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CS 106AX

Handout #03
September 26, 2025

Assignment #1—Simple JavaScript Programs

Due: October 3rd, 2025 at 5:00pm

Your job for this assignment is to implement programs to solve four programming problems. The starter code is a zip file which when expanded produces four folders, one for each problem. Each folder contains an HTML file that can more or less be ignored, except that you need to double-click it to load it into a browser (we recommend Chrome). You're to modify each JavaScript file in a simple, JavaScript-aware editor (we recommend Sublime, but you can really use anything you're comfortable with already). As you make changes to your JavaScript files, save and reload the companion HTML page to see how what you've written is working.

Problem 1: Validating Credit Card Numbers

When creditors like Visa, MasterCard, and American Express issue new credit cards, they ensure the numbers are valid according to ***Luhn's algorithm***. Luhn's algorithm can then be used by online retailers to immediately reject most mistyped or intentionally manufactured numbers.

Luhn's algorithm is a digit manipulation algorithm that works like this:

- Isolate every digit, starting from the right and moving left, doubling every second one. When this doubling produces a value greater than 9, subtract 9 from it. For example, 596825 would produce:
    - 5 on behalf of the 5 in the ones place
    - 2 * 2 = 4 on behalf of the 2 in the tens place
    - 8 on behalf of the 8 in the hundreds place
    - 2 * 6 - 9 = 3 on behalf of the 6 in the thousands place
    - 9 on behalf of the 9 in the ten thousands place
    - 2 * 5 – 9 = 1 on behalf of the 5 in the hundred thousands place
- Sum all of the transformed digits to produce the ***Luhn digit sum***. For example, the 596825 above has a Luhn digit sum of 5 + 4 + 8 + 3 + 9 + 1 = 30.
- If the Luhn digit sum ends in a 0, then and only then is the original number valid according to Luhn's algorithm. For example, 596825 is technically a valid credit card number according to Luhn's algorithm, whereas 596725 (where there's a 7 in place of that 8) is not.

Update the **`Luhn.js`** file to include your own implementation of the predicate function called **`isValid`**, which returns **`true`** if the credit card number looks to be valid according to Luhn's algorithm, and **`false`** otherwise. Launch **`luhn.html`** to exercise

your implementation using the supplied unit tests and any additional ones you want to add.

Problem 2: Hailstone Sequences

Douglas Hofstadter's Pulitzer-prize-winning book *Gödel, Escher, Bach* contains many interesting mathematical puzzles, many of which can be expressed in the form of computer programs. Of these, most require programming skills well beyond the second week of CS 106AX. However, in Chapter XII, Hofstadter mentions a wonderful problem that is well within the scope of the control statements we reviewed this past week. The problem can be expressed as follows:

> Pick some positive integer and call it *n*.
> If *n* is even, divide it by two.
> If *n* is odd, multiply it by three and add one.
> Continue this process until *n* is equal to one.

On page 401 of the Vintage edition, Hofstadter illustrates this process with the following example, starting with the number 15:

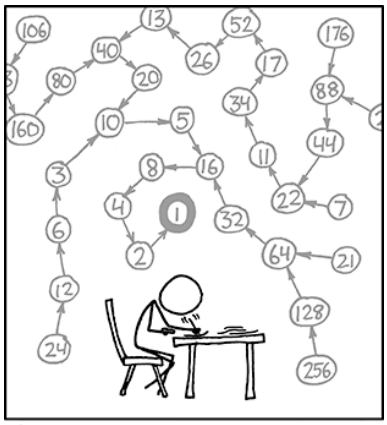
15	is odd, so I make 3*n*+1:	46
46	is even, so I take half:	23
23	is odd, so I make 3*n*+1:	70
70	is even, so I take half:	35
35	is odd, so I make 3*n*+1:	106
106	is even, so I take half:	53
53	is odd, so I make 3*n*+1:	160
160	is even, so I take half:	80
80	is even, so I take half:	40
40	is even, so I take half:	20
20	is even, so I take half:	10
10	is even, so I take half:	5
5	is odd, so I make 3*n*+1:	16
16	is even, so I take half:	8
8	is even, so I take half:	4
4	is even, so I take half:	2
2	is even, so I take half:	1

As you can see from this example, the numbers go up and down, but eventually—at least for all numbers that have ever been tried—comes down to end in 1. In some respects, this process is reminiscent of the formation of hailstones, which get carried upward by the winds over and over again before they finally descend to the ground. Because of this analogy, this sequence of numbers is usually called the ***Hailstone sequence,*** although it goes by many other names as well.

