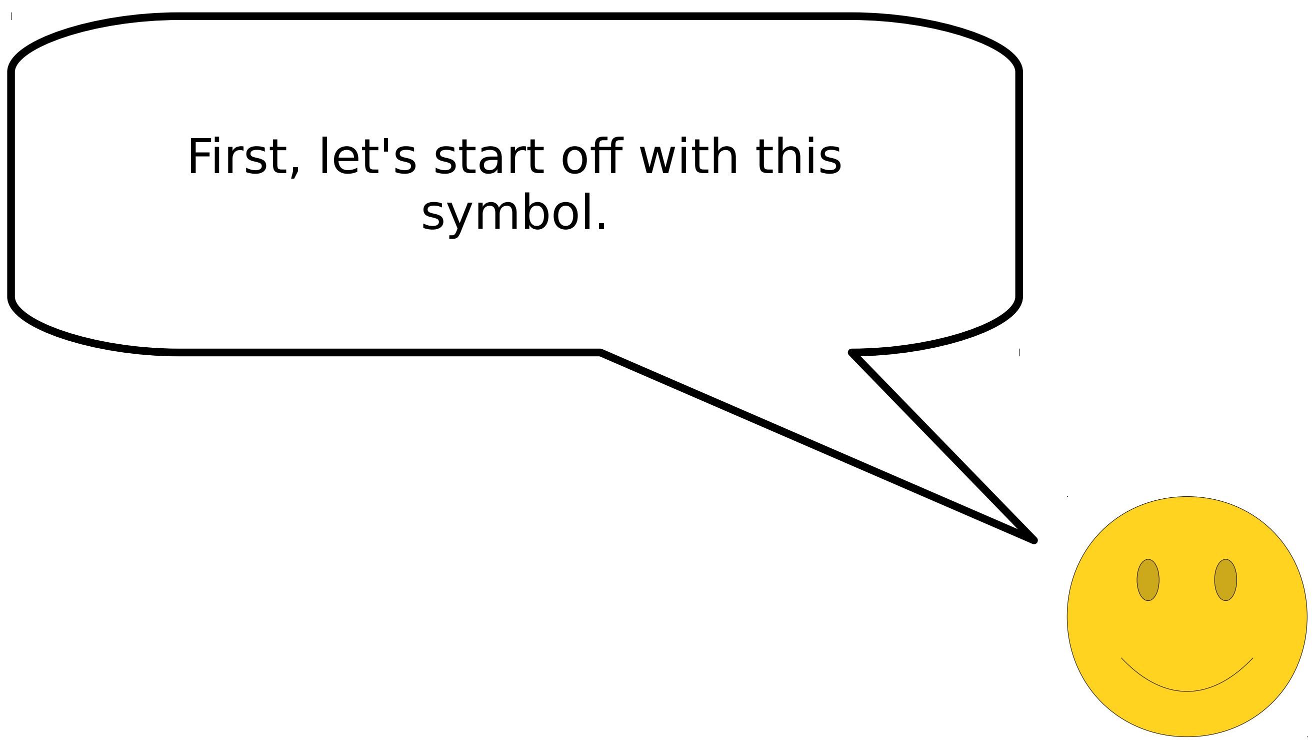

In our first lecture on sets and set theory, we introduced a bunch of new symbols and terminology.


This guide focuses on two of those symbols: ∈ and ⊆.


These symbols represent concepts that, while related, are different from one another and can take some practice to get used to.


If you're still a bit confused, don't worry! Let's take some time to review them and see how they work and how they differ.

$$\in$$



$$\in$$

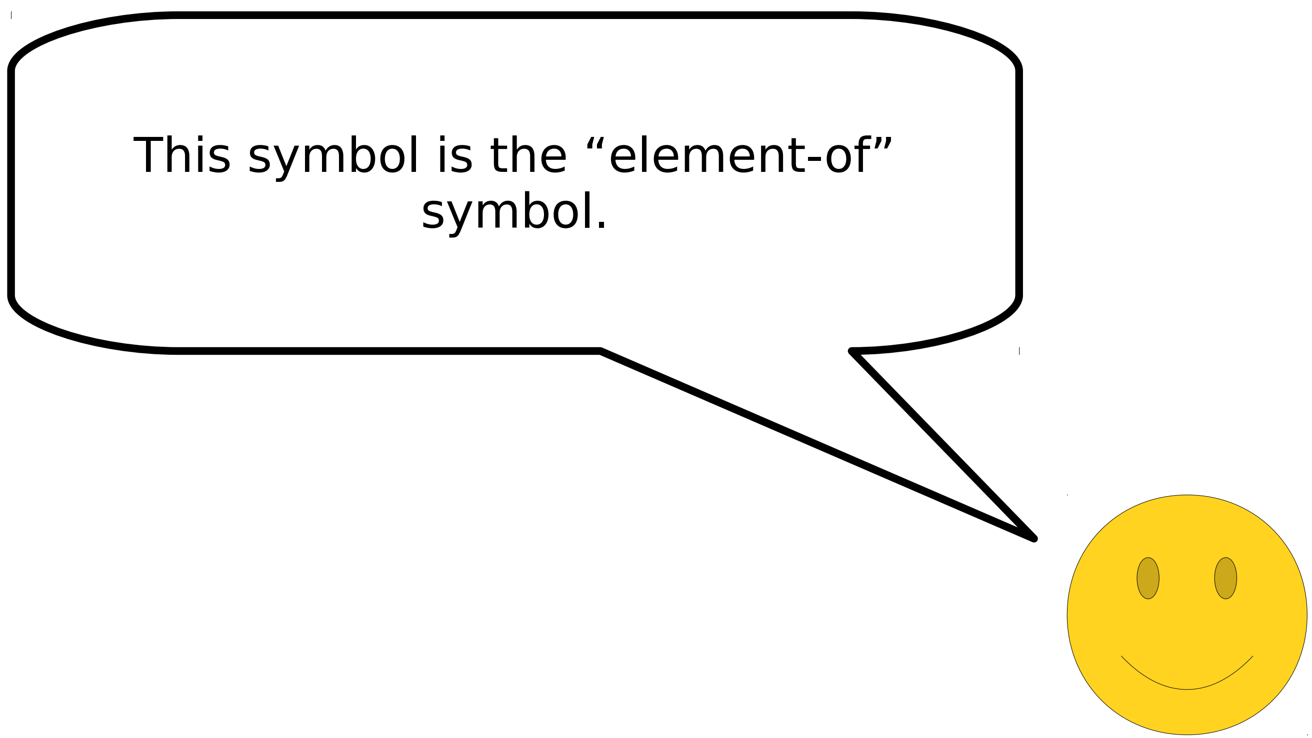
Before we begin, do you remember what this symbol means? If not, pull up your notes or look back at the first set of lecture slides.

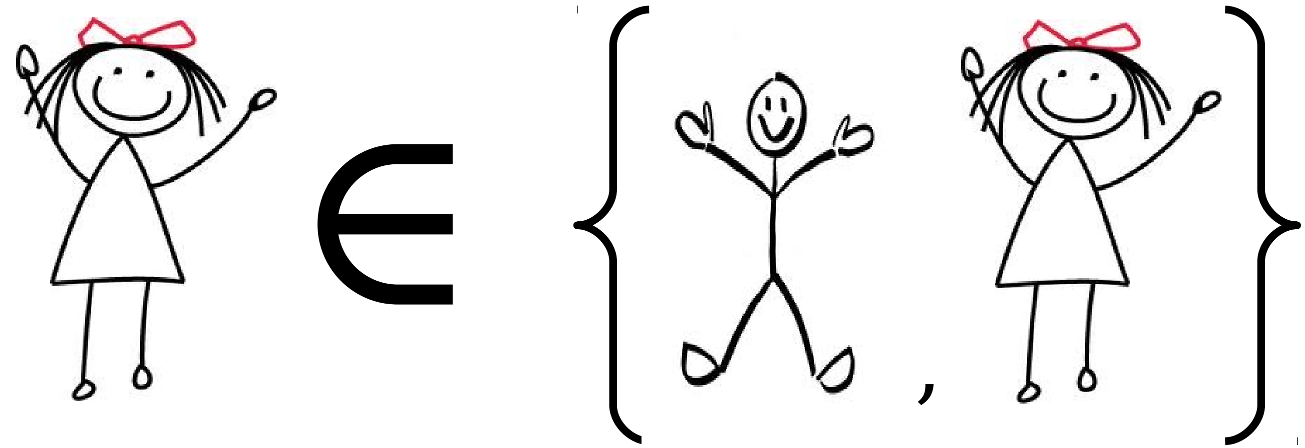
$$\in$$

Before we begin, do you remember what this symbol means? If not, pull up your notes or look back at the first set of lecture slides.

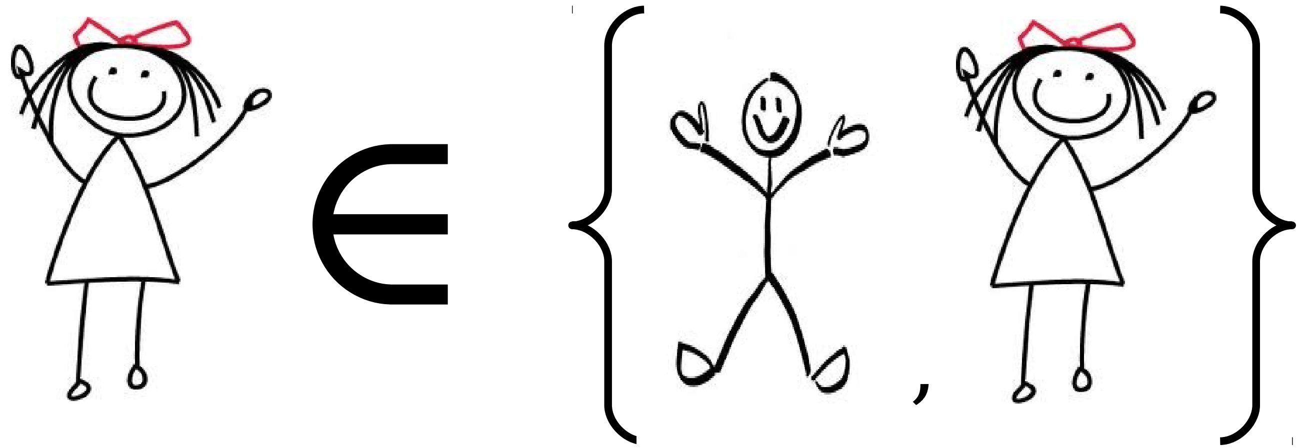
(If you haven't started taking notes in lecture, we really recommend it! It's extremely valuable.)

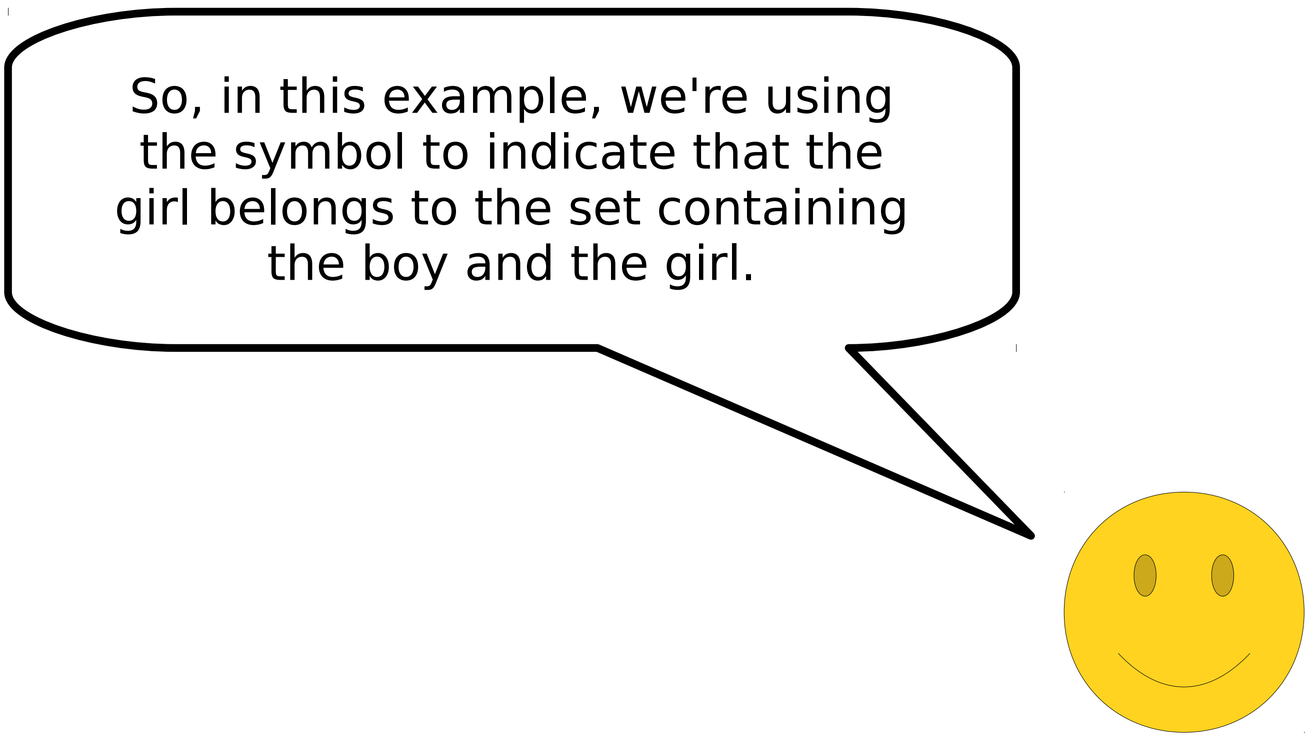
$$\in$$

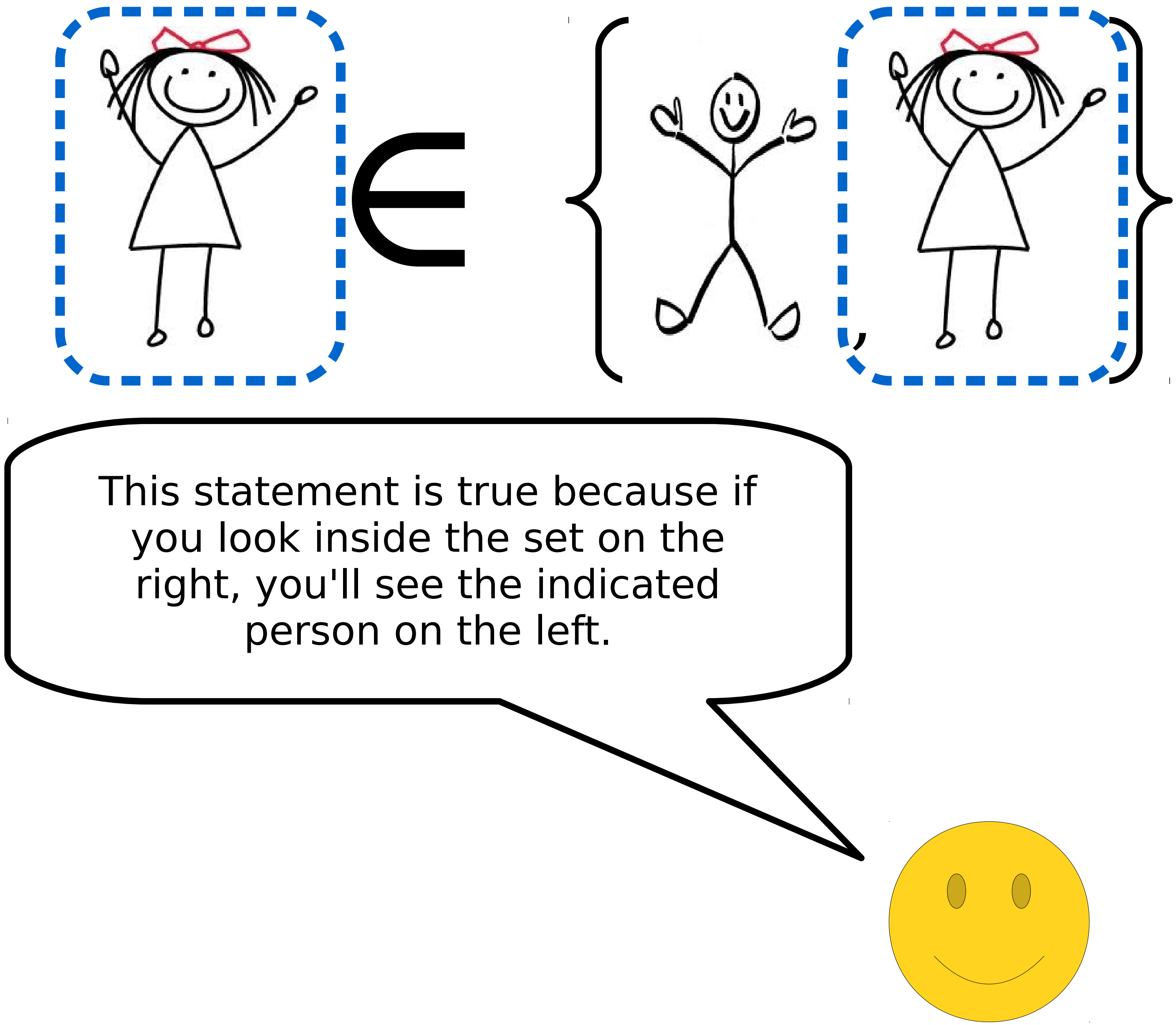


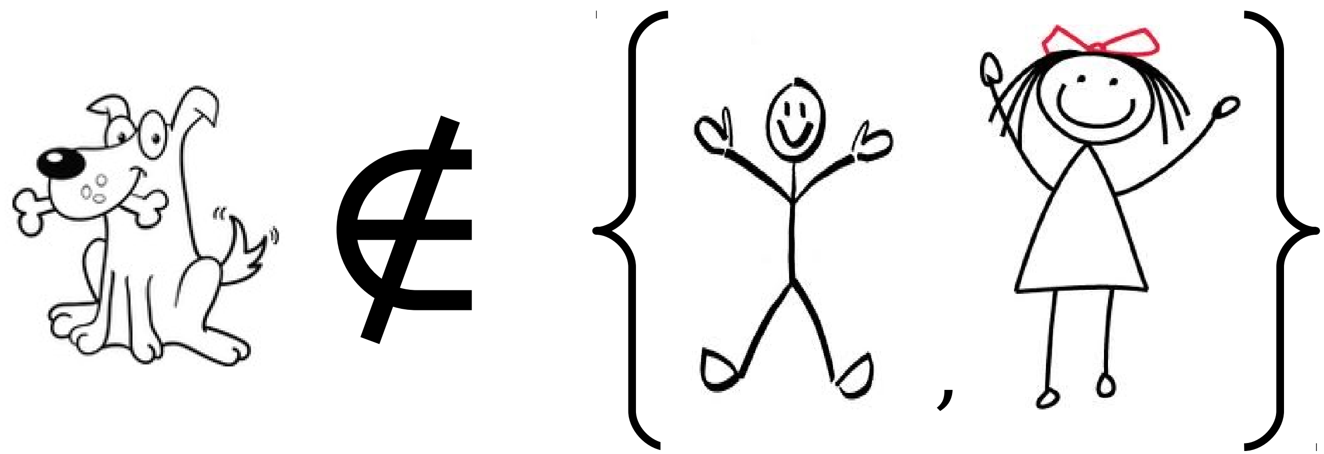


It's used to indicate that something is an ***element*** of a set.

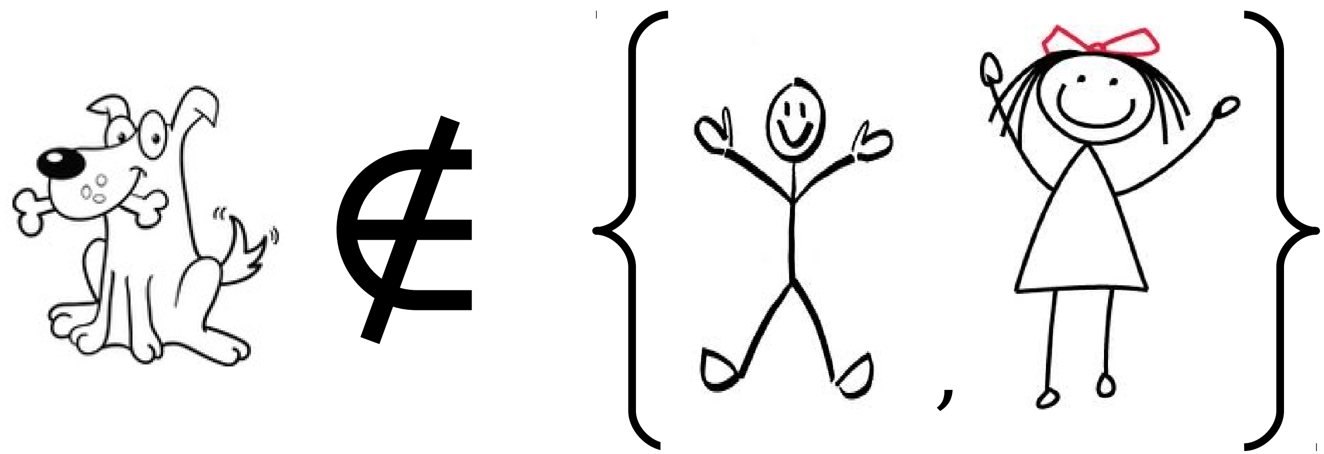


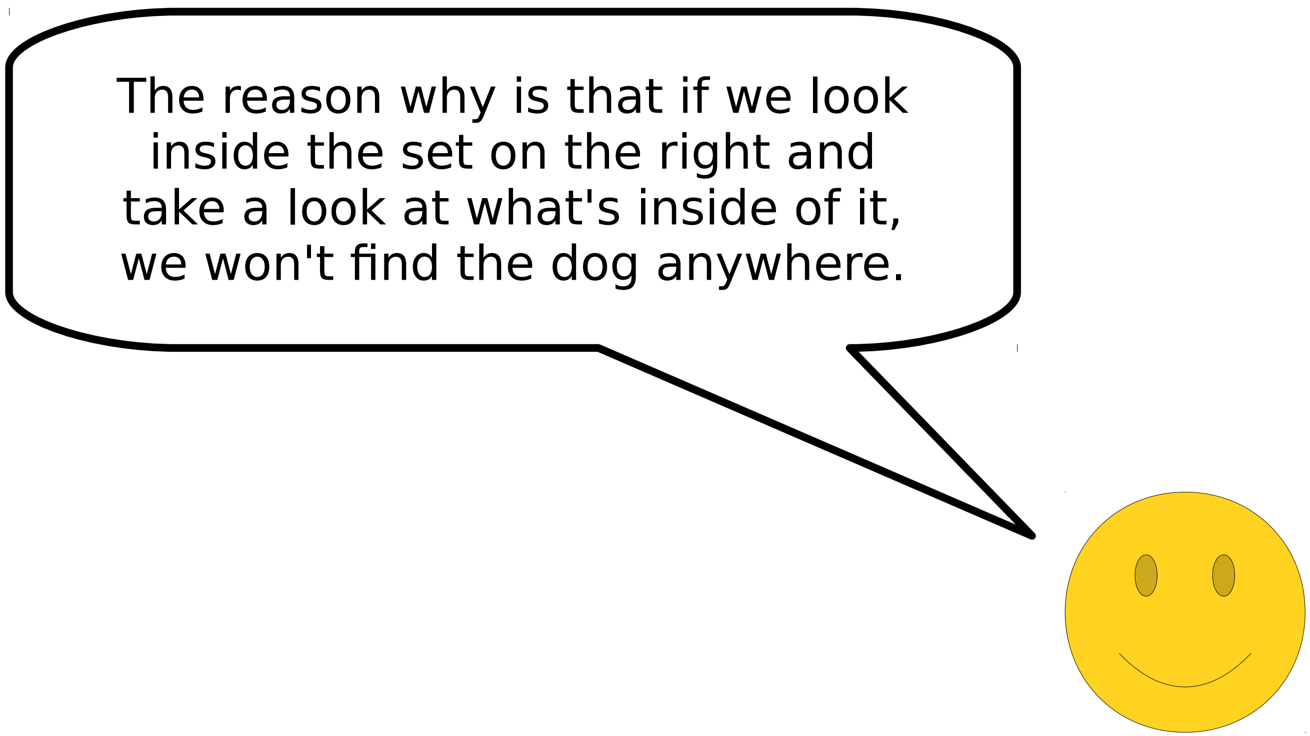

So, in this example, we're using the symbol to indicate that the girl belongs to the set containing the boy and the girl.

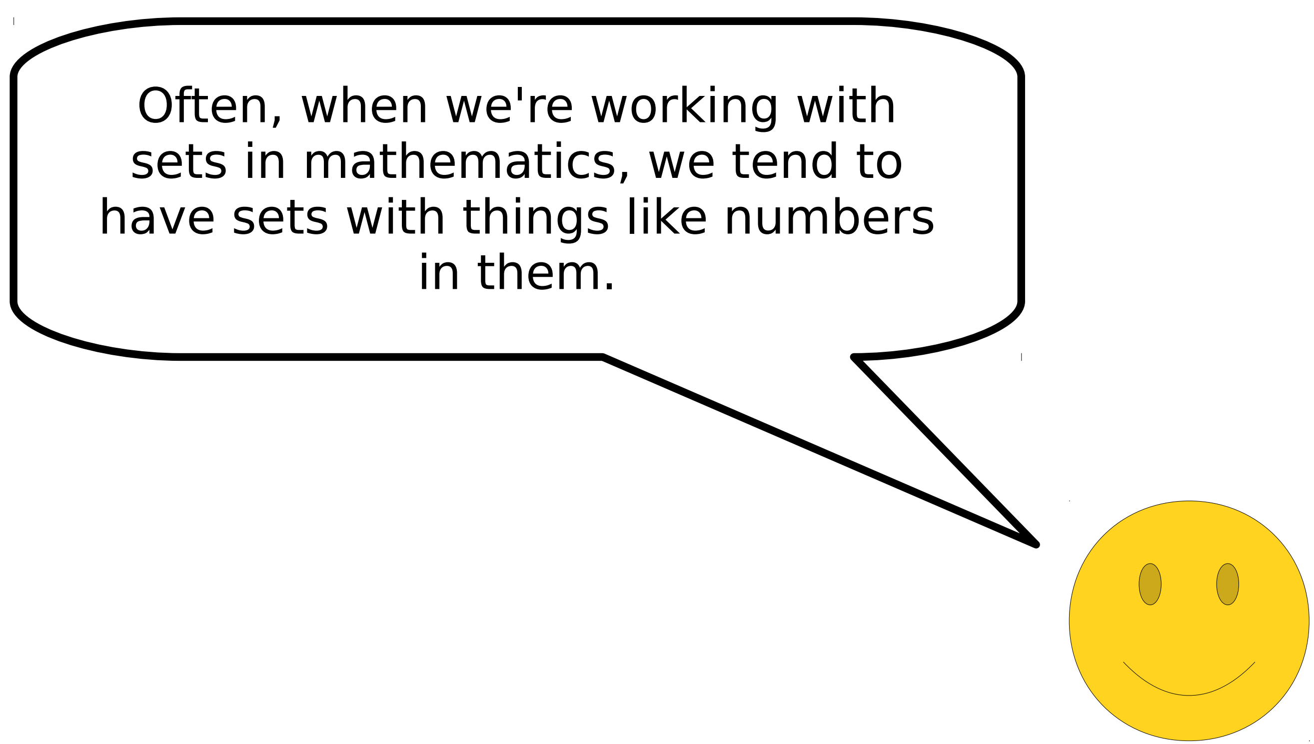

This statement is true because if you look inside the set on the right, you'll see the indicated person on the left.



On the other hand, the dog on the left is not an element of the set on the right.




The reason why is that if we look inside the set on the right and take a look at what's inside of it, we won't find the dog anywhere.


Often, when we're working with sets in mathematics, we tend to have sets with things like numbers in them.

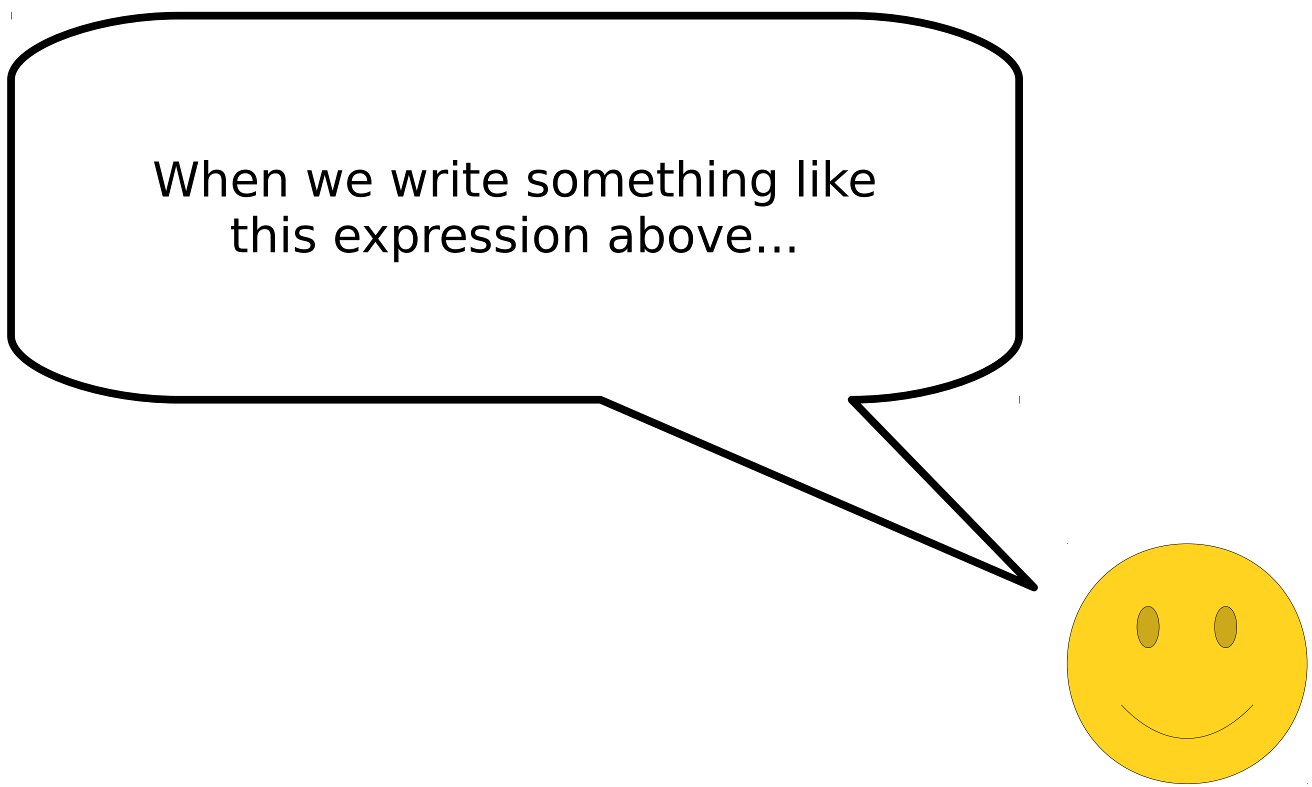
$$1 \in \{1, 2, 3, 4\}$$

So we'll typically see statements like this one, which is more mathematical in nature, even though the previous examples are perfectly correct uses of the $\in$ and $\notin$ symbols.

$\in$

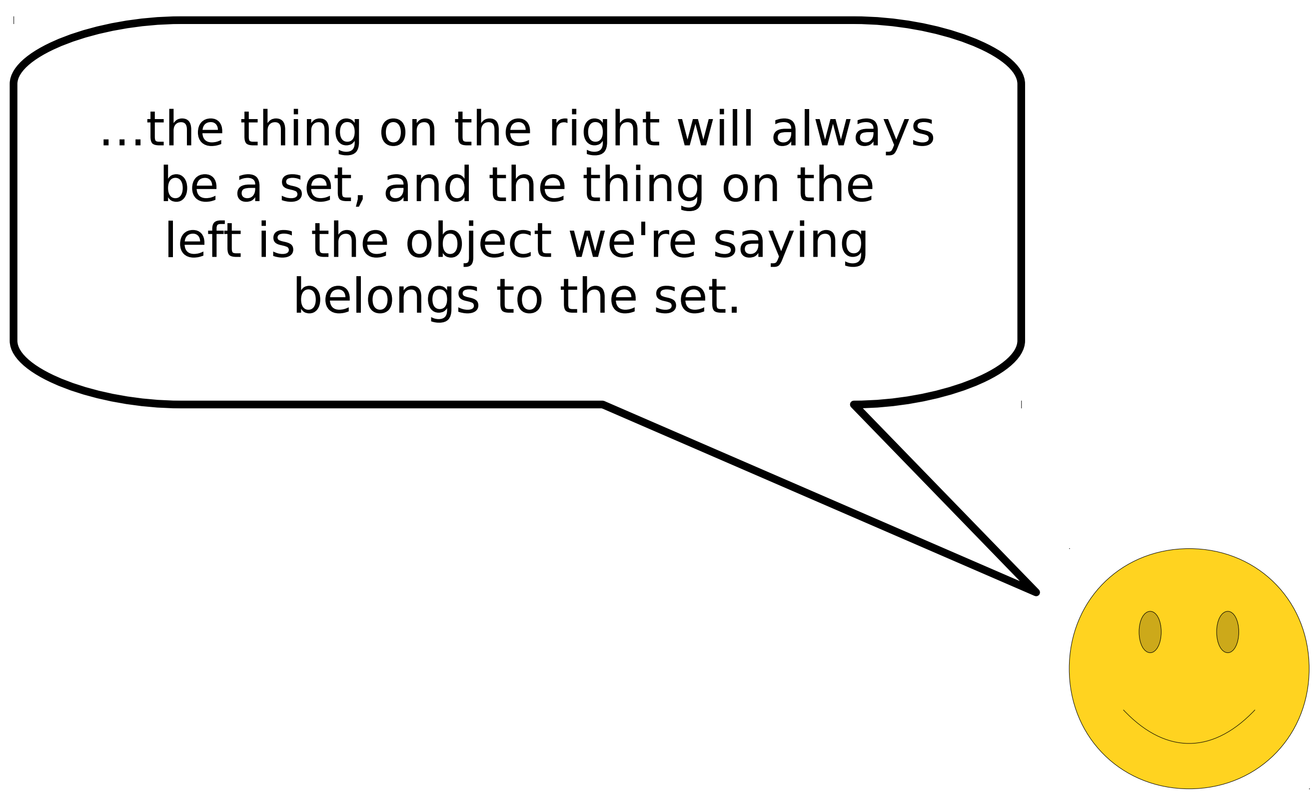
When using the ∈ symbol, it's important to keep in mind that there's a “sidedness” to it.

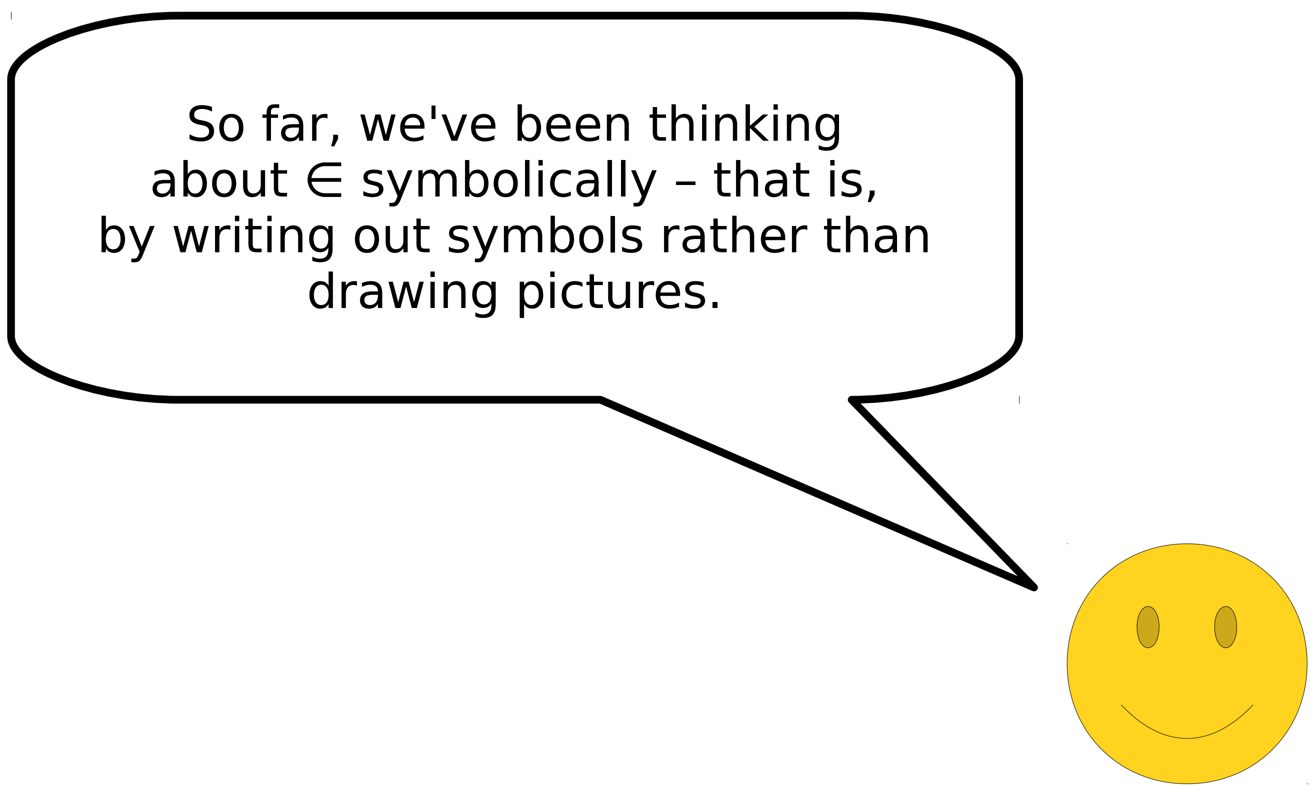
$$x \in S$$

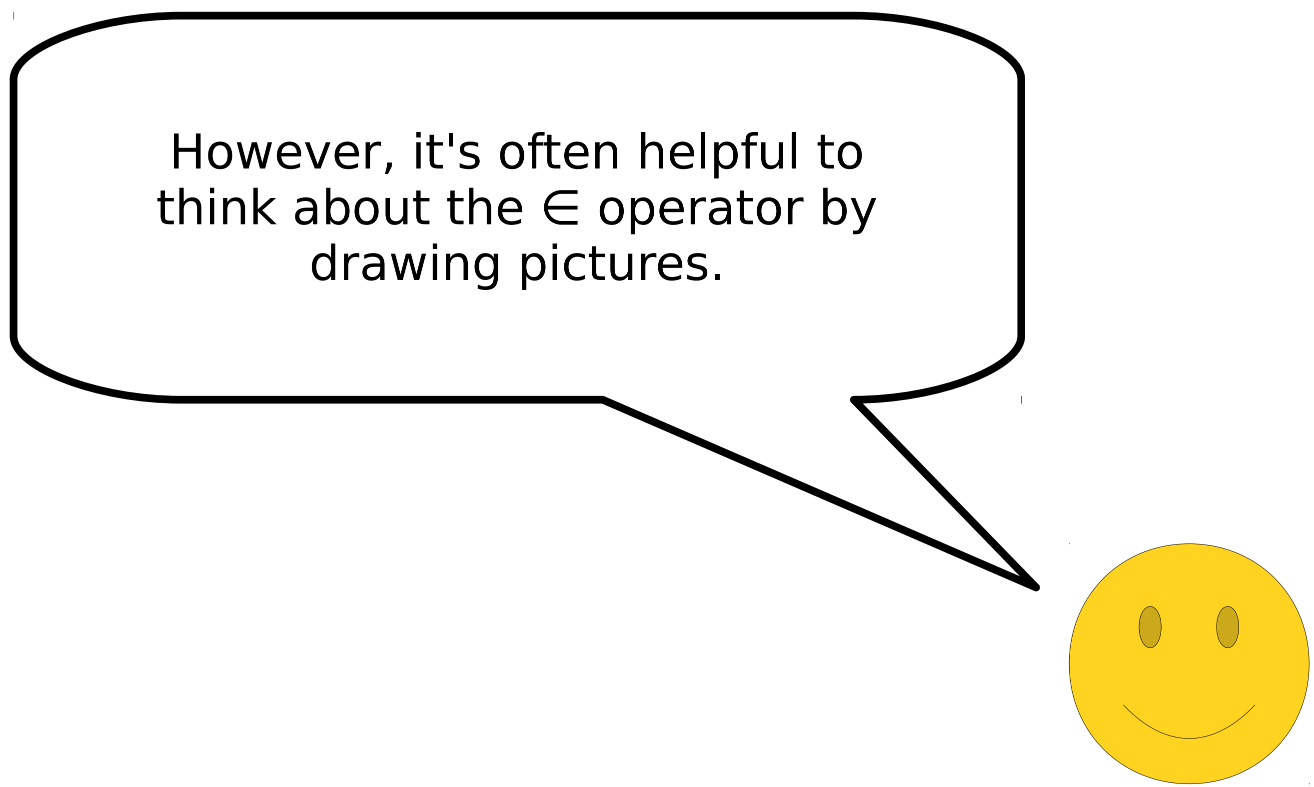


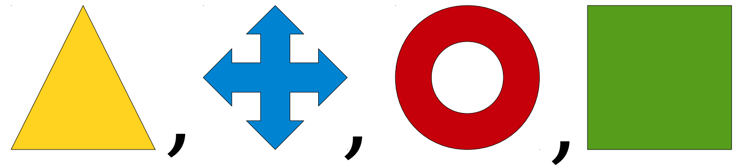
$$x \in S$$

This object is in this set

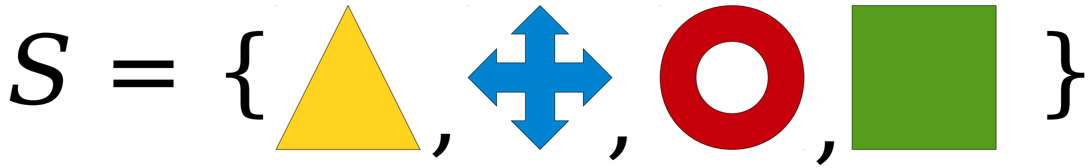


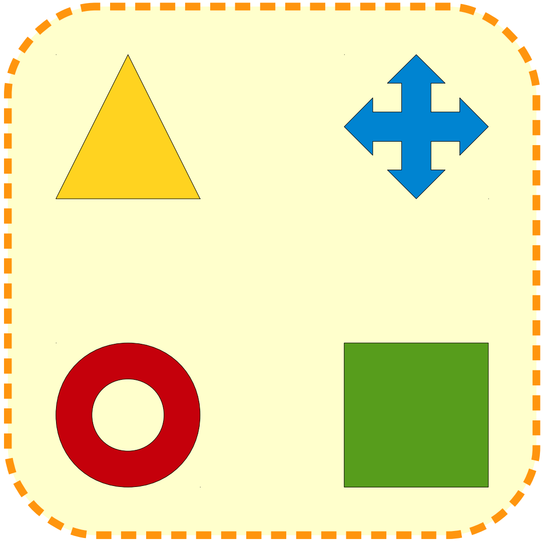

So far, we've been thinking
about ∈ symbolically – that is,
by writing out symbols rather than
drawing pictures.

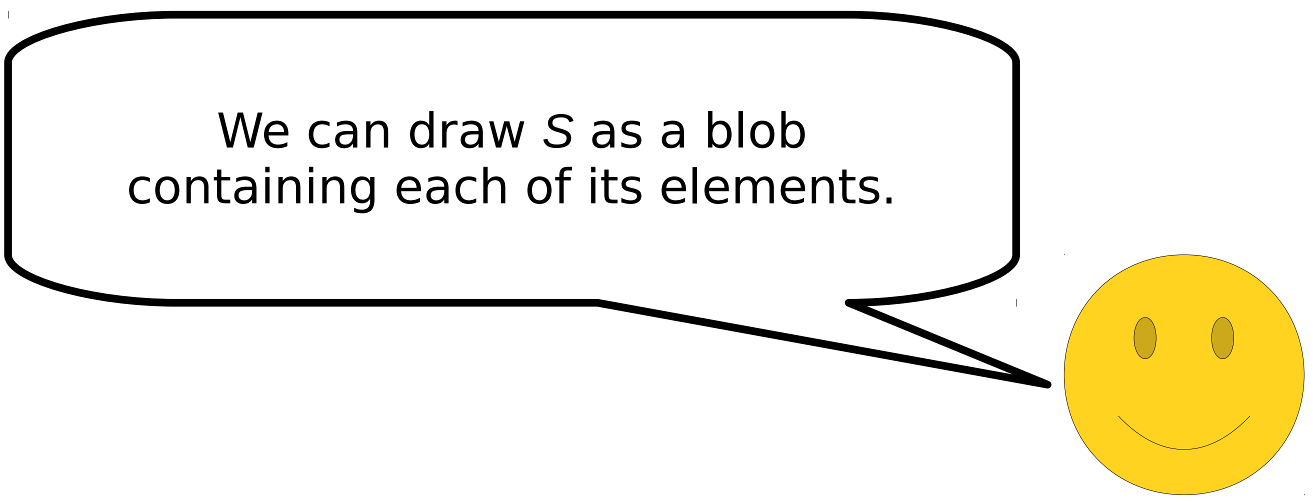

However, it's often helpful to think about the ∈ operator by drawing pictures.

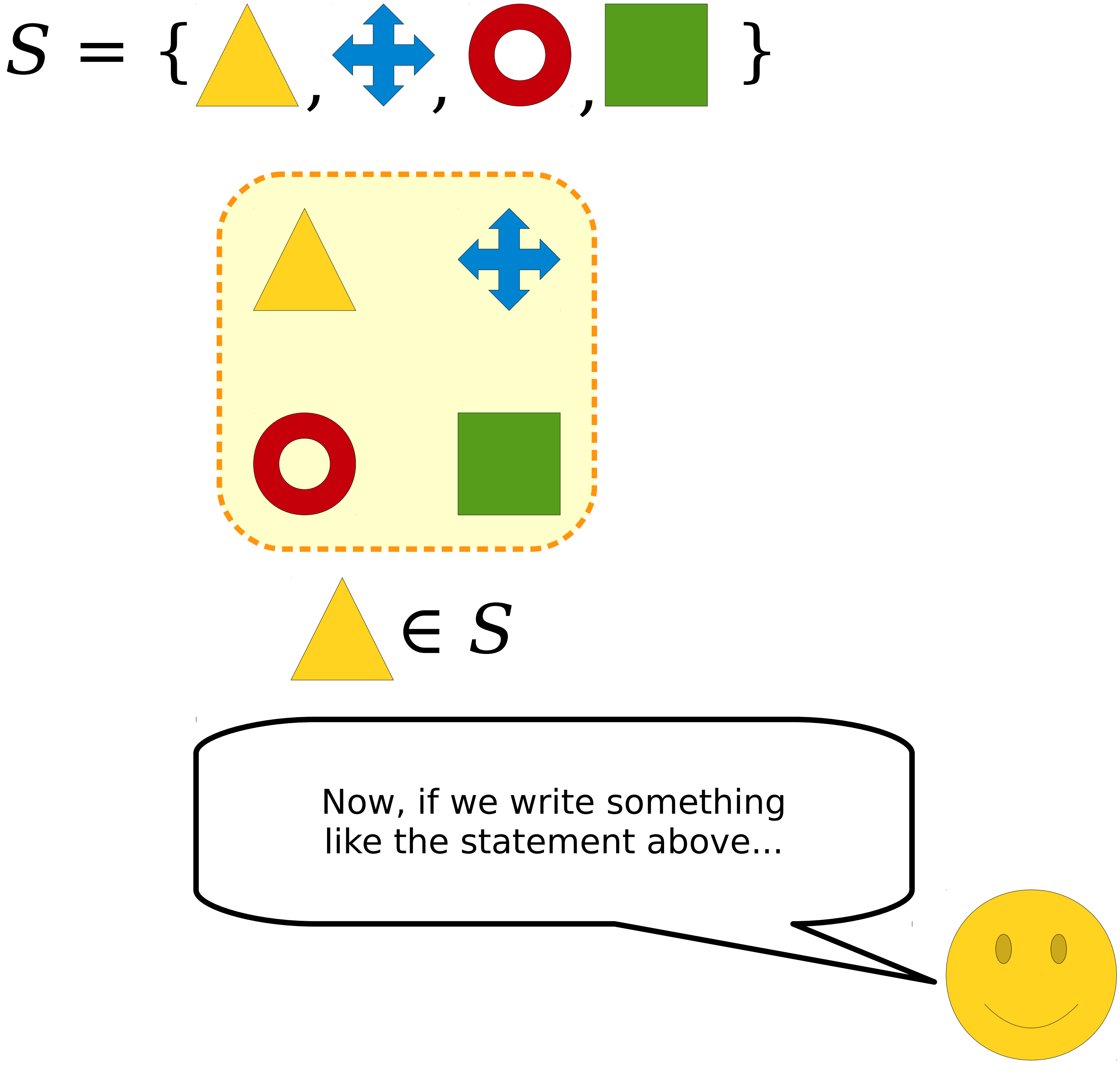
$S = \{$  $\}$

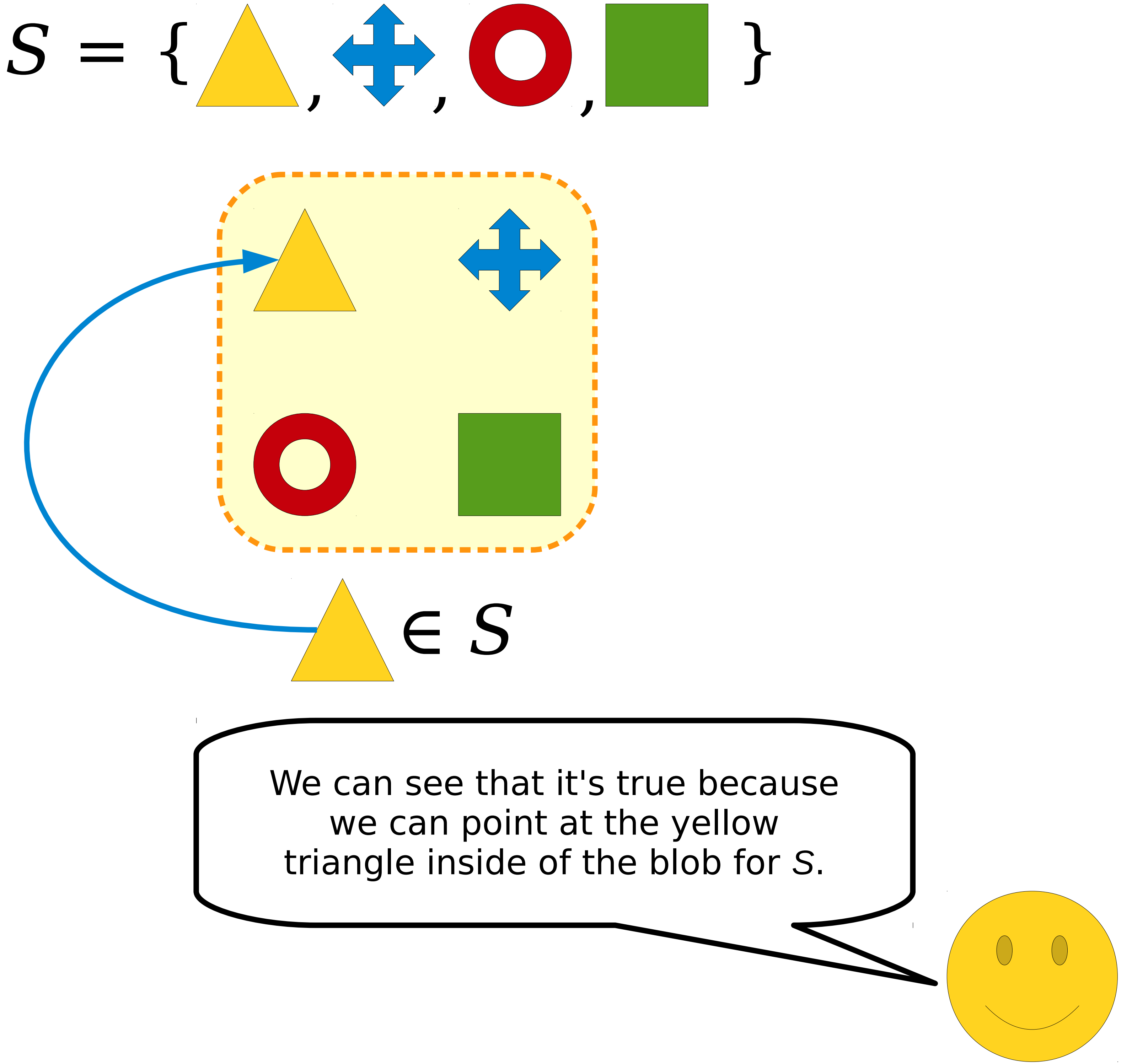
For example, let's imagine that we have this set S, which consists of shapes of different colors.

