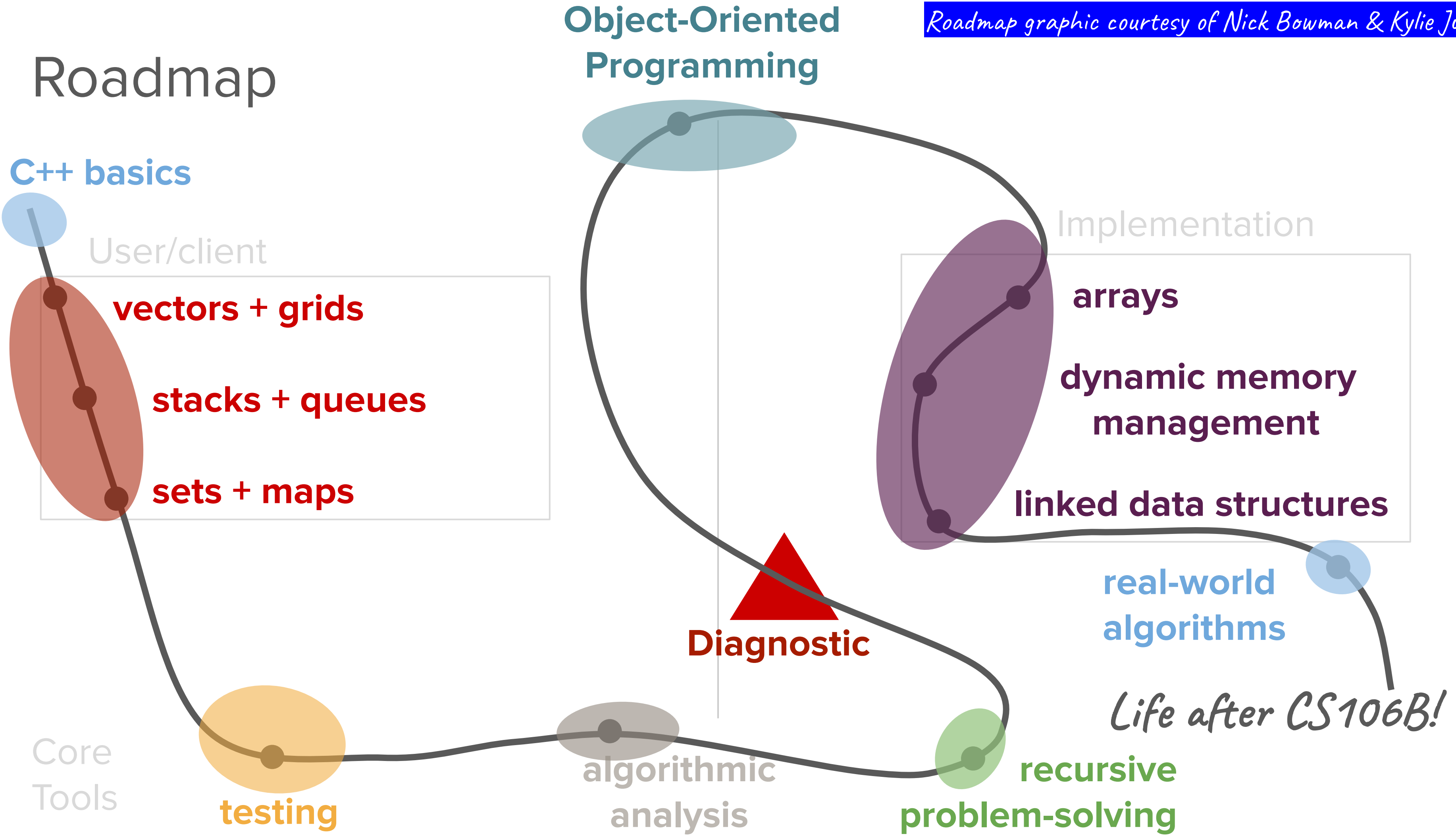
Beyond Efficiency: Algorithmic Analysis and Social Impact

What has been the most interesting application of recursion that you've encountered so far?