Write a function **`hailstone`** that takes an integer and then uses **`console.log`** to display the Hailstone sequence for that number, just as in Hofstadter's book, followed by a line showing the number of steps taken to reach 1. For example, your program should be able to produce a output that looks like this when **`hailstone(17)`** is called:

```
17 is odd, so I make 3n+1: 52
52 is even, so I take half: 26
26 is even, so I take half: 13
13 is odd, so I make 3n+1: 40
40 is even, so I take half: 20
20 is even, so I take half: 10
10 is even, so I take half: 5
5 is odd, so I make 3n+1: 16
16 is even, so I take half: 8
8 is even, so I take half: 4
4 is even, so I take half: 2
2 is even, so I take half: 1
The process took 12 steps to reach 1.
```

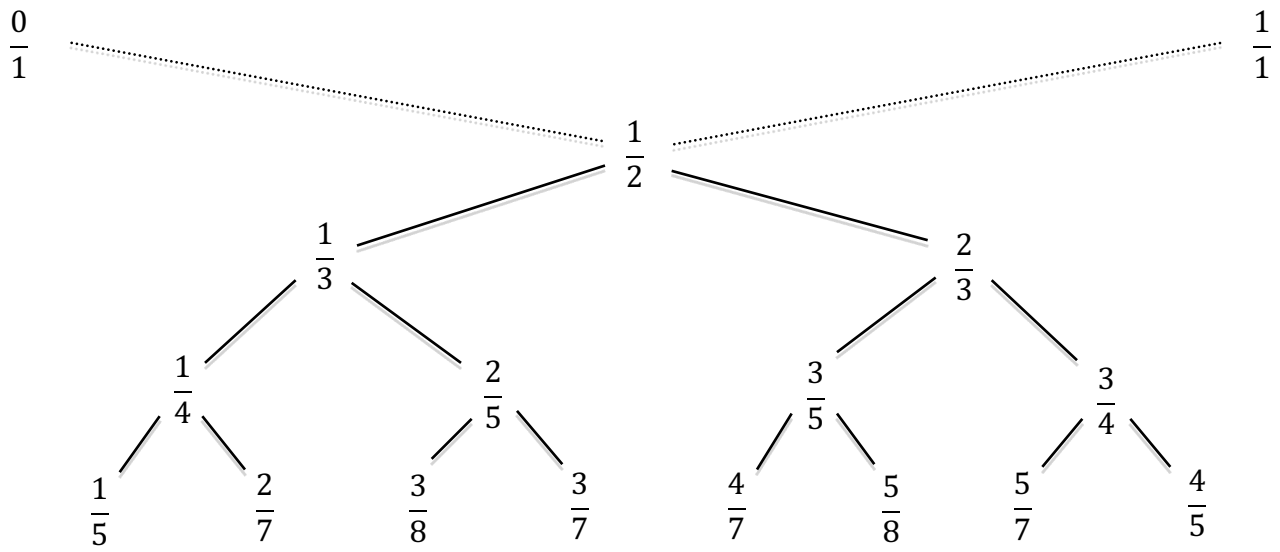
Aside: One fascinating thing about this problem is that no one has yet been able to prove that it always stops. The number of steps in the process can certainly get very large. How many steps, for example, does your program take when *n* is 27? The conjecture that this process always terminates is called the ***Collatz conjecture,*** and appears in the *XKCD* cartoon by Randall Munroe on the right.



THE COLLATZ CONJECTURE STATES THAT IF YOU PICK A NUMBER, AND IF IT'S EVEN DIVIDE IT BY TWO AND IF IT'S ODD MULTIPLY IT BY THREE AND ADD ONE, AND YOU REPEAT THIS PROCEDURE LONG ENOUGH, EVENTUALLY YOUR FRIENDS WILL STOP CALLING TO SEE IF YOU WANT TO HANG OUT.

Problem 3: Stern-Brocot Trees and Sequences

This exercise involves the following construction, which is an adaptation of something known as the Stern-Brocot tree:



Each fraction is $\frac{n_L+n_R}{d_L+d_R}$, where $\frac{n_L}{d_L}$ is the closest ancestor up and to the left, and $\frac{n_R}{d_R}$ is the closest ancestor up and to the right. $\frac{3}{7}$, for example, is produced from $\frac{2}{5}$ (first ancestor up and to the left) and $\frac{1}{2}$ (first ancestor up and to the right.)

