Cardinality and Graph Theory

Topics You Wanted More Practice With

- Binary relation/equivalence relation proofs
  - Guide to Proofs on Discrete Structures!!
  - The set-up stays the same; the inner part is problem-specific
  - Write out what you know and expand definitions
- Bijections and working with infinite sets [today]
- Diagonalization [this week’s 103 homework]
- Graph theory [today]
- Proofwriting technique

Let’s do a proof critique!

Theorem: In any graph with at least two nodes, there are at least two nodes with the same degree.

Proof: For a graph on $n \geq 2$ vertices, the possible degrees are 0 through n-1, since each vertex has at most one edge to every other vertex and none to itself. Thus if each vertex were to have a unique degree, there would have to be one with each possible degree. However, if one vertex has degree 0, then no other vertex can be adjacent to it, so in fact the maximum possible degree would be n-2. Since this is a contradiction, every simple graph on at least two vertices has two of equal degree.

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- ☐ ***Clearly articulate your start and end points.***
- ☐ ***Make each sentence "load-bearing."***
- ☐ ***Scope and properly introduce variables.***
- ☐ ***Make specific claims about specific variables.***
- ☐ ***Don't repeat definitions; use them instead.***
- ☐ ***Write in complete sentences and complete paragraphs.***
- ☐ ***Distinguish between proofs and disproofs.***