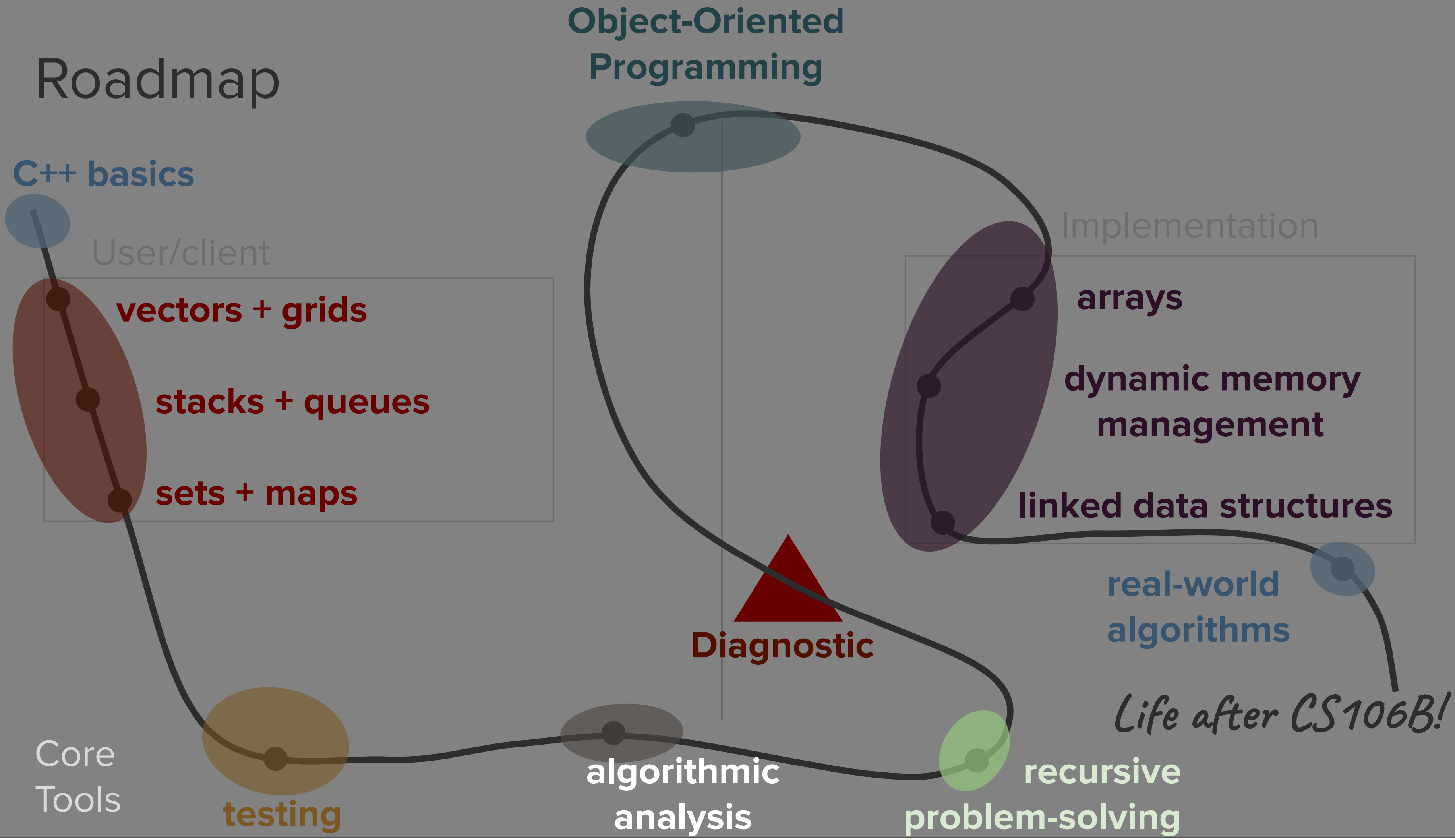
(put your answers the chat)