This manner of enumerating fractions has three interesting properties (stated without proof, but you can trust that they're correct):

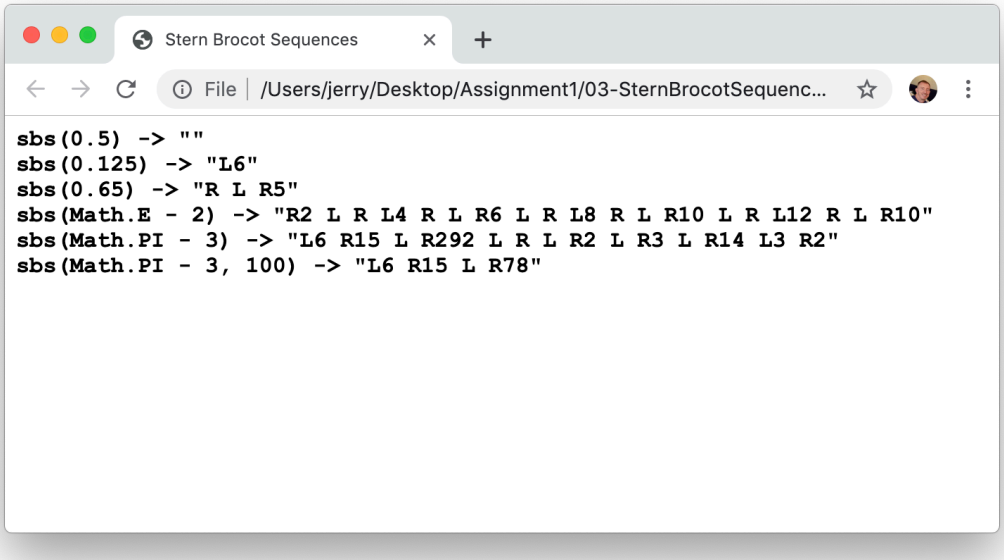
- each fraction generated by the construction is in reduced form,
- every single reduced fraction between 0 and 1 will eventually be formed, and
- $\frac{n_L}{d_L}$is always less than $\frac{n_L+n_R}{d_L+d_R}$, and $\frac{n_L+n_R}{d_L+d_R}$ is always less than $\frac{n_R}{d_R}$.

Each rational number between 0 and 1 can be expressed as a sequence of R's and L's. These R's and L's signal how one should descend the Stern-Brocot tree from $\frac{1}{2}$ to arrive at the fraction it represents. $\frac{1}{2}$ is represented by the empty sequence, $\frac{1}{3}$ is represented by L, and $\frac{3}{7}$ is represented by LRR. Whenever a letter appears two or more times in a run, we compress that run so that something like LLRLRRRRRLL is instead represented as L2 R L R5 L2. Irrational numbers like π - 3 don't appear in the tree, but rational numbers close to them do! If we keep descending through the tree until we hit some maximum

sequence length, then we use that Stern-Brocot sequence and just say that it's good enough.

For this problem, you're to implement a function called **sbs** (that's short for Stern-Brocot sequence) that accepts a positive, real number less than 1 and returns the Stern-Brocot sequence of R's and L's as outlined above. An optional second argument specifies the maximum number of Rs and Ls in the sequence. If that second argument is missing, it defaults to 500. For instance, the following test harness would produce the specified output, provided the implementation of **sbs** is solid:

```
function TestSternBrocotSequences() {
   console.log("sbs(0.5) -> " + sbs(0.5));
   console.log("sbs(0.125) -> " + sbs(0.125));
   console.log("sbs(0.65) -> " + sbs(0.65));
   console.log("sbs(Math.E - 2) -> " + sbs(Math.E - 2));
   console.log("sbs(Math.PI - 3) -> " + sbs(Math.PI - 3));
   console.log("sbs(Math.PI - 3, 100) -> " + sbs(Math.PI - 3, 100));
}
```