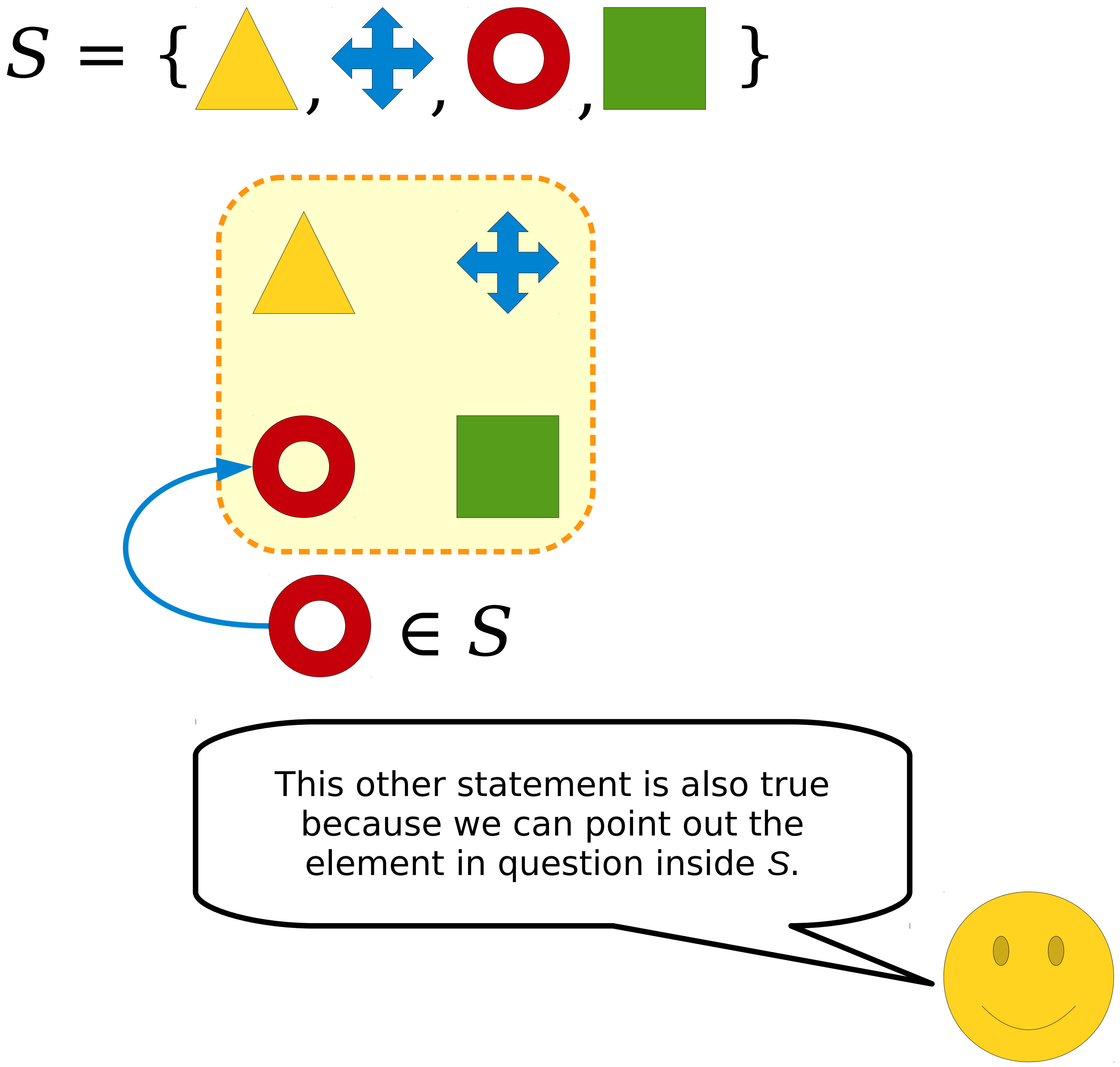


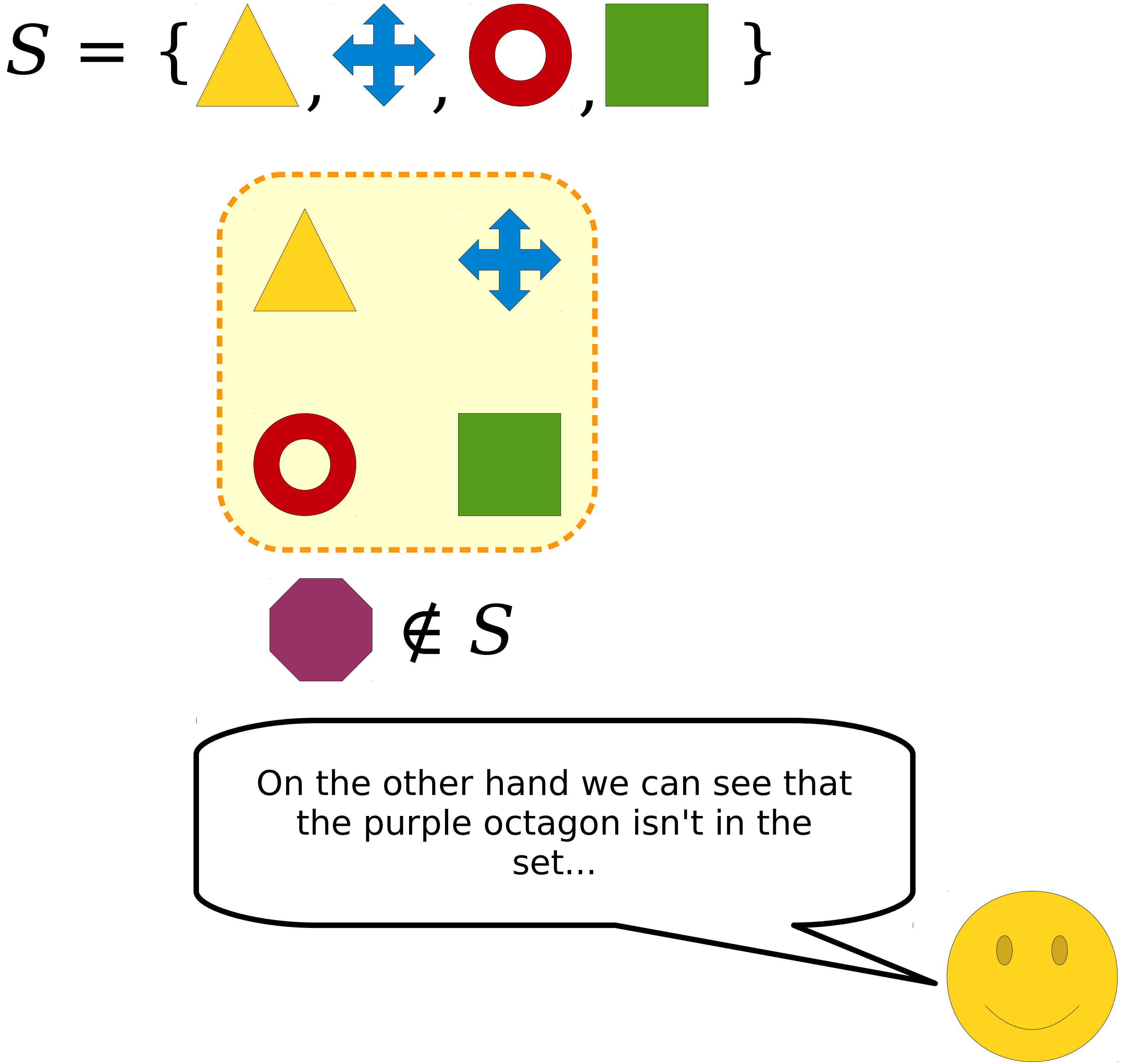


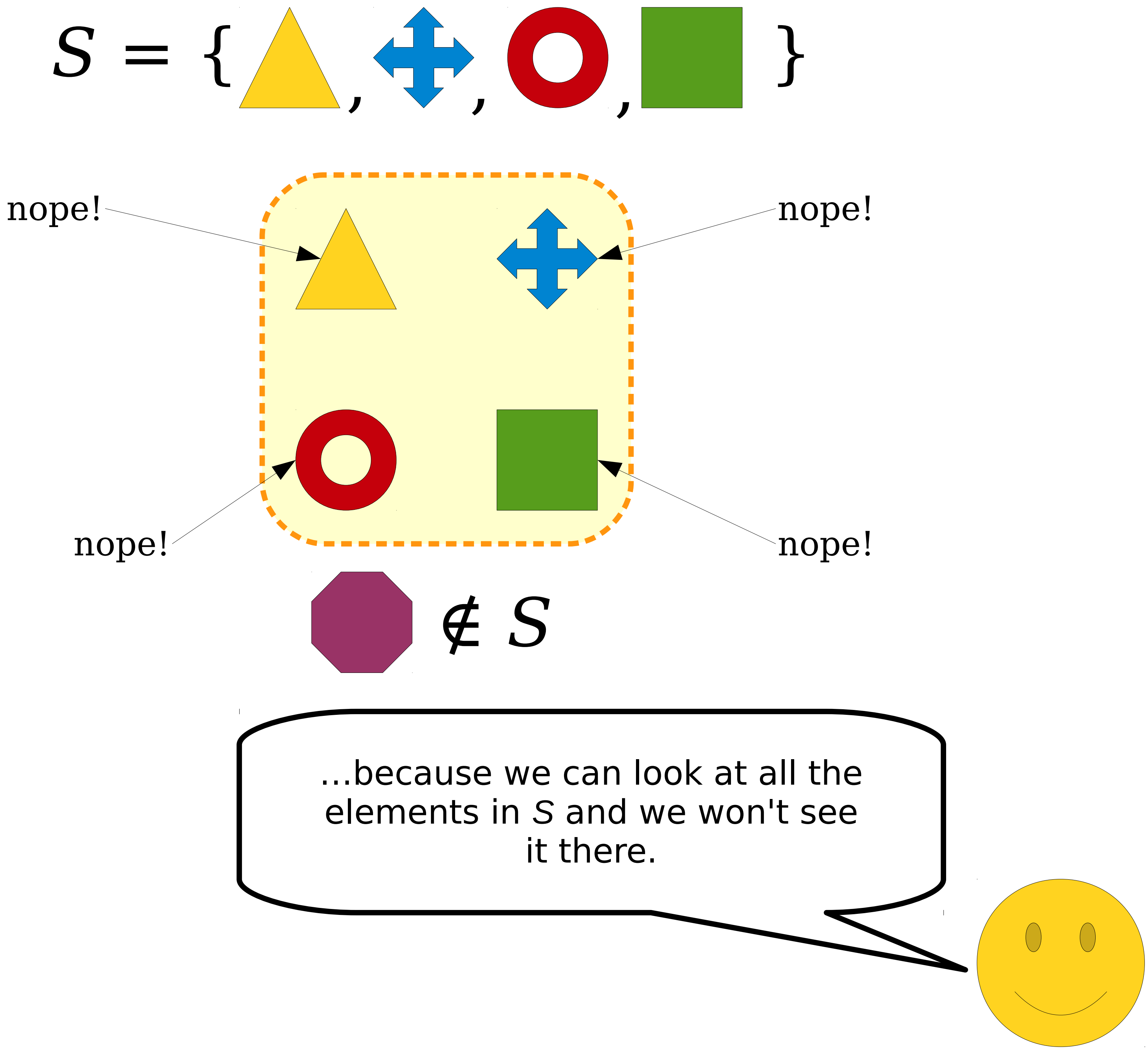

We can draw S as a blob
containing each of its elements.


S = { , , , }
∈ S
Now, if we write something like the statement above...


$S = \{$, , , $\}$
$\in S$
We can see that it's true because we can point at the yellow triangle inside of the blob for S.


S = { , , , }
∈ S
This other statement is also true
because we can point out the
element in question inside S.


S = { , , , }
∉ S
On the other hand we can see that the purple octagon isn't in the set...



S = { , , , }
nope!
nope!
nope!
nope!
∉ S
...because we can look at all the elements in S and we won't see it there.

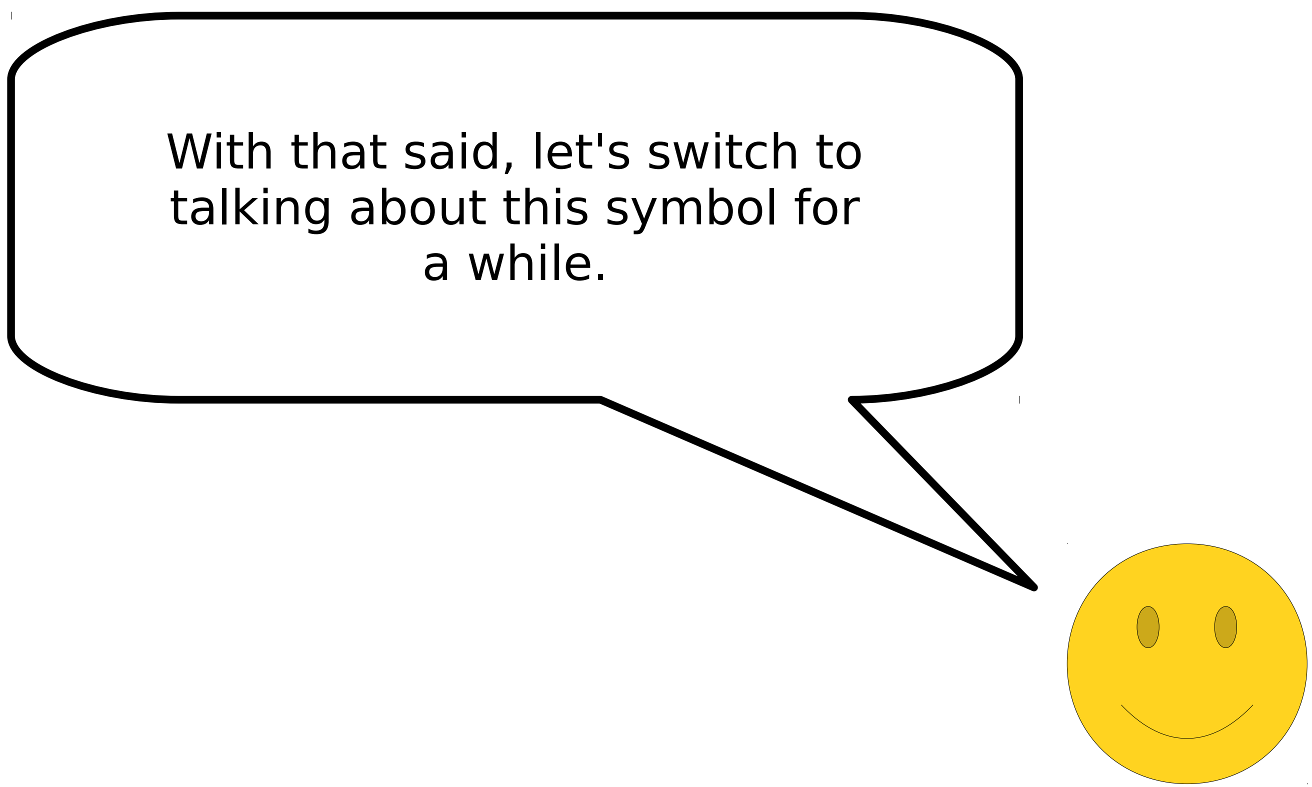

To recap things so far...

$$x \in S$$

This object is in this set

We use the ∈ symbol to indicate that some object belongs to some set.

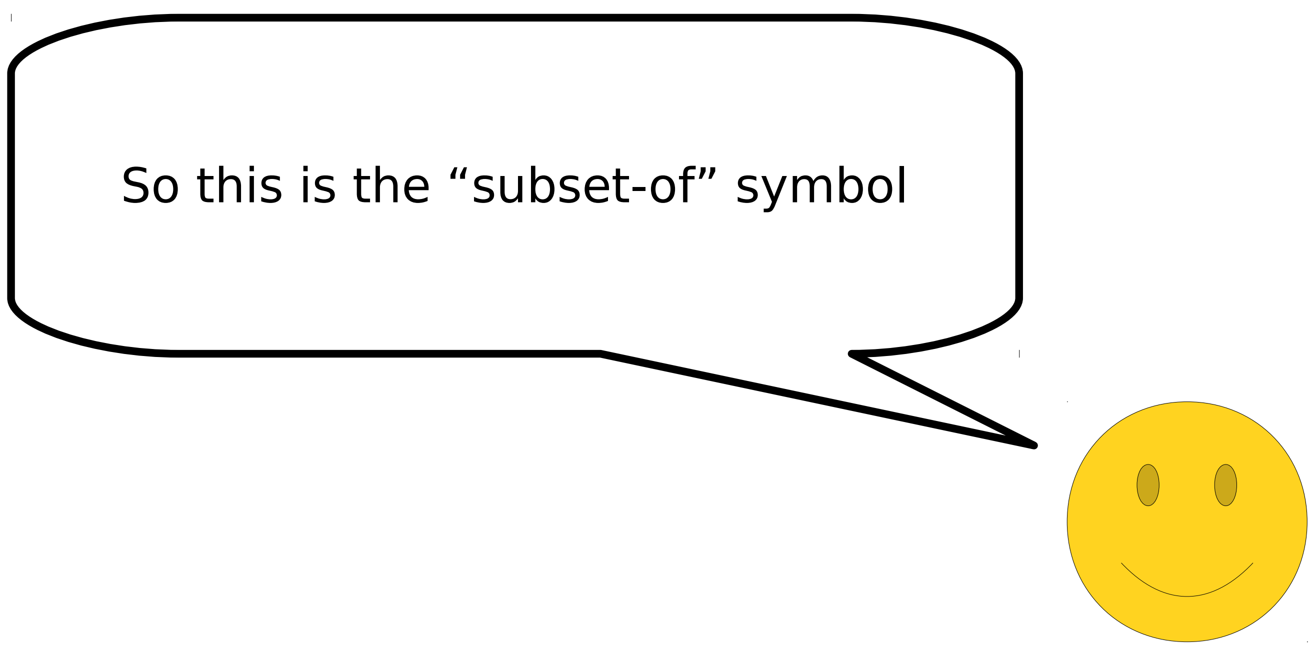
$$\subseteq$$


With that said, let's switch to talking about this symbol for a while.

$$\subseteq$$

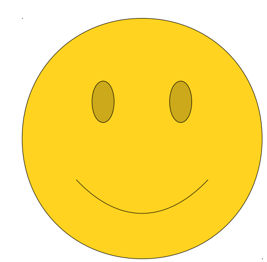
Before we talk about it, look at your notes and see what this symbol means. If you don't remember, quickly jump back to the lecture slides to get the answer.

$$\subseteq$$



$$S \subseteq T$$

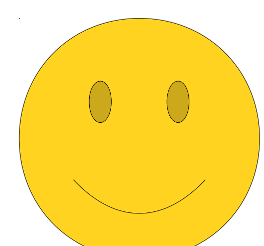
This expresses a relation that
can hold between two sets.

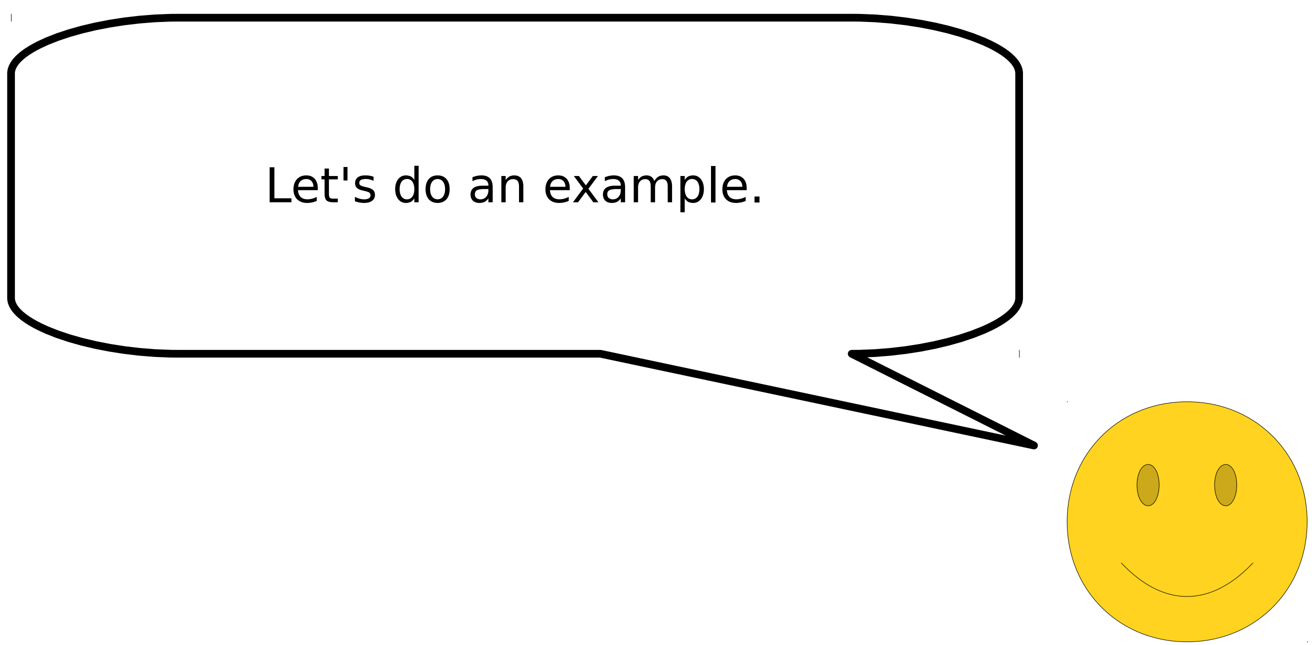


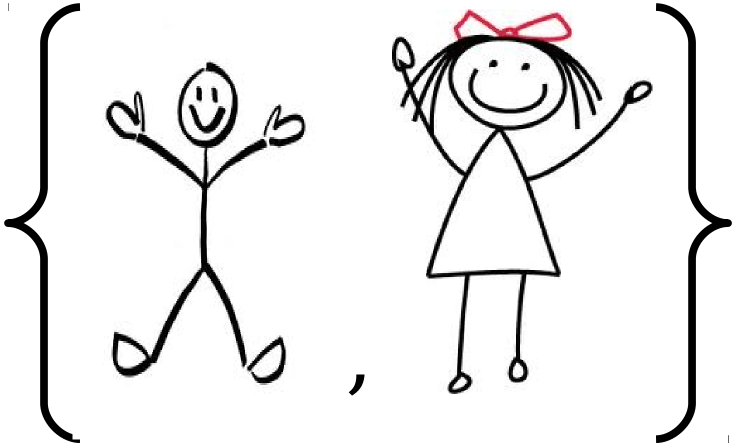
$$S \subseteq T$$

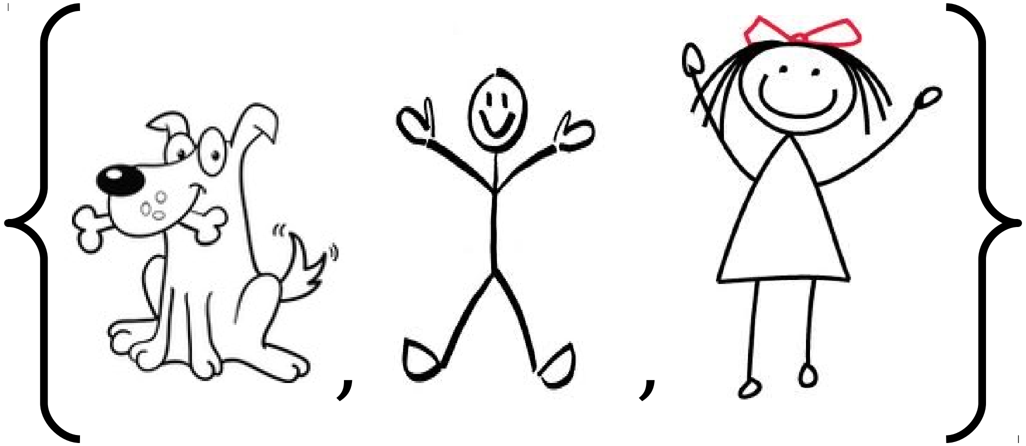
Every object in this set is in this set

Specifically, this statement means "every element of S is an element of T."

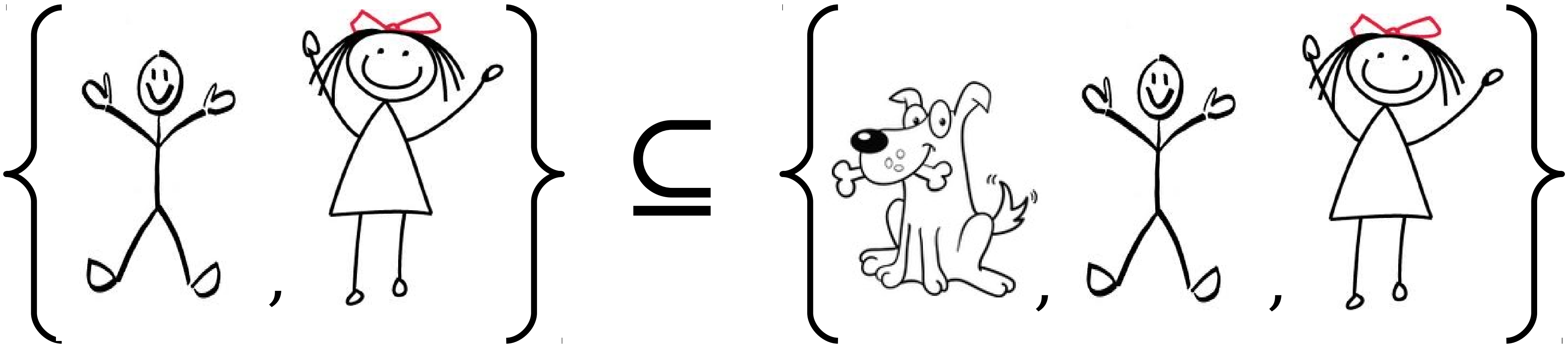


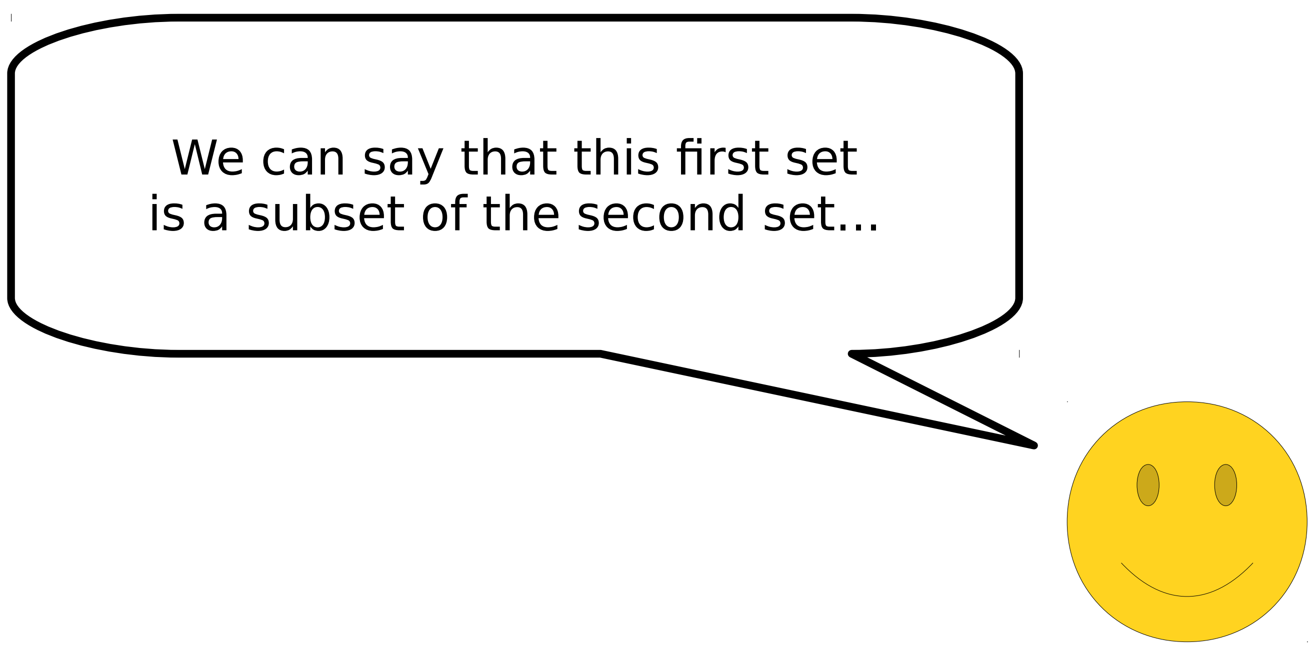

Let's do an example.

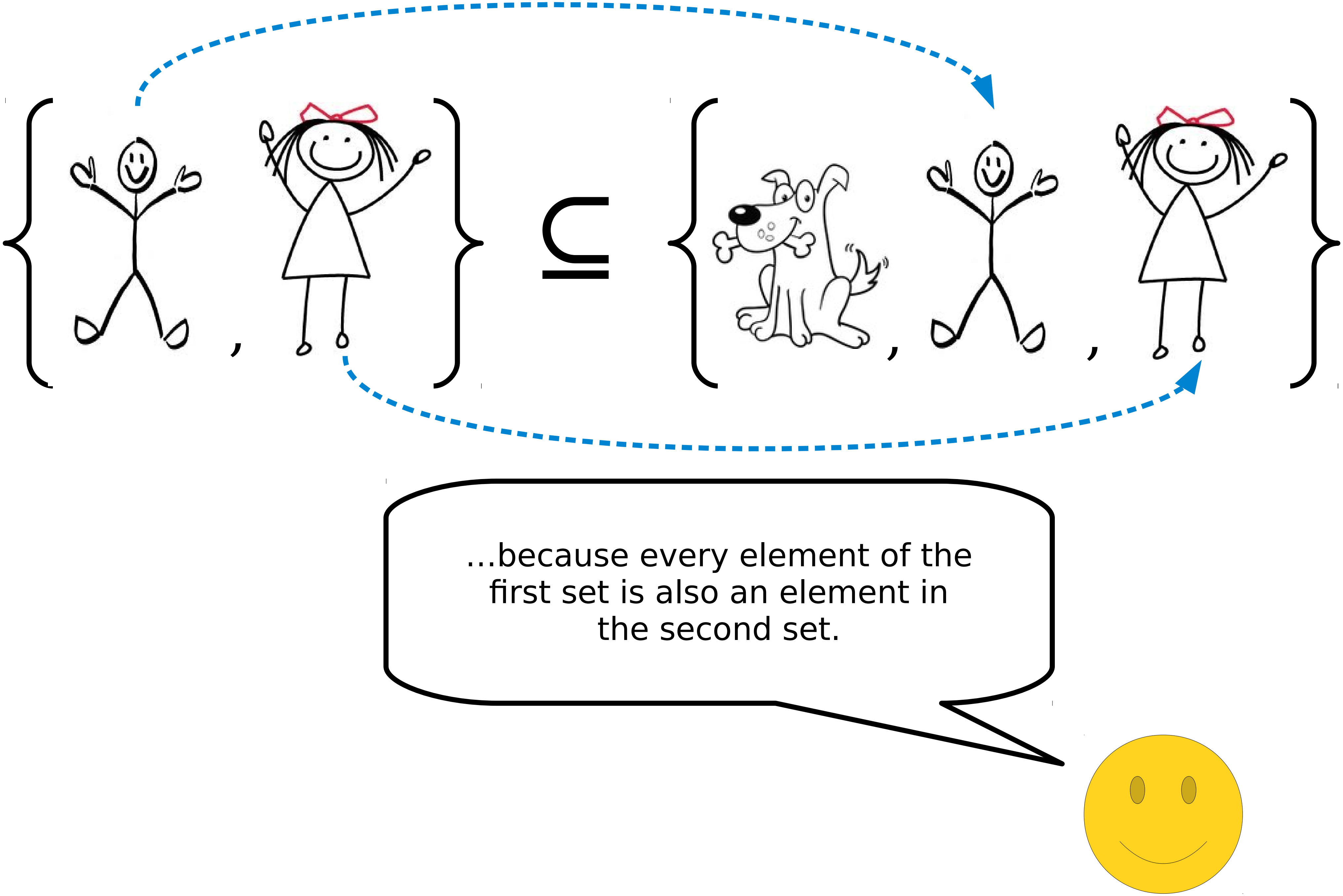


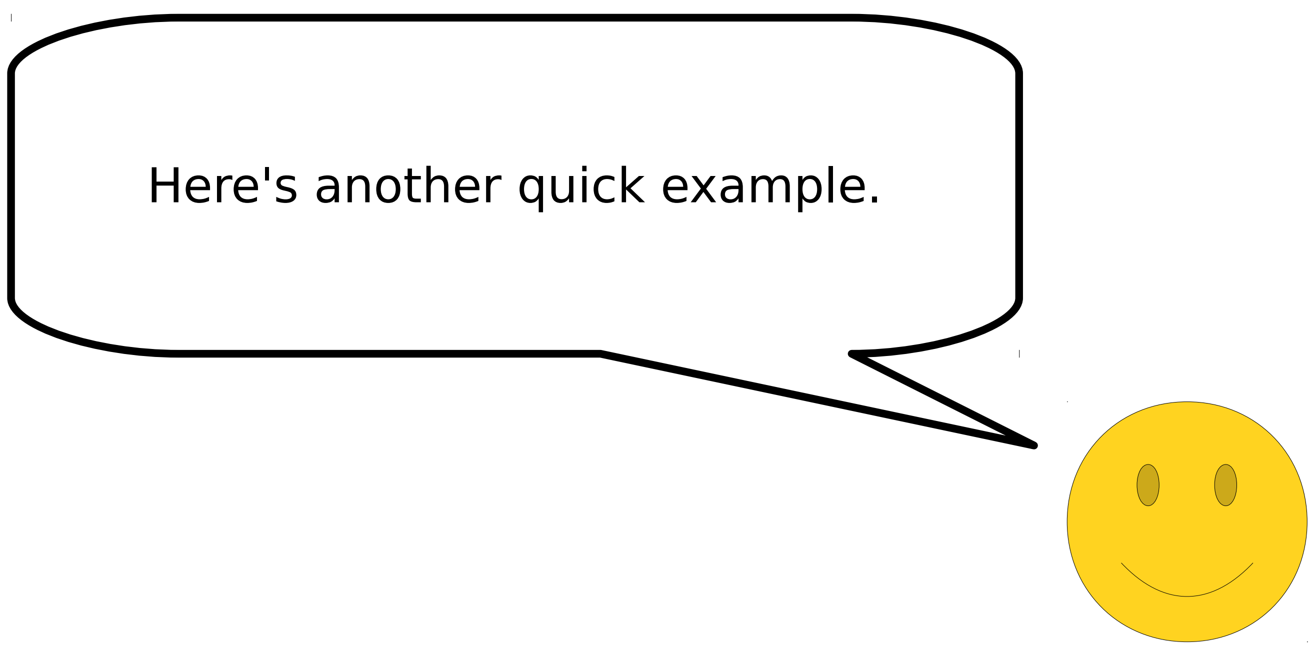


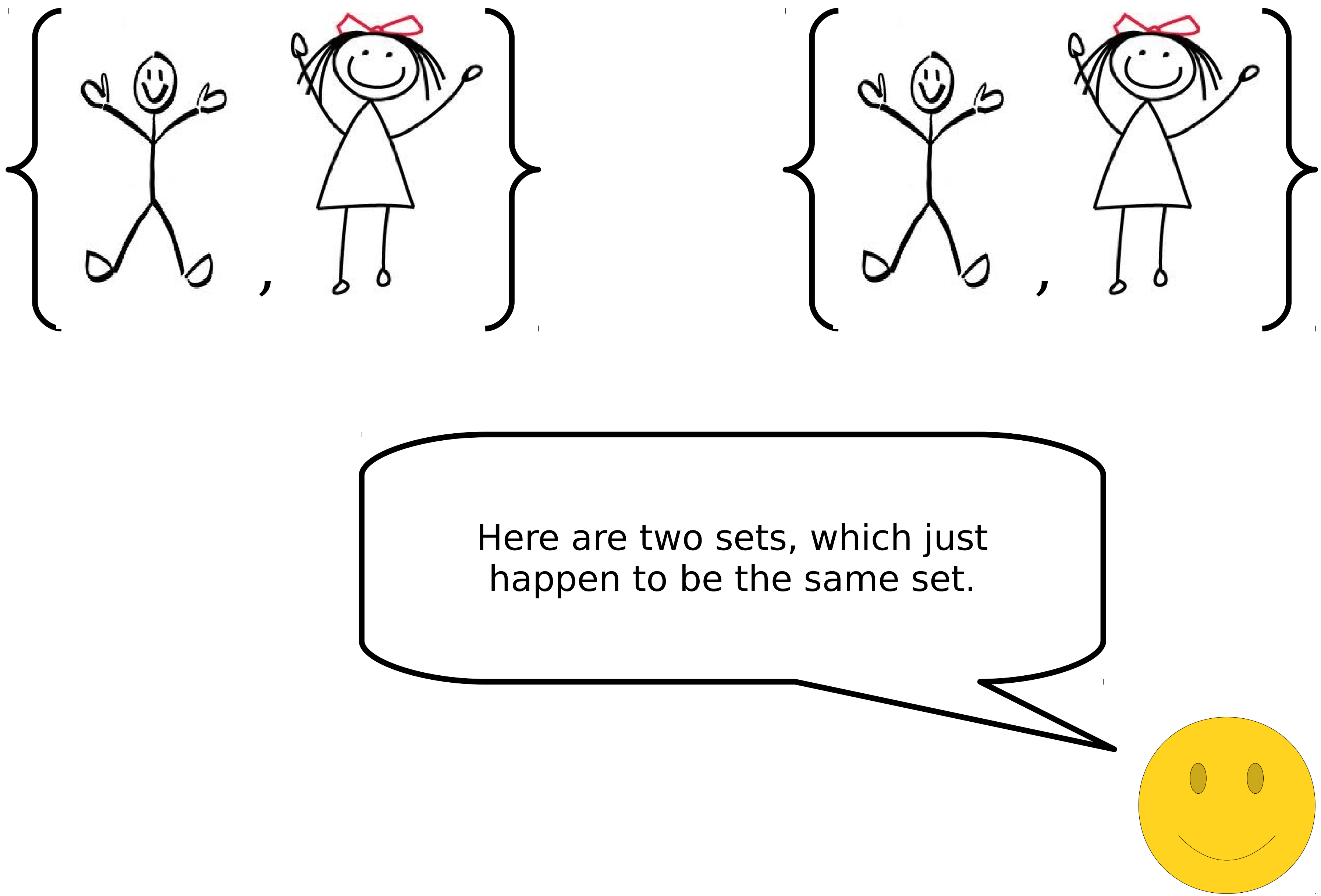
Here we have two different sets.

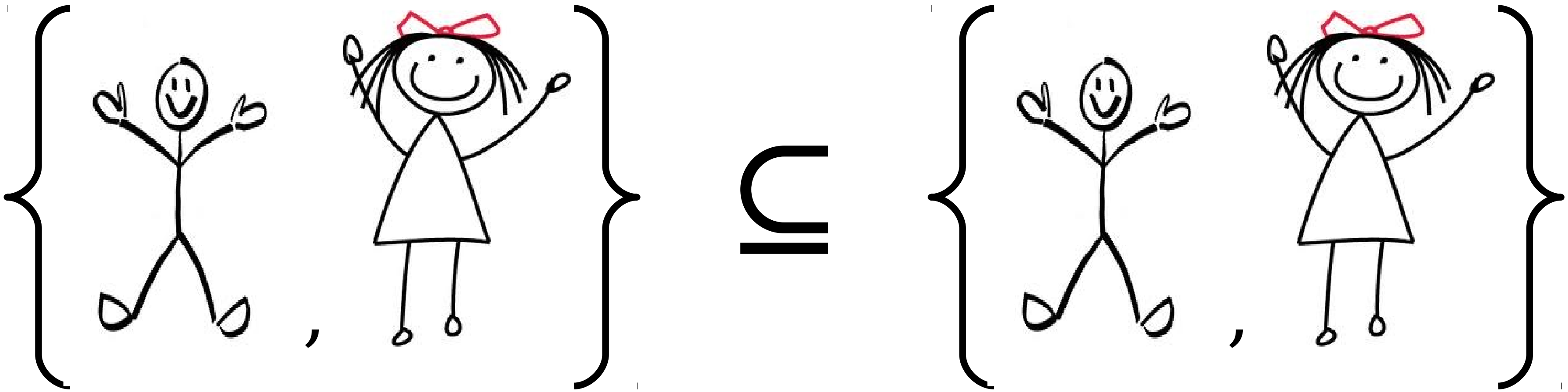



We can say that this first set
is a subset of the second set...

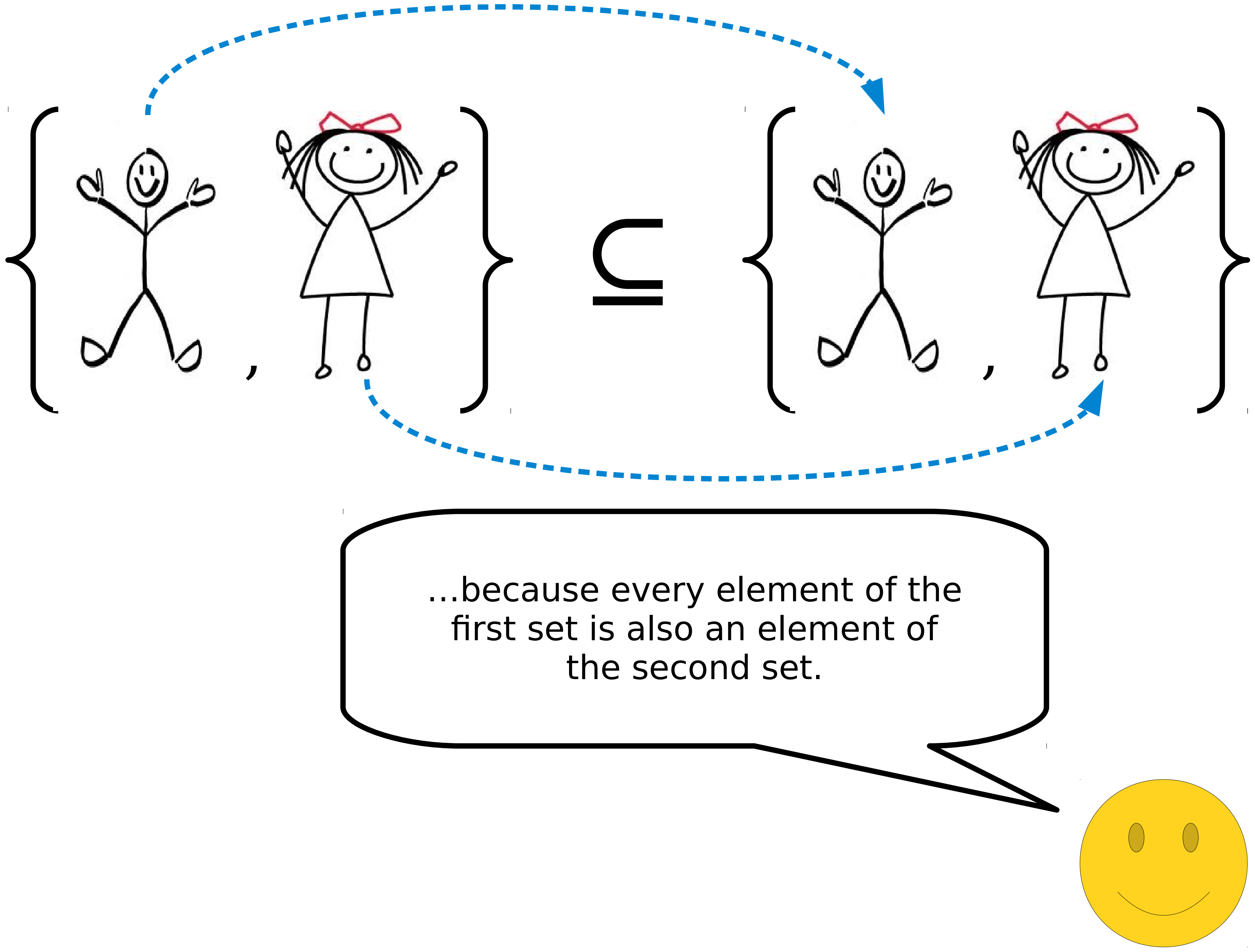

...because every element of the first set is also an element in the second set.

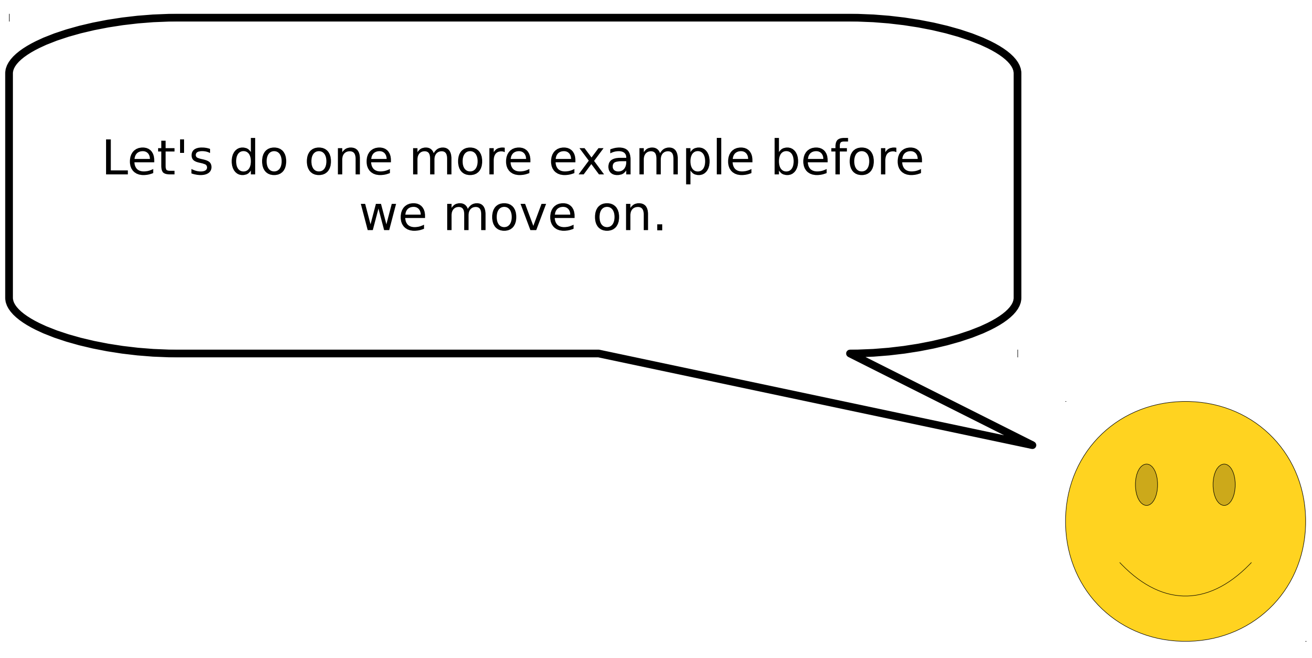

Here's another quick example.

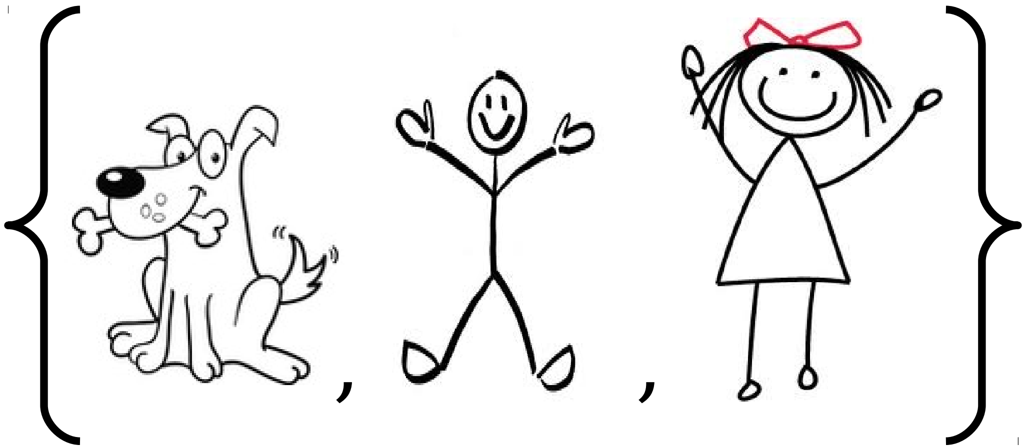

Here are two sets, which just happen to be the same set.