Roadmap

Roadmap graphic courtesy of Nick Bowman & Kylie Jue



Roadmap



Object-Oriented Programming
C++ basics
User/client
vectors + grids
stacks + queues
sets + maps
Implementation
arrays
dynamic memory management
linked data structures
real-world algorithms
Diagnostic
Life after CS106B!
Core Tools
testing
algorithmic analysis
recursive problem-solving

Today's question

What problems *should* we solve with recursive backtracking?

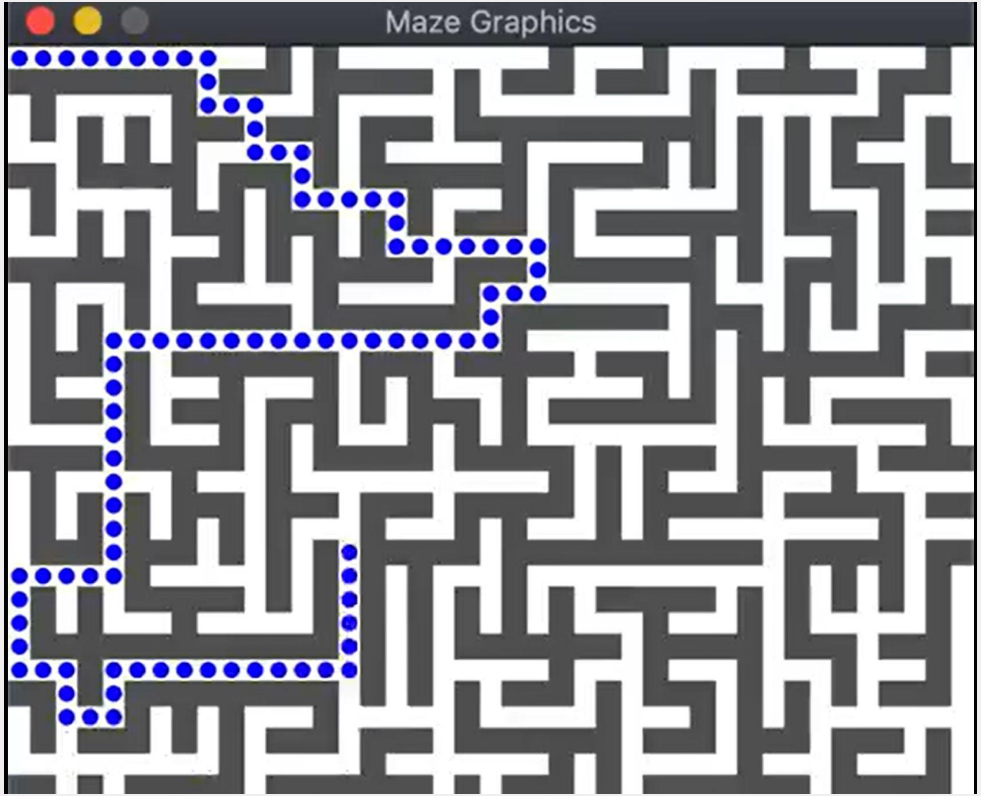
Today's topics

1. Review and Recursion Overview

2. Beyond Efficiency: Algorithmic Analysis and Social Impact

3. Optimization and Gerrymandering

Solving mazes with Depth- First Search (DFS)



What defines our maze decision tree?

- **Decision** at each step (each level of the tree):
  - Which valid move will we take?