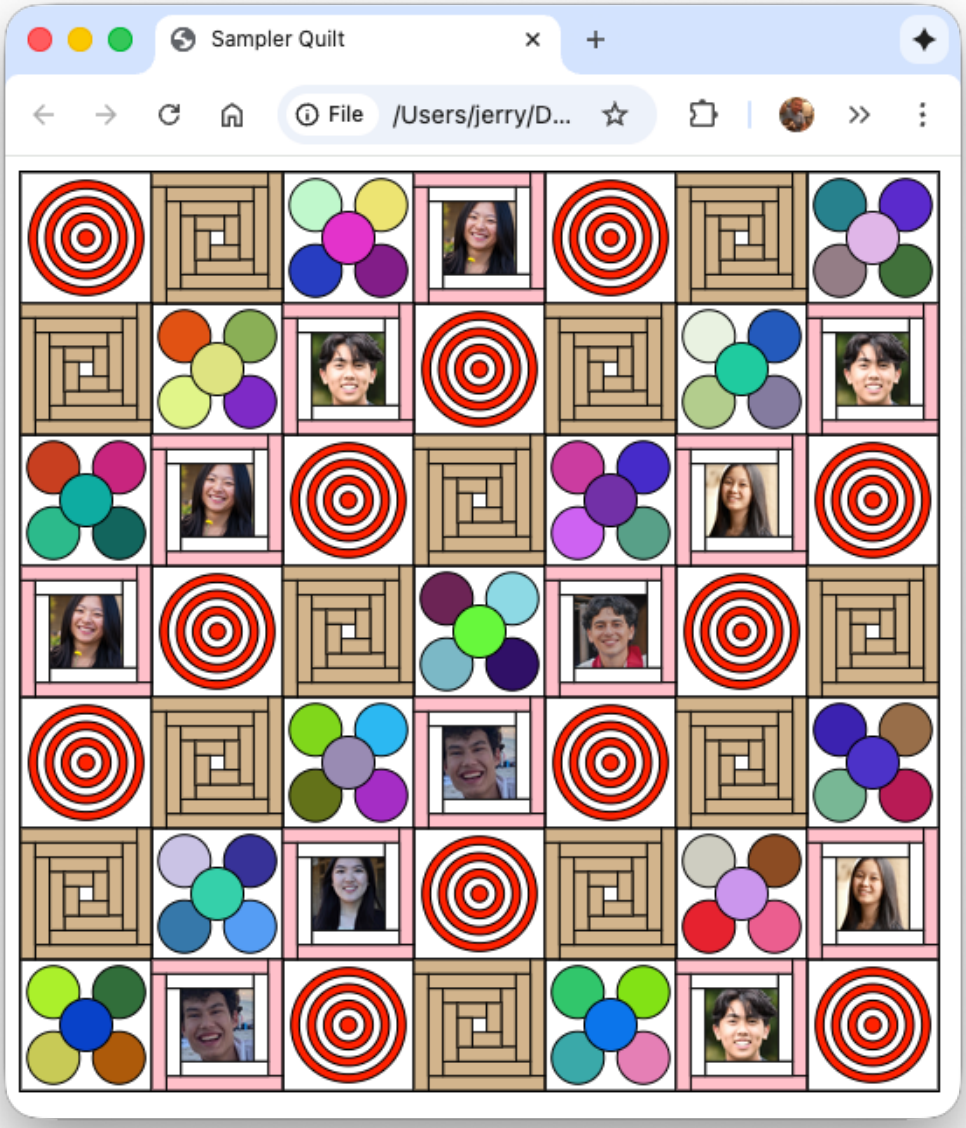


This is the algorithmically most intense function you need to write for Assignment 1, so don't be shy asking for help if you get stuck.

Problem 4: Sampler Quilt

Back in the early 1990s—long before JavaScript existed—Julie Zelenski and Katie Capps Parlante developed a graphics assignment that we used in our introductory courses for several years, and I've decided to revive it. ☺ The goal of the assignment was to draw a sampler quilt, which is composed of several different block types that illustrate a variety of quilting styles.

The sampler quilt should ultimately look like that presented below:



The quilt is 7 x 7, there are four different patch types, and the k^{th} row starts with the k^{th} patch type and rotates through the others, repeating as necessary.

- The bullseye patch is seven evenly spaced concentric circles alternating between red and white. The circle borders, like all borders in the quilt, is black.
- The log cabin patch draws several four, square frames of dovetailed logs—or rather, tan rectangles—such that the shorter dimension of each log equals the dimension of the white square at the center.
- The flower patch consists of five circles, all filled with randomly generated colors using the **`randomColor()`** function discussed in the textbook. The radius of each circle is one fifth the dimension of the patch itself, the fifth circle is centered in the patch and layered on top of the other four, each of which is centered in the patch's four main quadrants.
- The final patch—the i-love-my-section-leader patch—consists of a randomly selected image (with a URL of

  `https://cs106ax.stanford.edu/img/jenny.png`,

  where **`jenny`** is equally likely to be **`andy`**, **`diego`**, **`eugene`**, **`jenny`**, **`sabrina`**, or **`tina`**). The images are all exactly the size needed to fit snugly within a patch, though you layer two frames—the outer one pink, and the inner one white—on top of the image to make it look like a photo that's sitting at home in your family's living room. You'll want to read up on the **`GImage`** class in the textbook, and you'll want to leverage log cabin patch code you wrote to get the pink and white photo framing.

The point of the exercise is to come up with a clean decomposition and implementation that sensibly reuses code wherever possible. We don't care about the exact pixel alignments so much as we do code clarity and narrative. Don't sweat details that don't feel pedagogically important. And if you have questions, just ask us and we'll answer them.

Submitting Your Work

When you're ready to submit your work, visit the CS106AX course website and click on the Paperless button in the Resources section of the course web page. You should be led to an online tool where you can upload all of your completed programs. You don't need to submit the HTML files or the surrounding folders, since we already have those. You should, however, upload each of the four JavaScript files, since they contain all of your work!

Thanks everyone, and good luck completing your very first assignment! You're going to do great!!