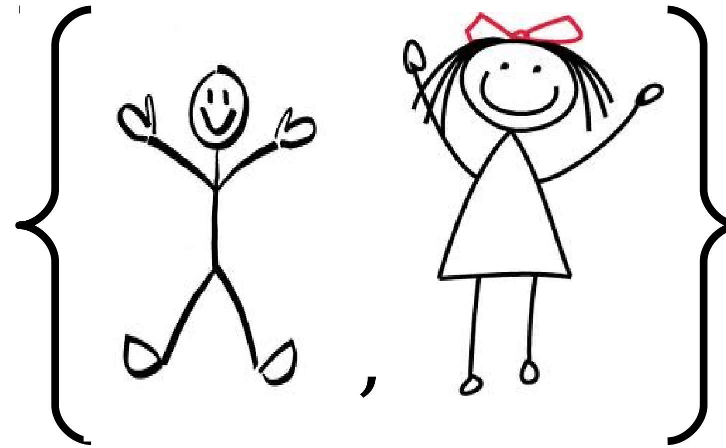


We say that the first set is a subset of the second set...

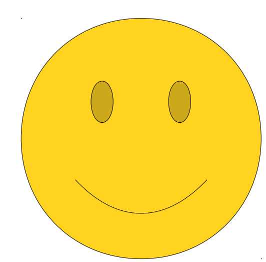

⊆
...because every element of the first set is also an element of the second set.

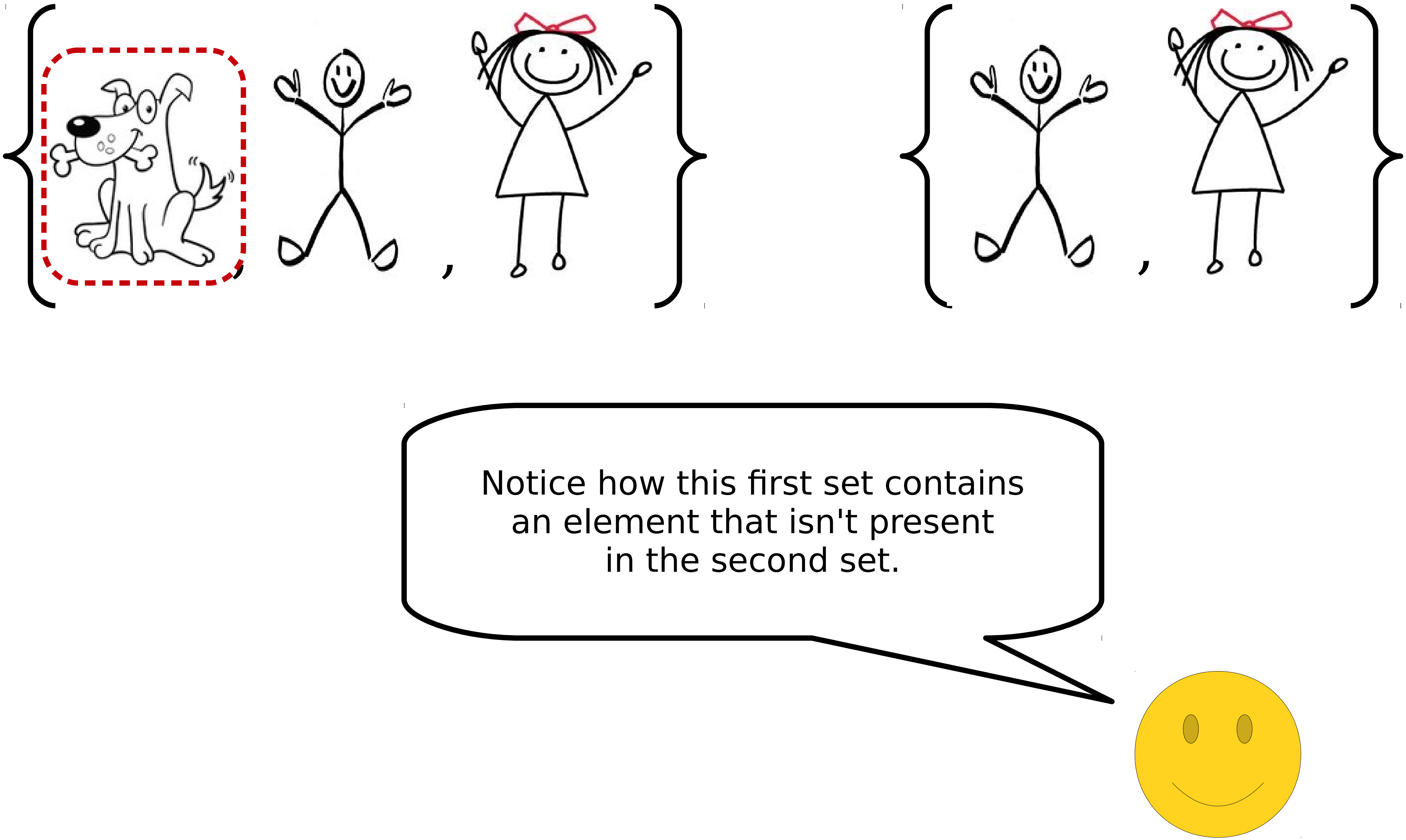

Let's do one more example before we move on.

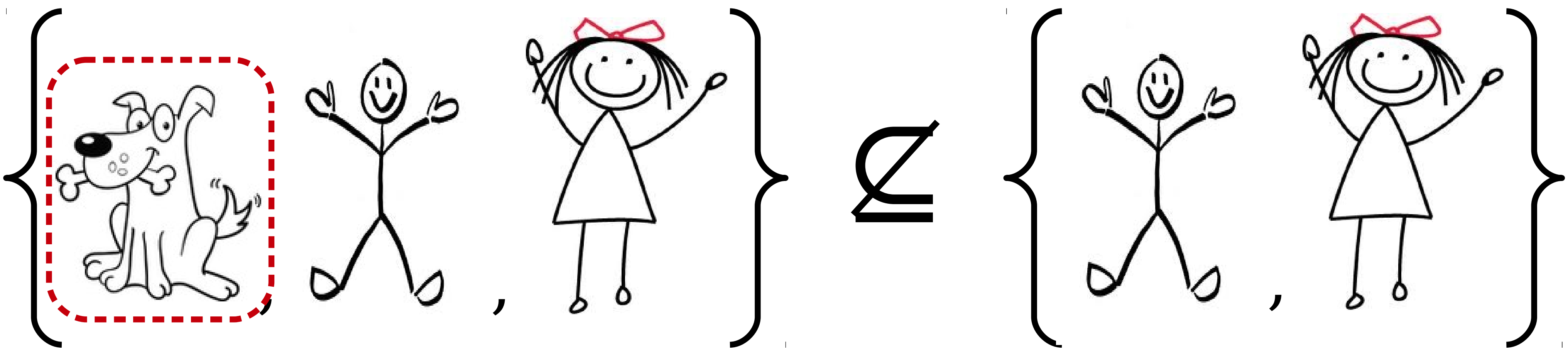




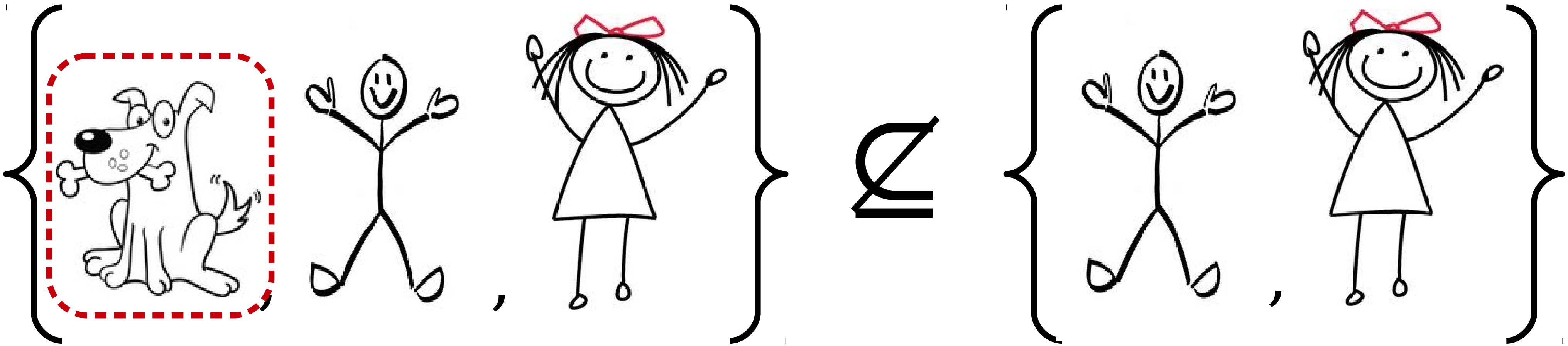
Here are two sets that we talked
about earlier, but presented in
the reverse order.



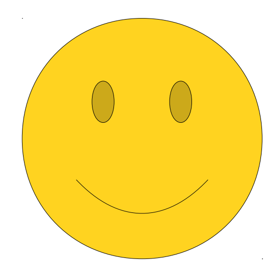

Notice how this first set contains an element that isn't present in the second set.

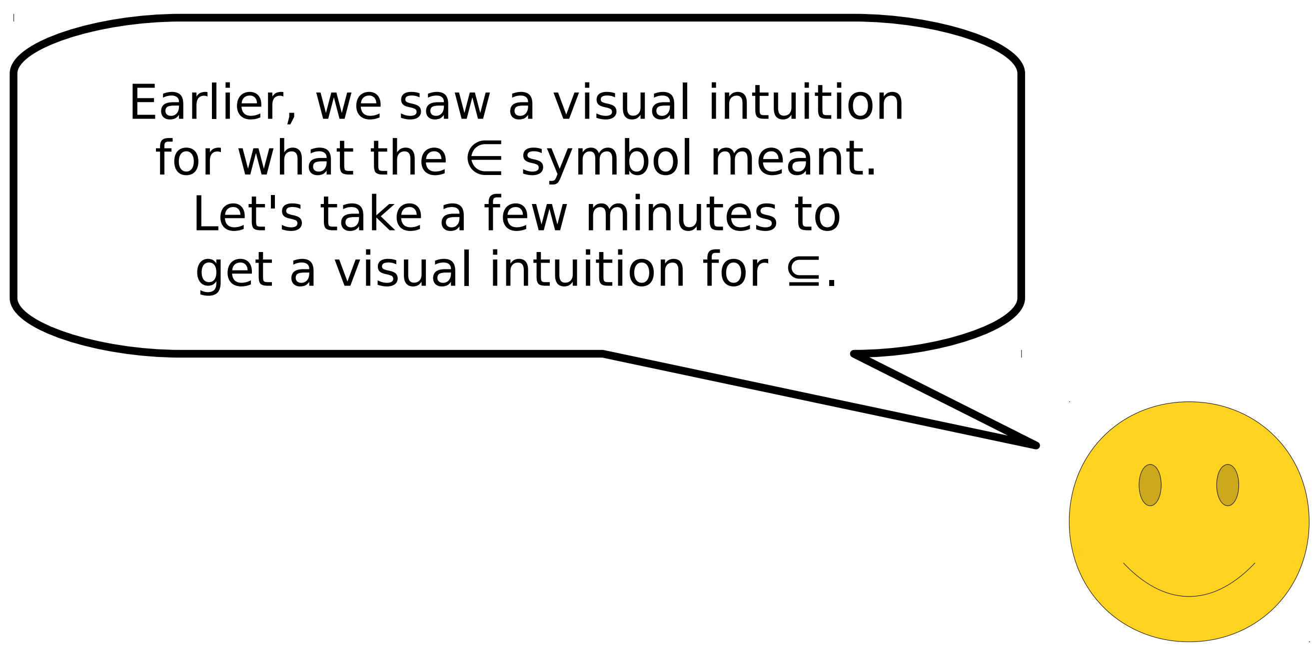


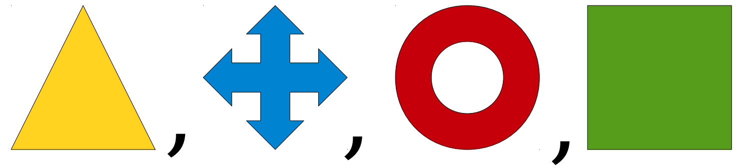
As a result, this first set is not a subset of the second set.

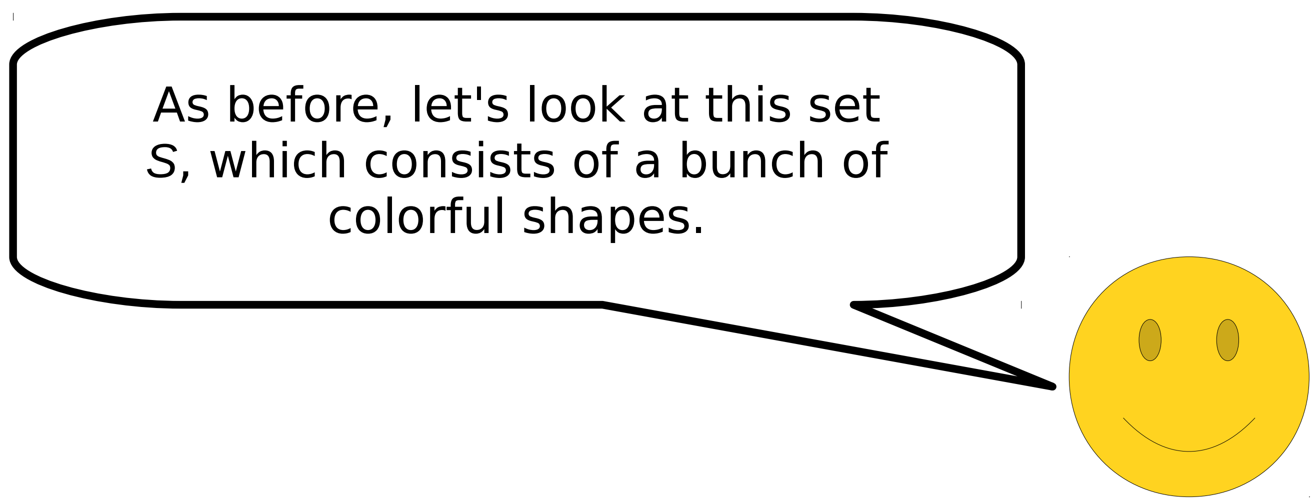


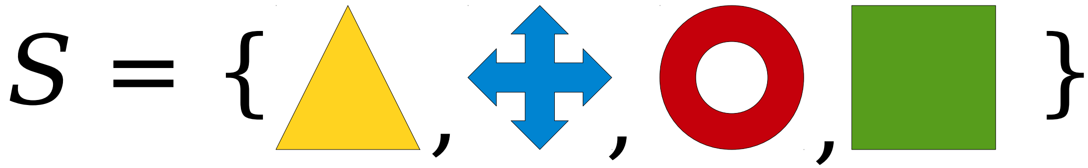
We denote this by using this special “not a subset” symbol. It's basically the subset symbol with a slash through it.

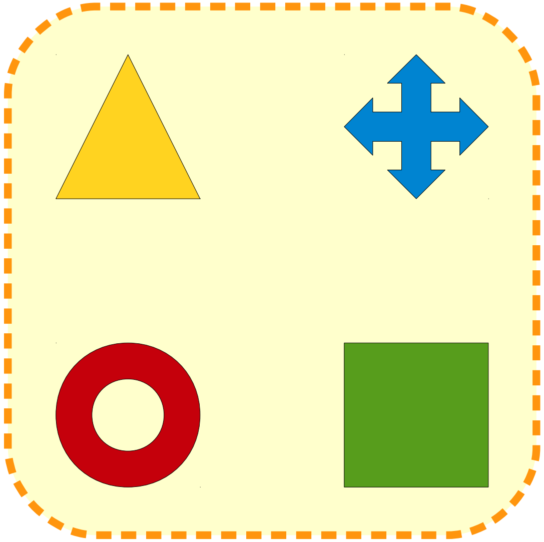


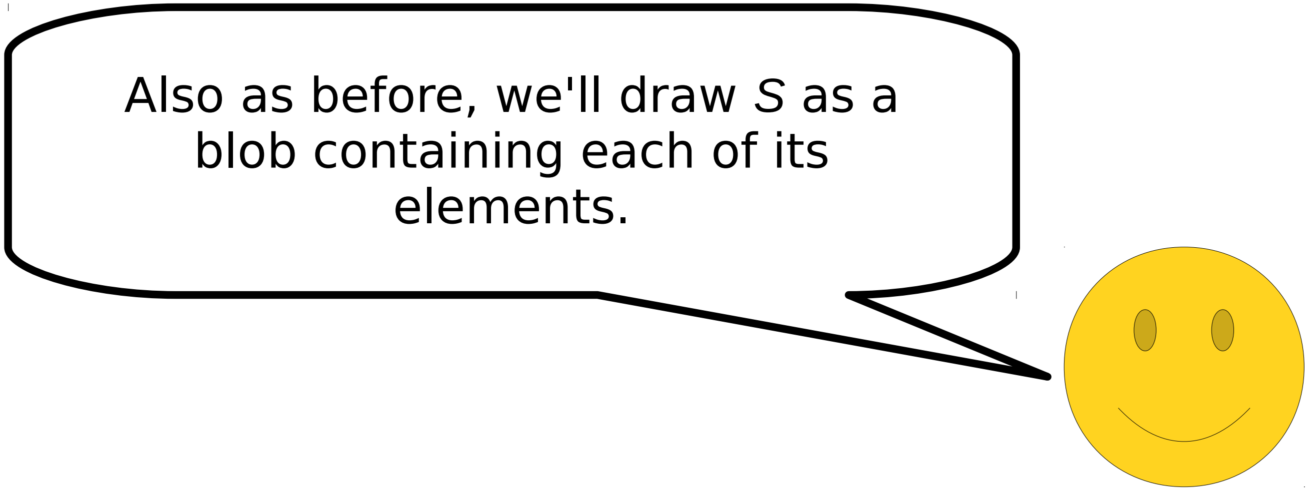

Earlier, we saw a visual intuition for what the ∈ symbol meant. Let's take a few minutes to get a visual intuition for ⊆.

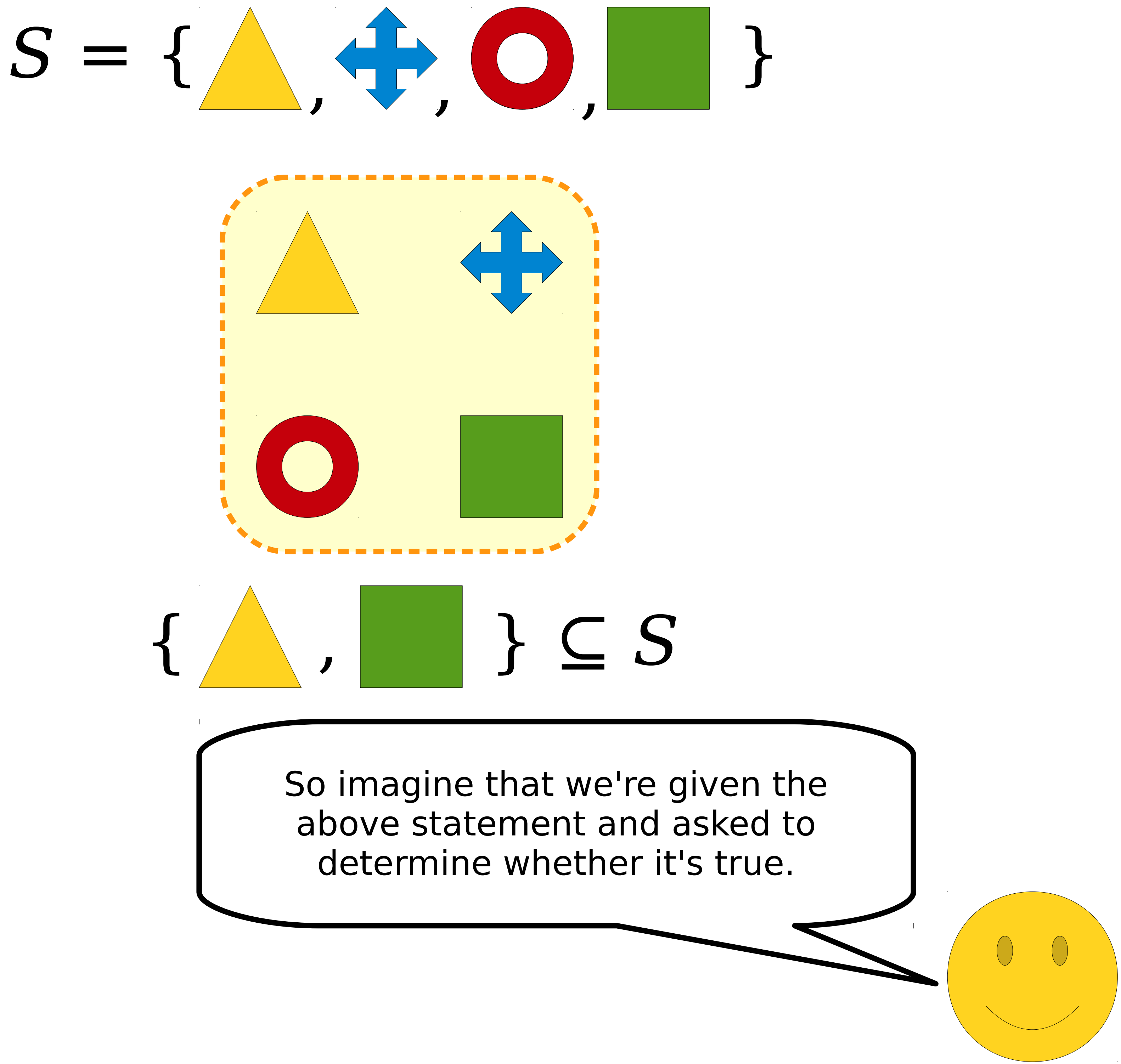
$S = \{$  $\}$

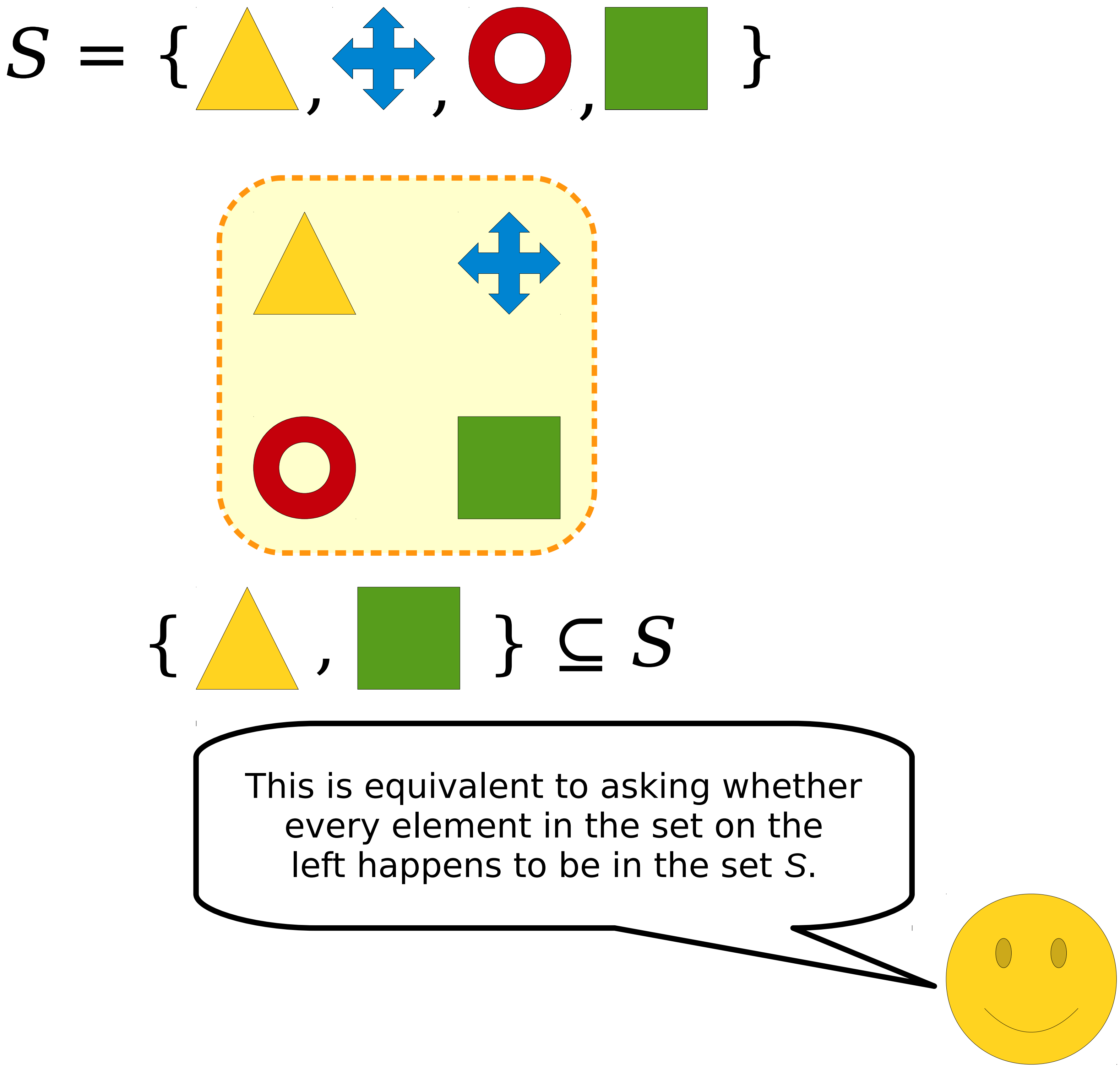

As before, let's look at this set
S, which consists of a bunch of
colorful shapes.

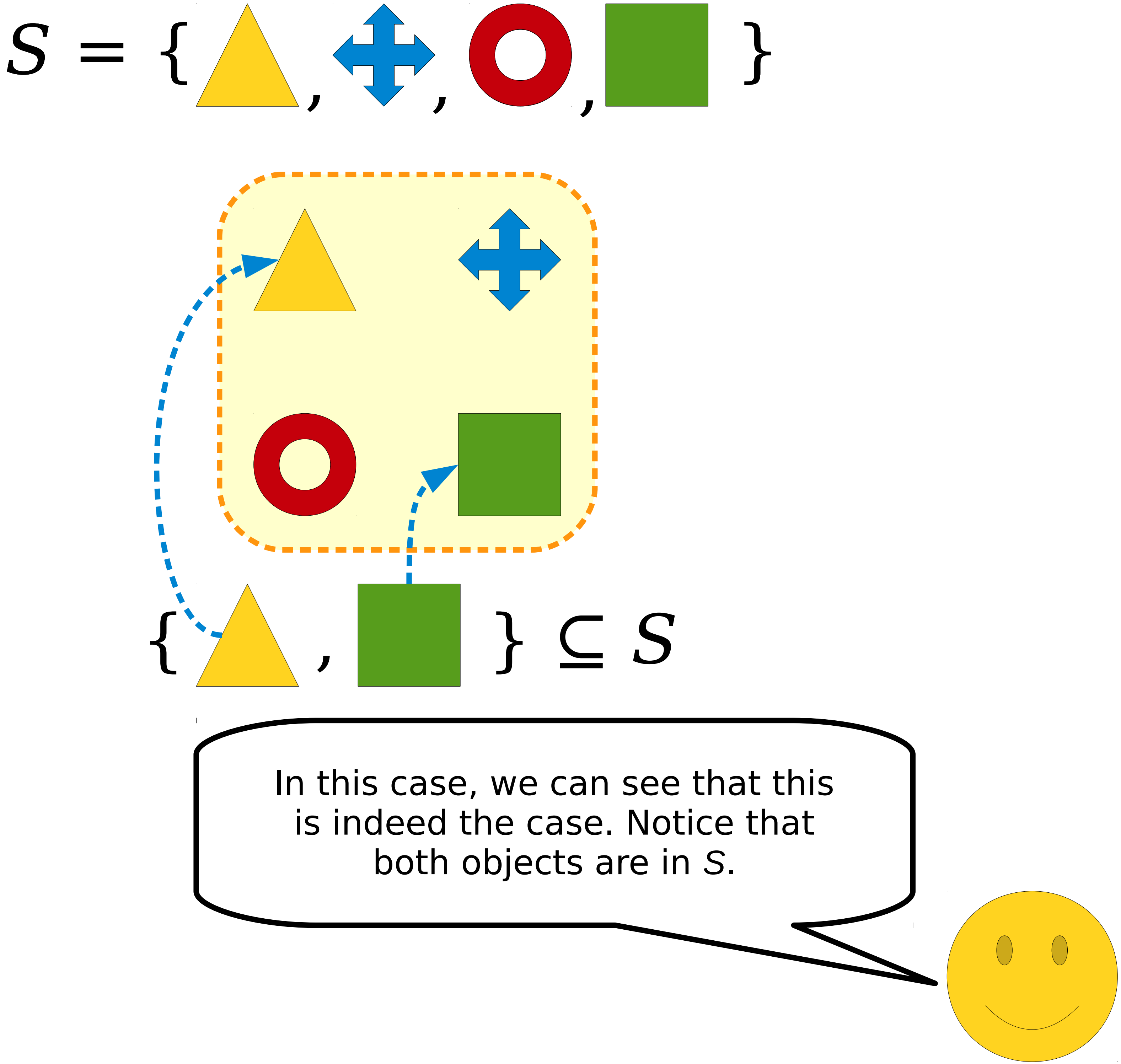

S = { , , , }

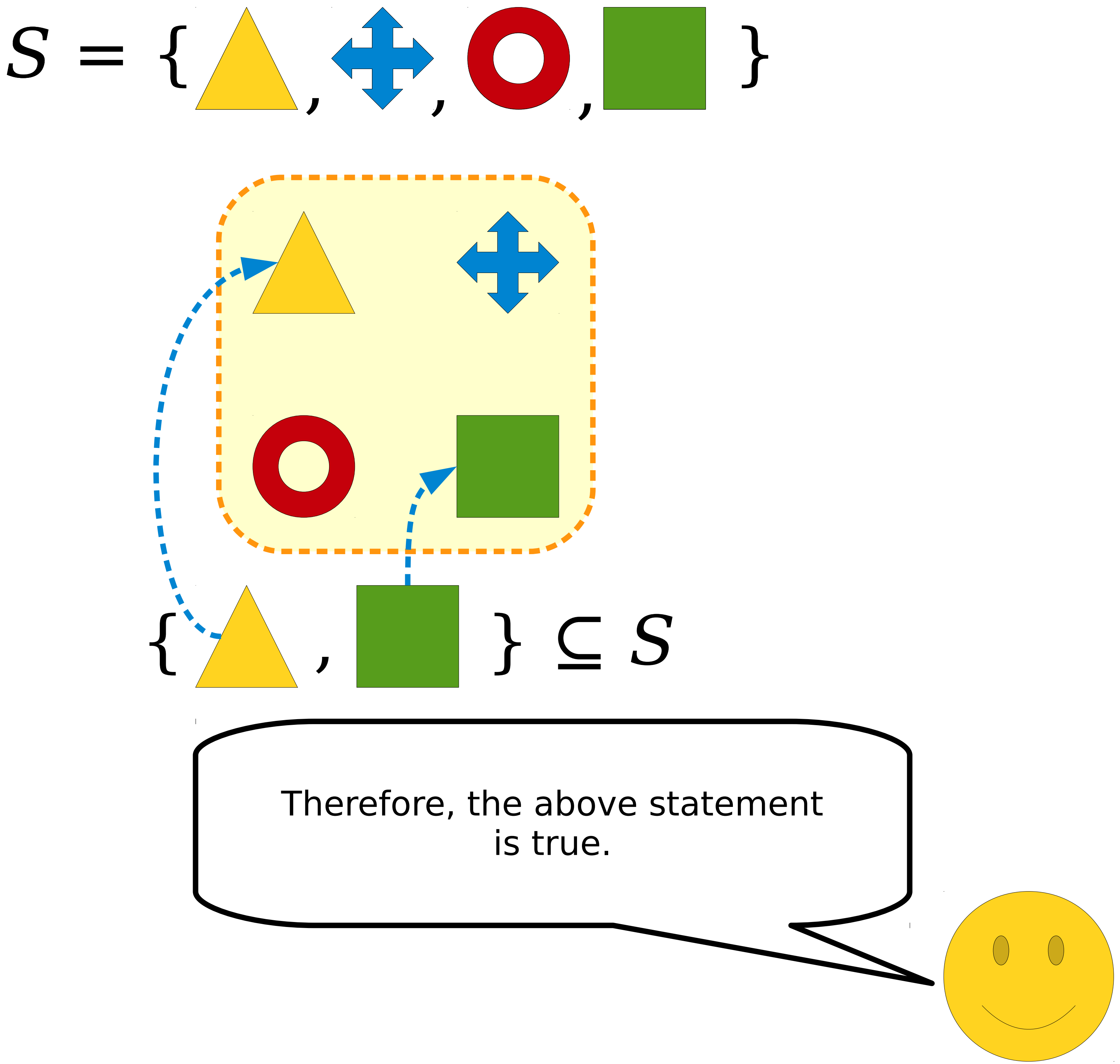


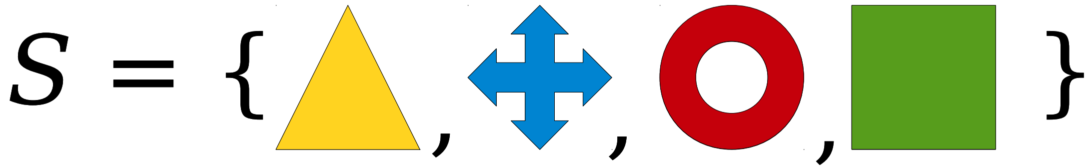

Also as before, we'll draw *S* as a blob containing each of its elements.

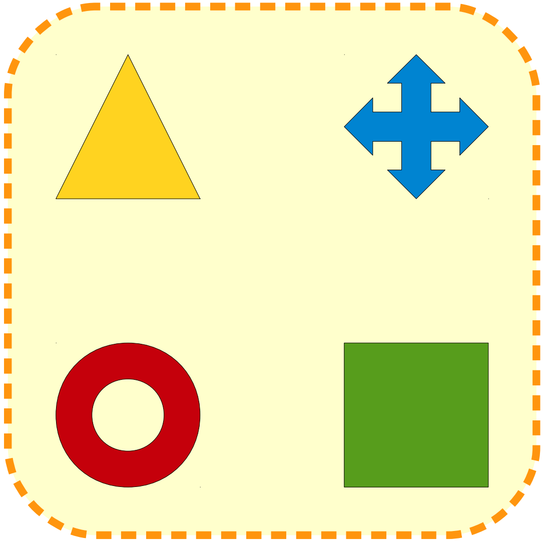

S = { , , , }
{ , } ⊆ S
So imagine that we're given the above statement and asked to determine whether it's true.


S = { , , , }
{ , } ⊆ S
This is equivalent to asking whether every element in the set on the left happens to be in the set S.

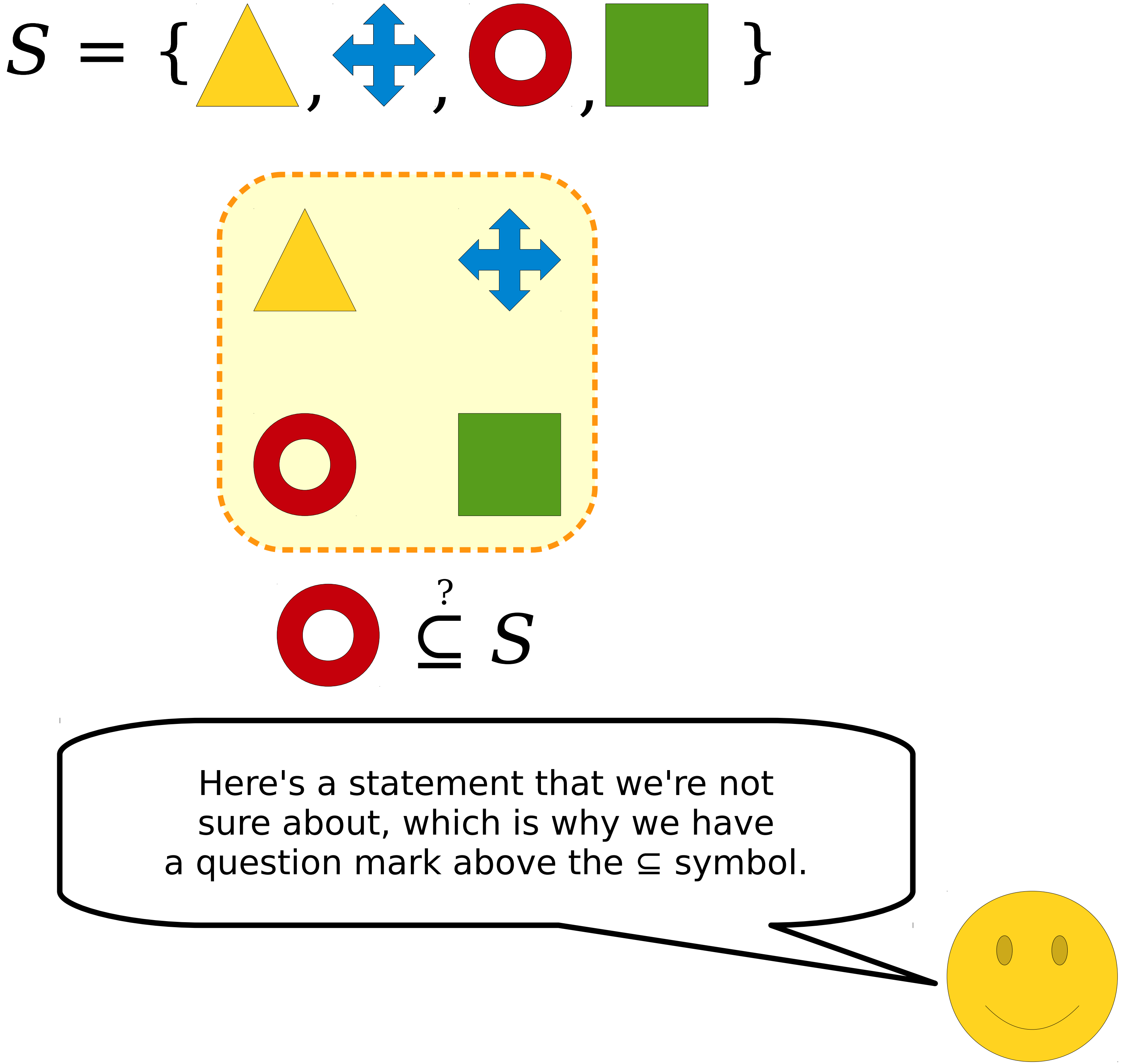

S = { , , , }
{ , } ⊆ S
In this case, we can see that this is indeed the case. Notice that both objects are in S.


S = { , , , }
{ , } ⊆ S
Therefore, the above statement is true.

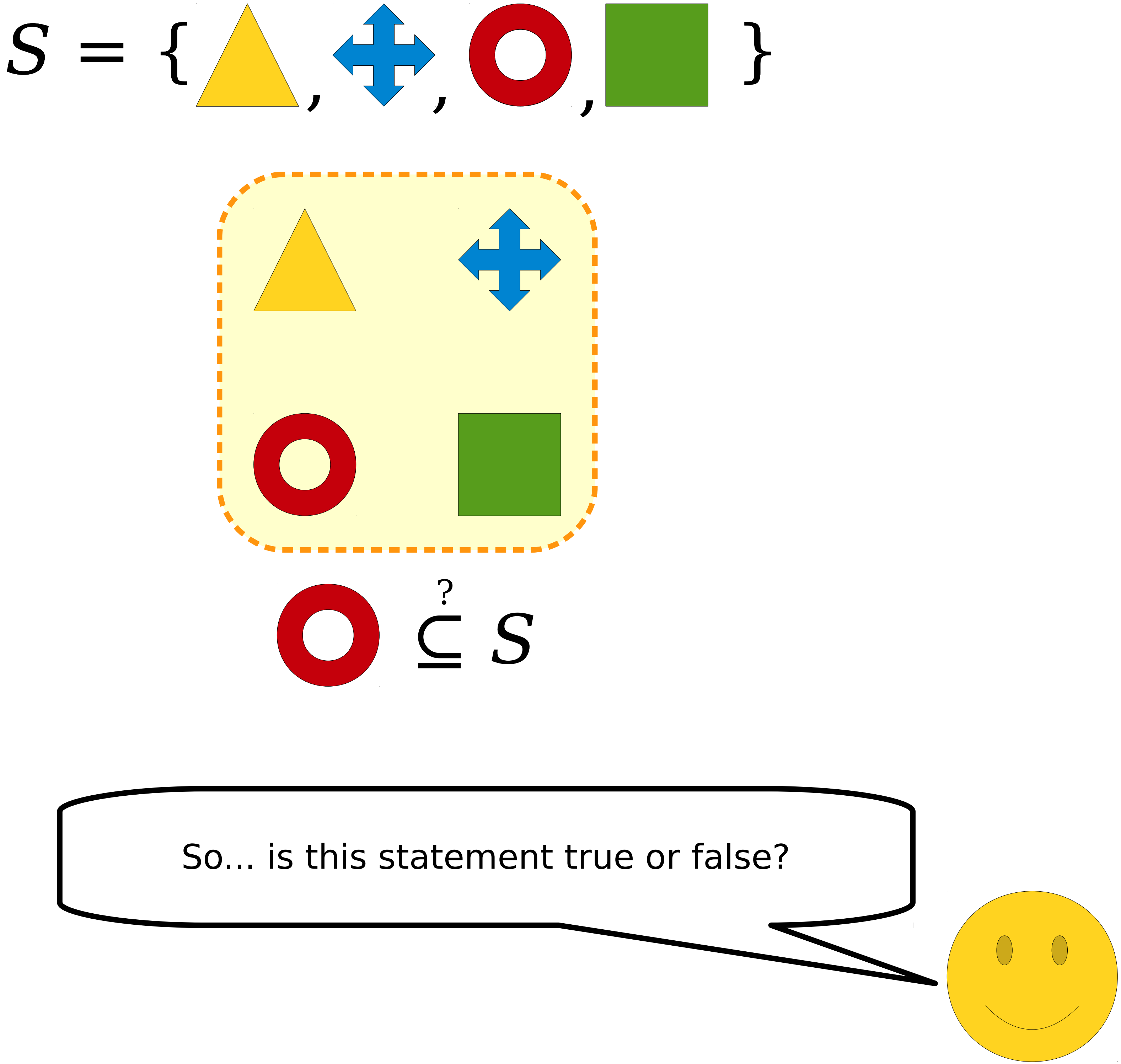

S = { , , , }

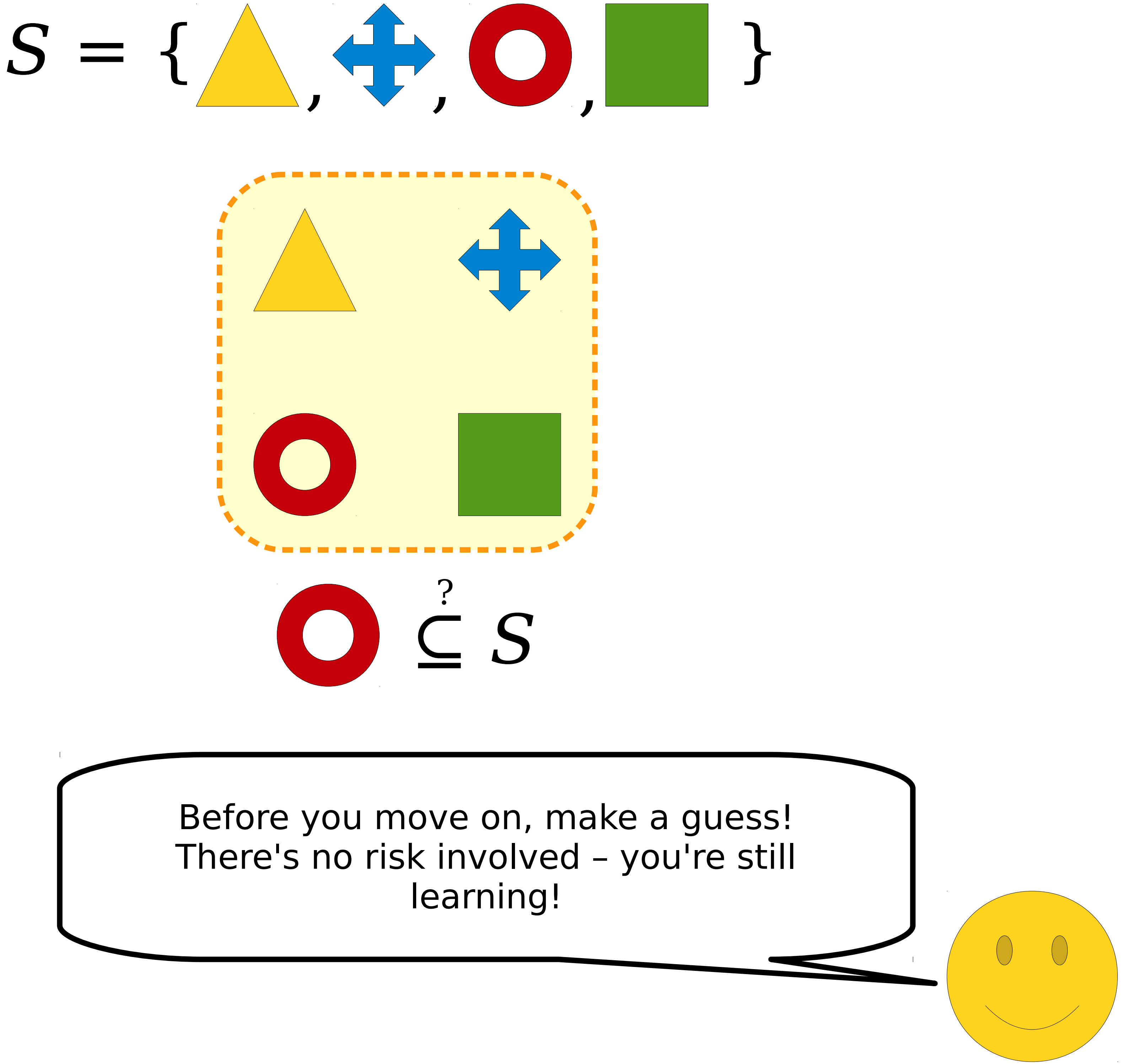


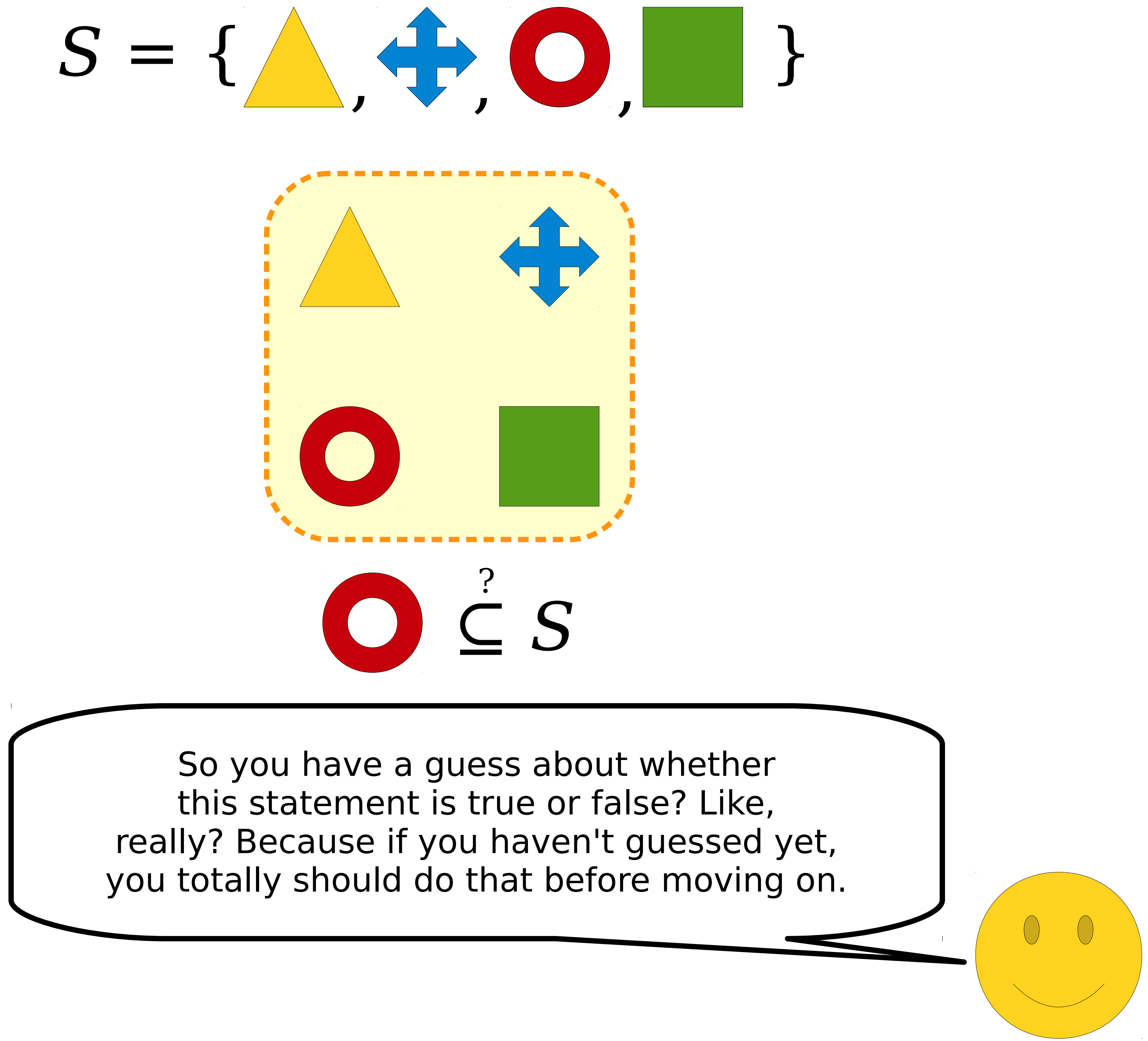

So let's try out a new statement.