- **Options** at each decision (branches from each node):
  - All valid moves (in bounds, not a wall, not previously visited) that are either North, South, East, or West of the current location

- Information we need to store along the way:
  - The path we've taken so far (a Stack we're building up)
  - Where we've already visited
  - Our current location

Pseudocode

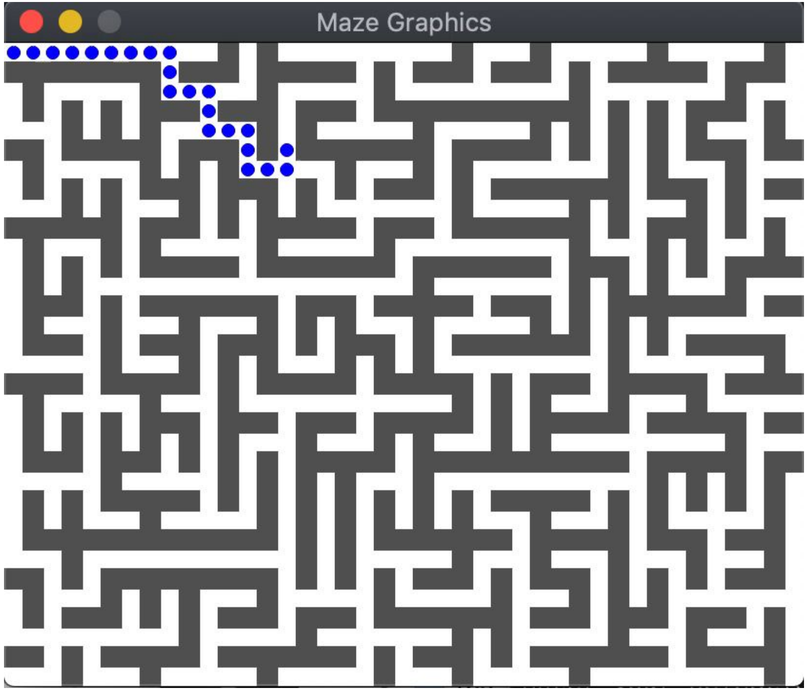
- Our helper function will have as **parameters**: the maze itself, the path we're building up, and the current location.
    - **Idea**: Use the boolean Grid (the maze itself) to store information about whether or not a location has been visited by flipping the cell to false once it's in the path (to avoid loops) ➜ This works with our existing **`generateValidMoves()`** function

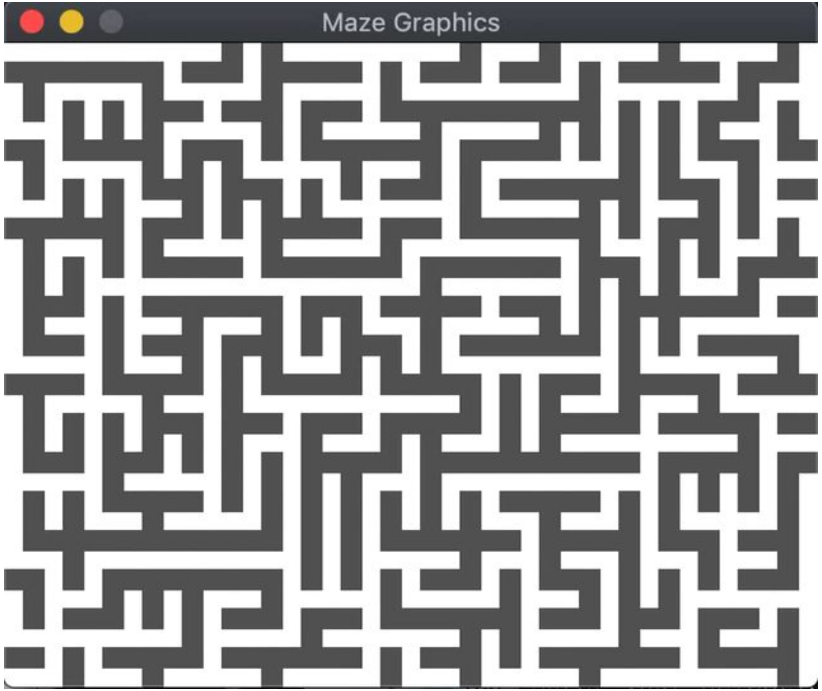
- **Recursive case**: Iterate over valid moves from **`generateValidMoves()`** and try adding them to our path
    - If any recursive call returns true, we have a solution
    - If all fail, return false

