Exercise 1. Compute the following without using computer software. You should find Fermat’s Little Theorem useful for some of these.
(a) The last decimal digit of $3^{1000}$.
(b) $3^{1000} \mod 31$.
(c) $3/16$ in $\mathbb{Z}_{31}$.

Exercise 2. Prove or disprove:
(a) $(A \cap B) \times C = (A \times C) \cap (B \times C)$
(b) $(A \times B) \cup (C \times D) = (A \cup C) \times (B \cup D)$
(c) $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

Exercise 3. Examples of relations:
(a) Find relations $R$ and $S$ on some set $A$, such that $R \circ S \neq S \circ R$.
(b) Find a relation $R$ on a finite set $A$, such that $R^n \neq R^{n+1}$ for every $n \in \mathbb{N}^+$.

Exercise 4. Give an example of a function $f : \mathbb{N} \to \mathbb{Z}$ that is:
(a) Neither injective nor surjective.
(b) Injective but not surjective.
(c) Surjective but not injective.
(d) Surjective and injective.

Exercise 5. Let $R$ and $S$ be equivalences on a set $A$. Decide which of the following are necessarily also equivalences on $A$; prove or give a counterexample. Then assume that $R$ and $S$ are partial orders on $A$ and decide which of the following are necessarily partial orders on $A$; again, prove or give a counterexample.
(a) $R \cap S$
(b) $R \cup S$
(c) $R \setminus S$
(d) $R \circ S$

Exercise 6. EXTRA CREDIT: Let $R$ be a relation on a set $A$, and $T$ be the transitive closure of $R$. Prove:
(a) $T$ is transitive.
(b) $T$ is the smallest transitive relation that contains $R$. (That is, if $U$ is a transitive relation on $A$ and $R \subseteq U$, then $T \subseteq U$.)
(c) If $|A| = n$ then
\[ T = \bigcup_{i=1}^{n} R^i. \]