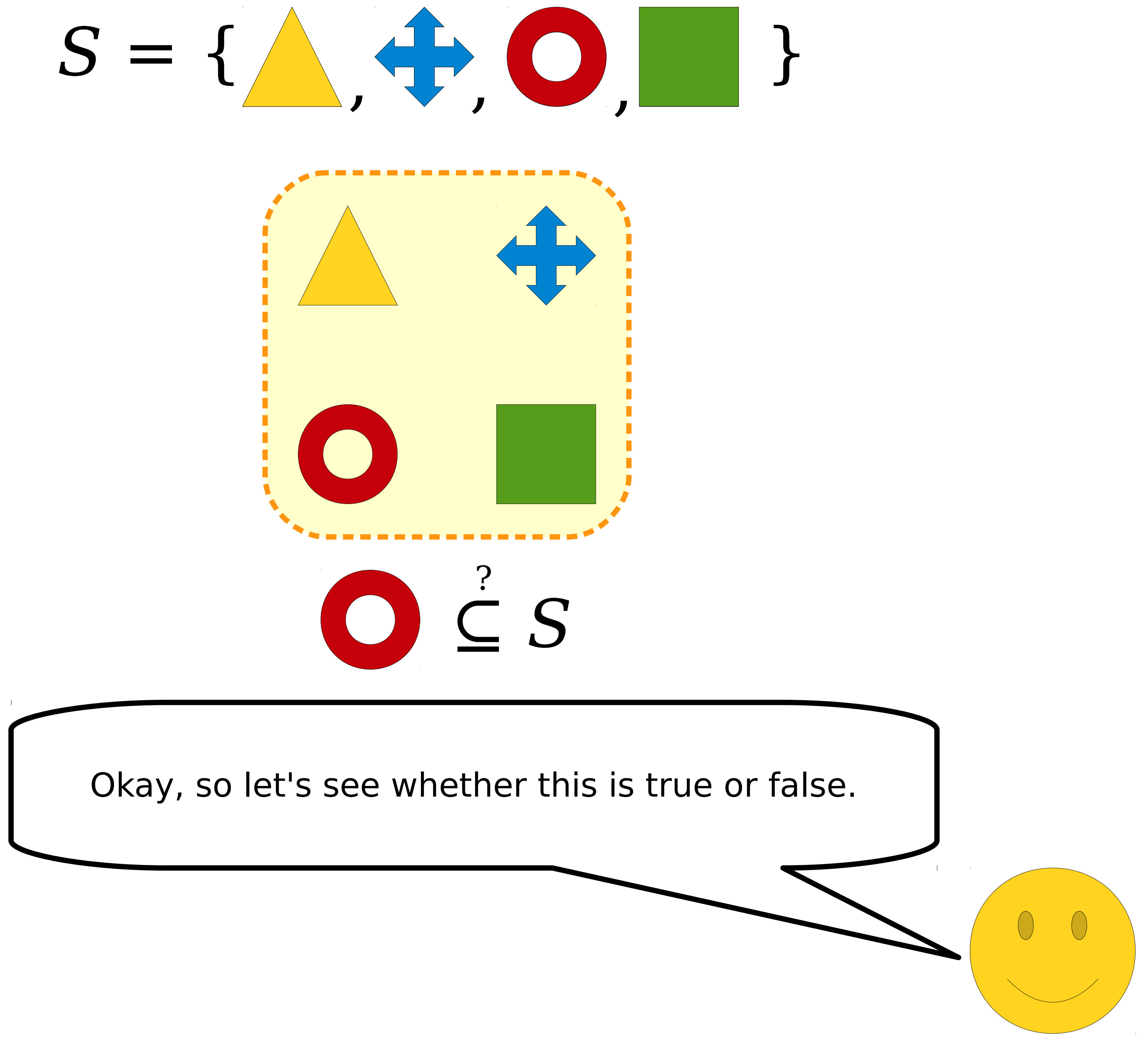


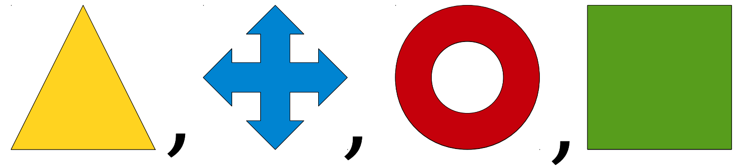
$S = \{ \; , \; , \; , \; \}$
$\overset{?}{\subseteq} S$
Here's a statement that we're not
sure about, which is why we have
a question mark above the ⊆ symbol.


S = { , , , }
?
⊆ S
So... is this statement true or false?


$S = \{ , , , \}$
$\stackrel{?}{\subseteq} S$
Before you move on, make a guess!
There's no risk involved – you're still learning!

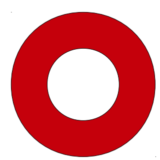

S = { , , , }
? ⊆ S
So you have a guess about whether
this statement is true or false? Like,
really? Because if you haven't guessed yet,
you totally should do that before moving on.


S = {
}
?
⊆ S
Okay, so let's see whether this is true or false.

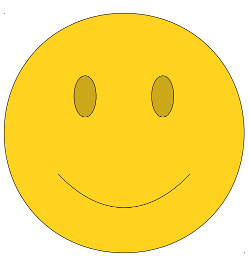
$S = \{$  $\}$

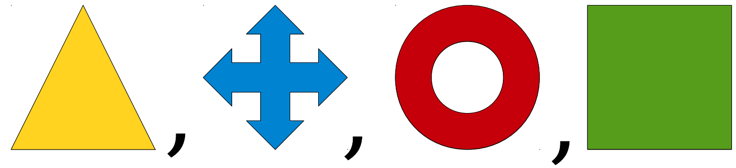
$S \subseteq T$

Every object in this set is in this set

 $\overset{?}{\subseteq} S$

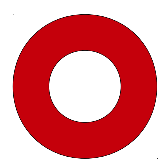
Earlier, we said that the ⊆ relation means that every object in the set on the left-hand side is an element of the set on the right-hand side.



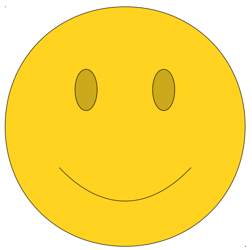
$S = \{$  $\}$

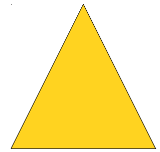
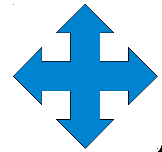
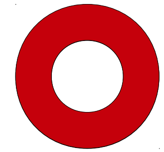
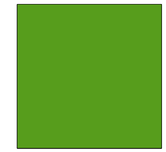
$$S \subseteq T$$

Every object in this set is in this set

 $\overset{?}{\subseteq} S$

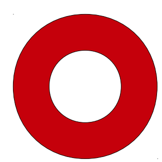
An important detail here is that the thing on the left-hand side of the ⊆ relation <u>has to be a set</u> for the relation to even make sense.



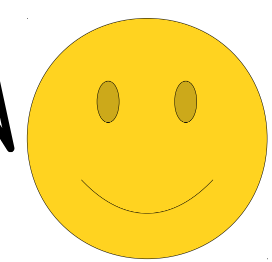
$S = \{$ , , ,  $\}$

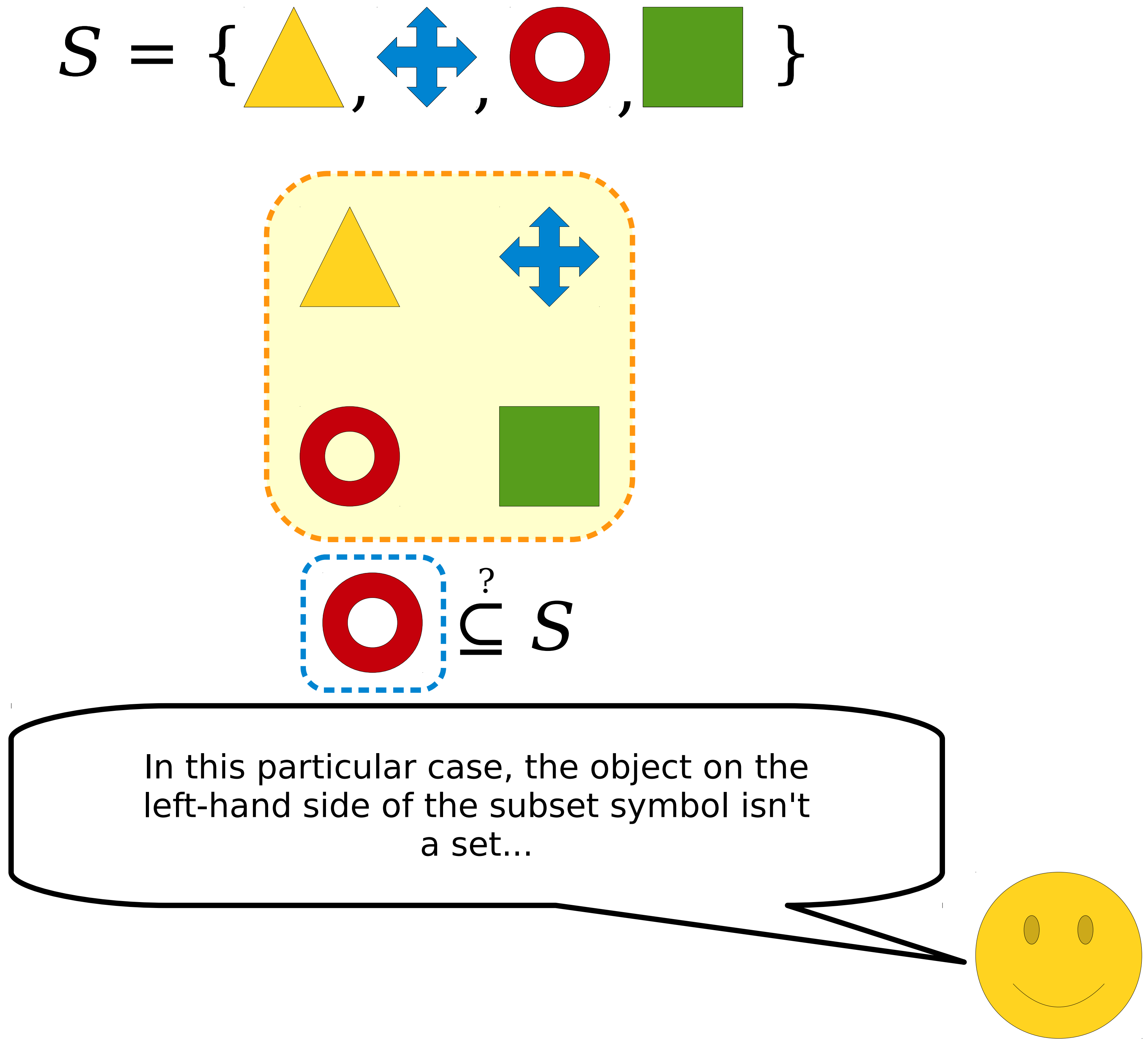
$$S \subseteq T$$

Every object in this set is in this set

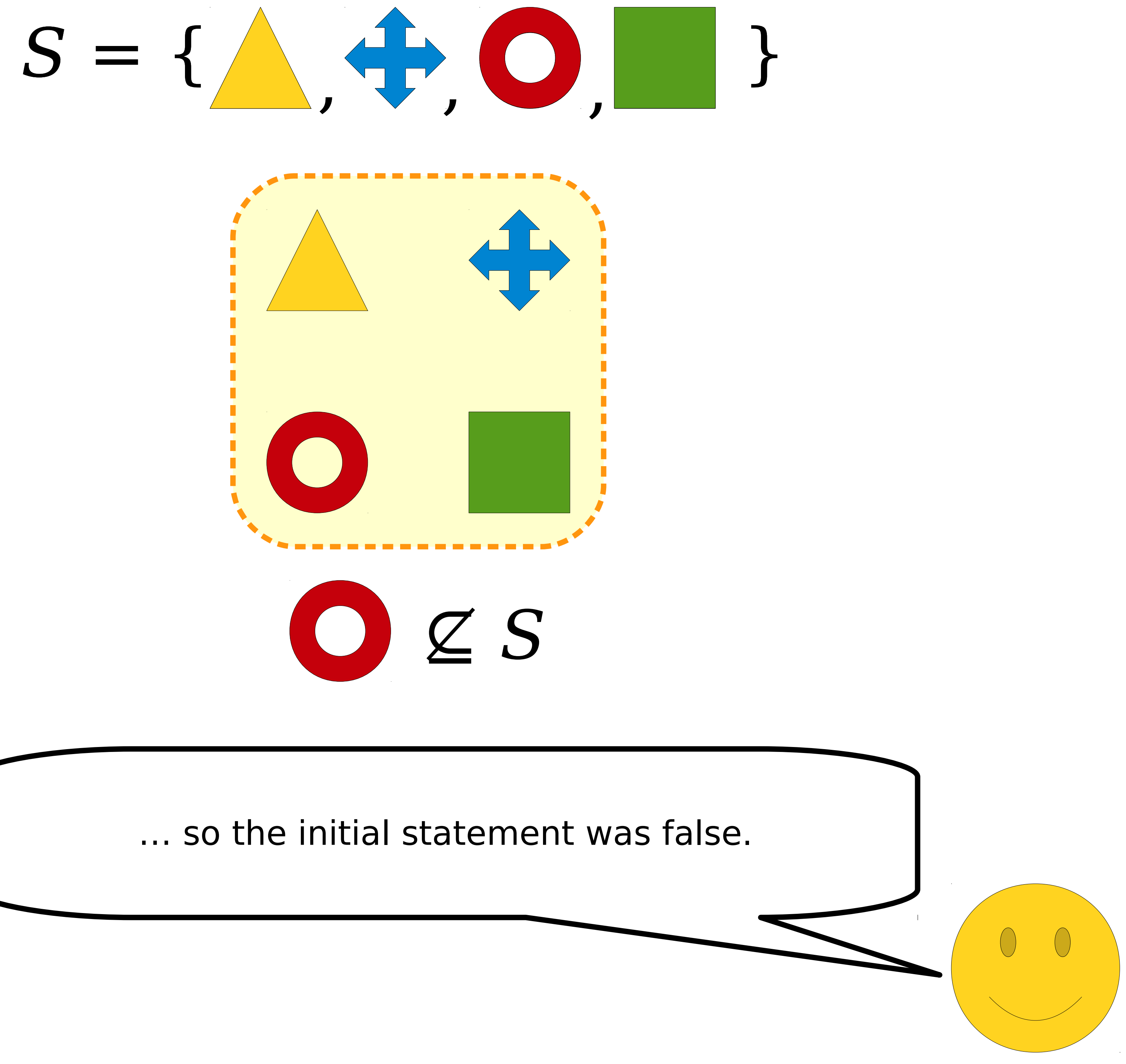
 $\overset{?}{\subseteq} S$

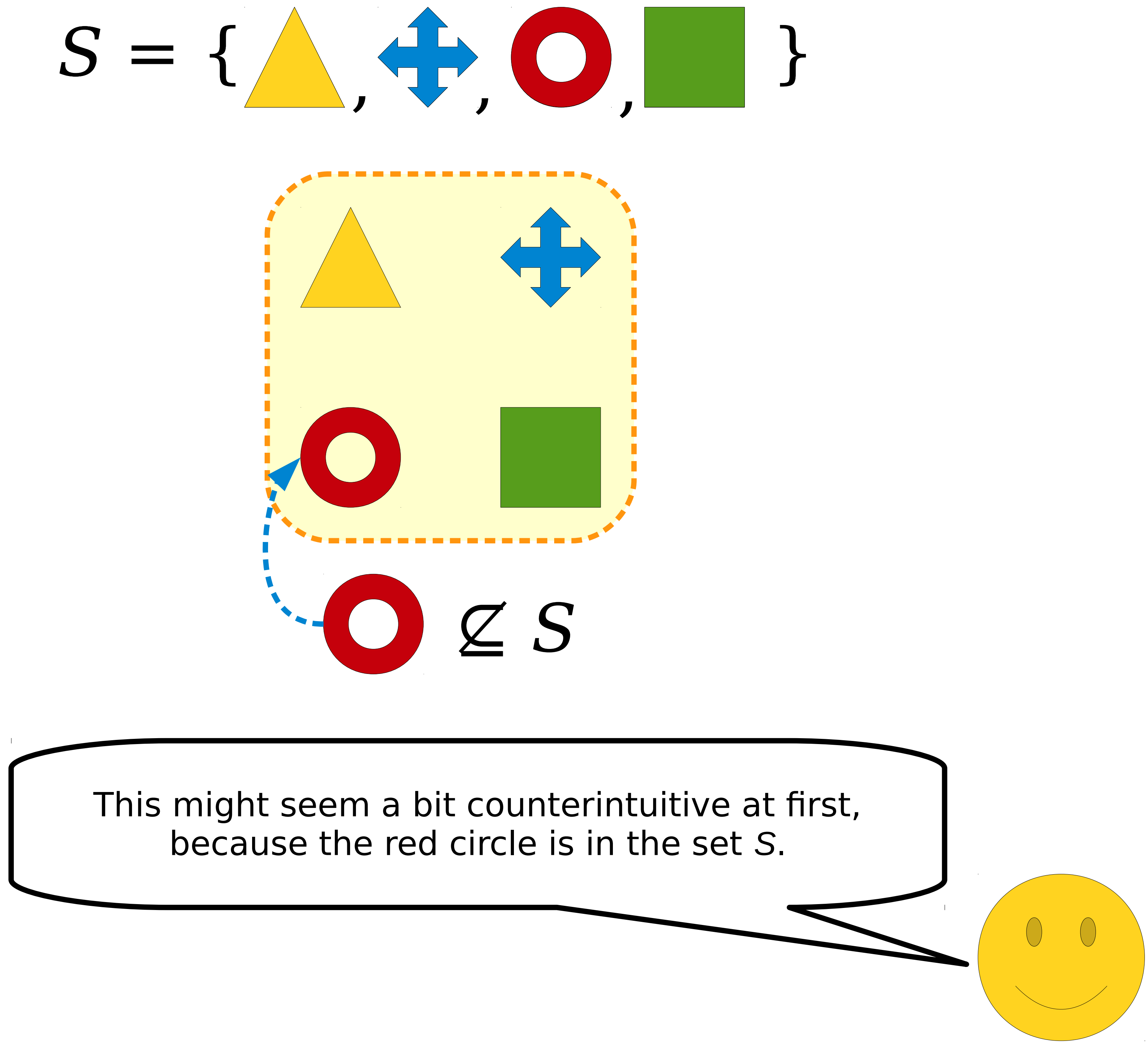
Otherwise, it would be like asking whether giraffe < 137 – it's a meaningless statement because you can't compare giraffes to numbers using less-than.

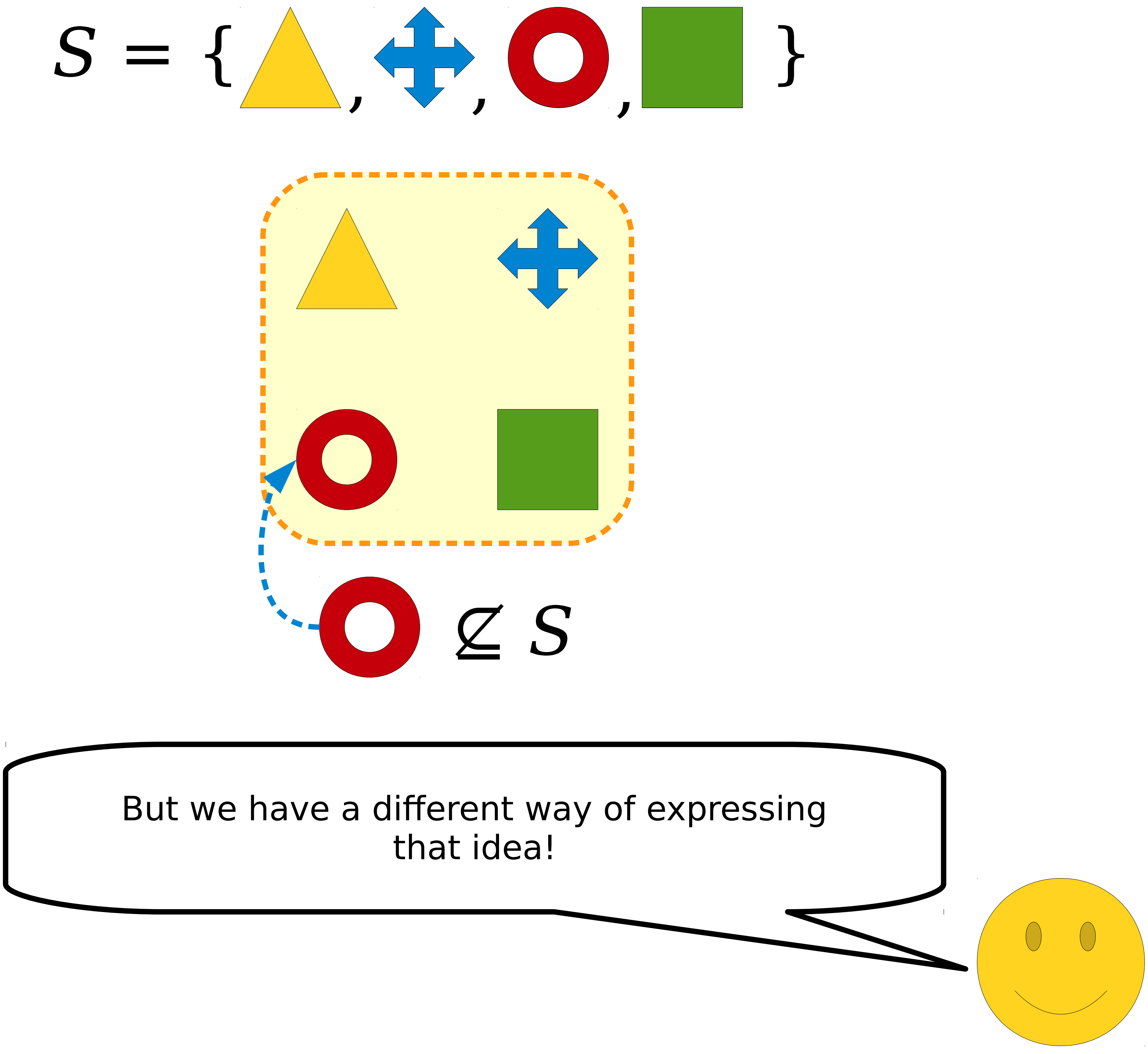


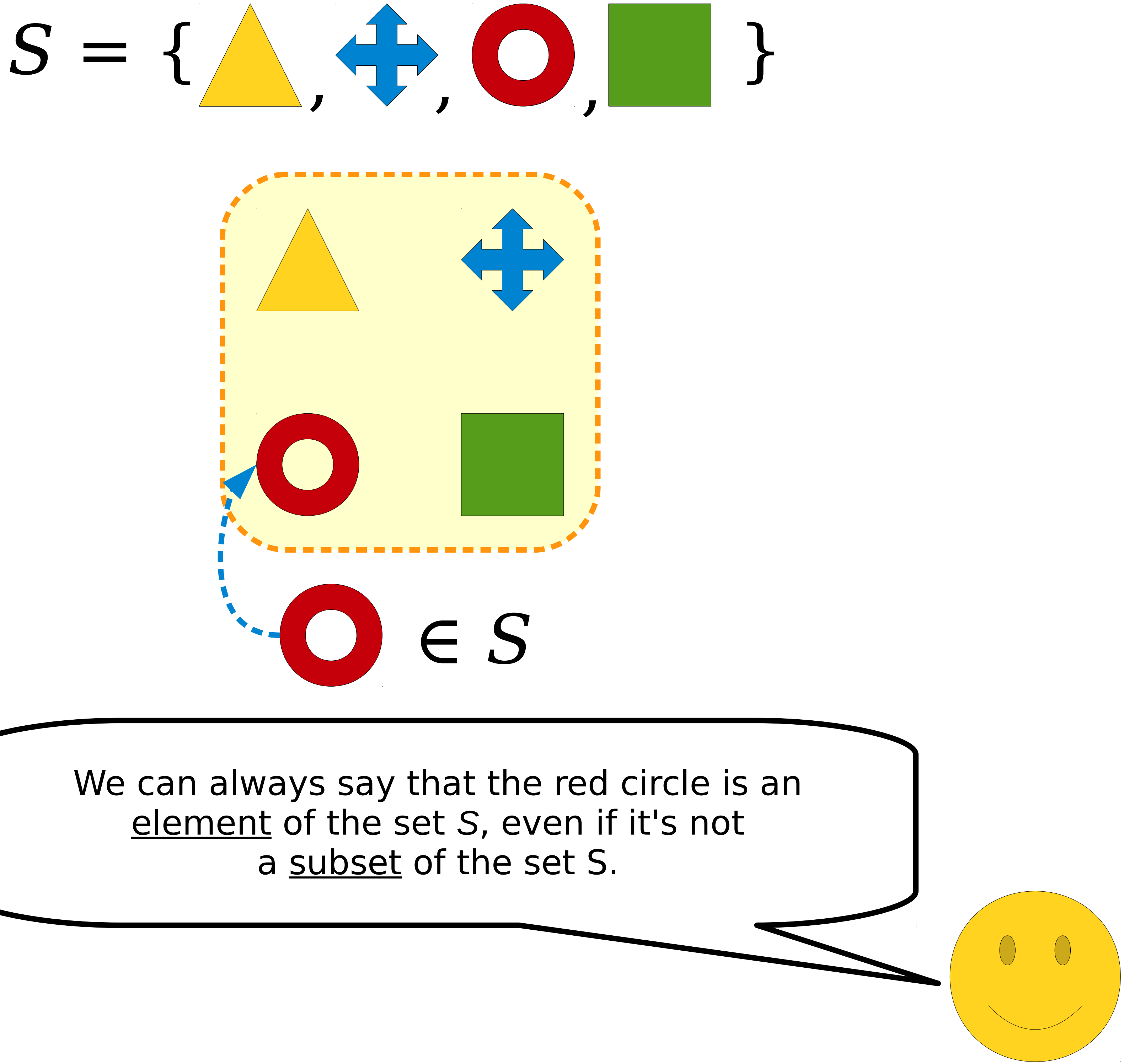


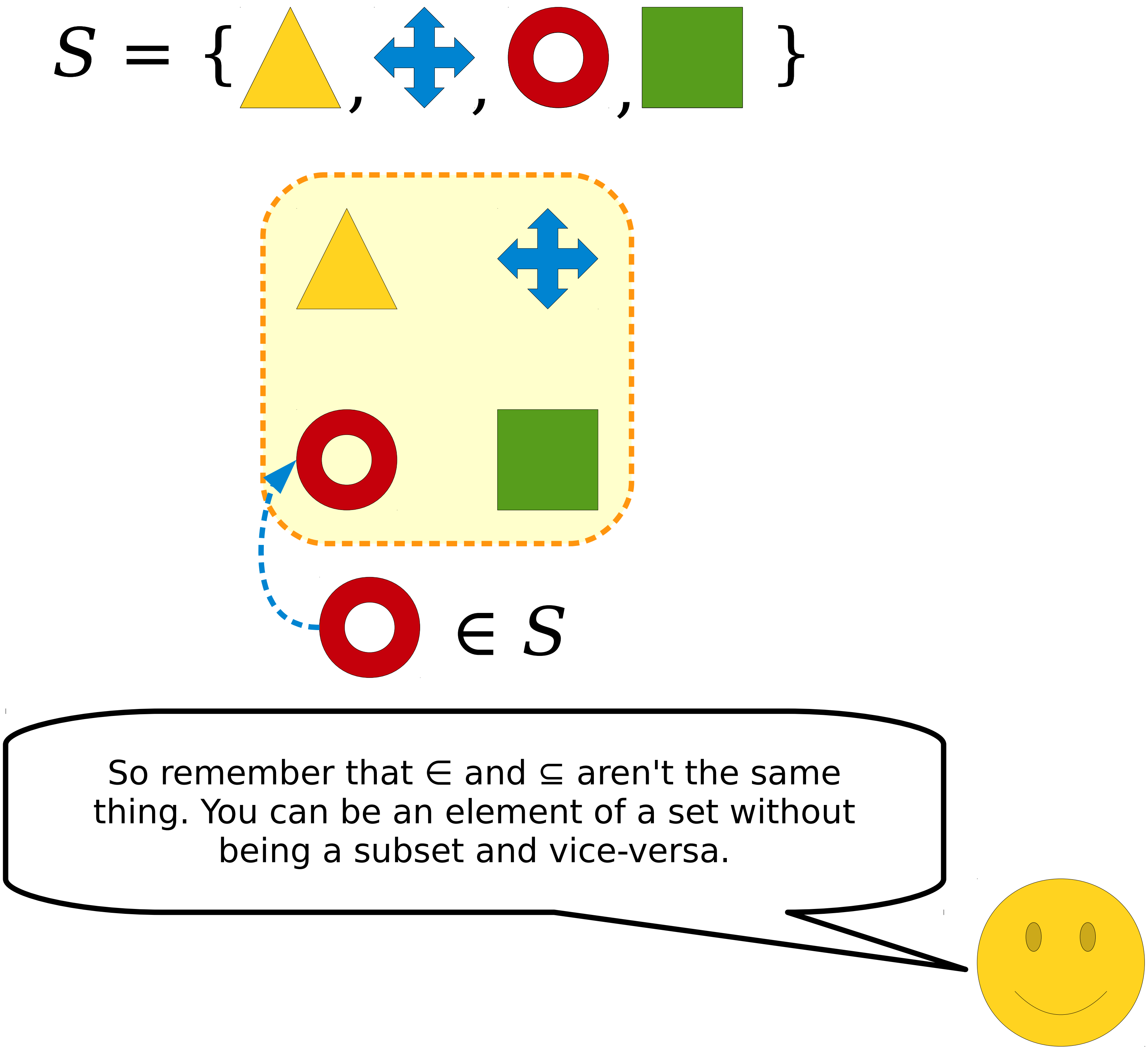
S = { , , , }
⊆? S
In this particular case, the object on the left-hand side of the subset symbol isn't a set...


$S = \{ \quad , \quad , \quad , \quad \}$
$\not\subseteq S$
... so the initial statement was false.

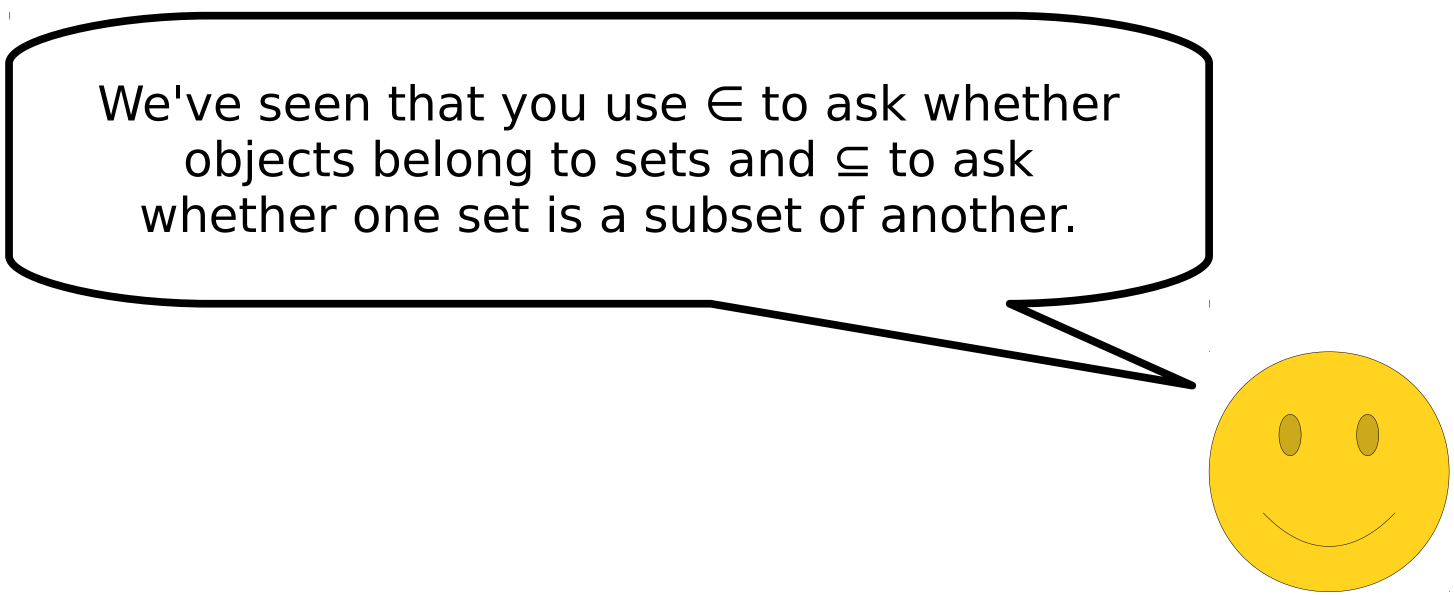

$S = \{$, , , $\}$
$\not\subseteq S$
This might seem a bit counterintuitive at first,
because the red circle is in the set S.

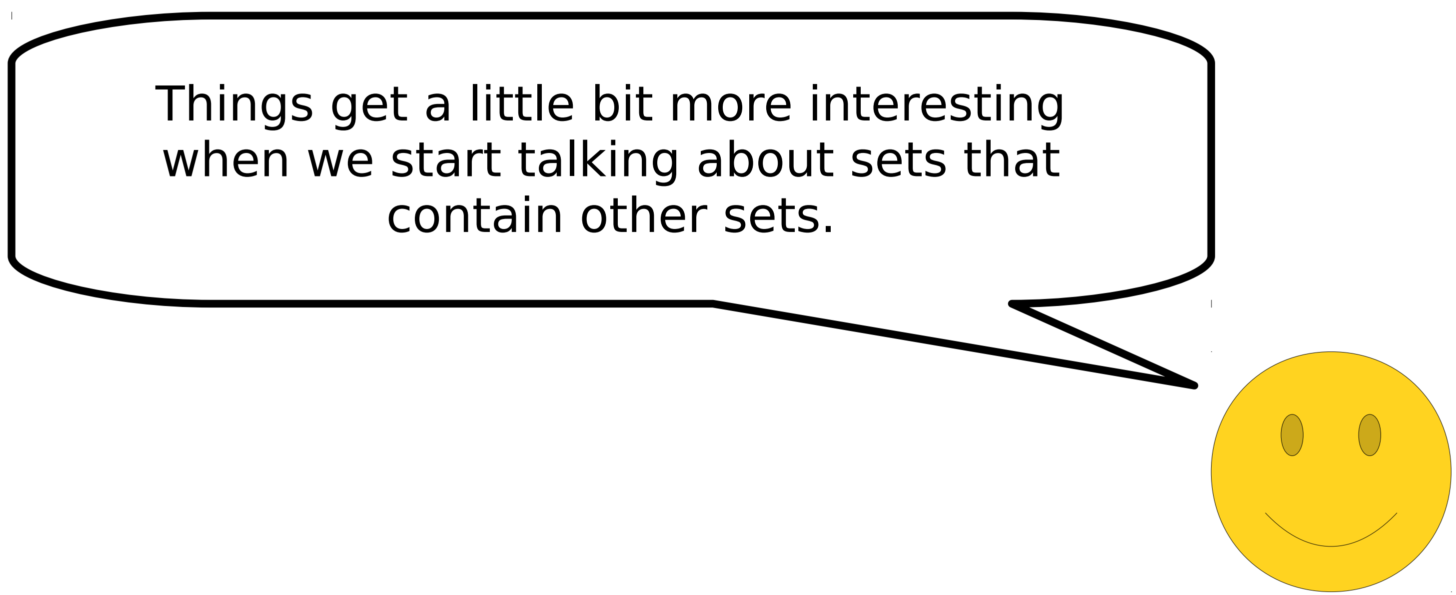

$S = \{$, , , $\}$
$\not\subseteq S$
But we have a different way of expressing that idea!


S = { , , , }
∈ S
We can always say that the red circle is an
element of the set S, even if it's not
a subset of the set S.


S = {   ,   ,   ,   }
∈ S
So remember that ∈ and ⊆ aren't the same thing. You can be an element of a set without being a subset and vice-versa.

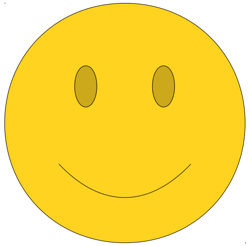

So now we've seen a little bit about the ∈ and ⊆ relations.


We've seen that you use ∈ to ask whether objects belong to sets and ⊆ to ask whether one set is a subset of another.

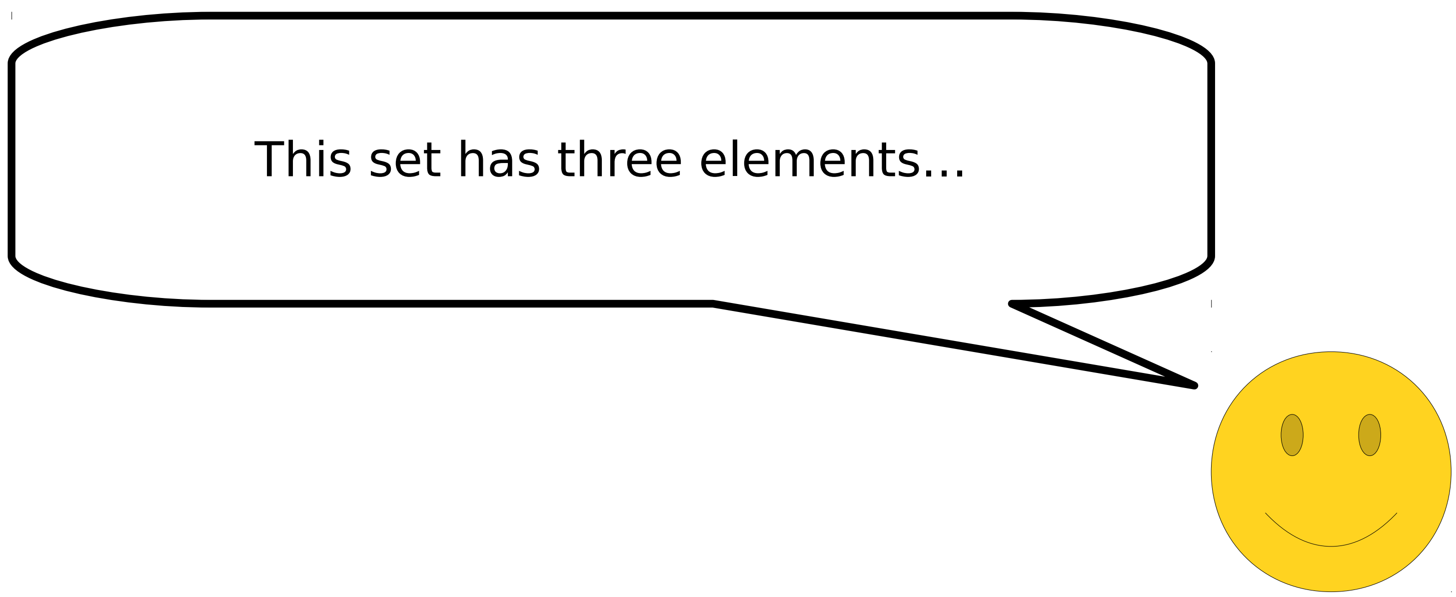

Things get a little bit more interesting when we start talking about sets that contain other sets.

$$\{\ 1,\ \{2,\ 3\},\ 4\ \}$$

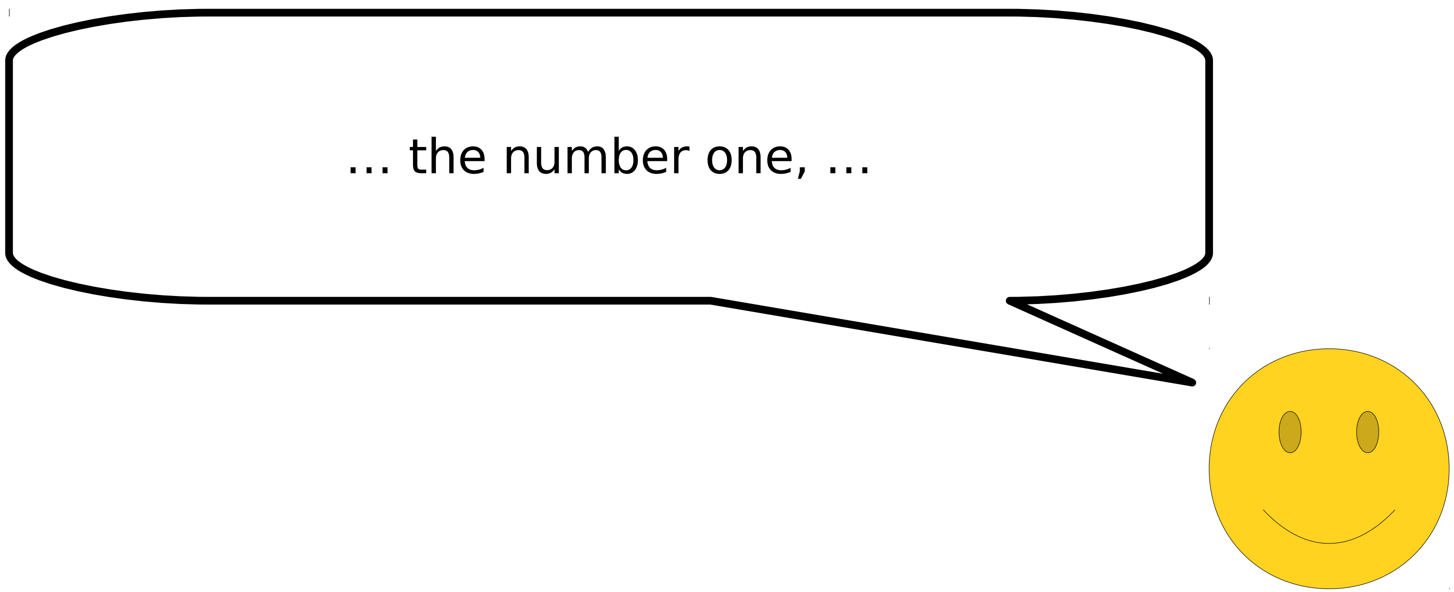
For example, take a look at this set.