- **Base case**: We can stop exploring when we've reached the exit ➜ return true if the current location is the exit

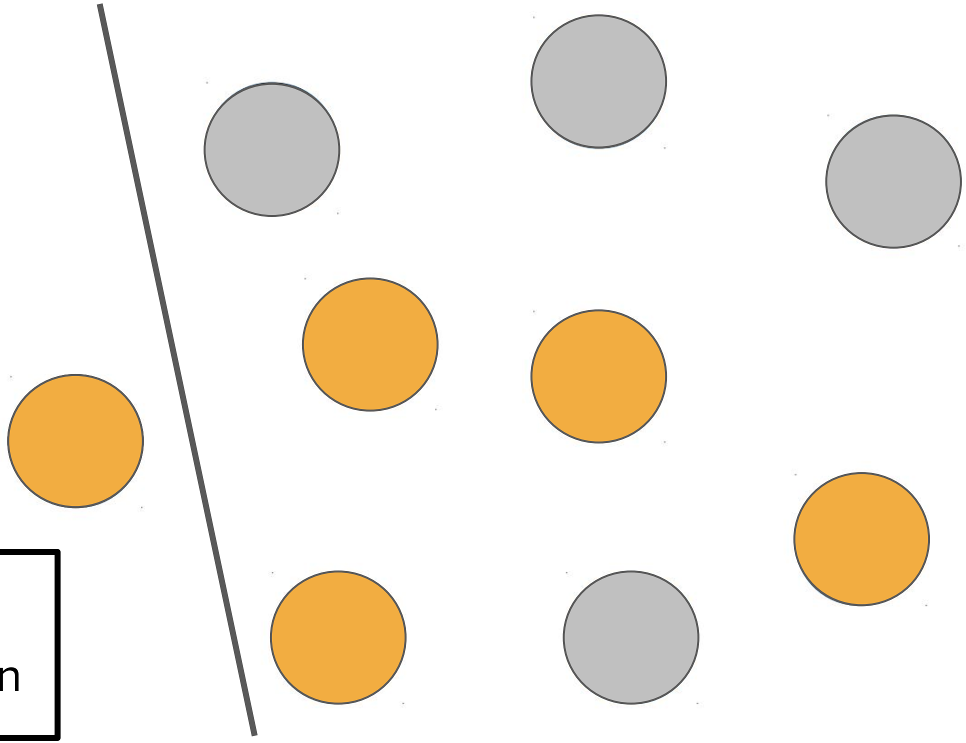
Breadth-First Search vs. Depth-First Search

Which do you think will be faster?





Generating Combinations



Option 2:
Include this person


You need at least five US Supreme Court justices to agree to set a precedent.
What are all the ways you can pick five justices of the US Supreme Court?

Combinations versus Subsets

- Making combinations is very similar to our recursive process for generating subsets!

- The differences:
    - We’re constraining the subsets’ size.
    - We’re building up a set of all valid subsets of that particular size (i.e. combinations).

- Instead of printing out subsets in our base case, we have to return individual sets in our base case and then build up and return our resulting set of sets in our recursive case

You now know how to use recursion to **view problems from a different perspective** that can lead to **short and elegant solutions.**

Organizing Your Recursive Toolbox

Two types of recursion

Basic recursion

- One repeated task that builds up a solution as you come back up the call stack
- The final base case defines the initial seed of the solution and each call contributes a little bit to the solution
- Initial call to recursive function produces final solution

Backtracking recursion

- Build up many possible solutions through multiple recursive calls at each step
- Seed the initial recursive call with an “empty” solution
- At each base case, you have a potential solution

We’ve seen lots of different backtracking strategies...

