Welcome Back!

Where We Are Now

- Week 1 covered these key topics:
  - Sets and set theory.
  - Direct proofs.
  - Proof by contradiction.
  - Proof by contrapositive.
  - Categories of statements: universal, existential, and implication.
  - How to negate statements.
- Your goal this week is to get as much practice as you can writing proofs and working around with these concepts. The practice problems for this week will help with this.

Things You Should Be Doing

- ***Attending lecture in person***. You will get *so much more* out of this class if you do, and it is significantly harder to fall behind.
- ***Taking your own notes in lecture***. This forces you to create your own independent understanding of the material.
- ***Working on the problem set***. Do not put this off! You need to work on it gradually over the course of the week.
  - ***… and doing so in a pair***. Working alone is harder and makes it trickier to get help when you need it.
- ***Reviewing the CS103 handouts***. We've given out a bunch of useful resources (e.g., the Proofwriting Checklist, Mathematical Vocabulary, & specific Guides); we recommend that you read them. Ideally you've done that by now; if not, make sure to go do that!
- ***Reading solution sets and asking questions***. You now have solutions to the PS1 checkpoint. Make sure that you have a complete understanding of how to solve each of those problems!

Things You Should Do Today

- Read Chapter Two of the course notes and (optionally) work through some of the exercises.
- Try to complete Problems 1 – 6 of the problem set by the end of the evening. Start playing around with Problems 7 and 8; those problems require some thought.
- Find a problem set partner if you don't already have one. (And hey – aren't you in a room full of people who might be good people to work with?)

Things You Should Do Tomorrow

- Look over your feedback on the PS1 checkpoint and make sure you understand all the feedback you get ***completely*** and ***unambiguously***. Ask the course staff for help, either on Piazza or in office hours, if you don't.
- Continue working on PS1. Try to have at least half of Problems 7 and 8 completed by then and have drafts of all your answers written up.
- Start reviewing your partner's answers and writing up a single, definitive set of answers that you're going to turn in.
- Stop by office hours to get feedback on your proofs and take that feedback seriously.
- (Also, complete the ***CS103A assignment*** after Wednesday's lecture—it is due Friday at 2:30 pm!)

Start working through the packet of problems we've provided. We'll reconvene as a group after a while.

A Few Choice Problems

Calling Back to Definitions

Theorem: If A and B are sets, then $\wp(A) \cap \wp(B) \subseteq \wp(A \cap B)$.

Theorem: If *A* and *B* are sets, then $\wp(A) \cap \wp(B) \subseteq \wp(A \cap B)$.

This is some *really* dense notation. Before we begin, let's write down what these terms mean. What do we know about them?

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Proof: Let A and B be sets.

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Proof: Let A and B be sets. We need to show that

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How do you prove that one set is a subset of another?

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Consider an arbitrary $S \in \wp(A) \cap \wp(B)$.

Can we say
anything about S
at this point?

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What do we need to prove about x to prove that $x \in A \cap B$?

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What do we already know about S?

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To prove that $S \subseteq A \cap B$, consider any $x \in S$. Since $x \in S$ and $S \subseteq A$, we know that $x \in A$.

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