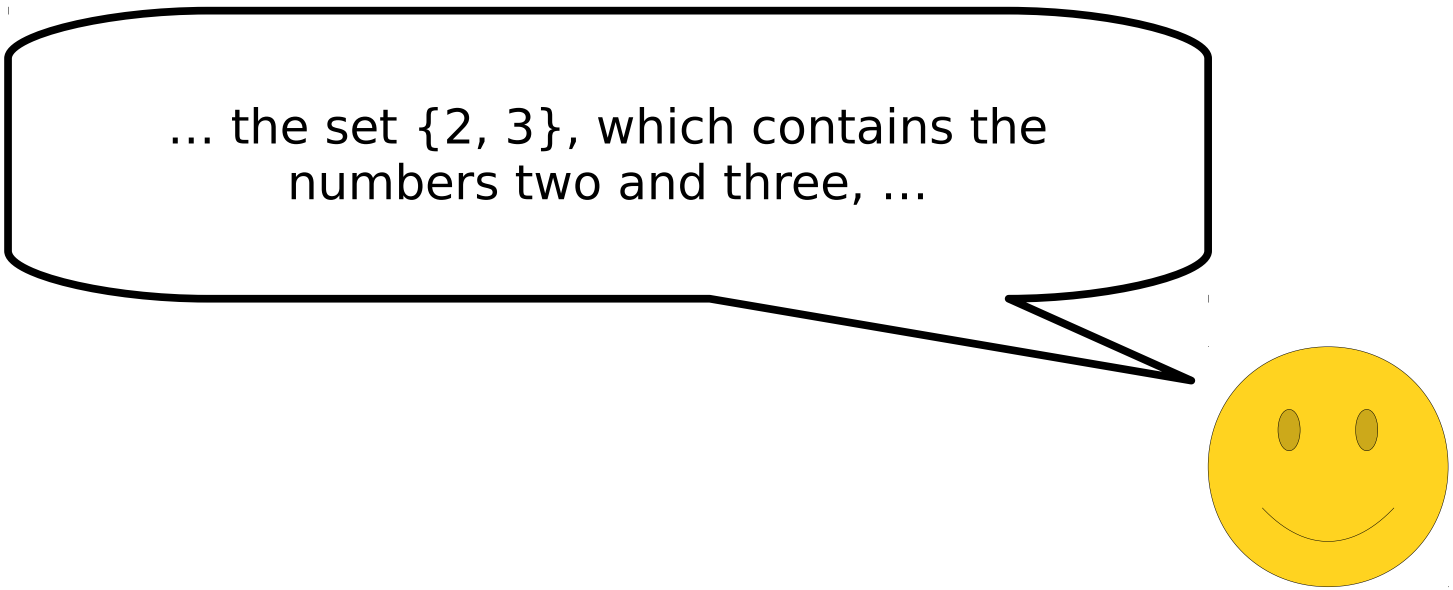
$$\{\ 1,\ \{2,\ 3\},\ 4\ \}$$



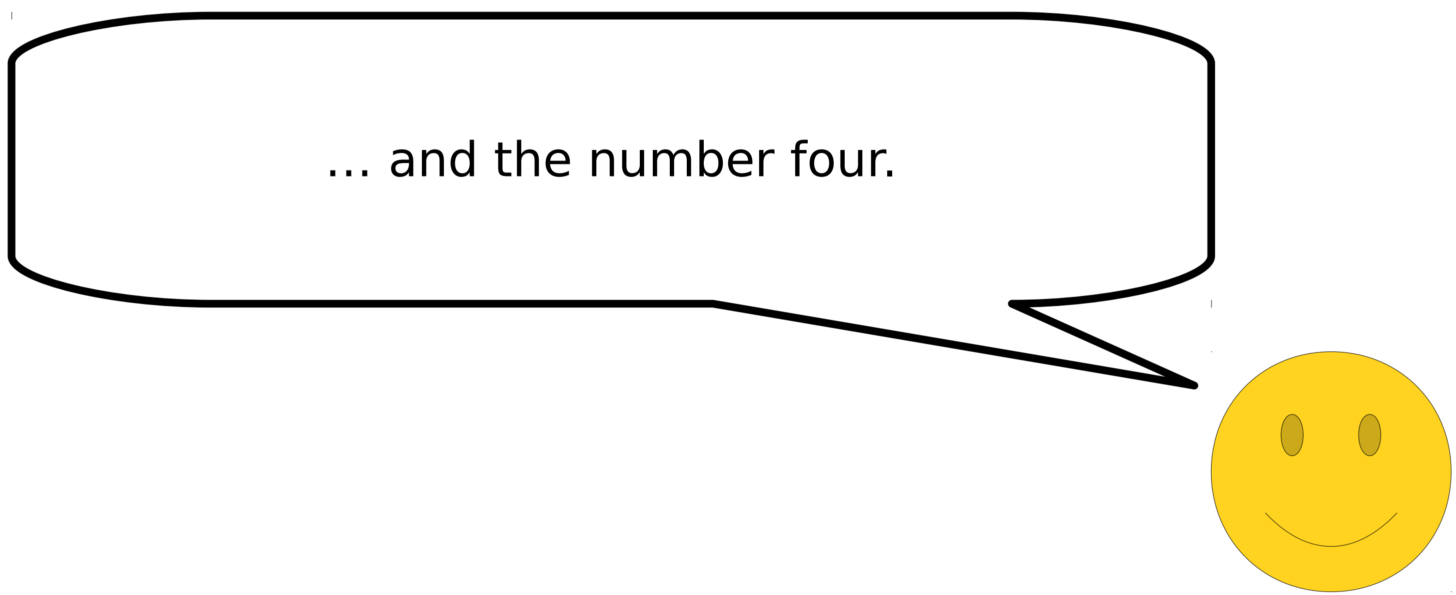
$$\{ 1, \{2, 3\}, 4 \}$$



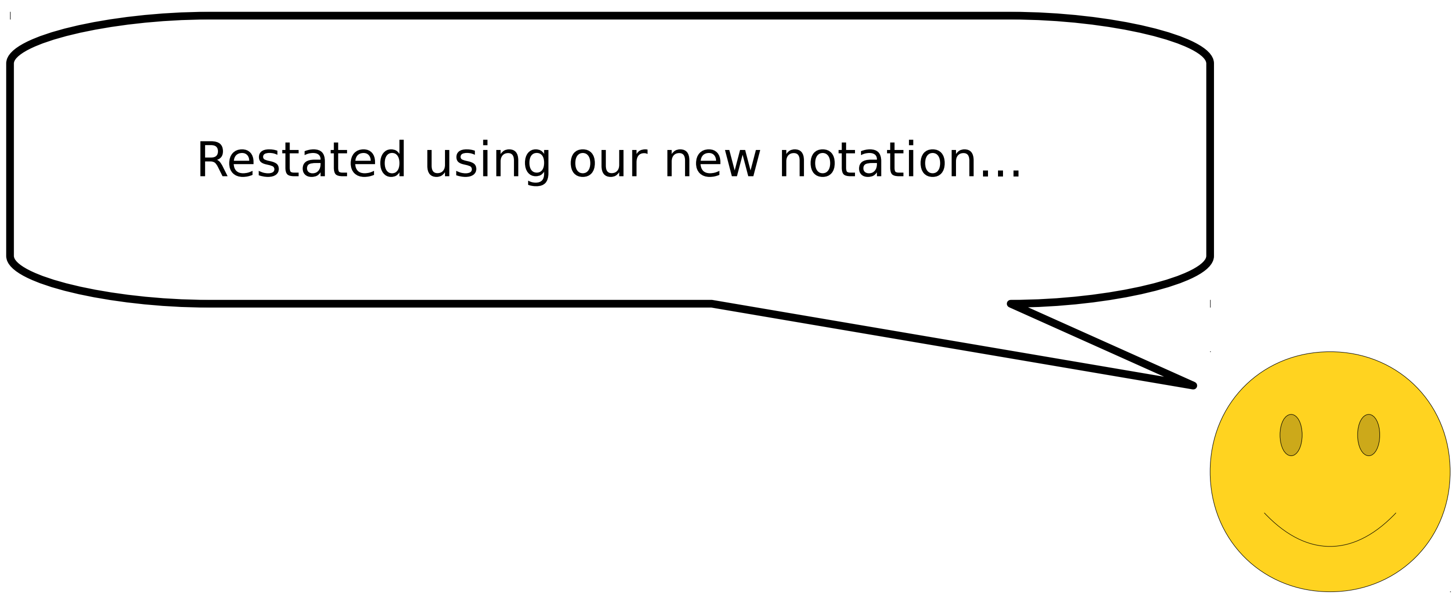
$$\{\ 1,\ \{2,\ 3\},\ 4\ \}$$



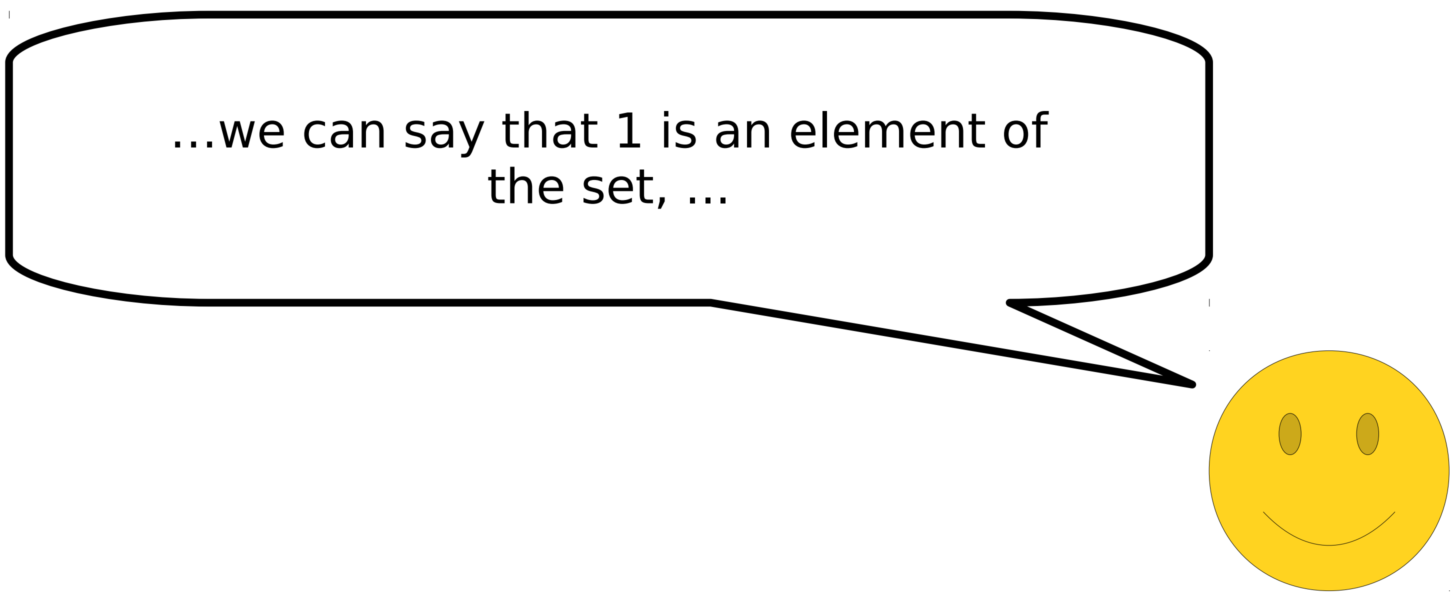
$$\{\ 1,\ \{2,\ 3\},\ 4\ \}$$



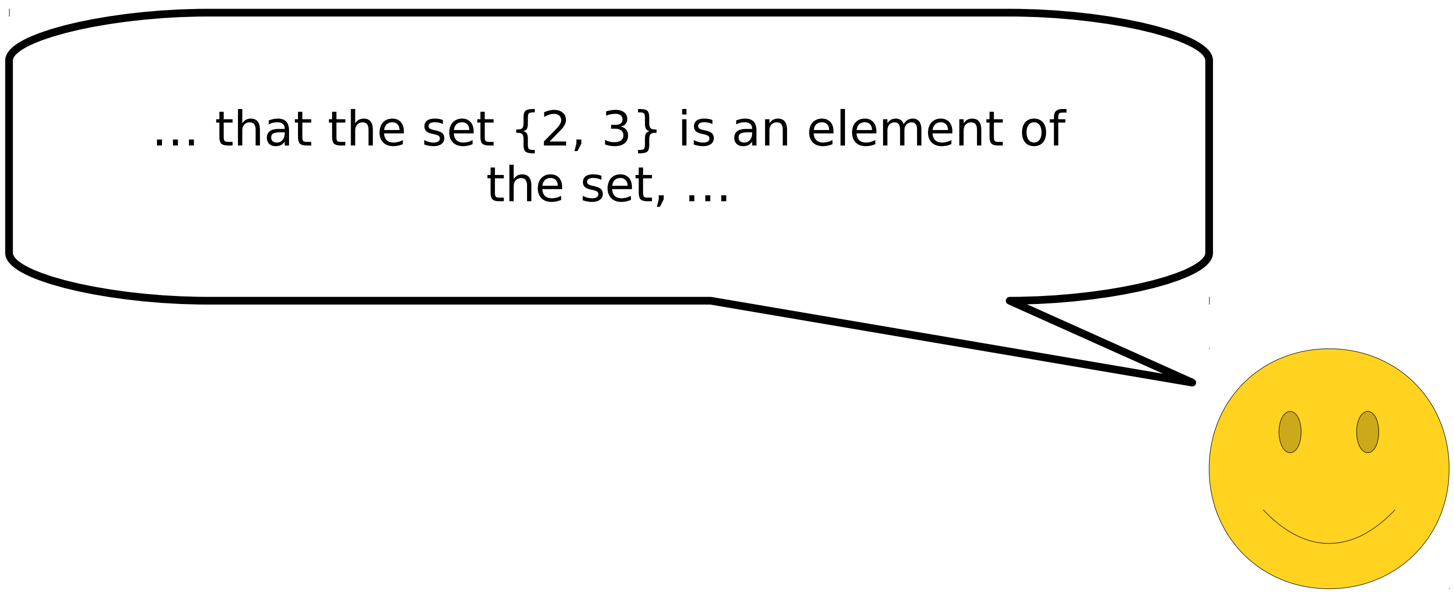
$$\{ 1, \{2, 3\}, 4 \}$$



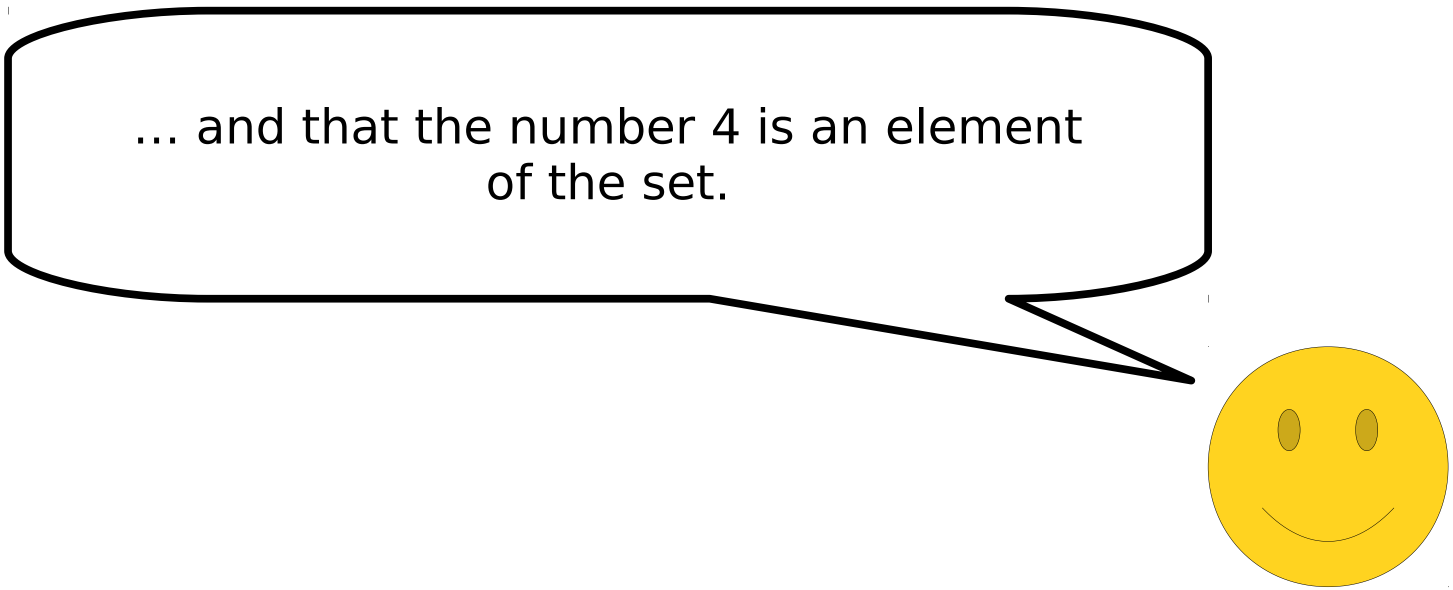
$$1 \in \{\, 1, \{2, 3\}, 4 \,\}$$



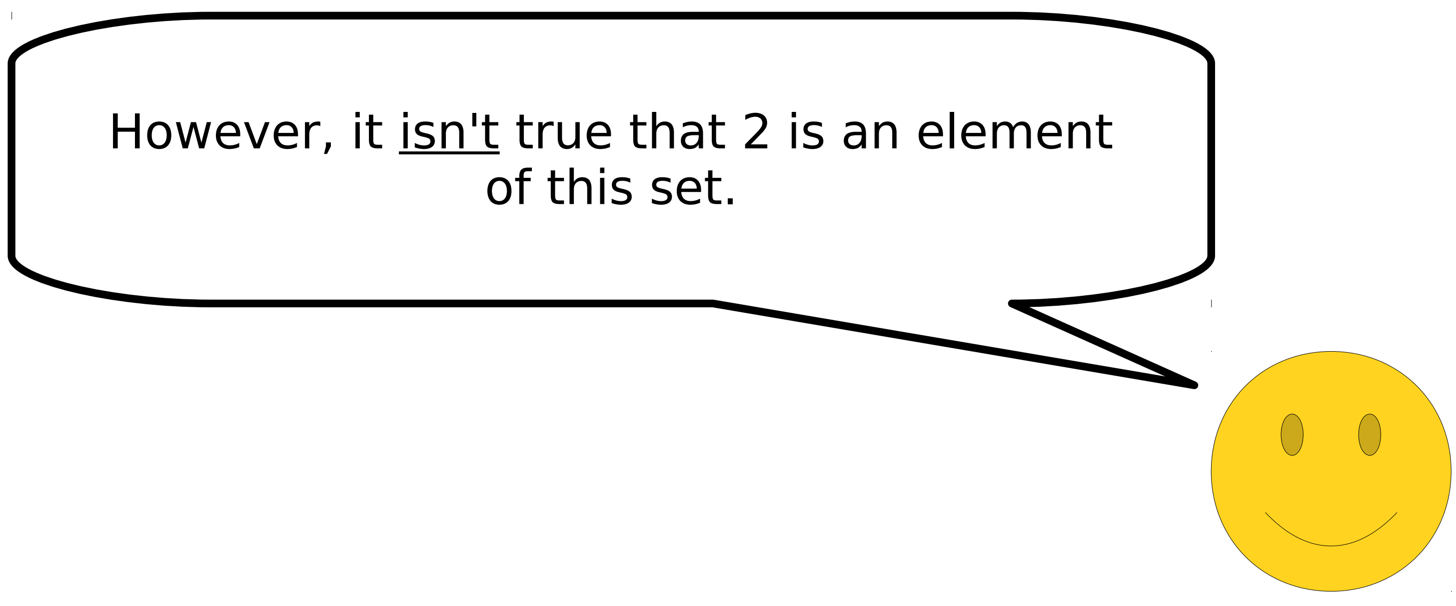
$$\{2, 3\} \in \{\ 1, \{2, 3\}, 4\ \}$$



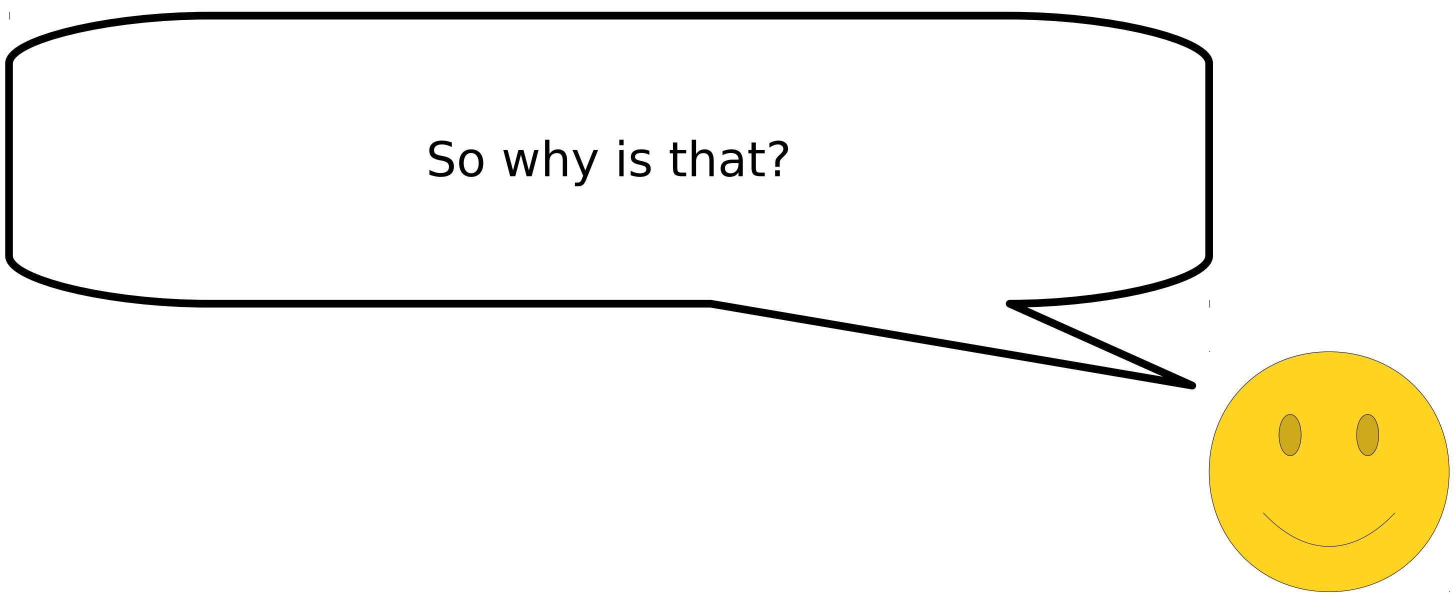
$$4 \in \{\ 1,\ \{2,\ 3\},\ 4\ \}$$



$$2 \notin \{\ 1,\ \{2,\ 3\},\ 4\ \}$$

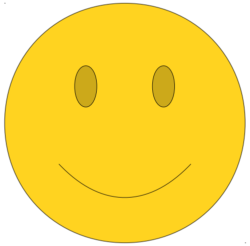


$$2 \notin \{ 1, \{2, 3\}, 4 \}$$

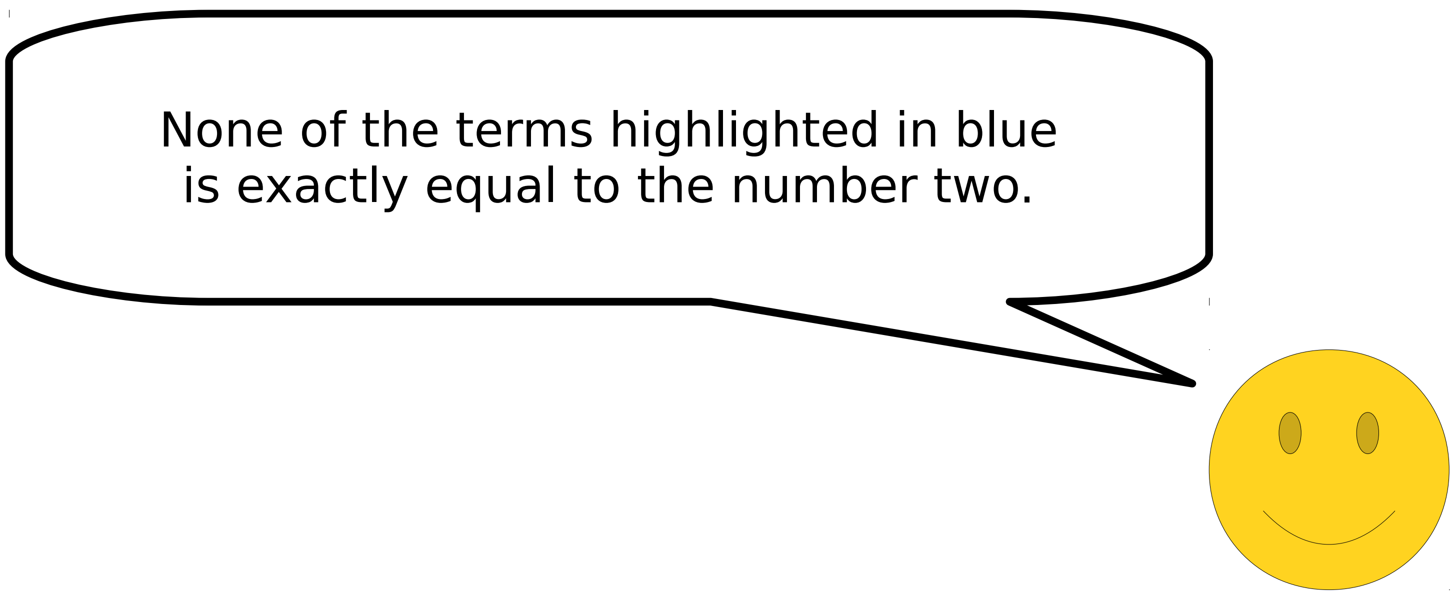


$$2 \notin \{ 1, \{2, 3\}, 4 \}$$

Remember, the set on the right has three different elements: the number 1, the set {2, 3}, and the number 4.



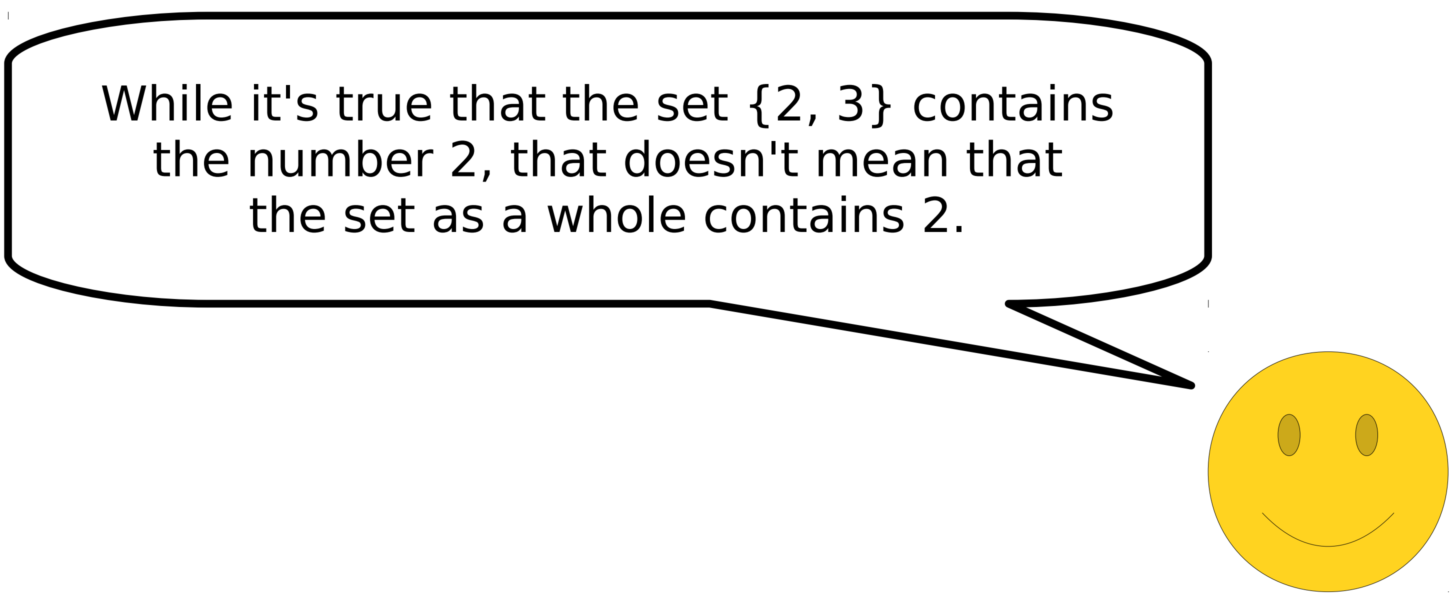
$$2 \notin \{\ 1,\ \{2,\ 3\},\ 4\ \}$$


None of the terms highlighted in blue
is exactly equal to the number two.

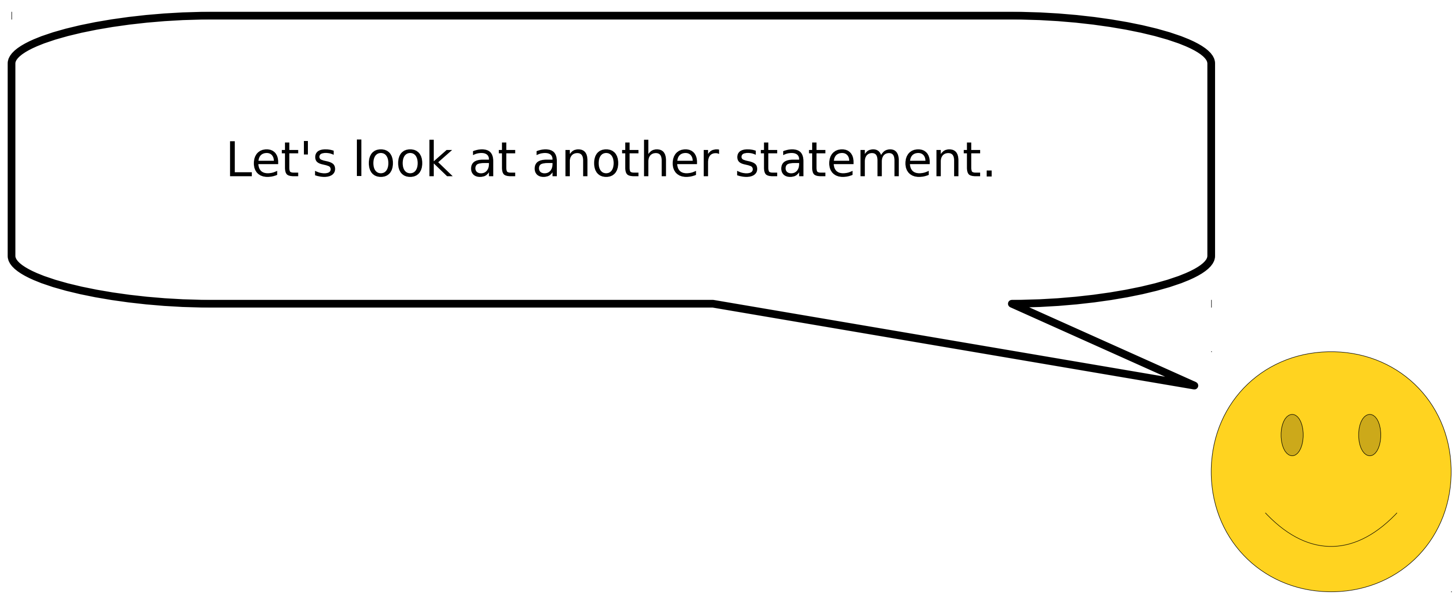
$$2 \notin \{ 1, \{2, 3\}, 4 \}$$

In other words, you can't point at something in the set and say “that thing right there is the number two.”

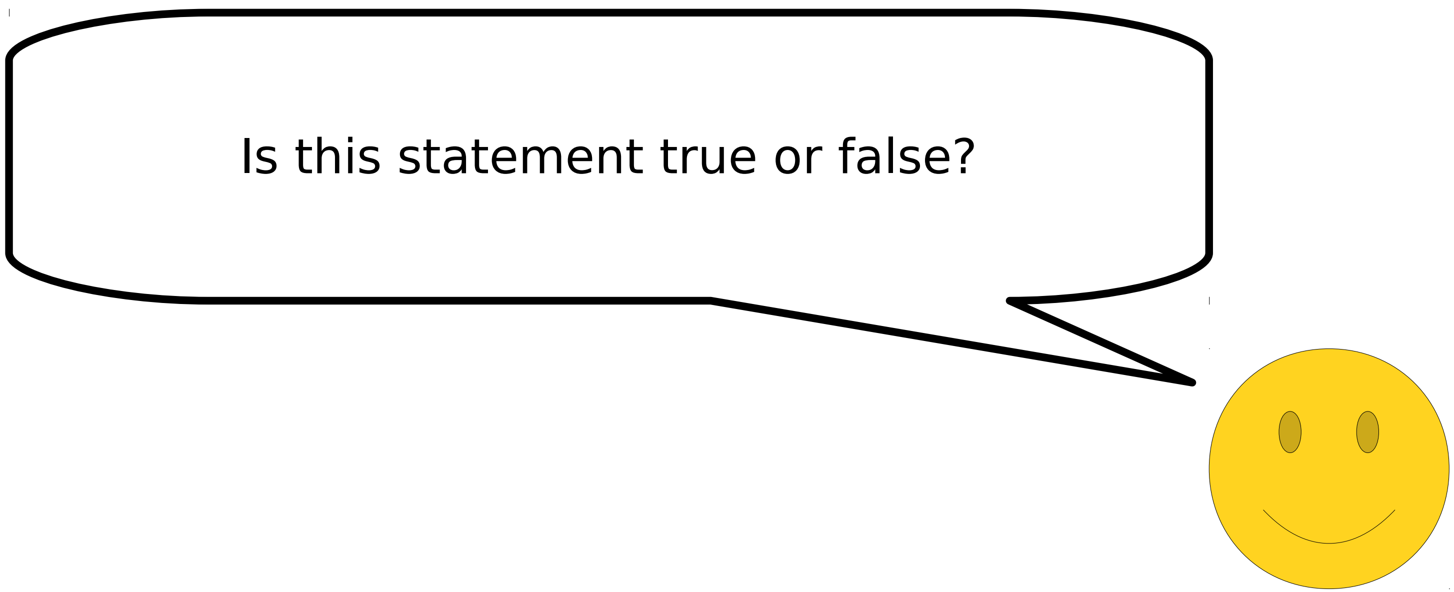
$$2 \notin \{\, 1,\ \{\boxed{2}, 3\},\ 4 \,\}$$



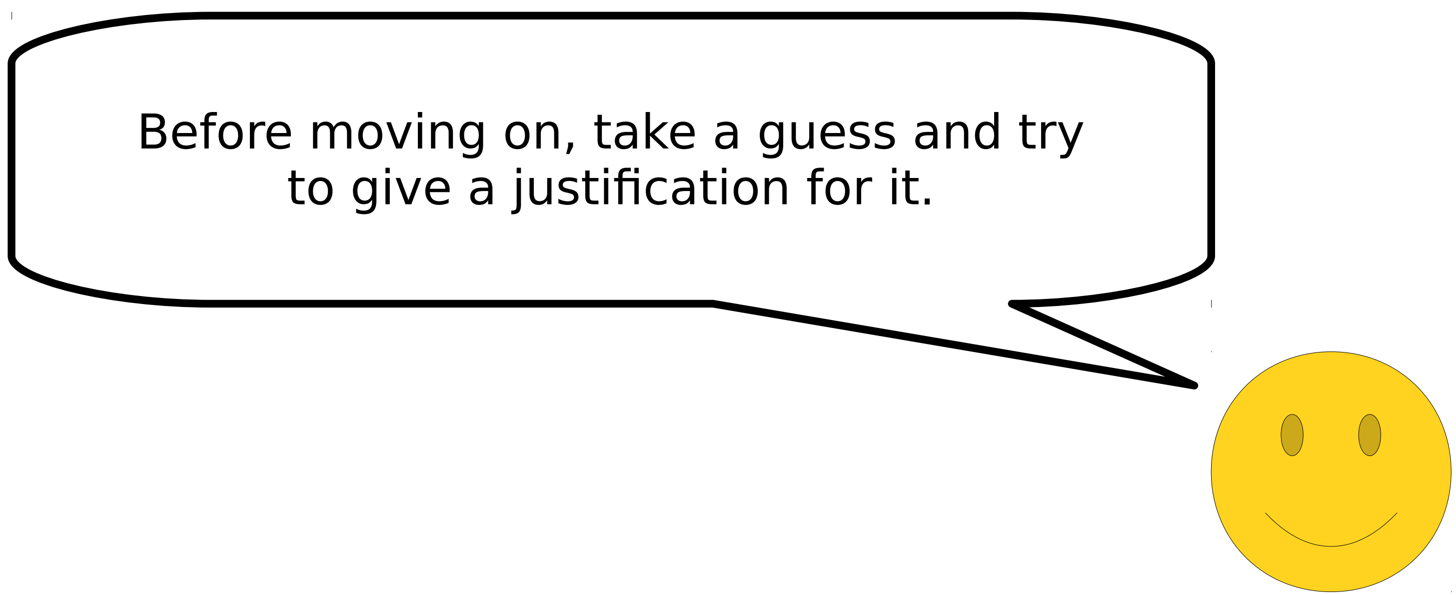
$$\{ 1, \{2, 3\}, 4 \}$$



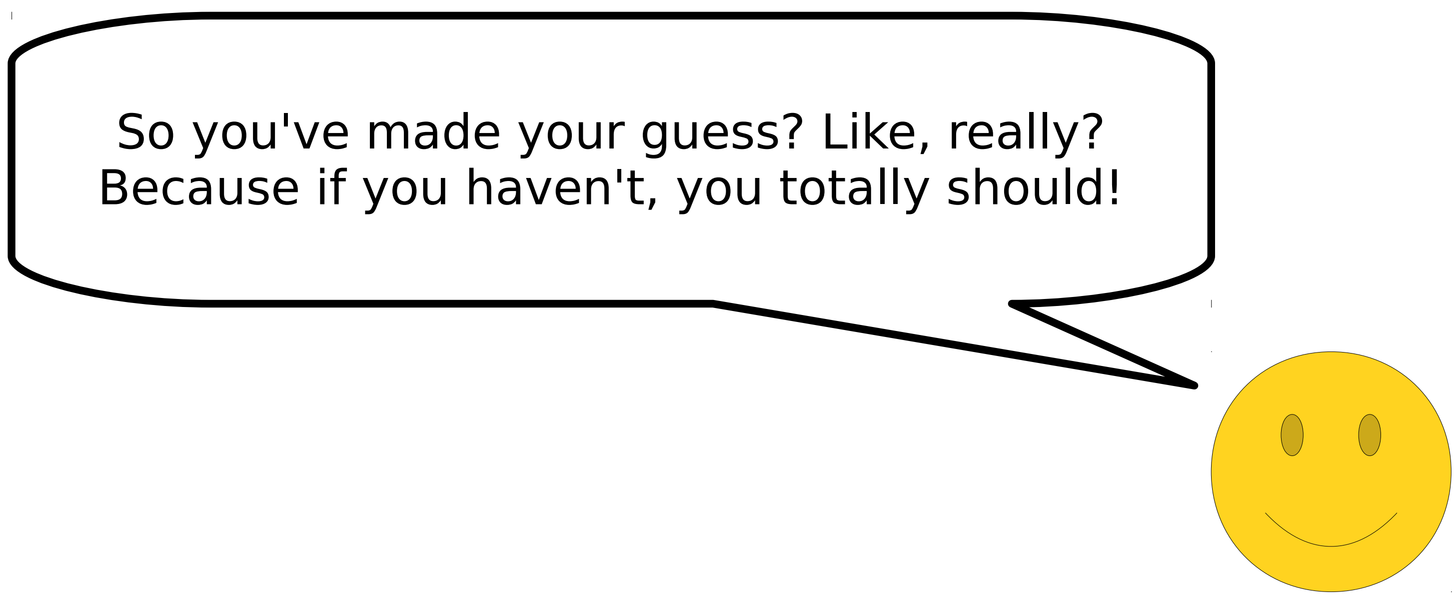
$$\{4\} \stackrel{?}{\in} \{\ 1,\ \{2,\ 3\},\ 4\ \}$$



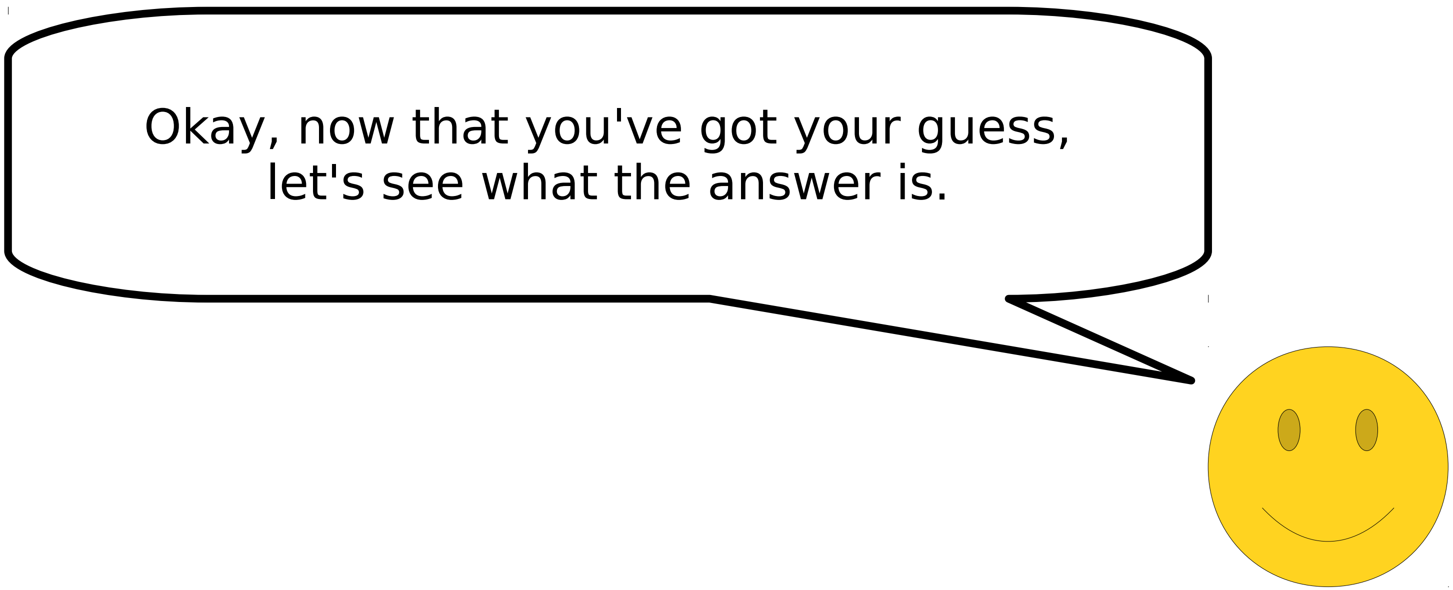
$$\{4\} \stackrel{?}{\in} \{\, 1, \{2, 3\}, 4 \,\}$$



$$\{4\} \overset{?}{\in} \{\ 1,\ \{2,\ 3\},\ 4\ \}$$



$$\{4\} \stackrel{?}{\in} \{ 1, \{2, 3\}, 4 \}$$

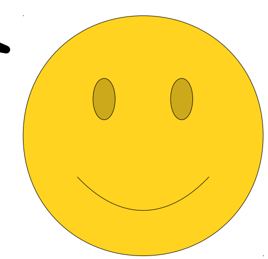


$$\{4\} \stackrel{?}{\in} \{ 1, \{2, 3\}, 4 \}$$

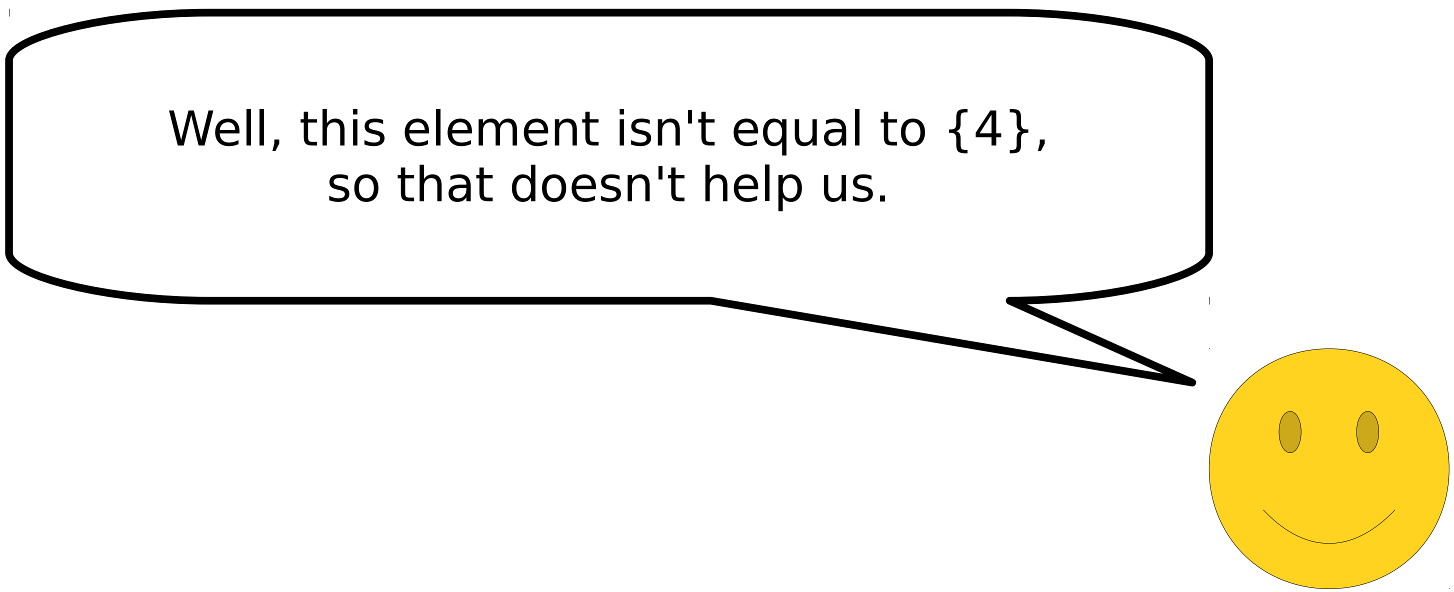
The question is whether {4}, the set containing the number 4, is an element of the set on the right-hand side.

$$\{4\} \stackrel{?}{\in} \{ 1, \{2, 3\}, 4 \}$$

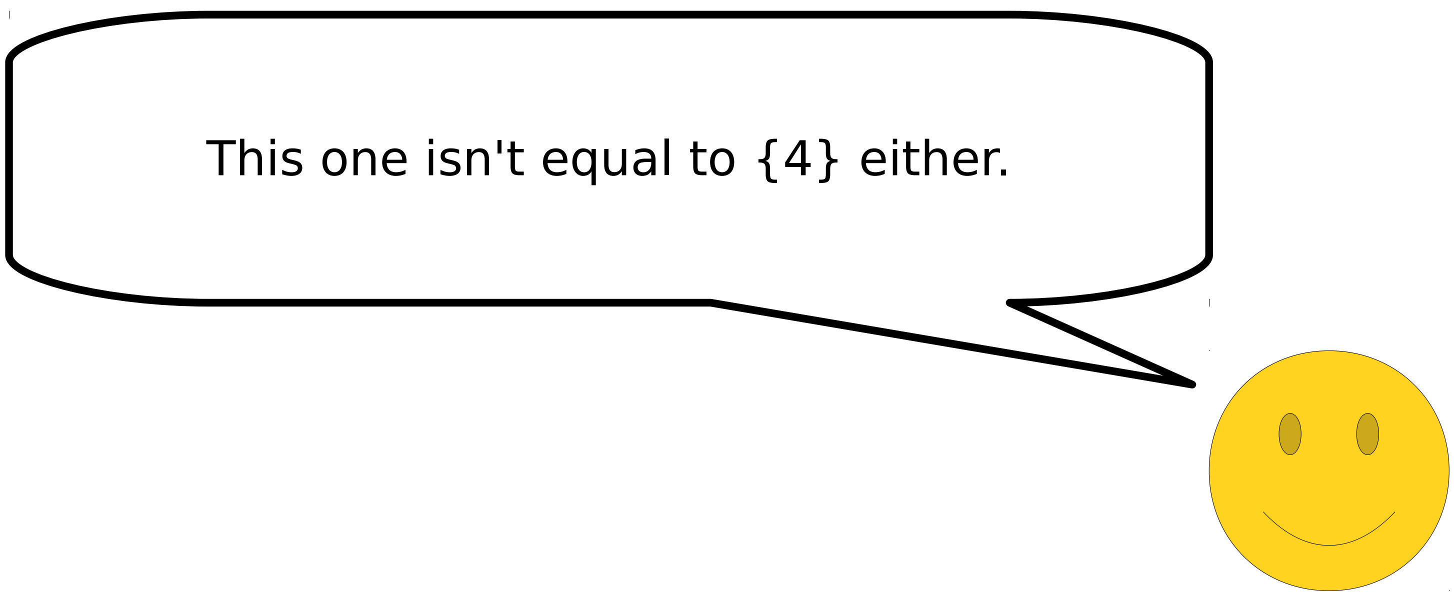
To answer this question, we can just go one element at a time through the set and see if any of them are {4}.



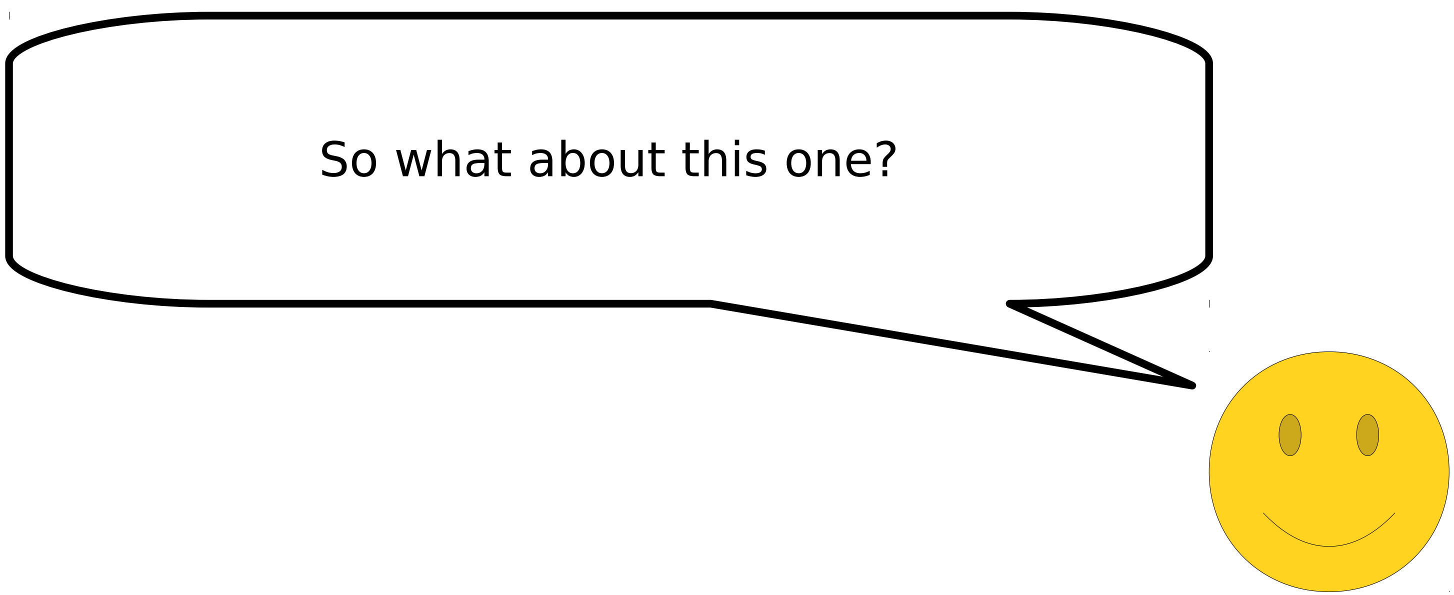
$$\{4\} \overset{?}{\in} \{\ 1, \{2, 3\}, 4\ \}$$



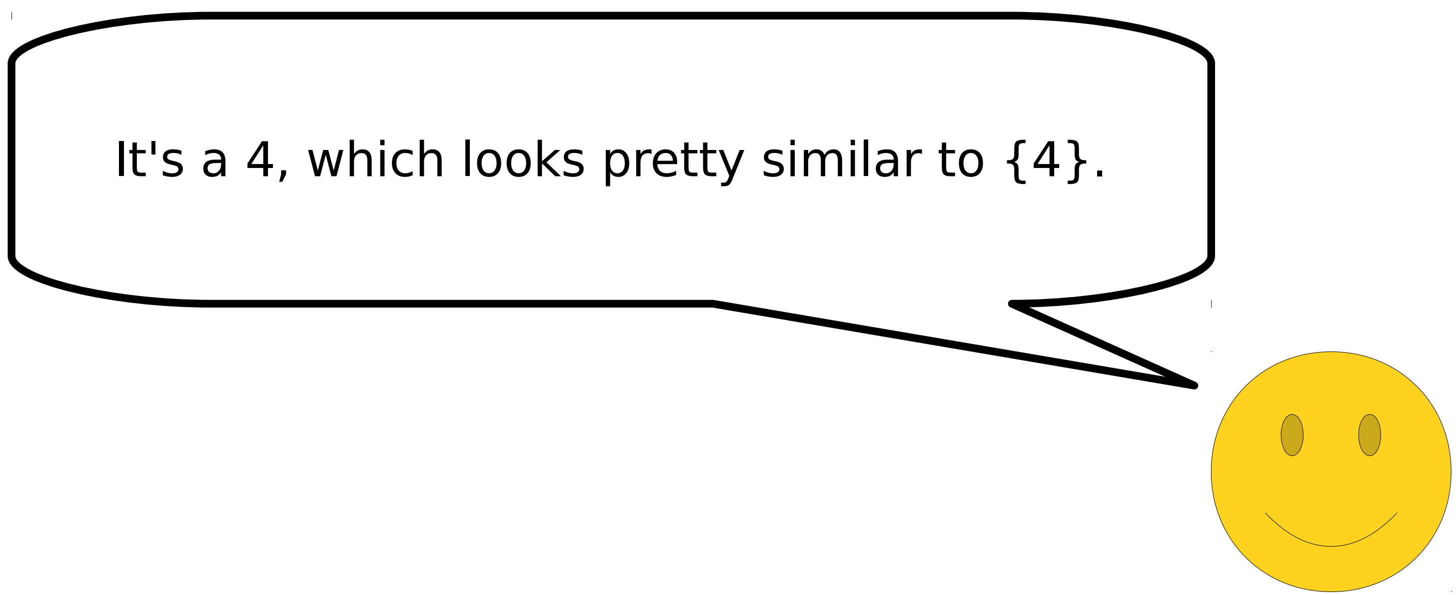
$$\{4\} \overset{?}{\in} \{\, 1,\ \{2,\ 3\},\ 4 \,\}$$



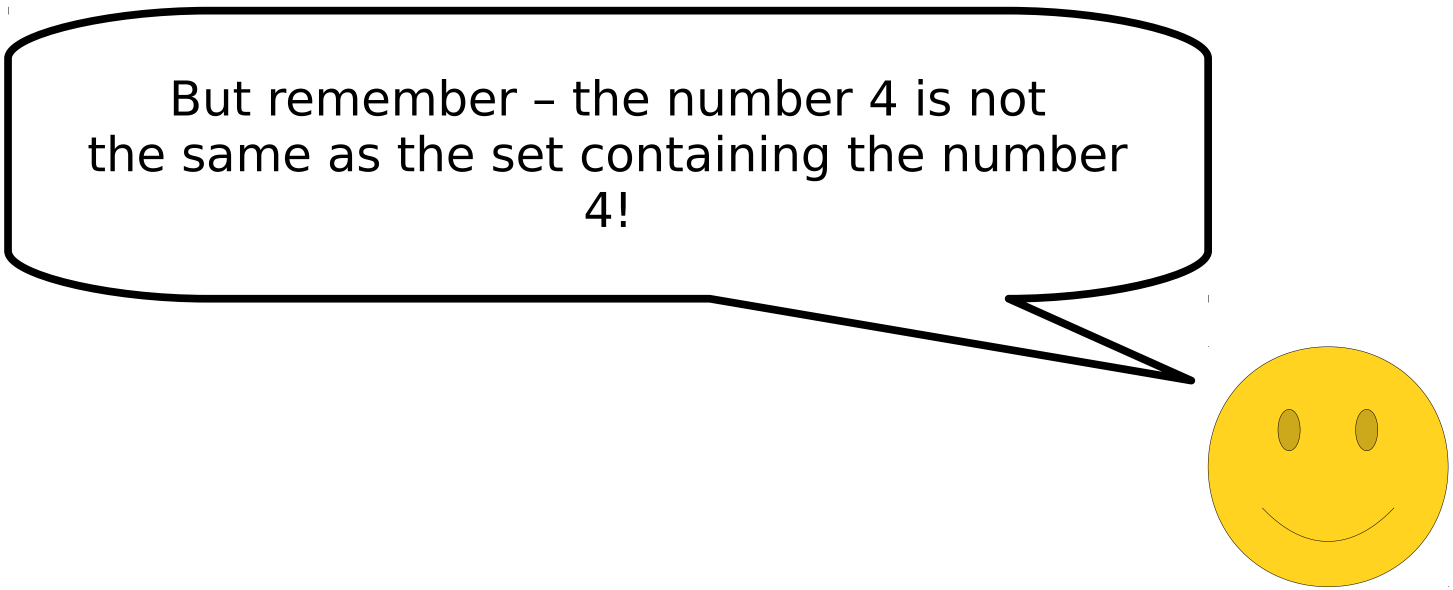
$$\{4\} \overset{?}{\in} \{\, 1,\ \{2,\ 3\},\ 4 \,\}$$



