Relations and Functions

Where We Are Now

- Our dive into discrete structures covered the following topics:

  - Binary relations.

  - Properties of relations: reflexivity, symmetry, transitivity

  - Equivalence relations.

  - Equivalence classes.

  - First-order definitions.

  - Functions.

  - Domains and co-domains.

  - Injections, surjections, and bijections.

  - Function composition.

- Your goal this week is to keep your proof skills sharp while building intuition around these new definitions.

Things You Should Do Today/Tomorrow

- Look over the feedback & solutions for PS2 (and the PS3 checkpoint) and make sure you understand all of it *completely* and *unambiguously*. Ask the course staff for help, either on Piazza or in office hours, if you don't.

- Continue working on PS3.

- Read the "Guide to Proofs on Discrete Structures" if you haven't already.

- Stop by office hours to get feedback on your proofs and take that feedback seriously.

- (Also, complete the *CS103A assignment* after Wednesday's lecture—it is due Friday at 2:30 pm)

Topics You Wanted More Practice With

- First-order translations
- Binary relations
- Proofwriting!

A binary relation R over a set A is called **_functional_** if

$$\forall x \in A. \ \forall y \in A. \ \forall z \in A. \ (xRy \land xRz \to y=z)$$

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How would you translate this into English?

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"For every $x,y,z \in A$, if xRy and xRz then $y=z$."

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How would you translate this into English?

"For every $x, y, z \in A$, if xRy and xRz then $y=z$."

"Every $x \in A$ relates to at most one entity in A."

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How would you set up a proof that some R is functional?

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ASSUME:

W.T.S.:

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ASSUME: Let $x, y,$ and z be elements of A
such that xRy and xRz.

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How would you set up a proof that some *R* is functional?

ASSUME: Let $x, y,$ and z be elements of A
such that xRy and xRz.

W.T.S.: We will prove that $y=z$.