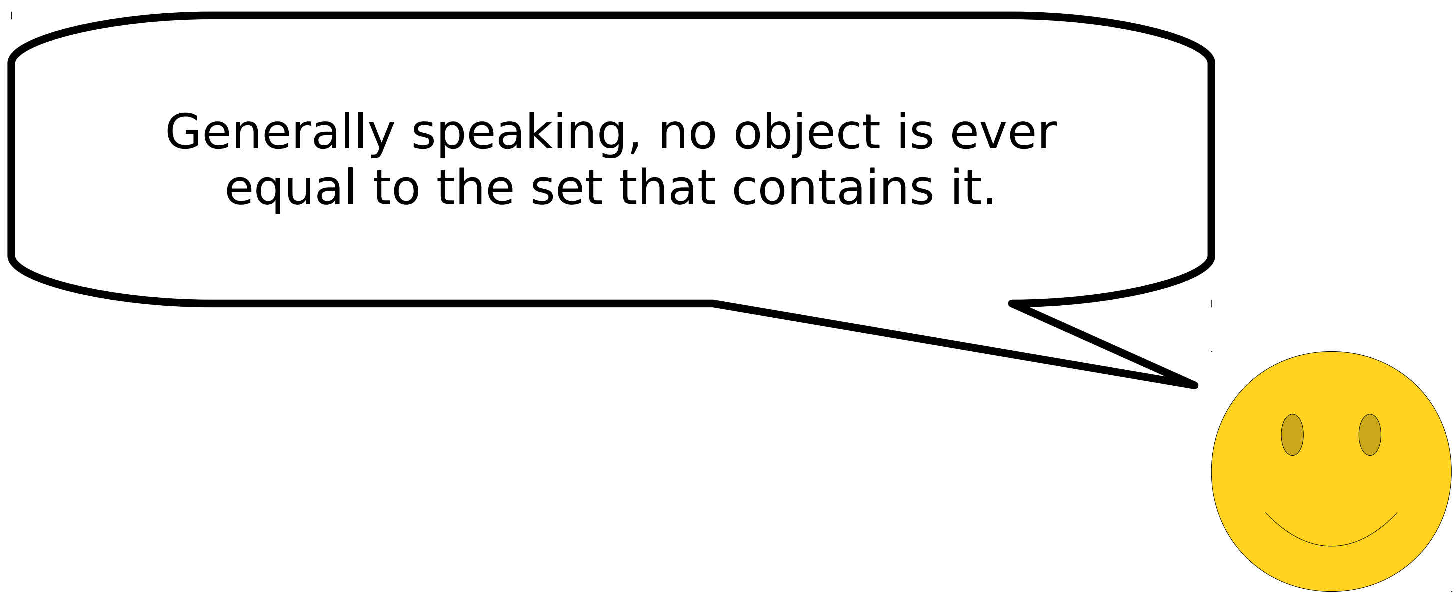
$$\{4\} \overset{?}{\in} \{\, 1,\ \{2,\ 3\},\ 4 \,\}$$



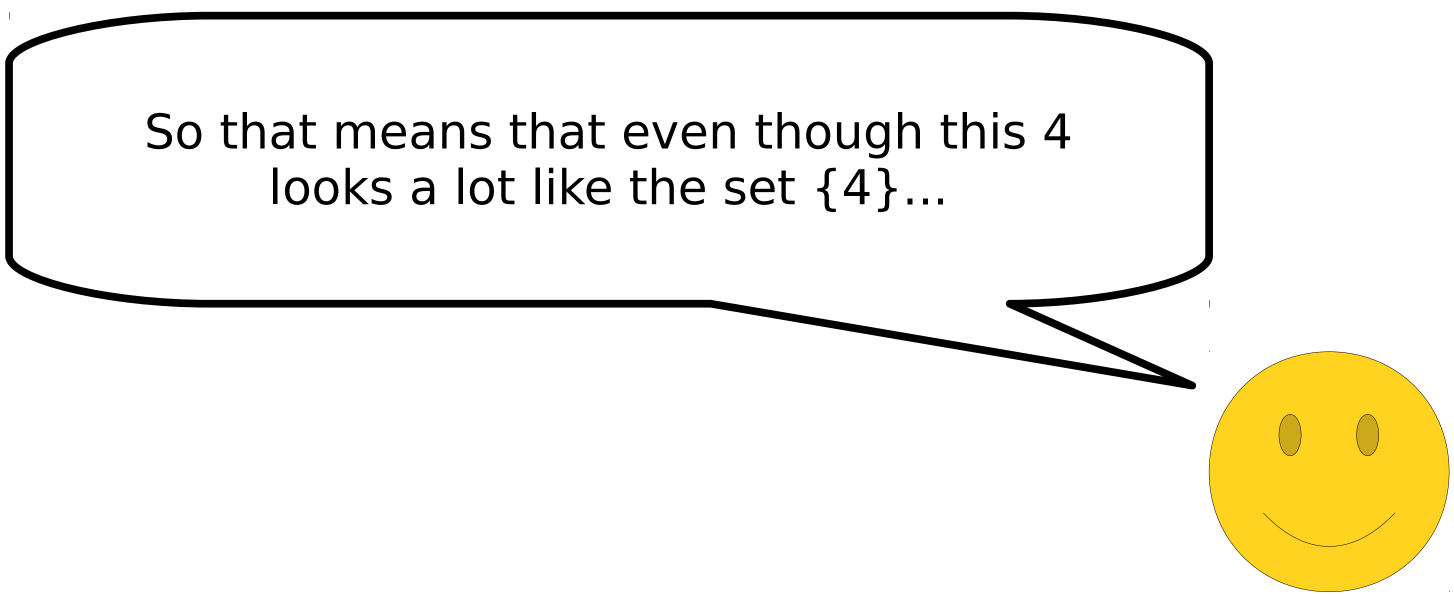
$$\{4\} \overset{?}{\in} \{ 1, \{2, 3\}, 4 \}$$



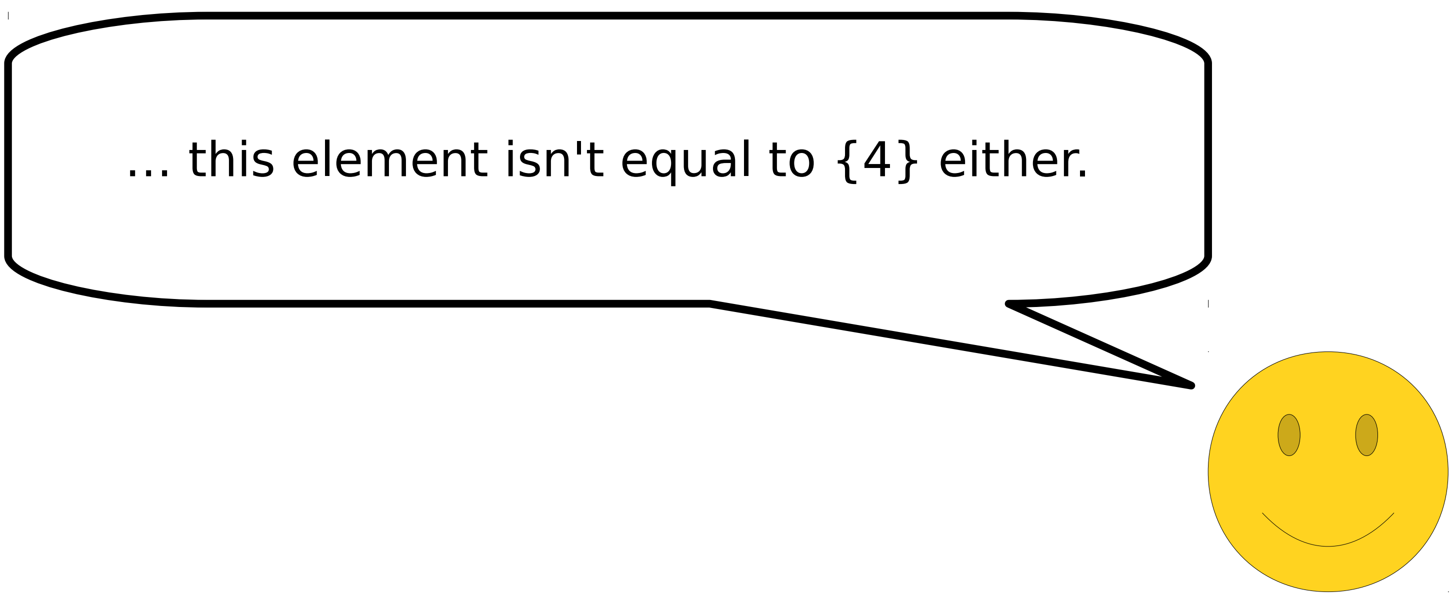
$$\{4\} \overset{?}{\in} \{\ 1,\ \{2,\ 3\},\ 4\ \}$$



$$\{4\} \stackrel{?}{\in} \{\ 1,\ \{2,\ 3\},\ 4\ \}$$

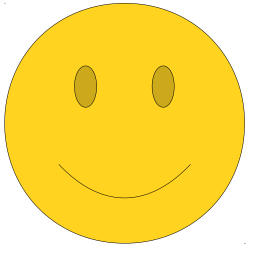


$$\{4\} \overset{?}{\in} \{\ 1,\ \{2,\ 3\},\ 4\ \}$$



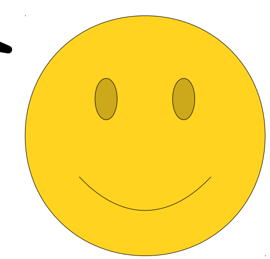
$$\{4\} \overset{?}{\in} \{\ 1,\ \{2,\ 3\},\ 4\ \}$$

So we've gone through all the elements of the set, trying to see if any of them are equal to {4}, and it looks like none of them are.

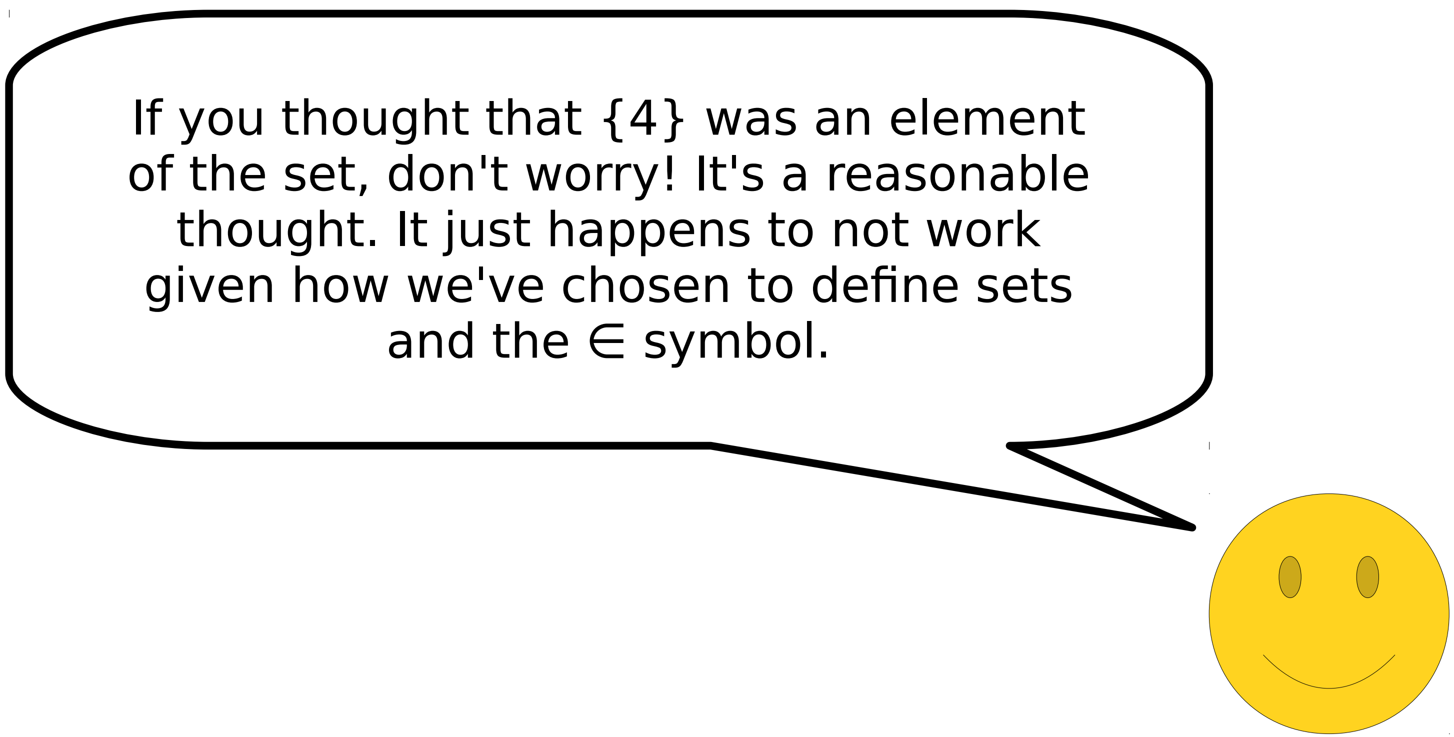


$$\{4\} \notin \{\ 1,\ \{2,\ 3\},\ 4\ \}$$

That means that {4} is not an element of the set.

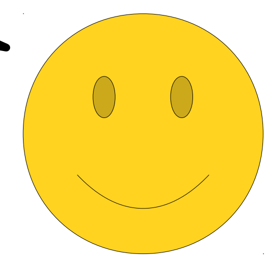


$$\{4\} \notin \{ 1, \{2, 3\}, 4 \}$$

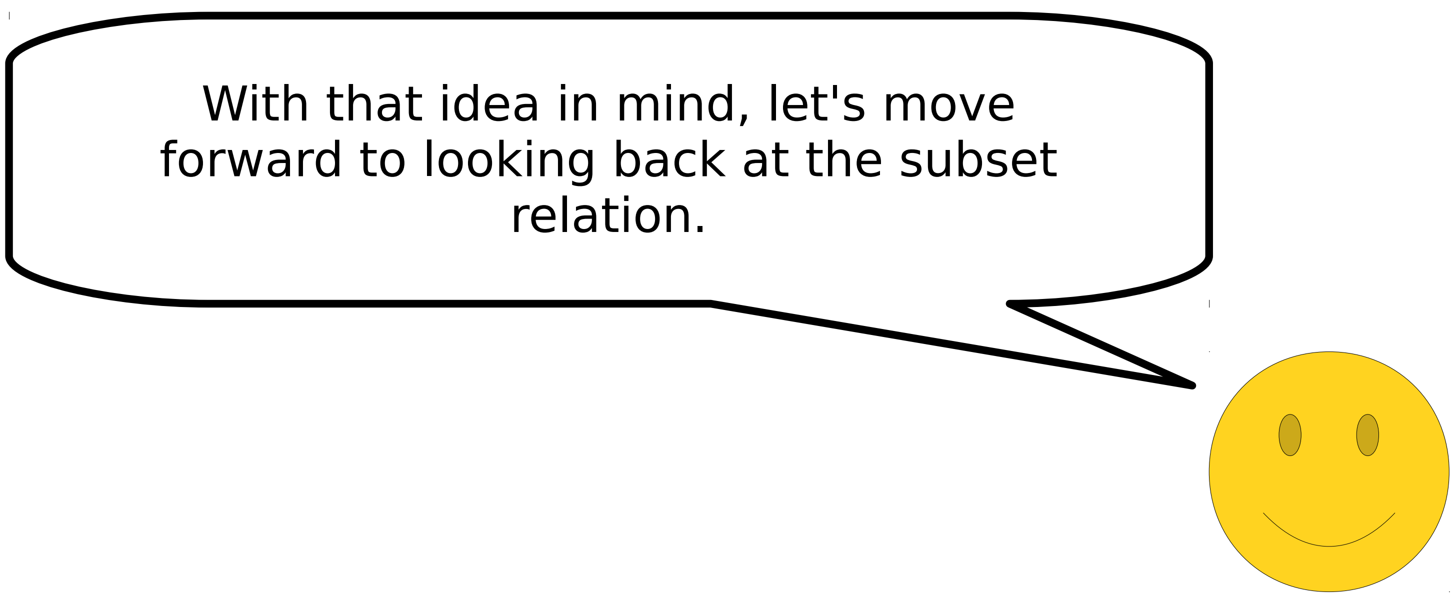

If you thought that {4} was an element of the set, don't worry! It's a reasonable thought. It just happens to not work given how we've chosen to define sets and the ∈ symbol.

$$\{4\} \notin \{\ 1,\ \{2,\ 3\},\ 4\ \}$$

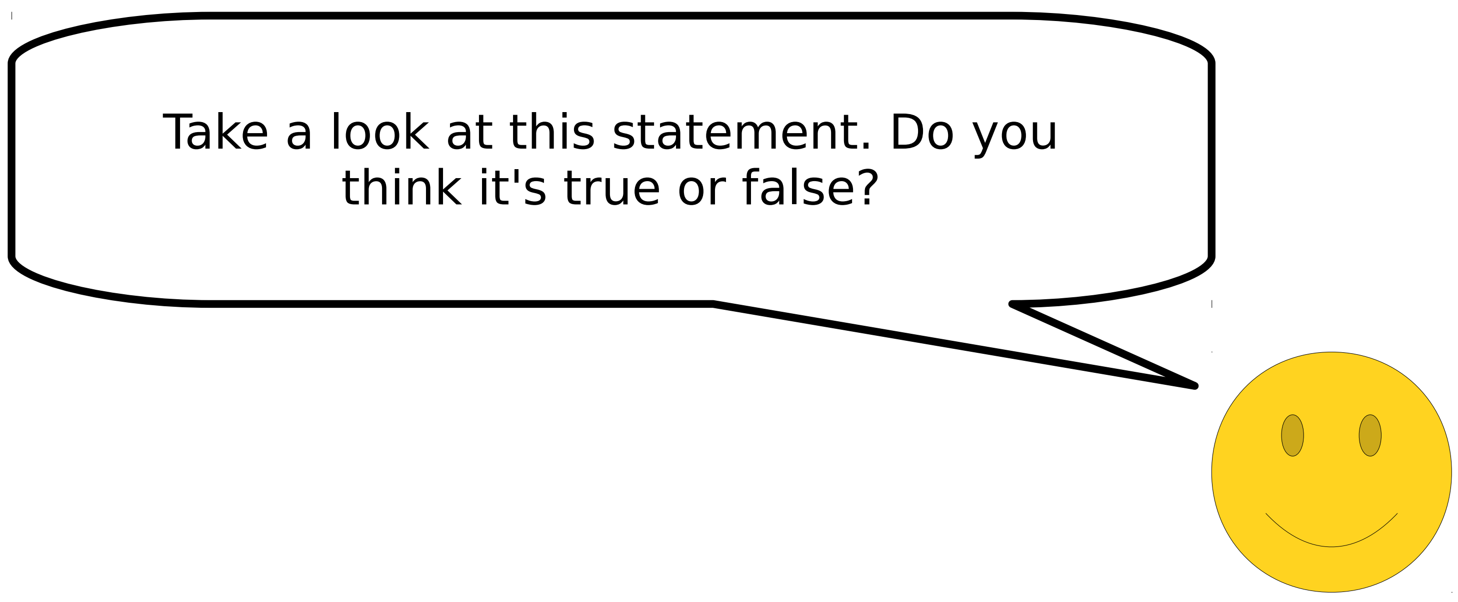
Going forward, remember that x and {x} are different things – x is an individual object, while {x} is a set containing the object x.



$$\{\ 1,\ \{2,\ 3\},\ 4\ \}$$

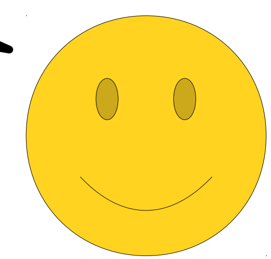

With that idea in mind, let's move forward to looking back at the subset relation.

$$\{1\} \stackrel{?}{\subseteq} \{\ 1,\ \{2,\ 3\},\ 4\ \}$$

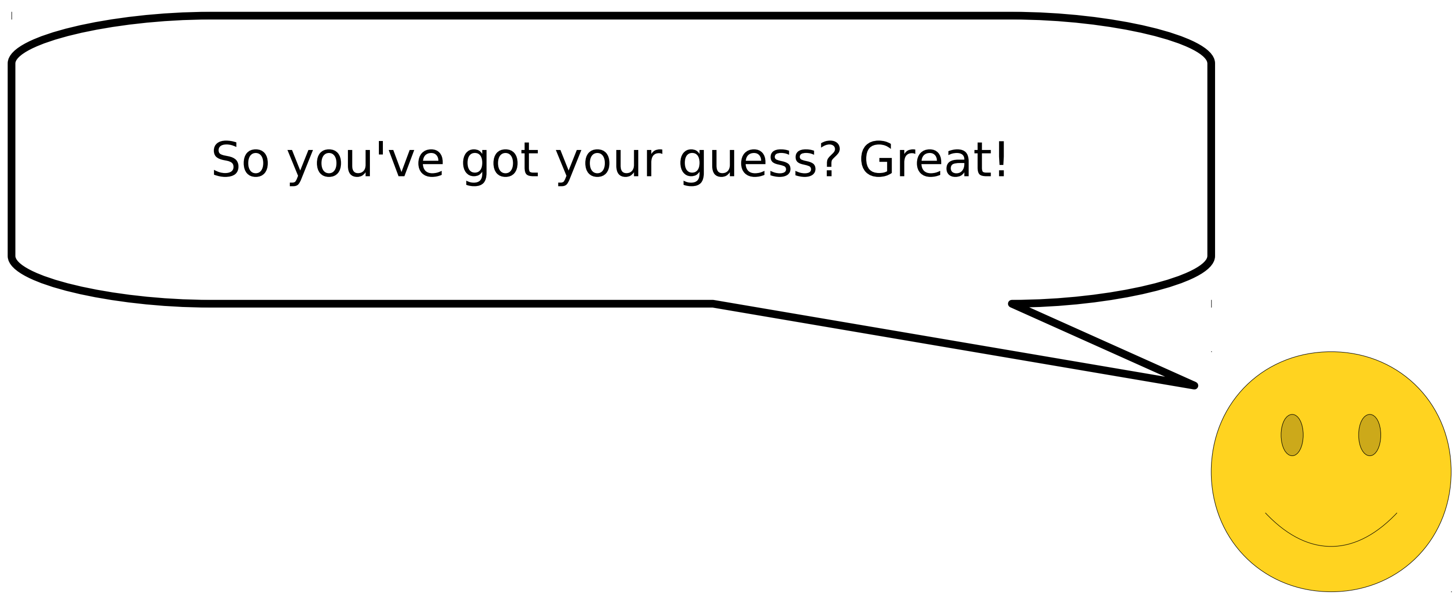


$$\{1\} \overset{?}{\subseteq} \{\ 1,\ \{2,\ 3\},\ 4\ \}$$

As before, take a minute to come up with a guess before moving on. You'll be happier if you do.



$$\{1\} \overset{?}{\subseteq} \{ 1, \{2, 3\}, 4 \}$$

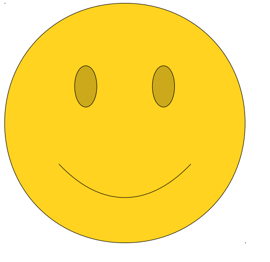


$$\{1\} \stackrel{?}{\subseteq} \{\ 1,\ \{2,\ 3\},\ 4\ \}$$

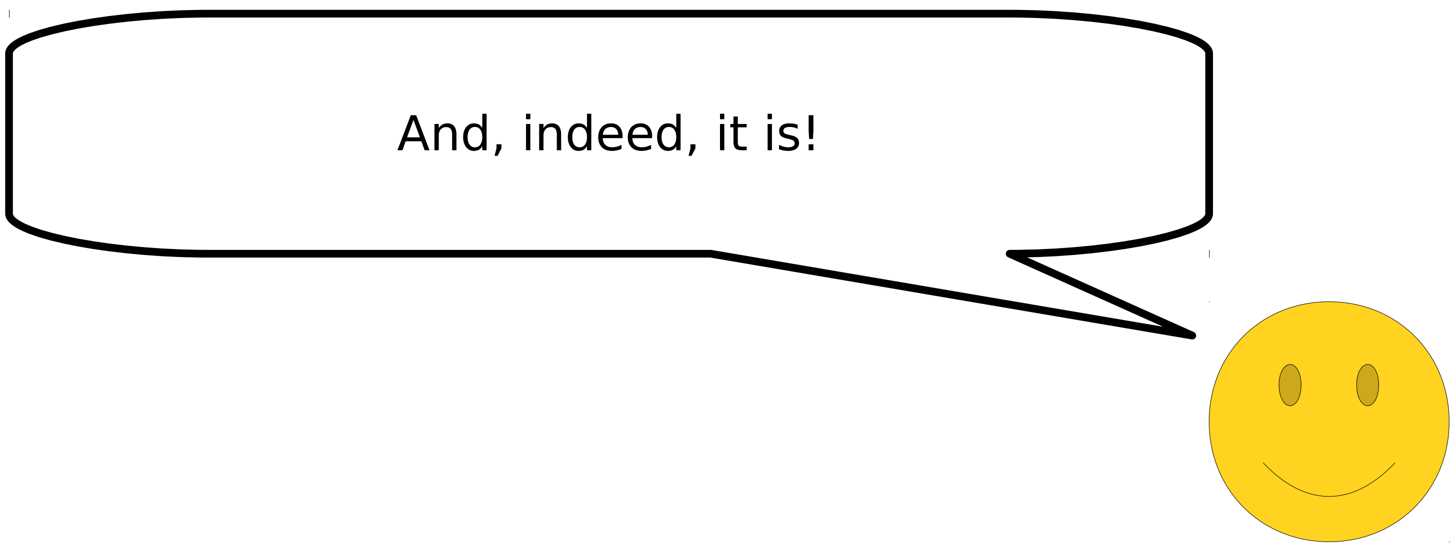
To determine whether {1} is a subset of {1, {2,3}, 4}, we need to ask whether every element of {1} is also an element of {1, {2,3}, 4}

$$\{1\} \stackrel{?}{\subseteq} \{\ 1,\ \{2,\ 3\},\ 4\ \}$$

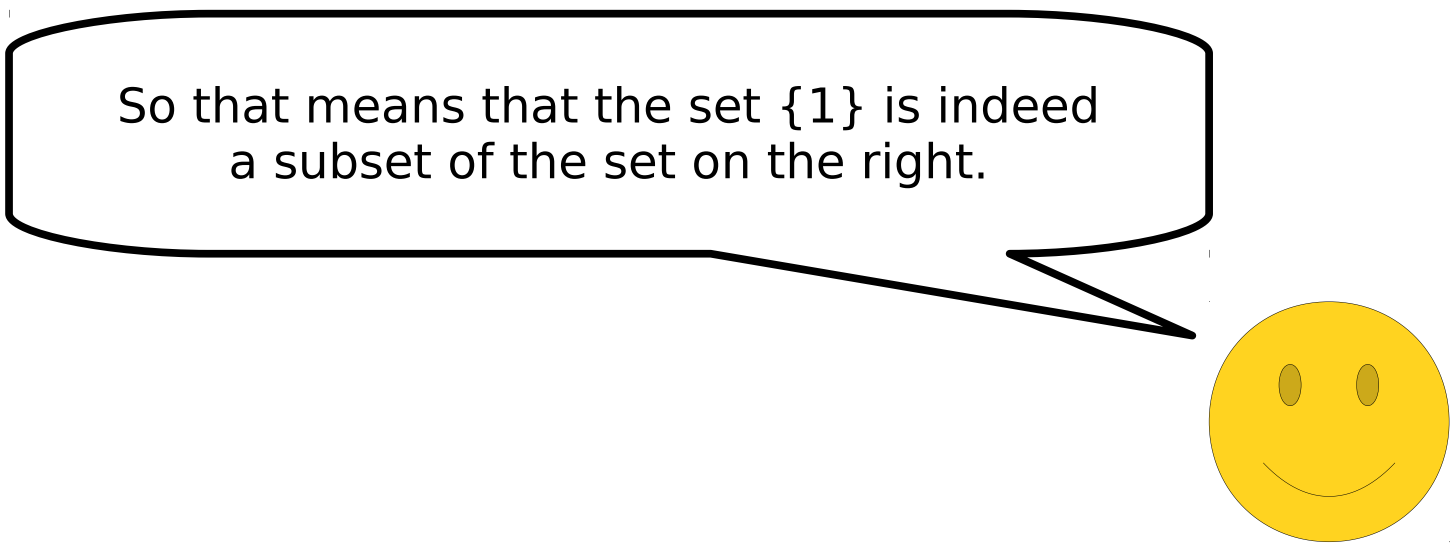
The set {1} only has one element (the number 1), so we just need to see whether the number 1 is contained in the set on the right.



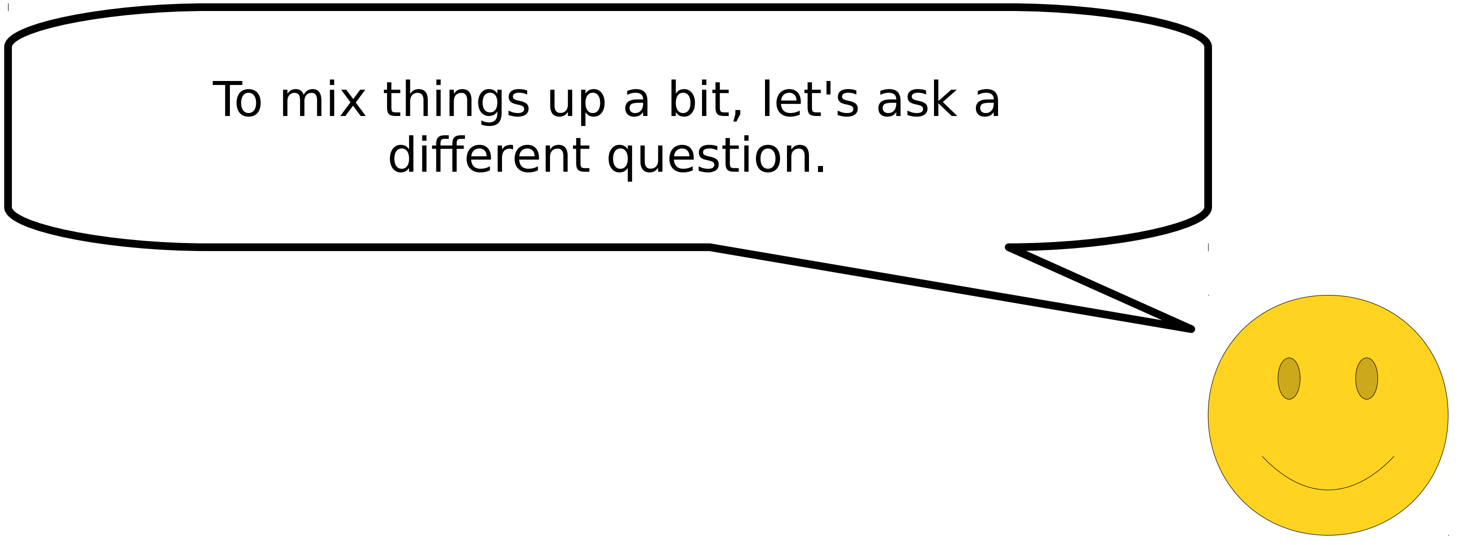
$$\{1\} \overset{?}{\subseteq} \{\ 1, \{2, 3\}, 4\ \}$$



$$\{1\} \subseteq \{\ 1,\ \{2,\ 3\},\ 4\ \}$$

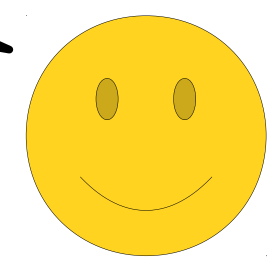


$$\{1\} \subseteq \{\ 1,\ \{2,\ 3\},\ 4\ \}$$

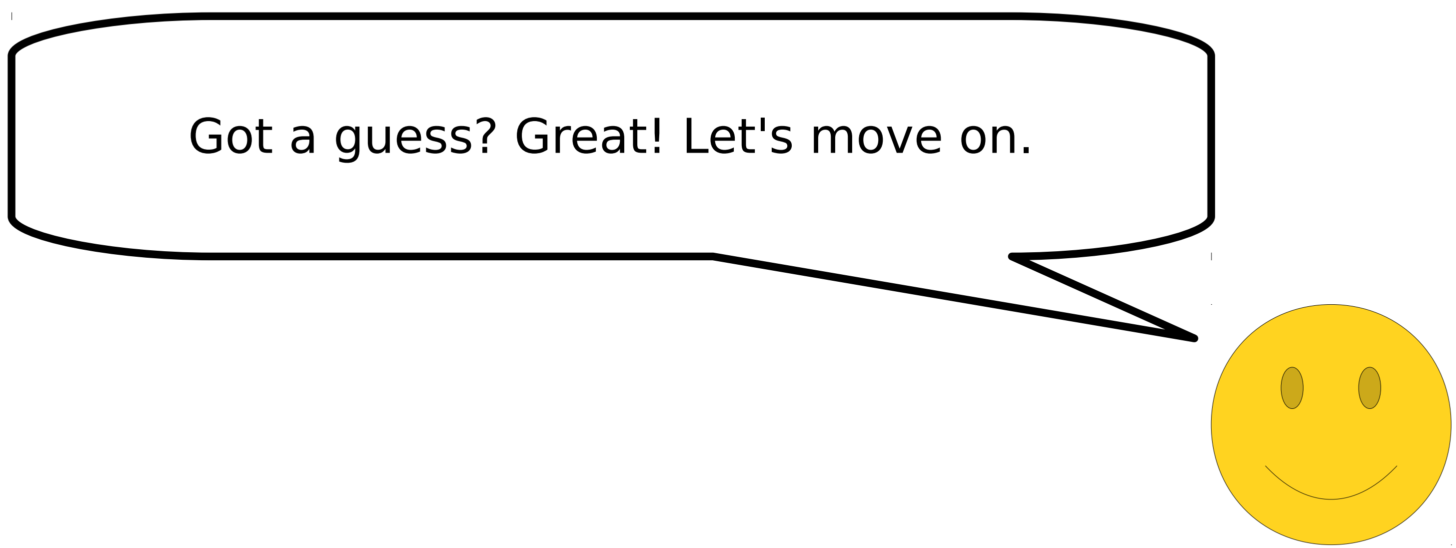


$$\{1\} \stackrel{?}{\in} \{ 1, \{2, 3\}, 4 \}$$

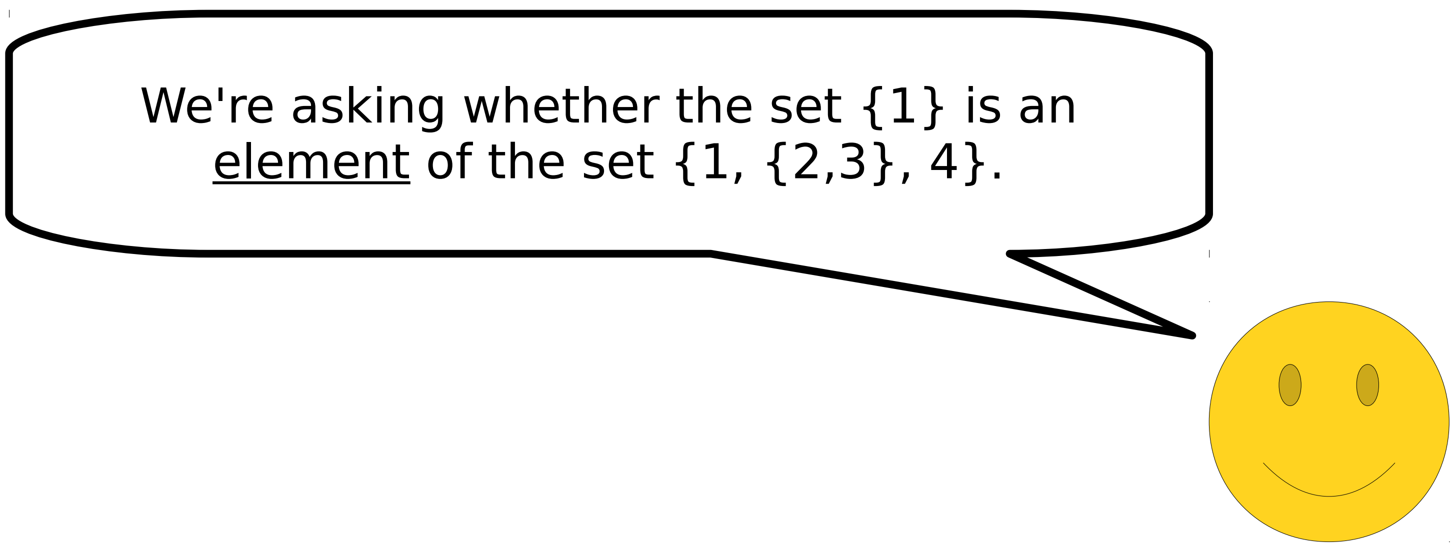
Is this new statement true? As before, take a minute to think it over.



$$\{1\} \overset{?}{\in} \{\ 1,\ \{2,\ 3\},\ 4\ \}$$

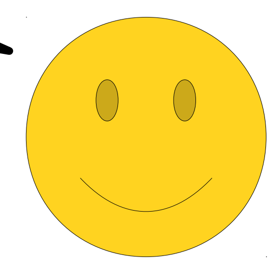


$$\{1\} \stackrel{?}{\in} \{\, 1,\ \{2,\ 3\},\ 4\ \}$$



$$\{1\} \overset{?}{\in} \{\, 1,\ \{2,\ 3\},\ 4 \,\}$$

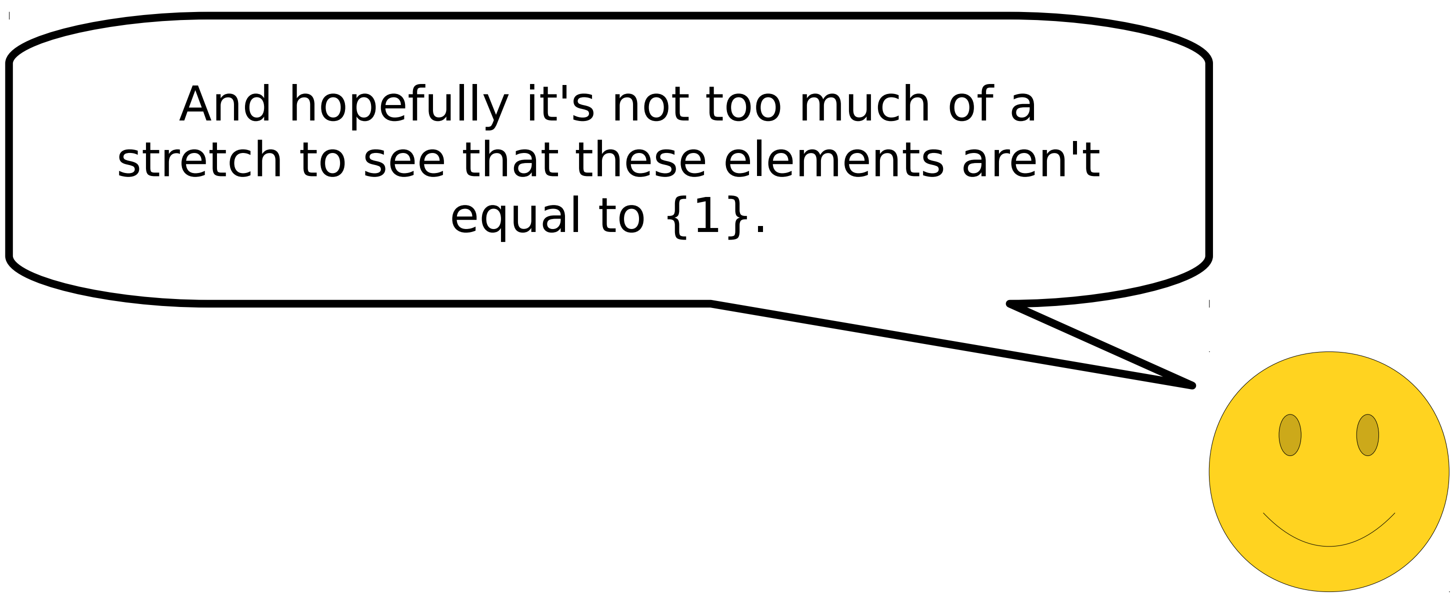
That means that we need to see if anything in the set {1, {2,3}, 4} is equal to the set {1}.



$$\{1\} \overset{?}{\in} \{\, 1,\ \{2,\ 3\},\ 4 \,\}$$

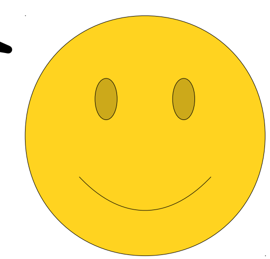
Using our reasoning from before, we know that 1 and {1} aren't the same thing, so this element isn't the set {1}.

$$\{1\} \overset{?}{\in} \{ 1, \{2, 3\}, 4 \}$$

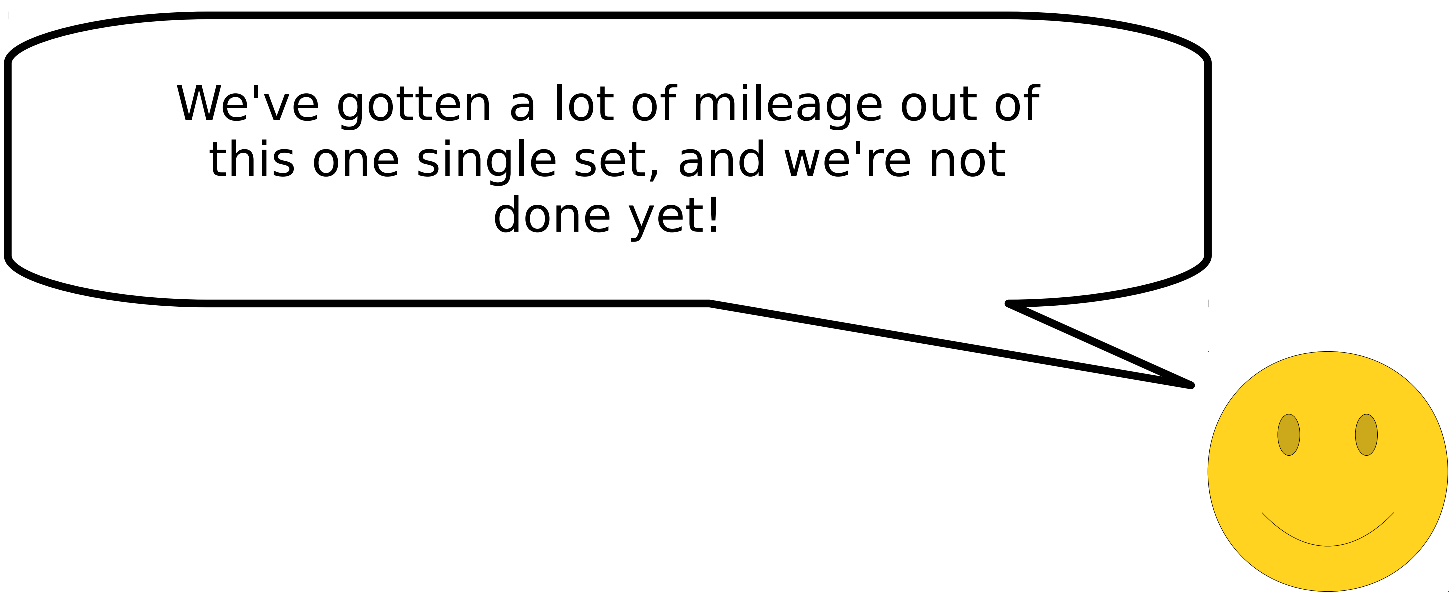


$$\{1\} \notin \{\ 1,\ \{2,\ 3\},\ 4\ \}$$

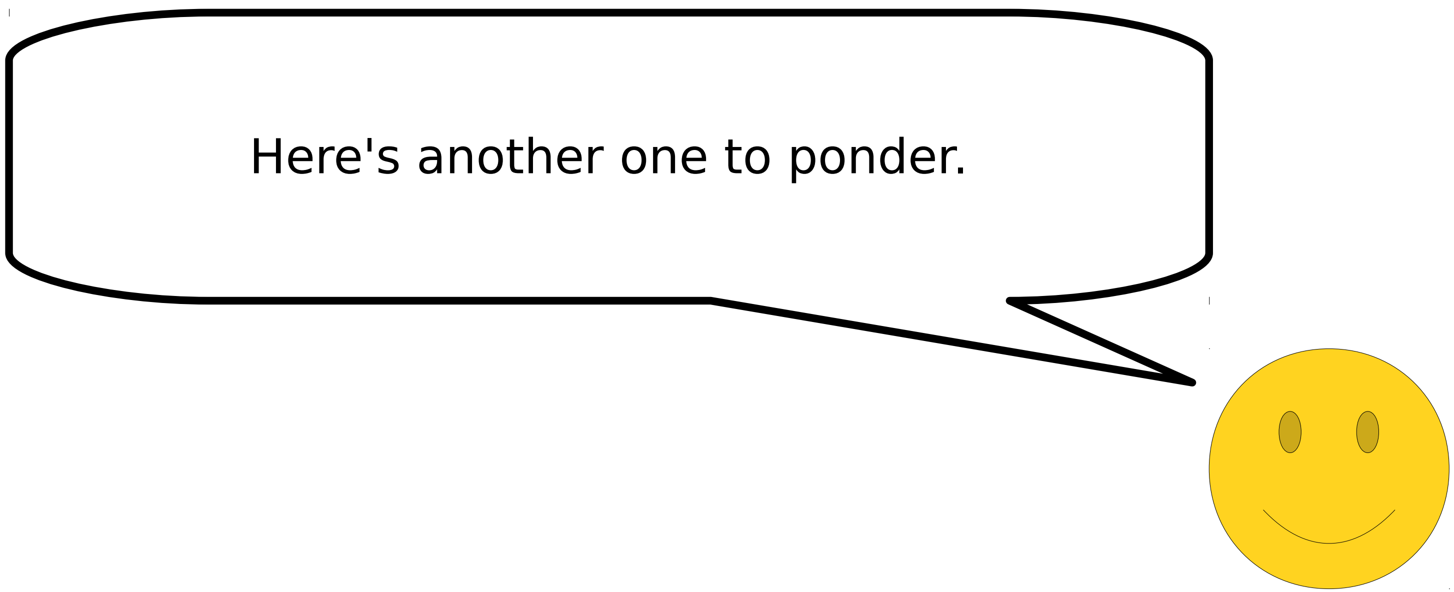
So overall we see that nothing in the set on the right is equal to {1}, so {1} is <u>not</u> an element of {1, {2,3}, 4}.



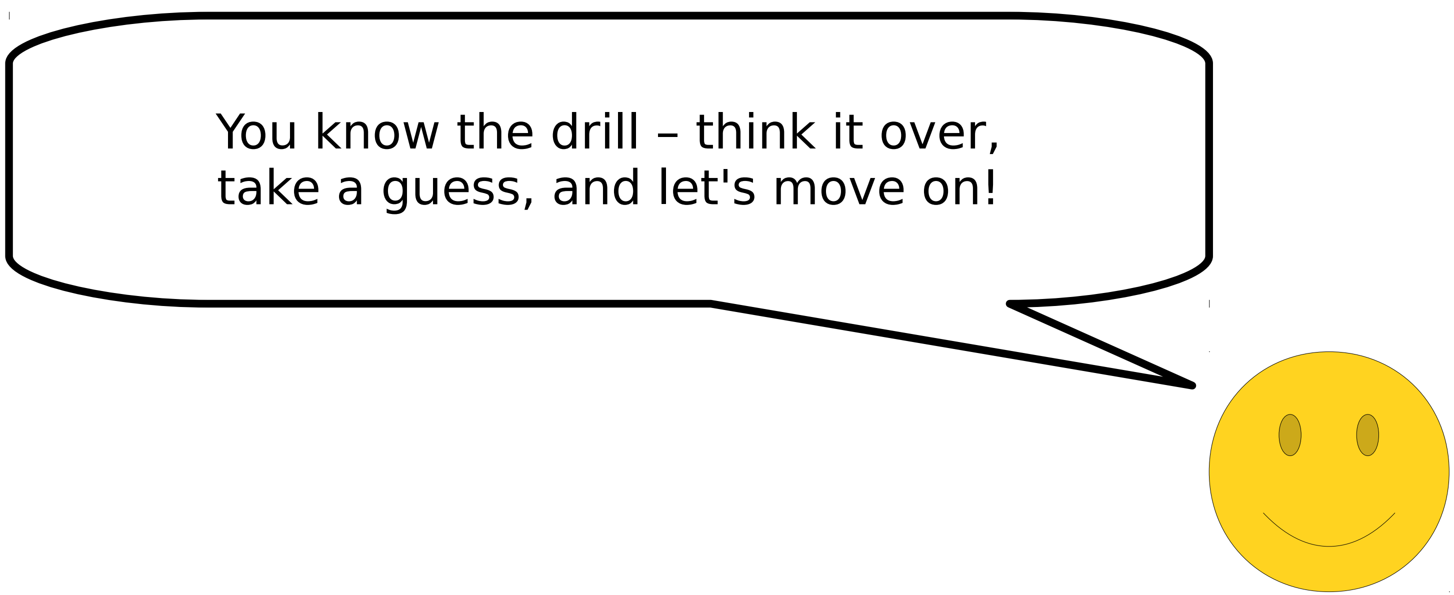
$$\{\ 1,\ \{2,\ 3\},\ 4\ \}$$


We've gotten a lot of mileage out of this one single set, and we're not done yet!

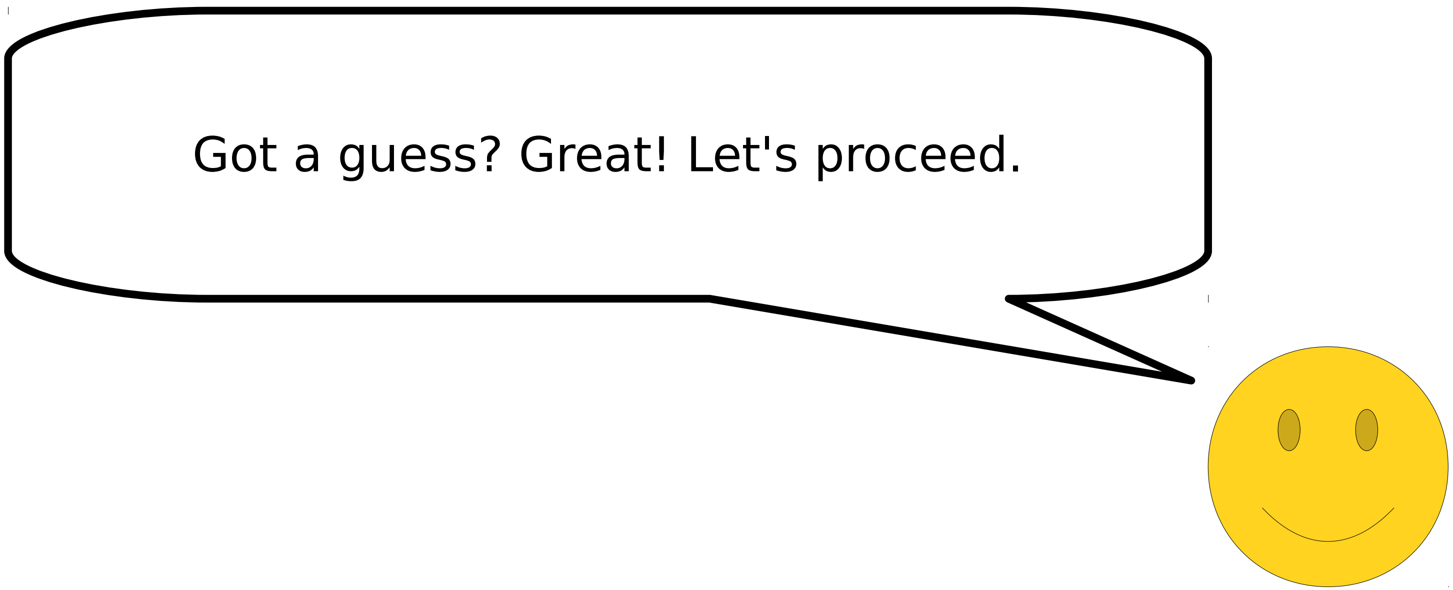
$$\{2,\ 3\} \overset{?}{\subseteq} \{\ 1,\ \{2,\ 3\},\ 4\ \}$$



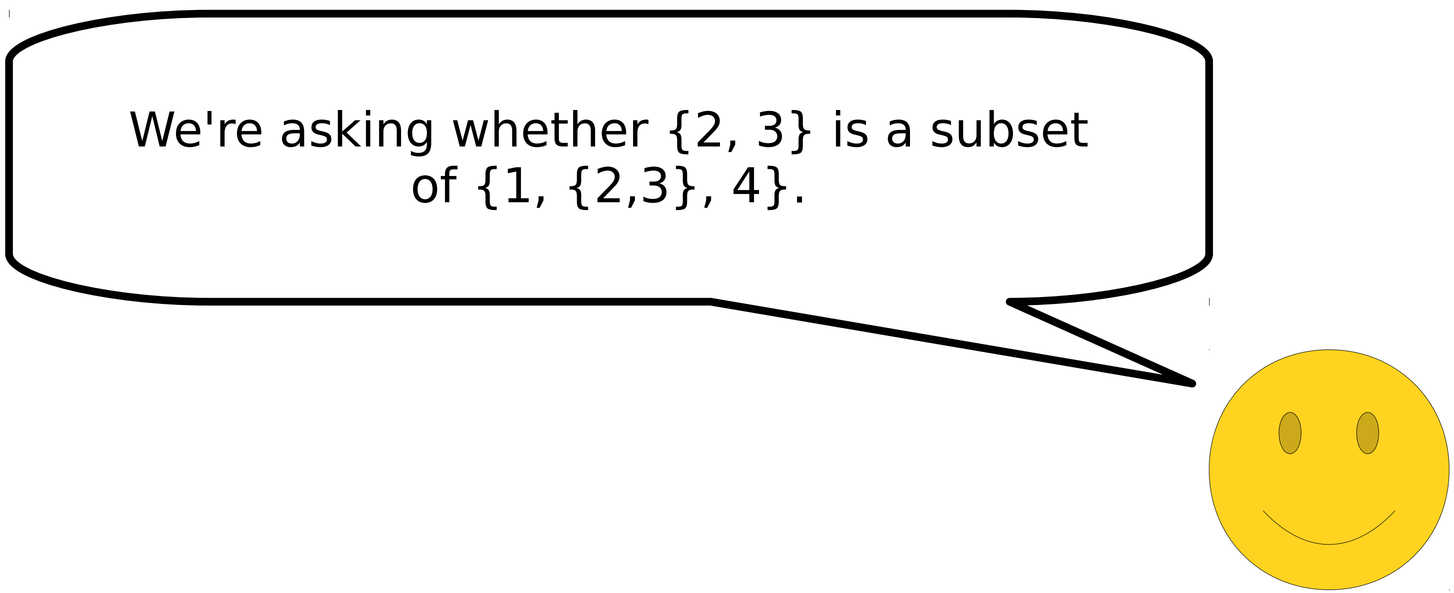
$$\{2, 3\} \overset{?}{\subseteq} \{ 1, \{2, 3\}, 4 \}$$



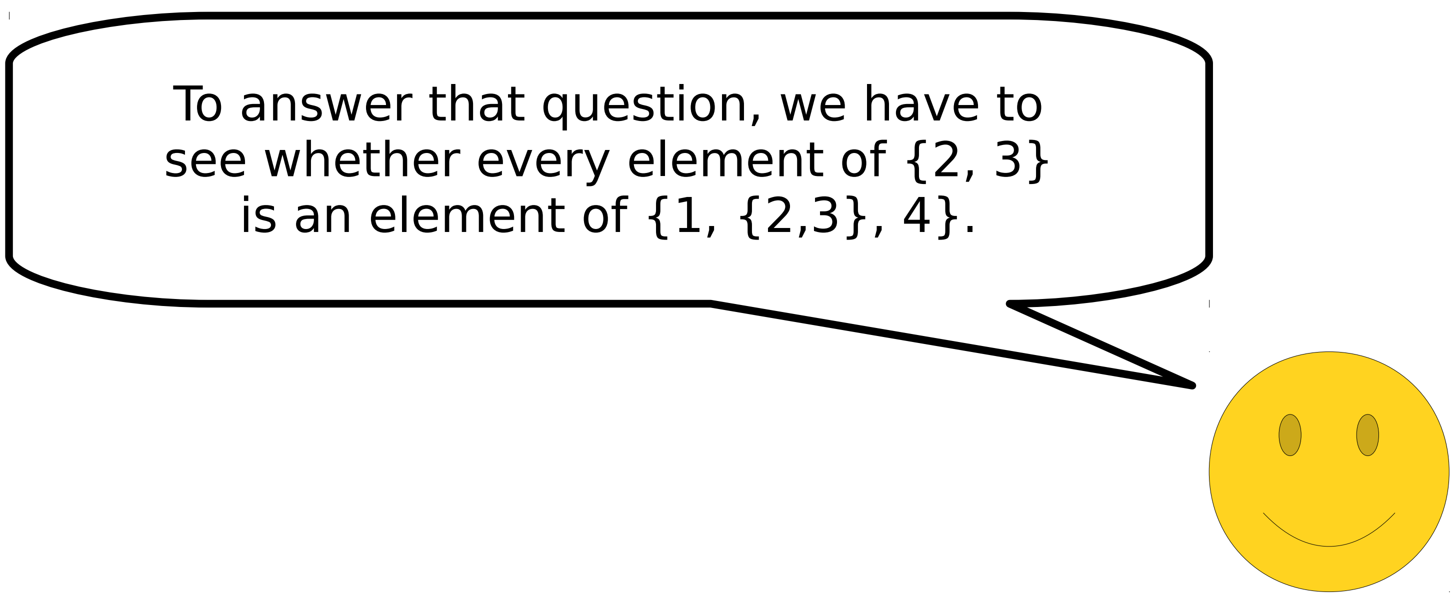
$$\{2, 3\} \stackrel{?}{\subseteq} \{ 1, \{2, 3\}, 4 \}$$



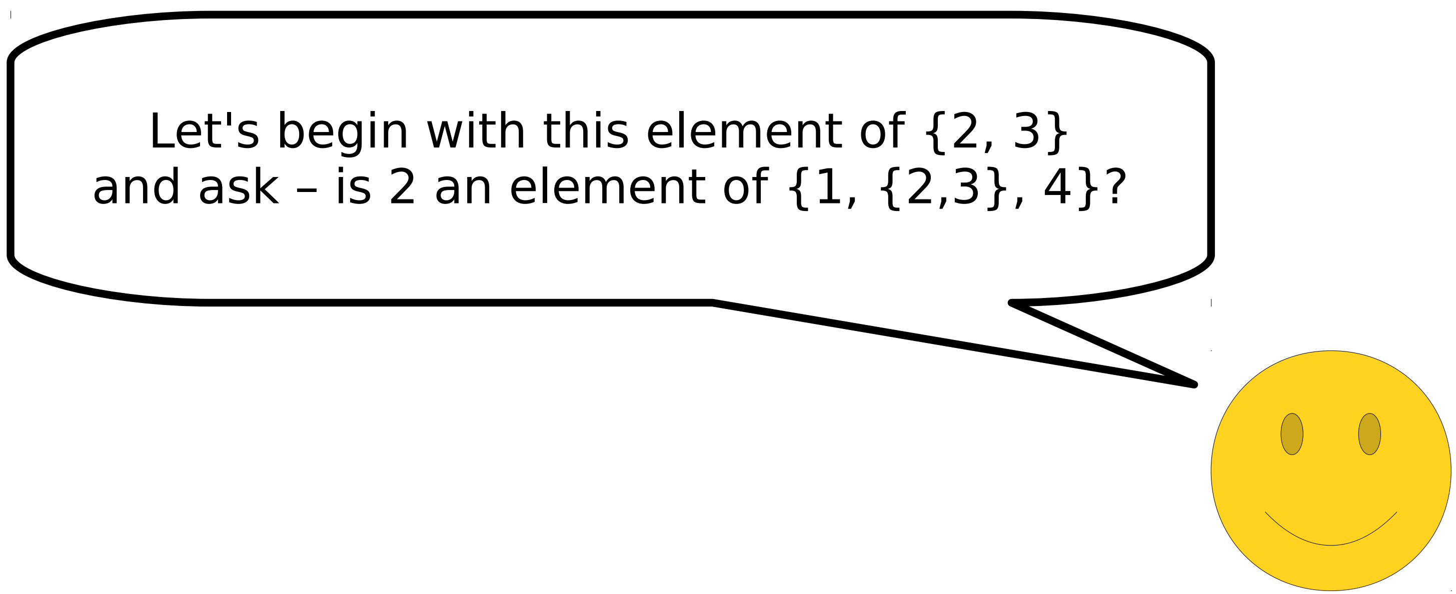
$$\{2, 3\} \overset{?}{\subseteq} \{ 1, \{2, 3\}, 4 \}$$



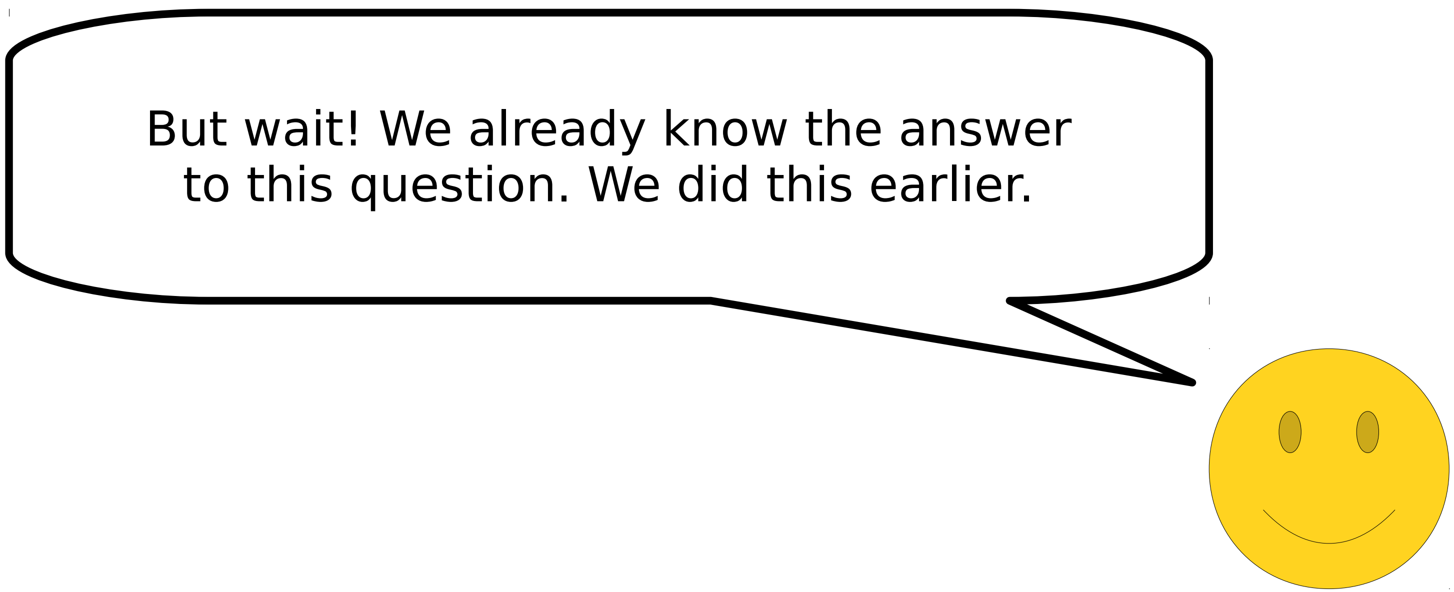
$$\{2,\ 3\} \overset{?}{\subseteq} \{\ 1,\ \{2,\ 3\},\ 4\ \}$$



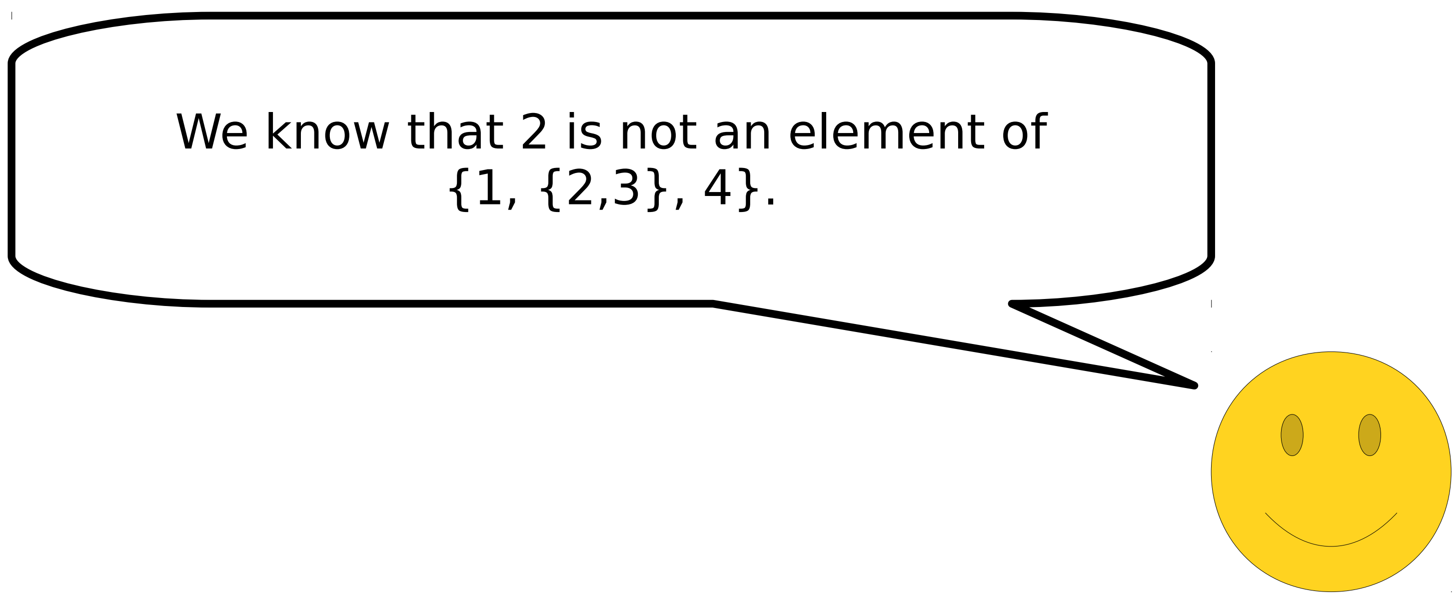
$$\{2,\ 3\} \overset{?}{\subseteq} \{\ 1,\ \{2,\ 3\},\ 4\ \}$$



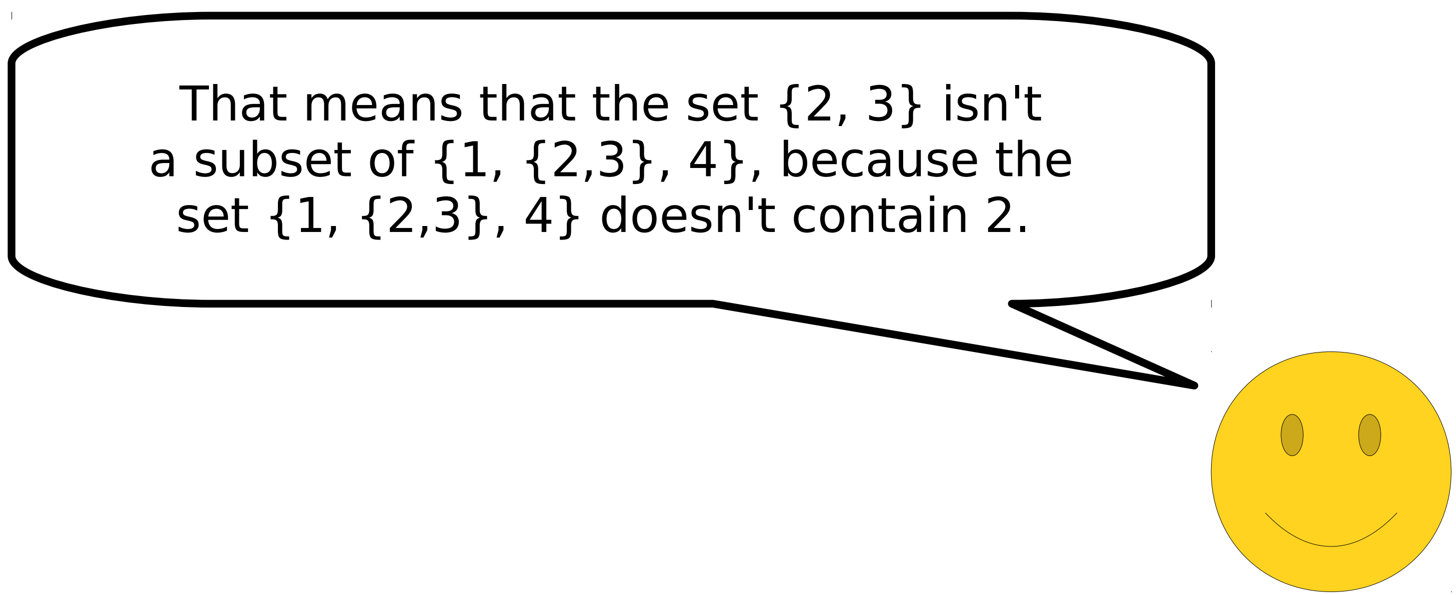
$$\{2,\ 3\} \stackrel{?}{\subseteq} \{\ 1,\ \{2,\ 3\},\ 4\ \}$$



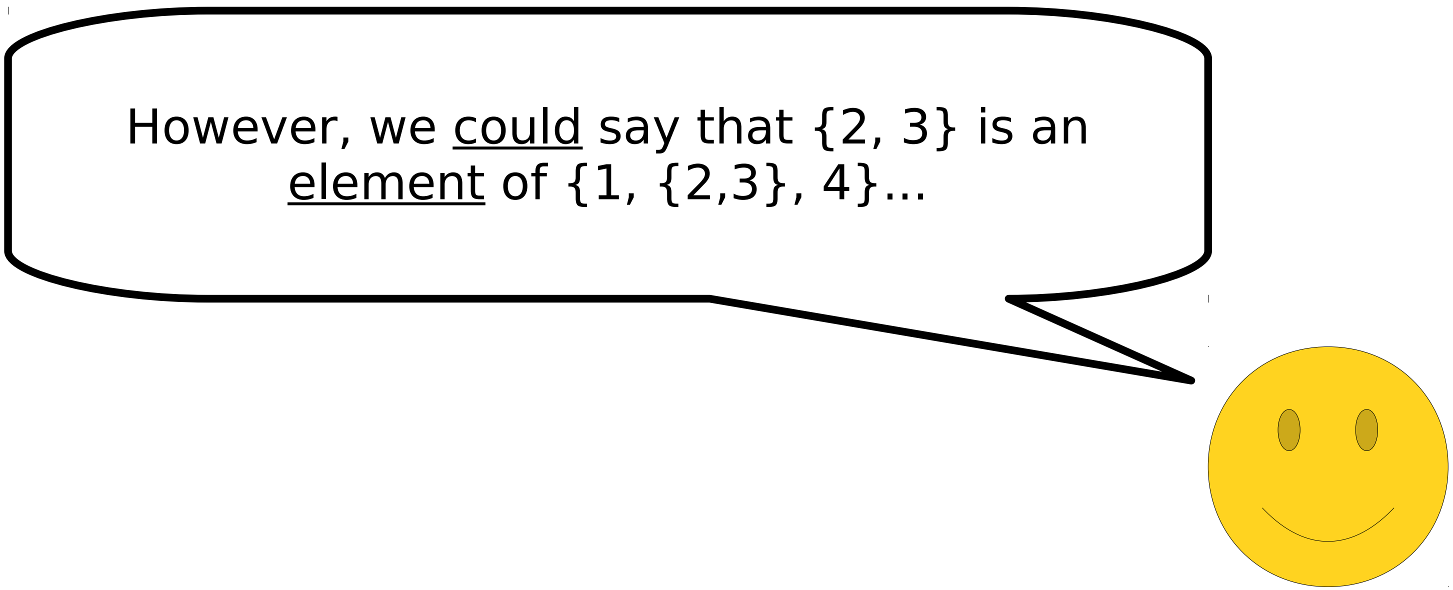
$$\{2,\ 3\} \overset{?}{\subseteq} \{\ 1,\ \{2,\ 3\},\ 4\ \}$$



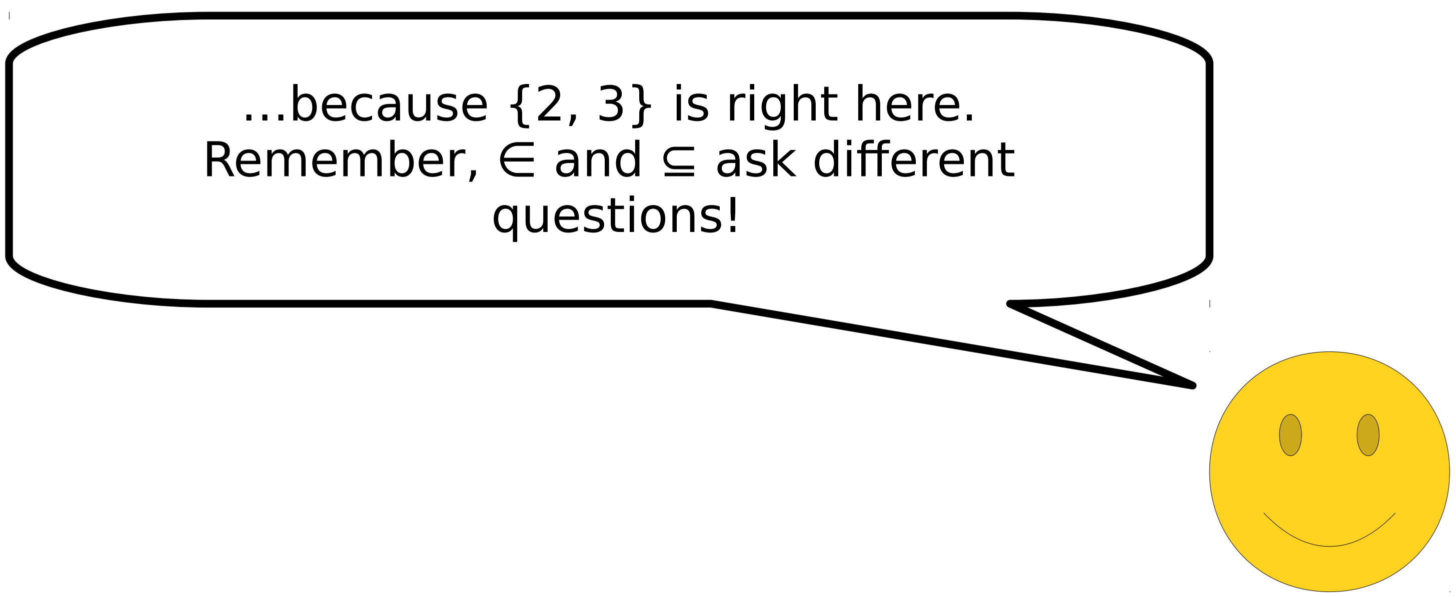
$$\{2,\ 3\} \not\subseteq \{\ 1,\ \{2,\ 3\},\ 4\ \}$$

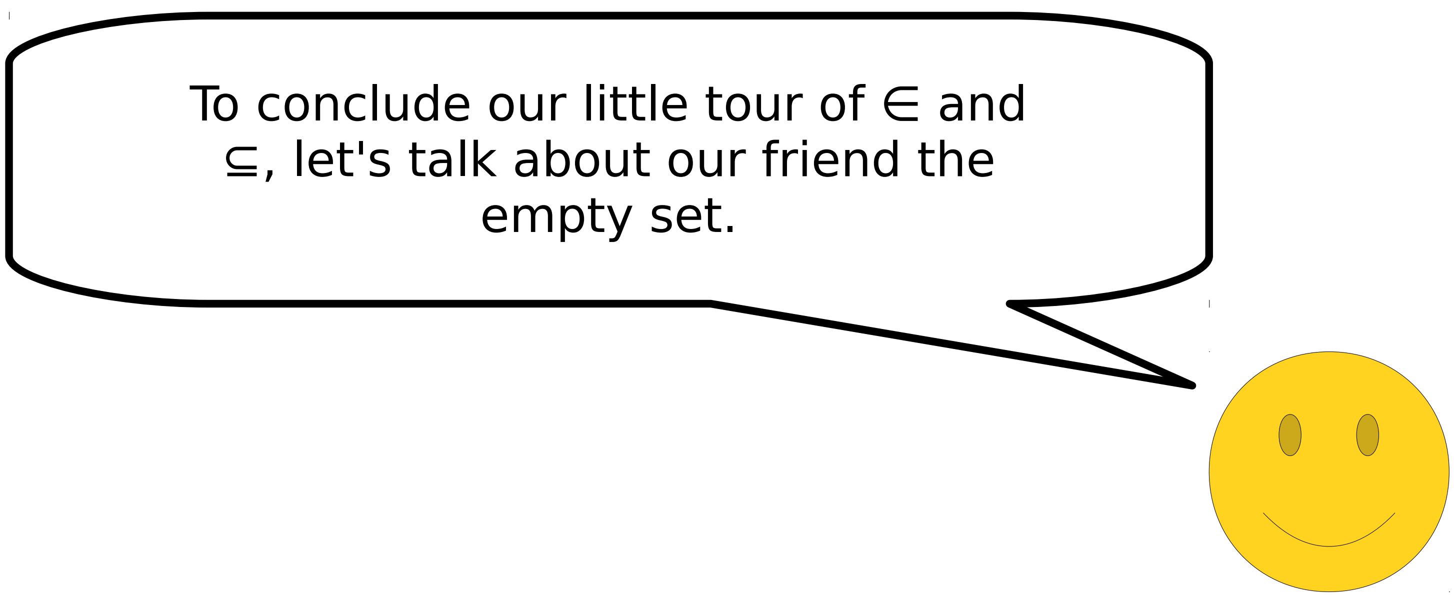


$$\{2,\ 3\} \in \{\ 1,\ \{2,\ 3\},\ 4\ \}$$

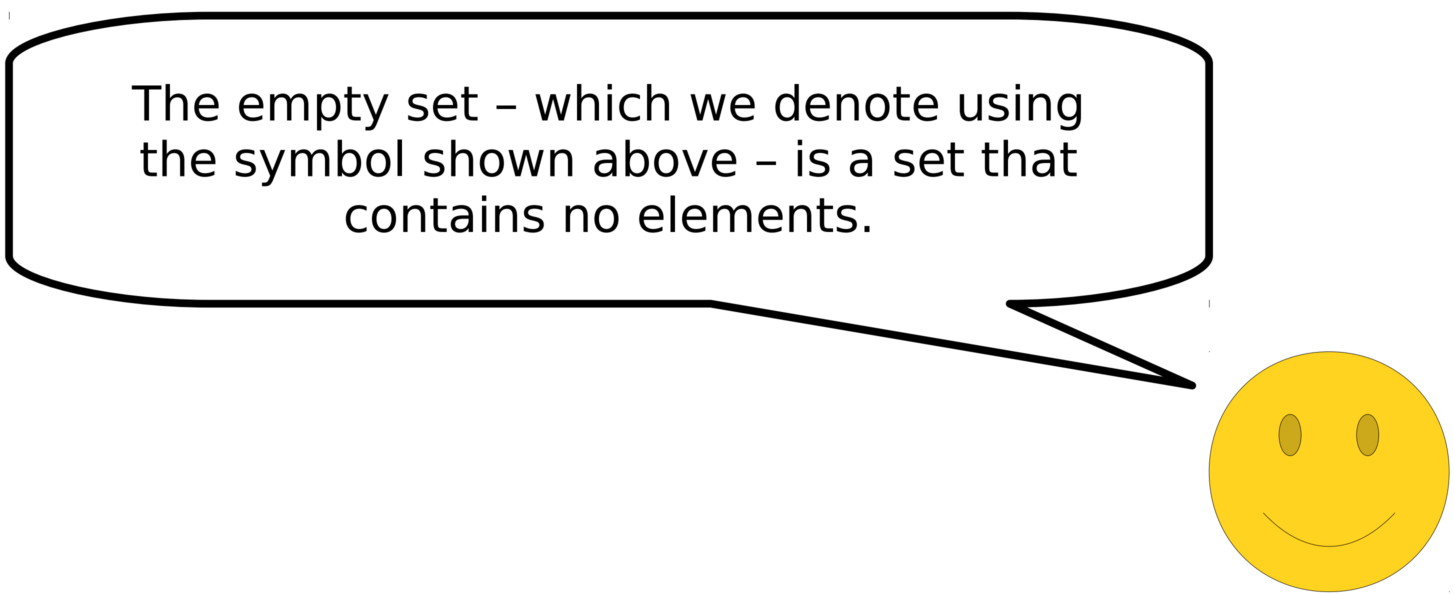


$$\{2, 3\} \in \{\ 1,\ \{2, 3\},\ 4\ \}$$


...because {2, 3} is right here.
Remember, ∈ and ⊆ ask different questions!

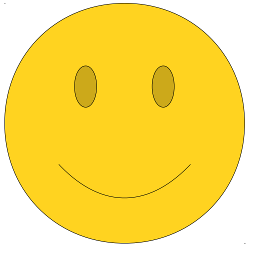

To conclude our little tour of ∈ and ⊆, let's talk about our friend the empty set.

$\emptyset$

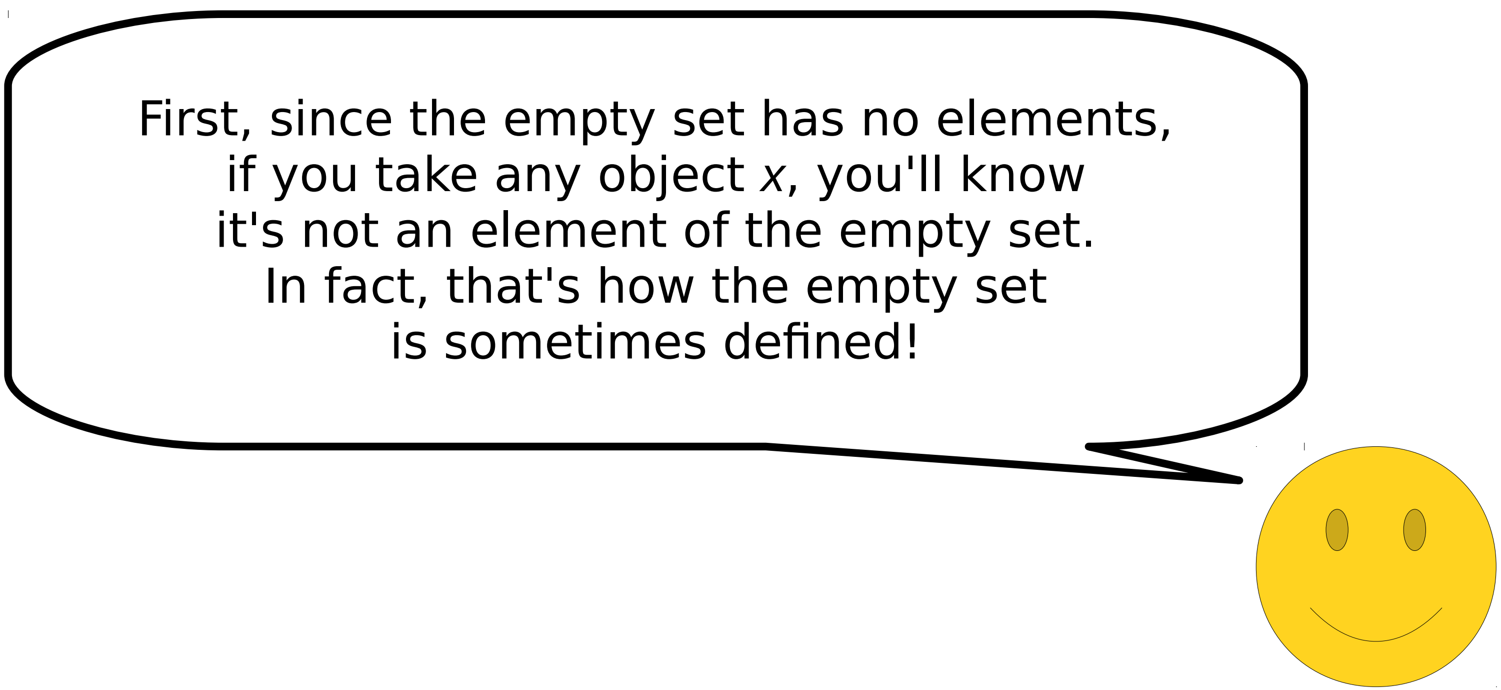

The empty set – which we denote using the symbol shown above – is a set that contains no elements.

$\varnothing$

The empty set has a few nice properties
that are worth keeping in mind.



$$x \notin \emptyset$$

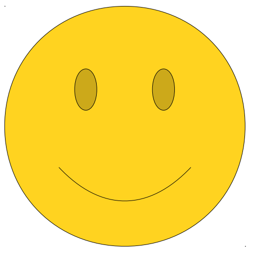


$$\emptyset \subseteq S$$

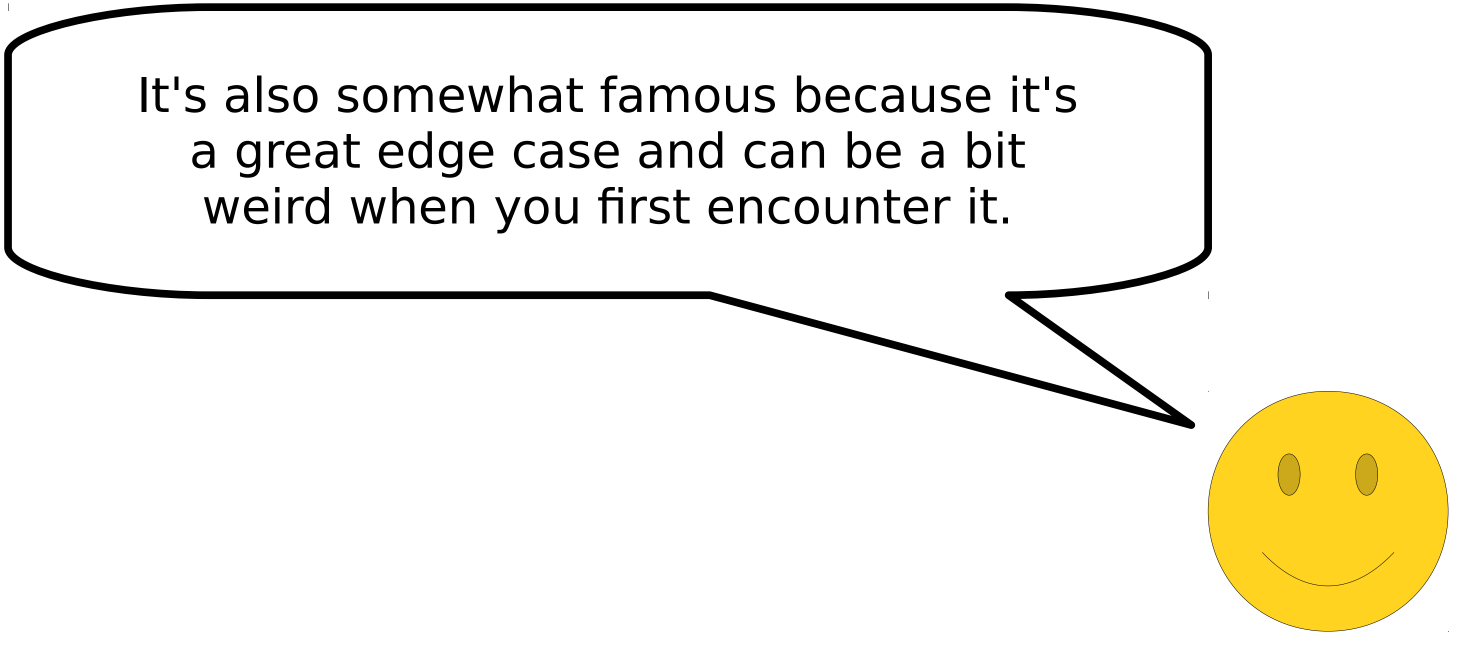
Second, as mentioned in class, we mentioned that the empty set is a subset of <u>any</u> set S. This is a consequence of <u>vacuous truth</u>.

$\emptyset$

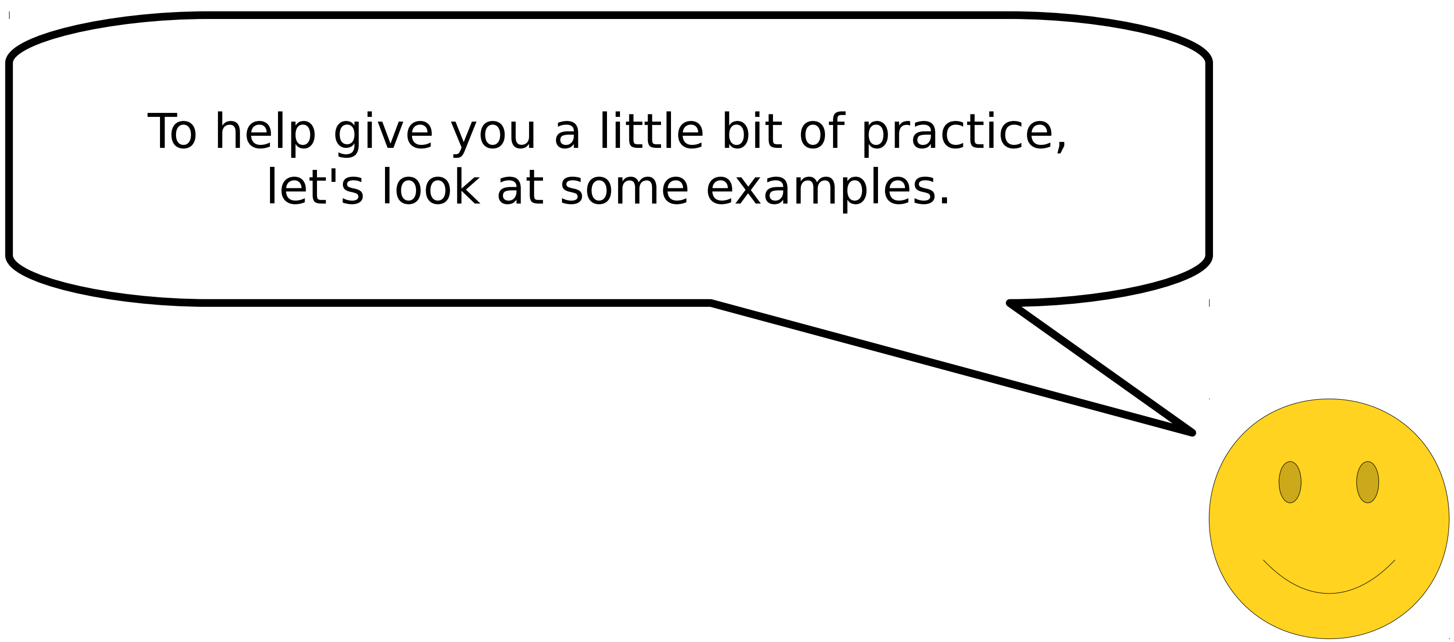
Although the empty set is probably most famous because of these two properties, it's just a set like any other and it obeys all the regular rules of the $\in$ and $\subseteq$ relations.



Ø

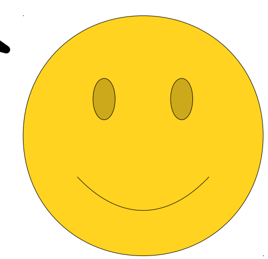

It's also somewhat famous because it's a great edge case and can be a bit weird when you first encounter it.

Ø

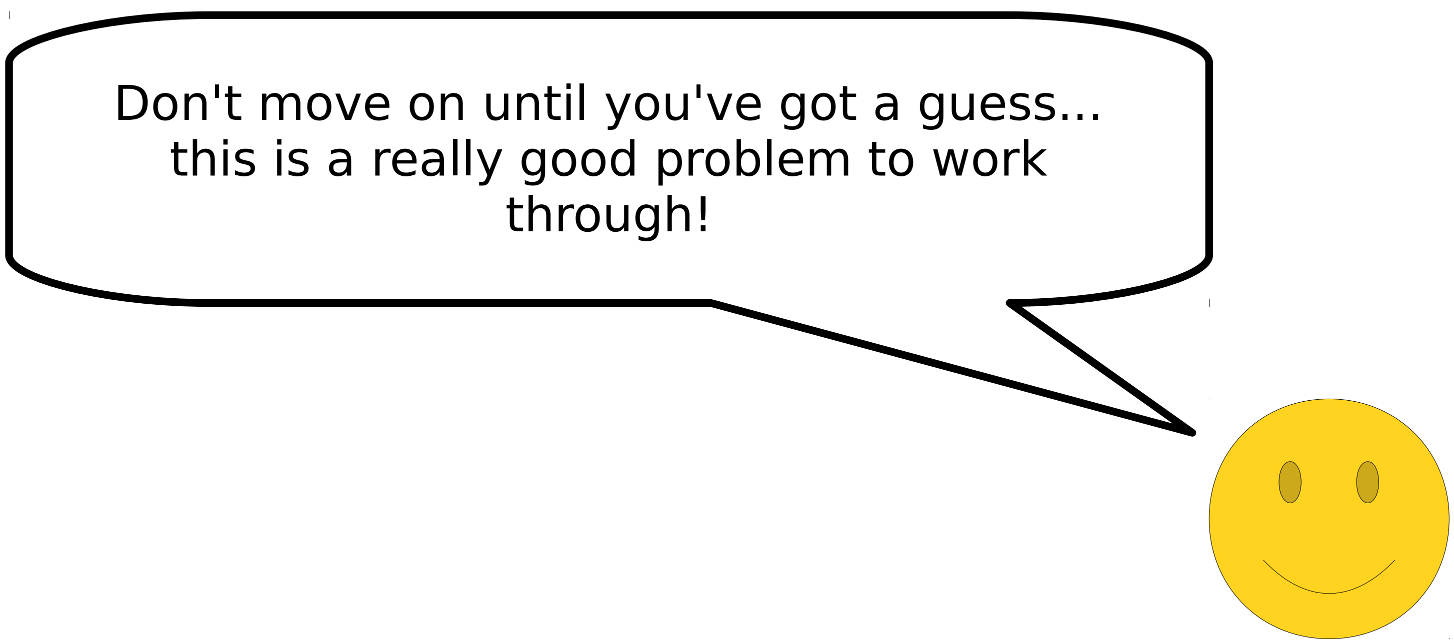

To help give you a little bit of practice,
let's look at some examples.

$$\emptyset \stackrel{?}{\in} \{ 1, \{2, 3\}, 4 \}$$

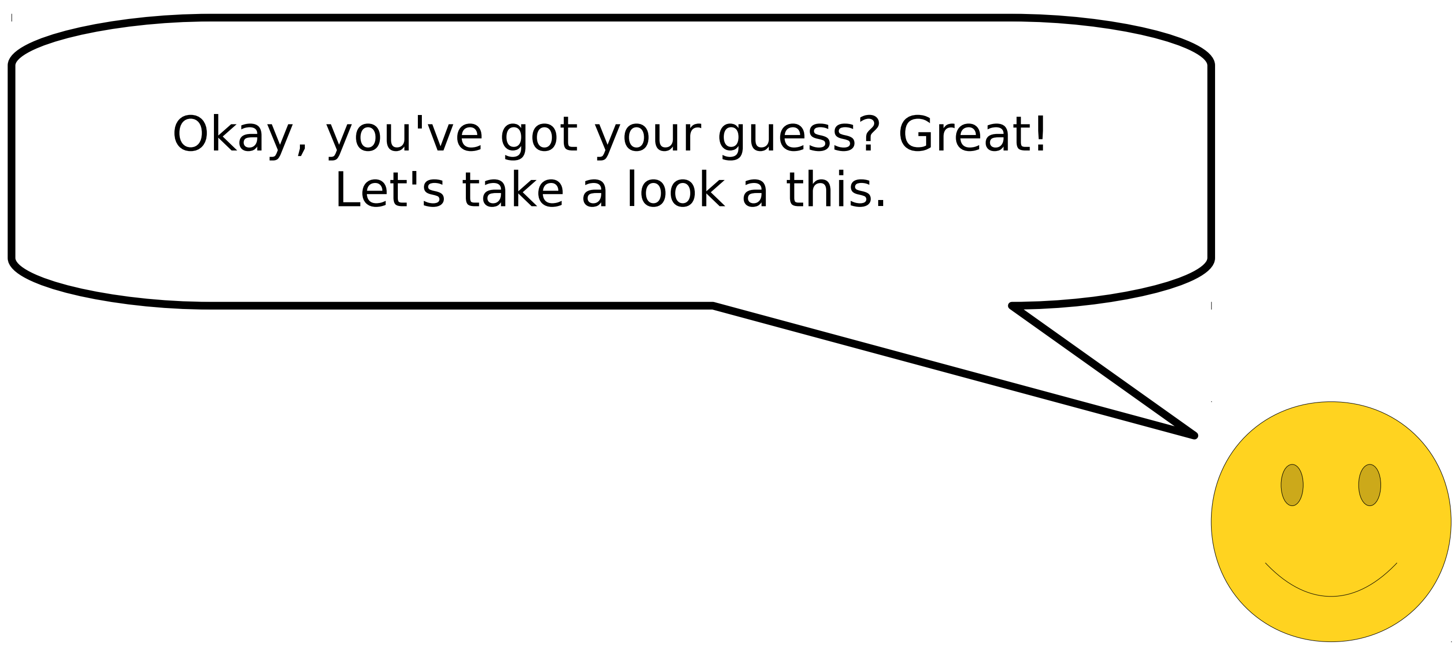
First, take a look at this statement. Is this statement true or false?



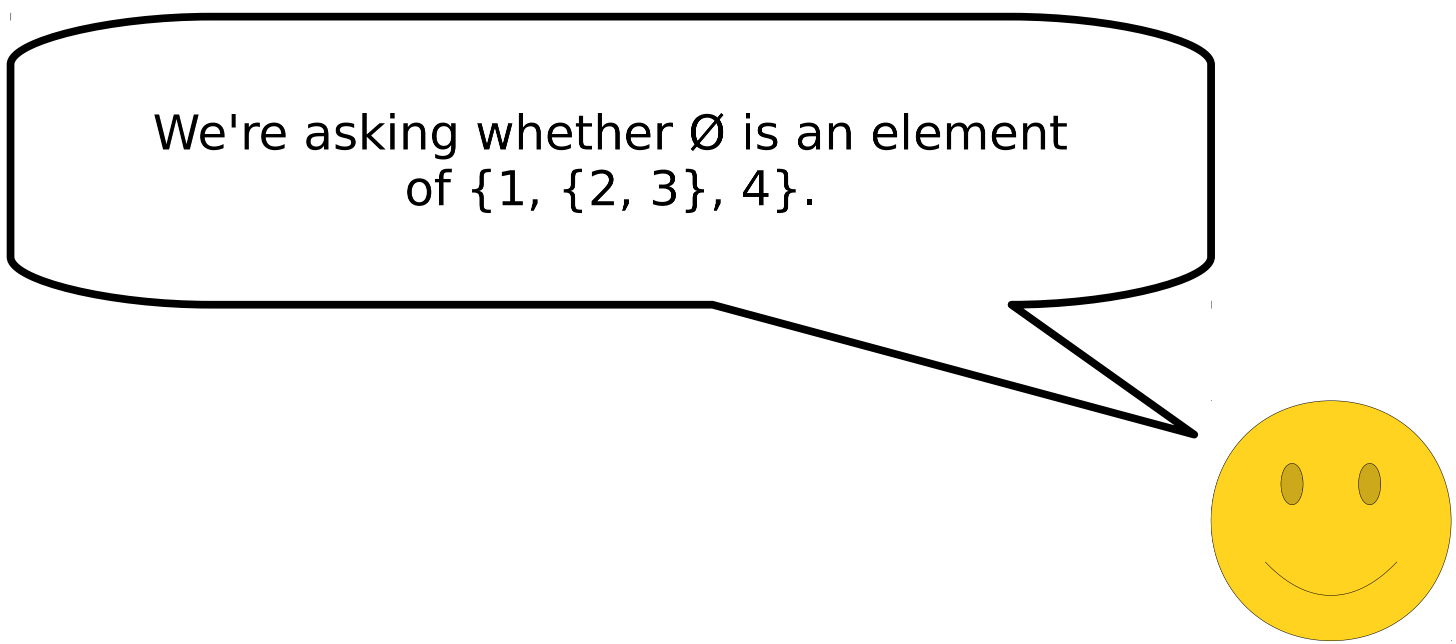
$$\emptyset \overset{?}{\in} \{\ 1,\ \{2,\ 3\},\ 4\ \}$$



$$\emptyset \overset{?}{\in} \{ 1, \{2, 3\}, 4 \}$$

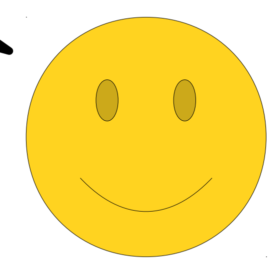


$$\emptyset \overset{?}{\in} \{\ 1,\ \{2,\ 3\},\ 4\ \}$$

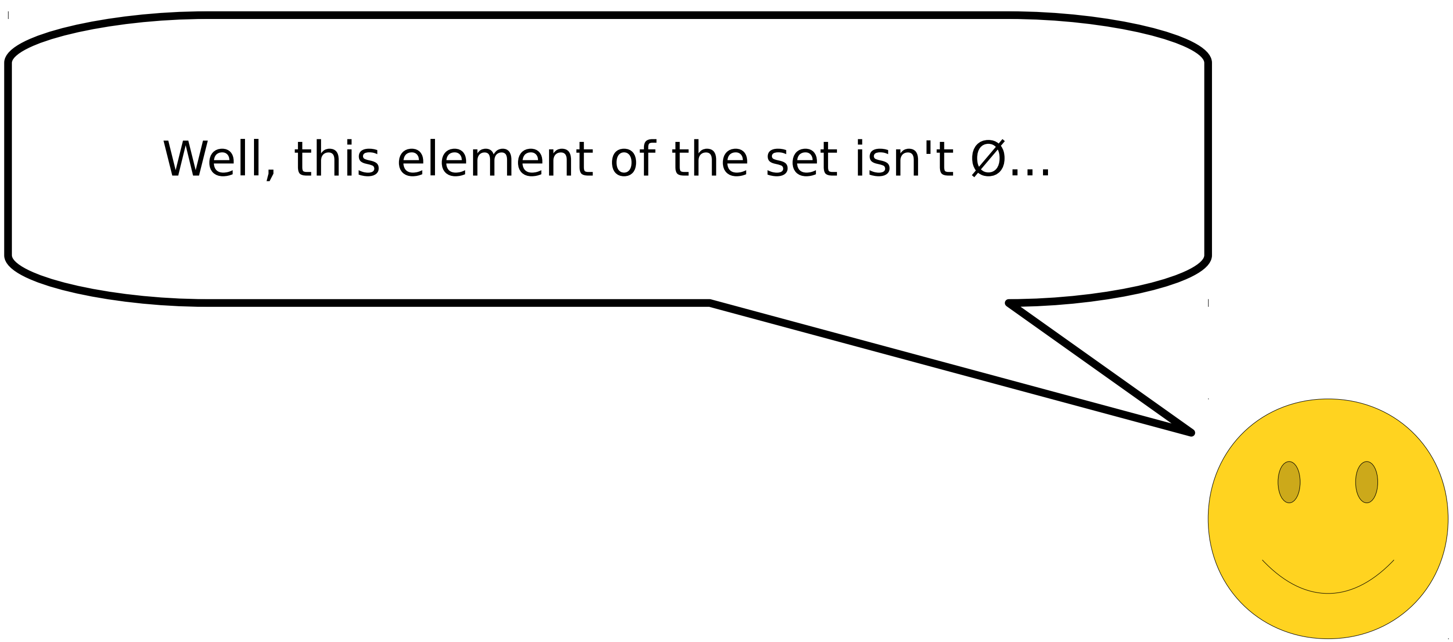


$$\emptyset \overset{?}{\in} \{\ 1,\ \{2,\ 3\},\ 4\ \}$$

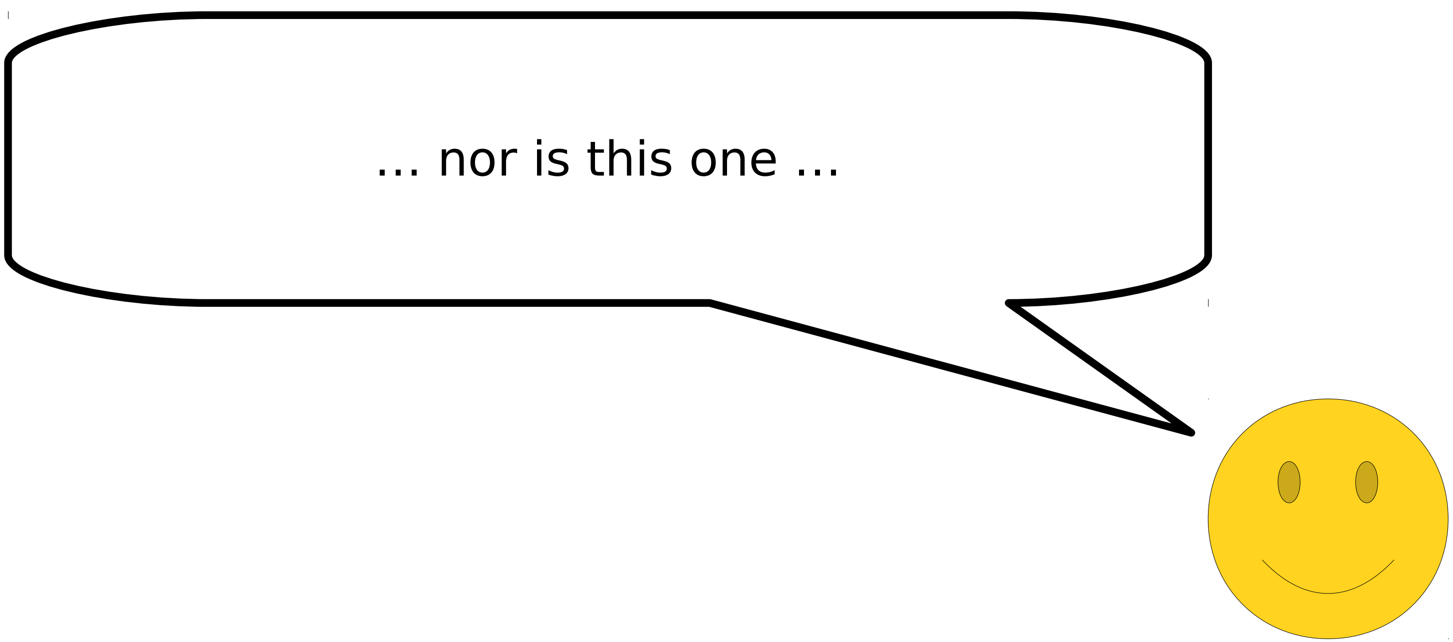
That means we need to see whether any of the elements of {1, {2,3}, 4} happen to be equal to Ø.



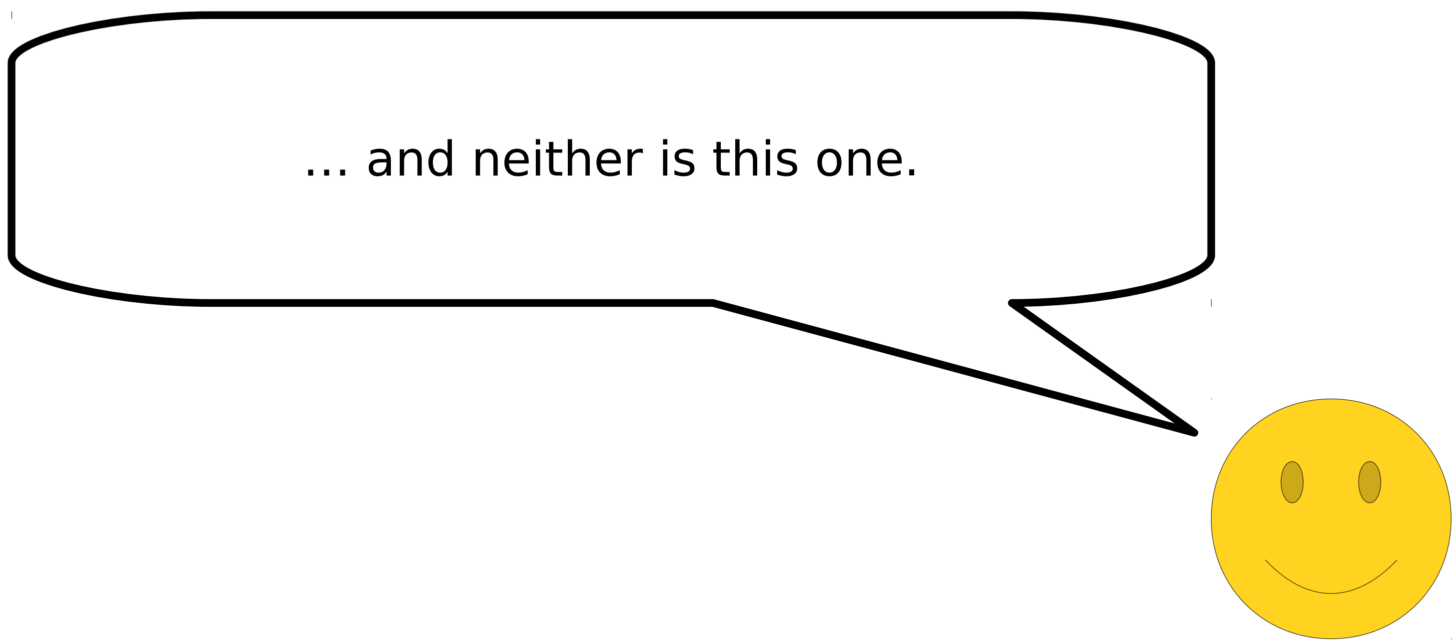
$$\emptyset \overset{?}{\in} \{\ 1, \{2, 3\}, 4\ \}$$



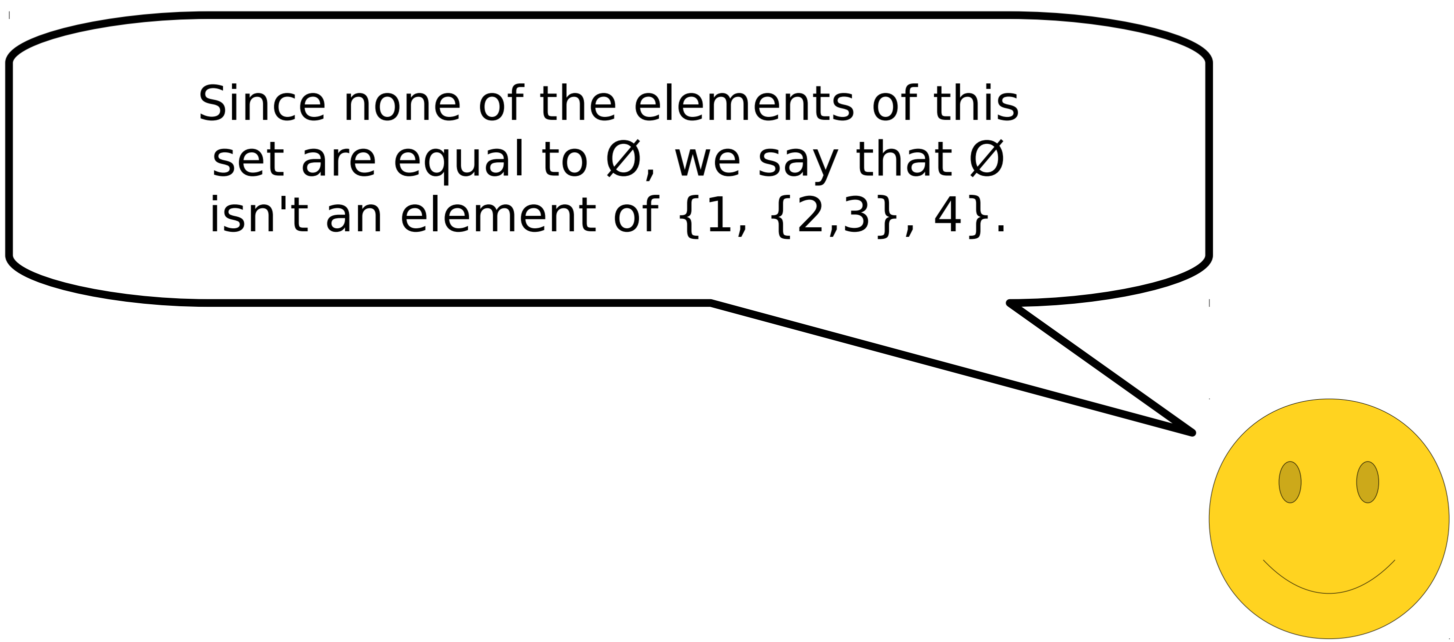
$$\emptyset \overset{?}{\in} \{\ 1,\ \{2,\ 3\},\ 4\ \}$$



$$\varnothing \overset{?}{\in} \{\ 1,\ \{2,\ 3\},\ 4\ \}$$

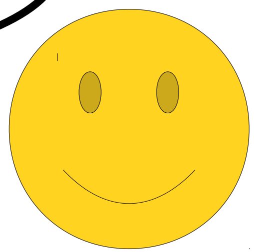


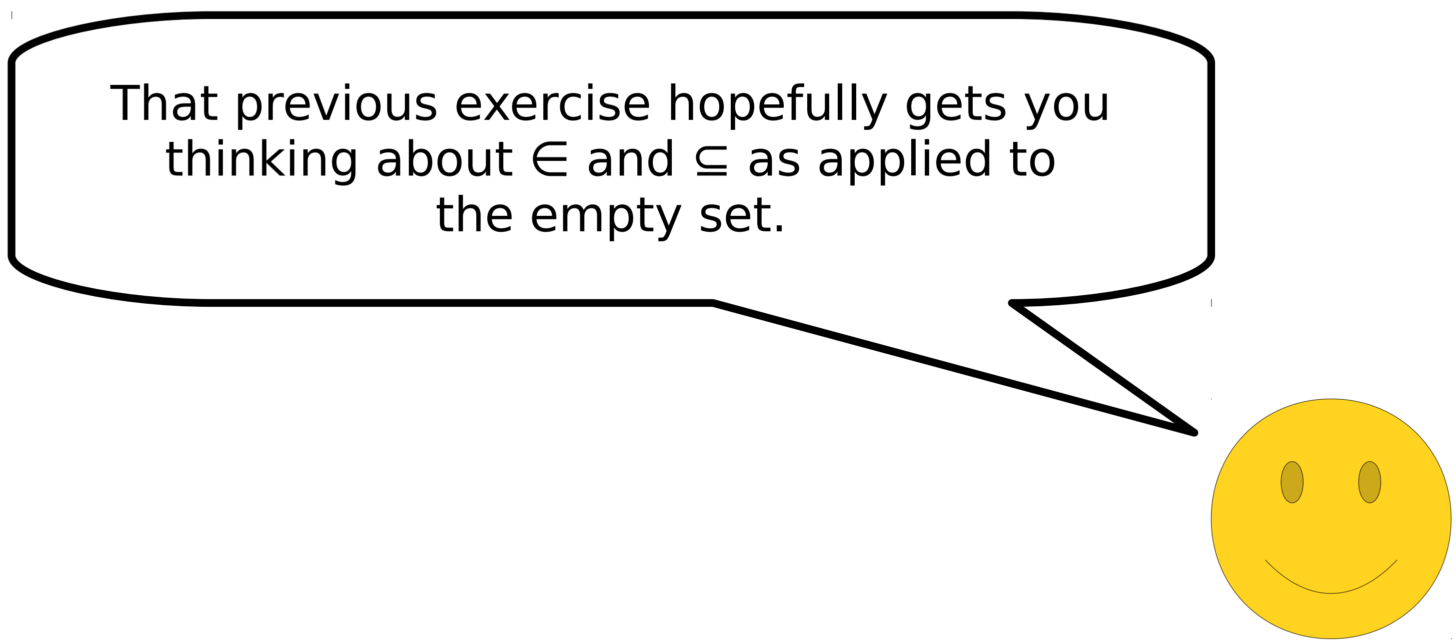
$$\emptyset \notin \{\ 1,\ \{2,\ 3\},\ 4\ \}$$



$$\varnothing \subseteq \{ 1, \{2, 3\}, 4 \}$$

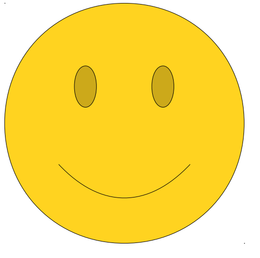
Now, we <u>could</u> correctly say that the empty set is a <u>subset</u> of {1, {2,3}, 4}, because it's a subset of every set. But, as you've seen before, ∈ and ⊆ represent different concepts, so that doesn't mean that Ø would be an element of this set.



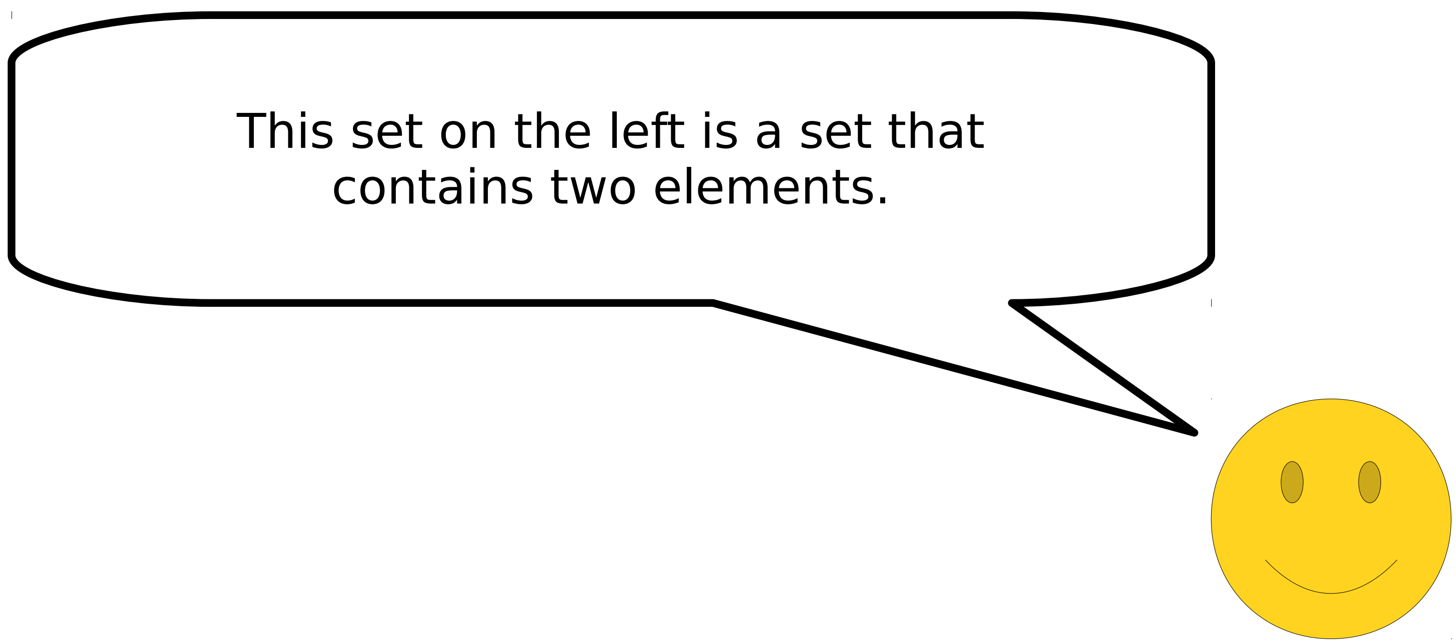

That previous exercise hopefully gets you thinking about ∈ and ⊆ as applied to the empty set.

$$\emptyset \overset{?}{\in} \{ \emptyset, 137 \}$$

Let's take a look at the above statement.
What exactly is going on here?

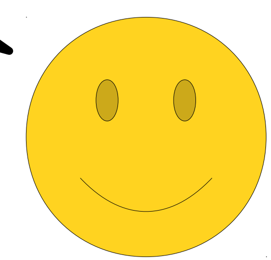


$$\emptyset \overset{?}{\in} \{ \emptyset, 137 \}$$

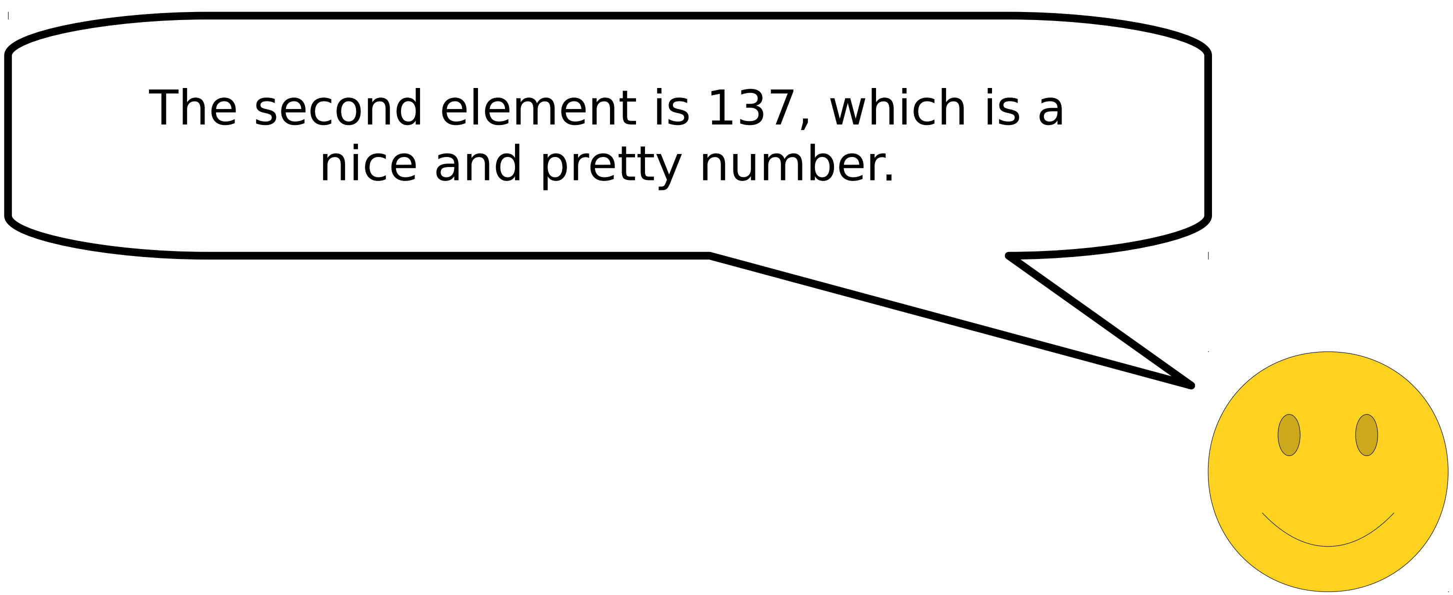


$$\varnothing \overset{?}{\in} \{ \varnothing, 137 \}$$

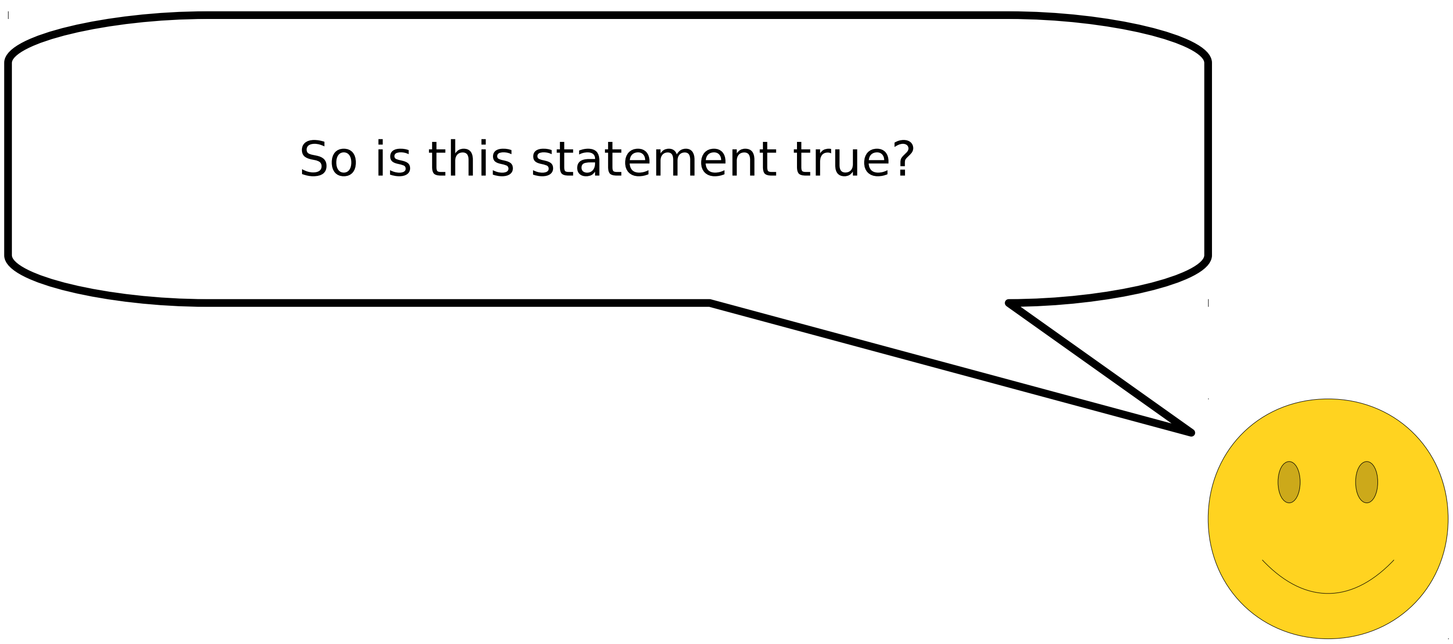
The first element is the empty set. You've seen examples of sets containing other sets before, and in this case this set just happens to contain Ø.



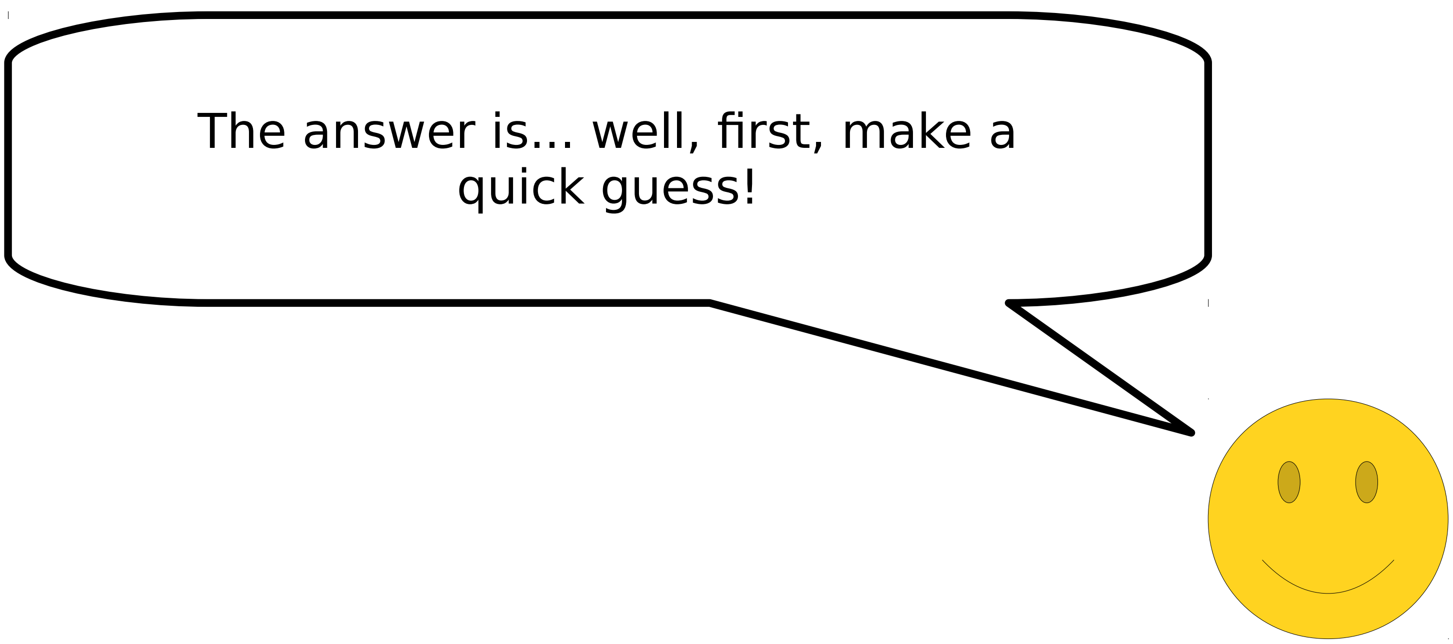
$$\emptyset \overset{?}{\in} \{ \emptyset, 137 \}$$



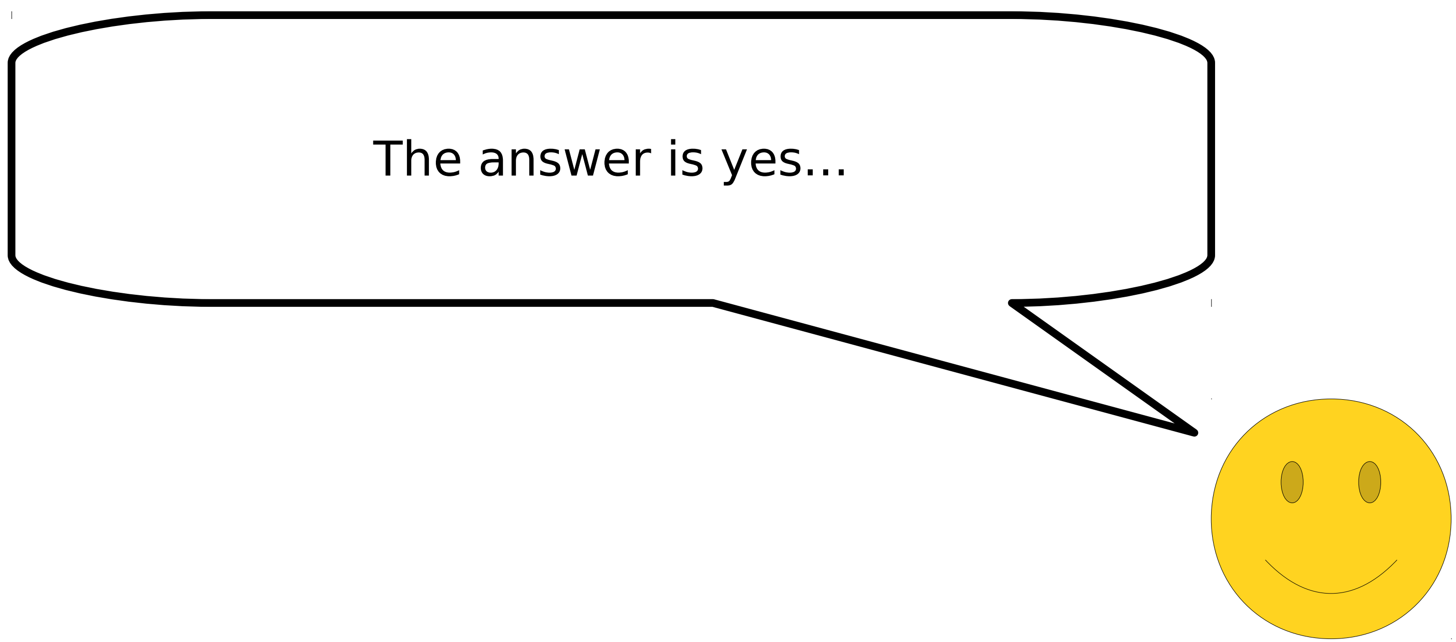
$$\emptyset \overset{?}{\in} \{ \emptyset, 137 \}$$



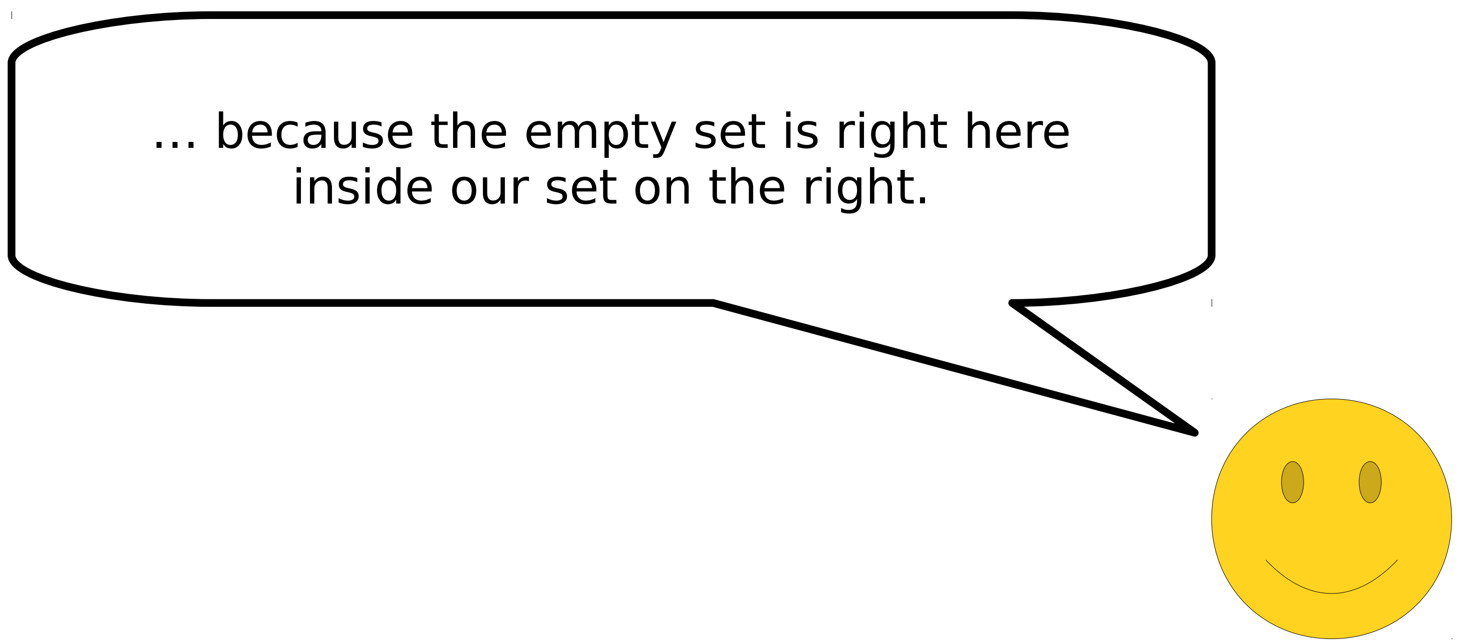
$$\emptyset \overset{?}{\in} \{ \emptyset, 137 \}$$



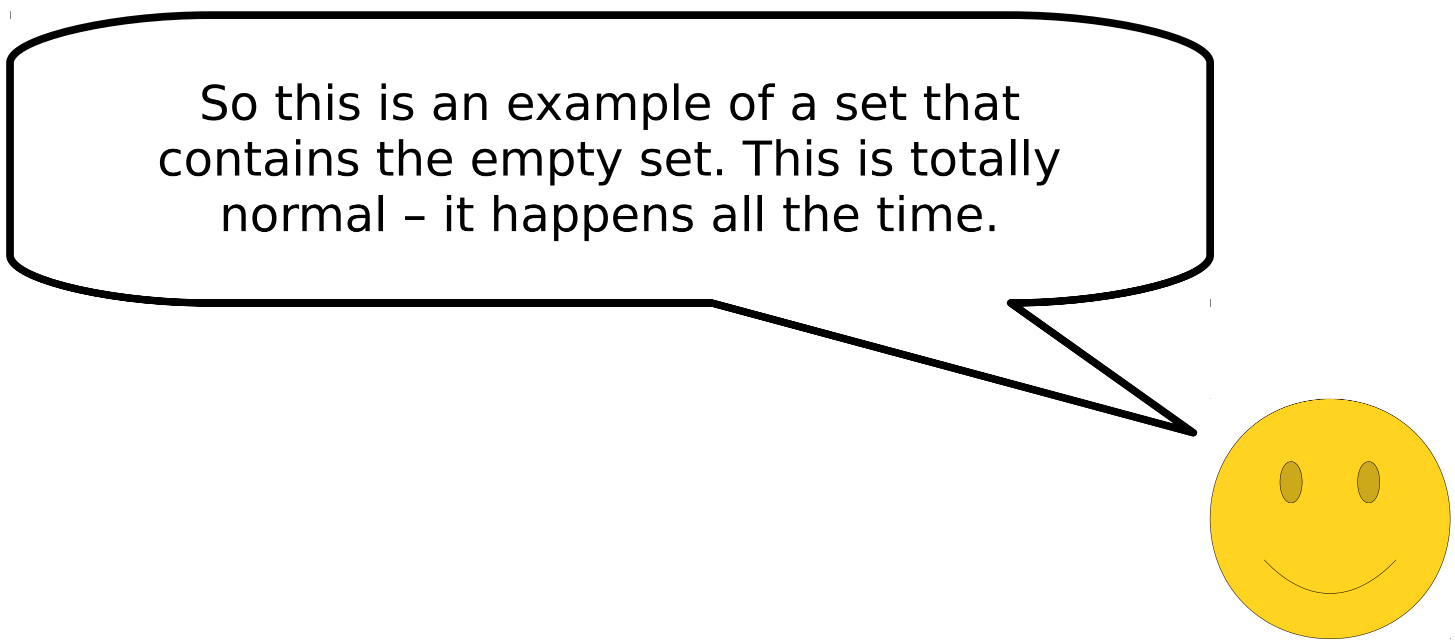
$$\emptyset \in \{\ \emptyset, 137\ \}$$



$$\varnothing \in \{\, \varnothing, 137 \,\}$$



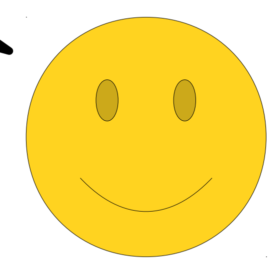
$$\emptyset \in \{ \emptyset, 137 \}$$



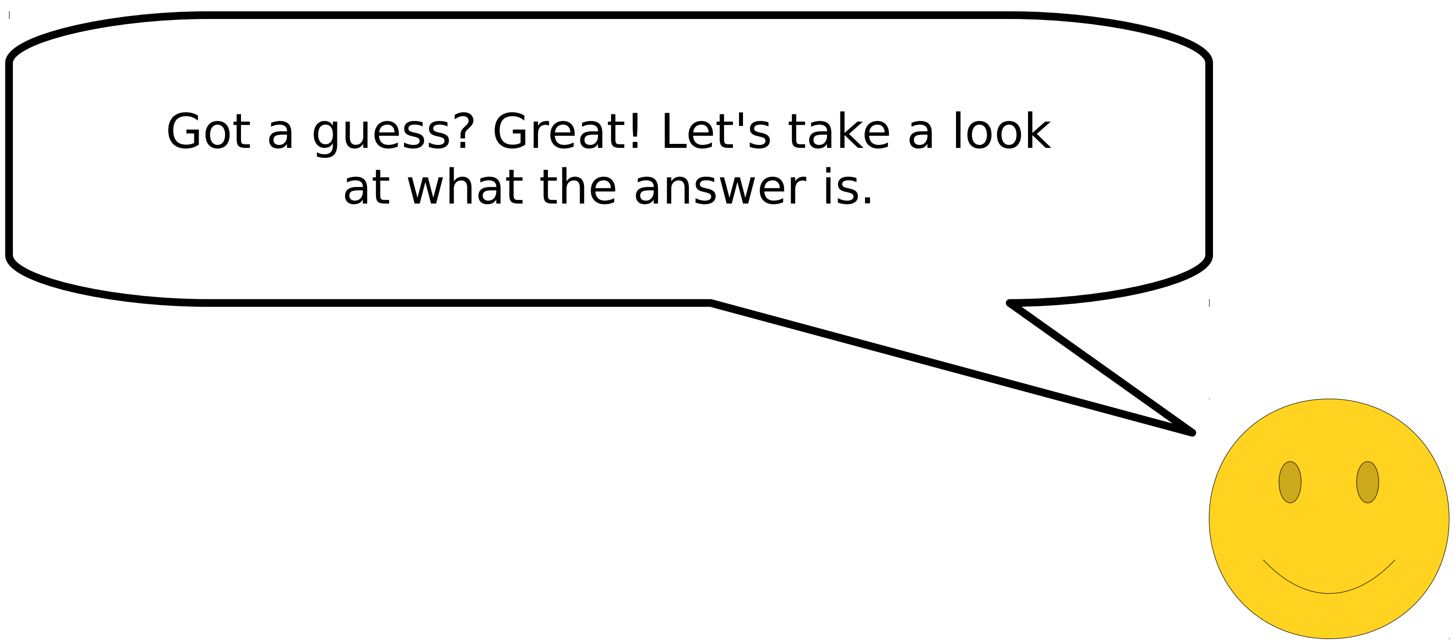

I've been talking for a while now, so let's do one final exercise and then call it a day.

$$\{\emptyset, 4\} \stackrel{?}{\subseteq} \{ 1, \{2, 3\}, 4 \}$$

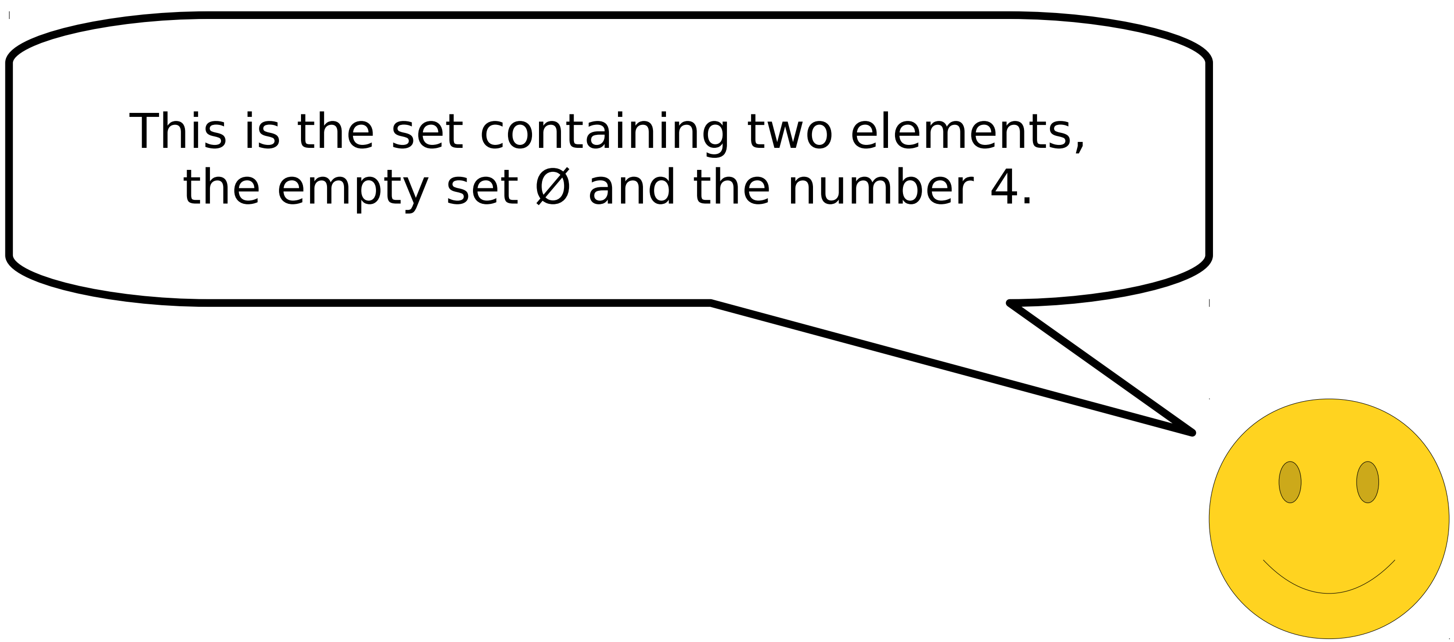
Take a minute to think about whether this statement is true or false.



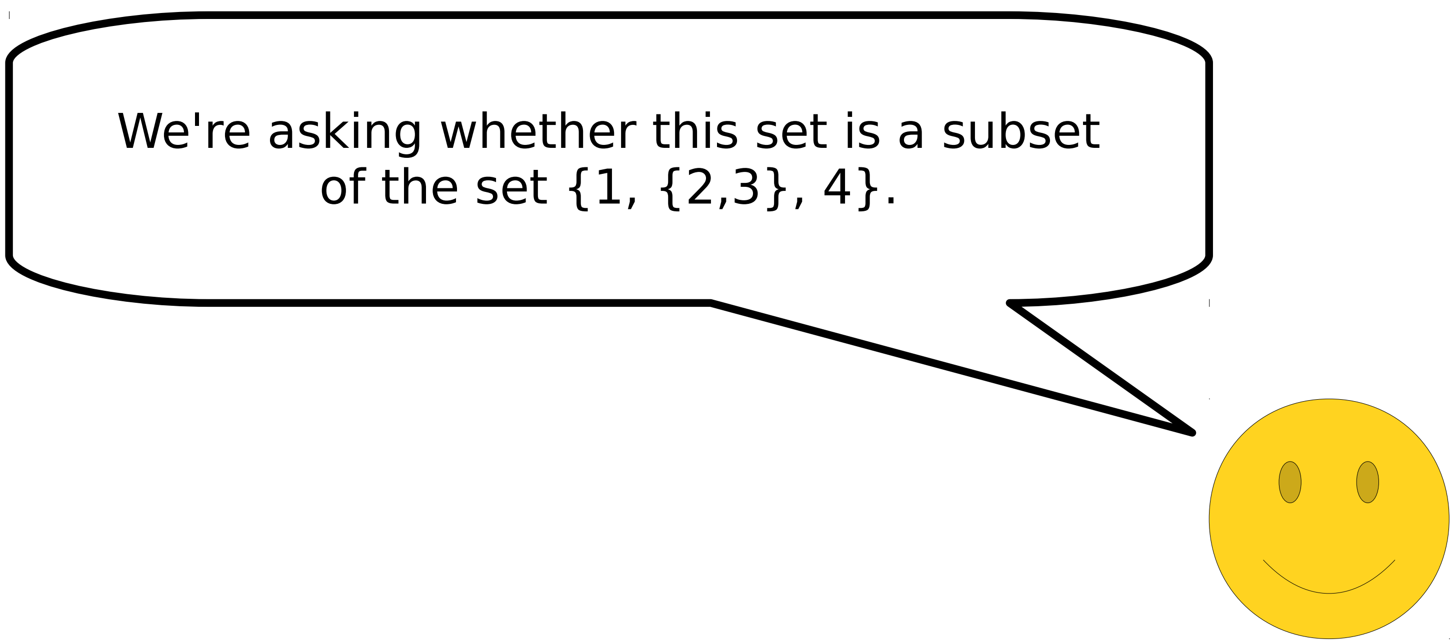
$$\{\emptyset, 4\} \overset{?}{\subseteq} \{ 1, \{2, 3\}, 4 \}$$



$$\{\varnothing, 4\} \overset{?}{\subseteq} \{\ 1,\ \{2,\ 3\},\ 4\ \}$$

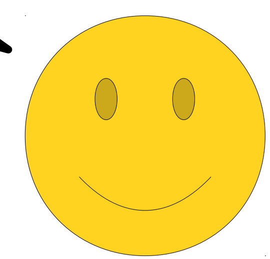


$$\{\emptyset, 4\} \overset{?}{\subseteq} \{ 1, \{2, 3\}, 4 \}$$

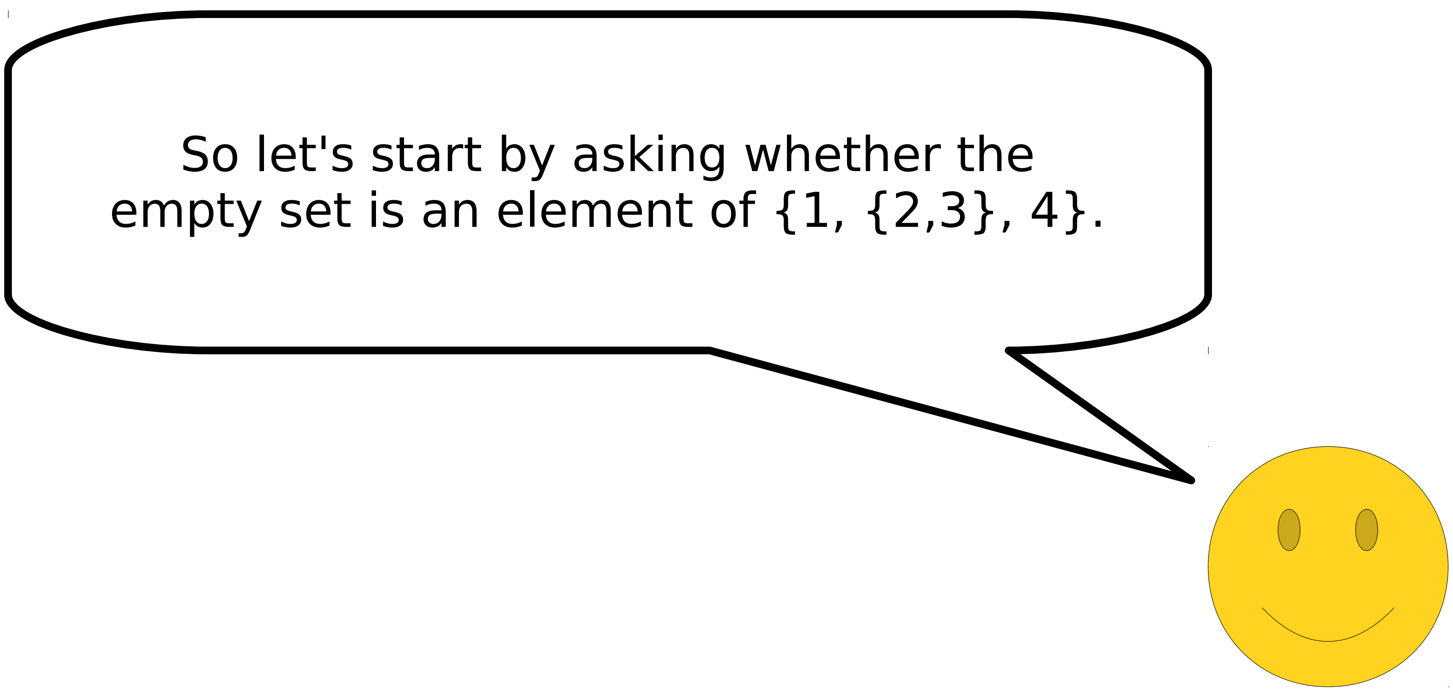


$$\{\emptyset, 4\} \overset{?}{\subseteq} \{\ 1,\ \{2,\ 3\},\ 4\ \}$$

As you've seen before, this means that
we need to check whether every element
of the set on the left happens to be
an element of the set on the right.

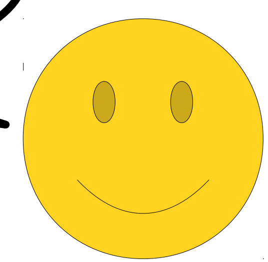


$$\{\emptyset, 4\} \stackrel{?}{\subseteq} \{ 1, \{2, 3\}, 4 \}$$



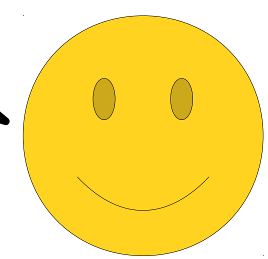
$$\{\emptyset, 4\} \overset{?}{\subseteq} \{ 1, \{2, 3\}, 4 \}$$

Something to notice here is that even though the “top-level” question asks about the ⊆ relation, right now we're asking whether this particular thing (the empty set) is an <u>element</u> of another set, and that involves the ∈ relation.



$$\{\varnothing, 4\} \overset{?}{\subseteq} \{\ 1, \{2, 3\}, 4\ \}$$

And, as we saw before, we know that
the empty set is not an element of
{1, {2,3}, 4}, because none of the elements
of that set are equal to Ø.



$$\{\emptyset, 4\} \not\subseteq \{\ 1,\ \{2,\ 3\},\ 4\ \}$$

As a result, we can say that the set on
the left is not a subset of the set
on the right, even though the above
statement involves Ø and the ⊆ relation.


Hope this helps!
Please feel free to ask questions if you have them.

Did you find this useful? If so, let us know! We can go and make more guides like these.