Questions to ask yourself when planning your strategy:

- What does my decision tree look like? (decisions, options, what to keep track of)
- What are our base and recursive cases?
- What’s the provided function prototype and requirements? Do we need a helper function?
- Do we care about returning or keeping track of the path we took to get to our solution?
- Which of our three use cases does our problem fall into? (generate/count all solutions, find one solution/prove its existence, pick one best solution)
- What are we returning as our solution? (a boolean, a final value, a set of results, etc.)
- What are we building up as our “many possibilities” in order to find our solution? (subsets, permutations, combinations, or something else)

Backtracking recursion: **Exploring many possible solutions**

Overall paradigm: choose/explore/unchoose

Two ways of doing it

- **Choose explore undo**
  - Uses pass by reference; usually with large data structures
  - Explicit unchoose step by "undoing" prior modifications to structure
  - E.g. Generating subsets (one set passed around by reference to track subsets)
- **Copy edit explore**
  - Pass by value; usually when memory constraints aren't an issue
  - Implicit unchoose step by virtue of making edits to copy
  - E.g. Building up a string over time

Three use cases for backtracking

1. Generate/count all solutions (enumeration)
2. Find one solution (or prove existence)
3. Pick one best solution

General examples of things you can do:

- Permutations
- Subsets
- Combinations
- etc.

You've seen how to use recursive backtracking to **determine whether something is possible** and, if so, to **find some way to do it.**

You've seen how to use recursive backtracking to **enumerate all objects of some type**, which you can use to find the **optimal solution to a problem.**

The Knapsack Problem

Pseudocode

- **Recursive case**:
  - Select an unconsidered item based on the index.
  - Recursively calculate the values both with and without the item.
  - Return the higher value.

- **Base cases**:
  - No remaining capacity in the knapsack ➜ return 0 (not a valid combination with weight <= 5)
  - No more items to choose from ➜ return current value

Let’s see the code!

Takeaways

- Finding the best solution to a problem (optimization) can often be thought of as an additional layer of complexity/decision making on top of the recursive enumeration we've seen before

- For "hard" problems, the best solution can only be found by enumerating all possible options and selecting the best one.

- Creative use of the return value of recursive functions can make applying optimization to an existing function straightforward.

Learning goals

After completing this assignment, you will be able to…

- Appreciate the elegance and power of recursive problem-solving and identify problems that are well-suited to be solved recursively.
- Understand and trace how data is stored and altered across multiple recursive function calls.
- Identify and carry out techniques for testing and debugging recursive functions.
- Break down a problem into a collection of smaller, self-similar tasks.
- Develop a recursive algorithm by dividing a problem into one or more base cases and one or more recursive cases.
- Apply the general frameworks of recursive backtracking to solve problems that cannot easily be solved using an iterative approach.
- Implement more advanced recursive algorithms to solve problems that cannot be easily solved using an iterative approach

Assignment parts

This assignment consists of a collection of multiple different recursive exercises, grouped into two parts.

- ### Part 1: Fundamental Recursion

  The first collection of recursive problems walks you through some fundamental recursion problems that increase in difficulty and complexity over time. These problems will serve as the foundational practice for you to begin to grapple with the core tenants of recursive problem-solving. The material needed to solve these problems was covered in lecture on Tuesday-Thursday (7/6-7/8).

  - #### Fundamental Recursion Warmup

    Practice with unit tests and debugging on core recursive functions.

  - #### Balanced Operators

    Determine whether a snippet of code has properly matched pairs of bracketing characters.

  - #### Sierpinksi Fractal

    Draw a beautiful self-similar fractal triangle.

  - #### Merging Sorted Sequences

    Implement an efficient divide-and-conquer algorithm for merging a collection of sorted sequences.

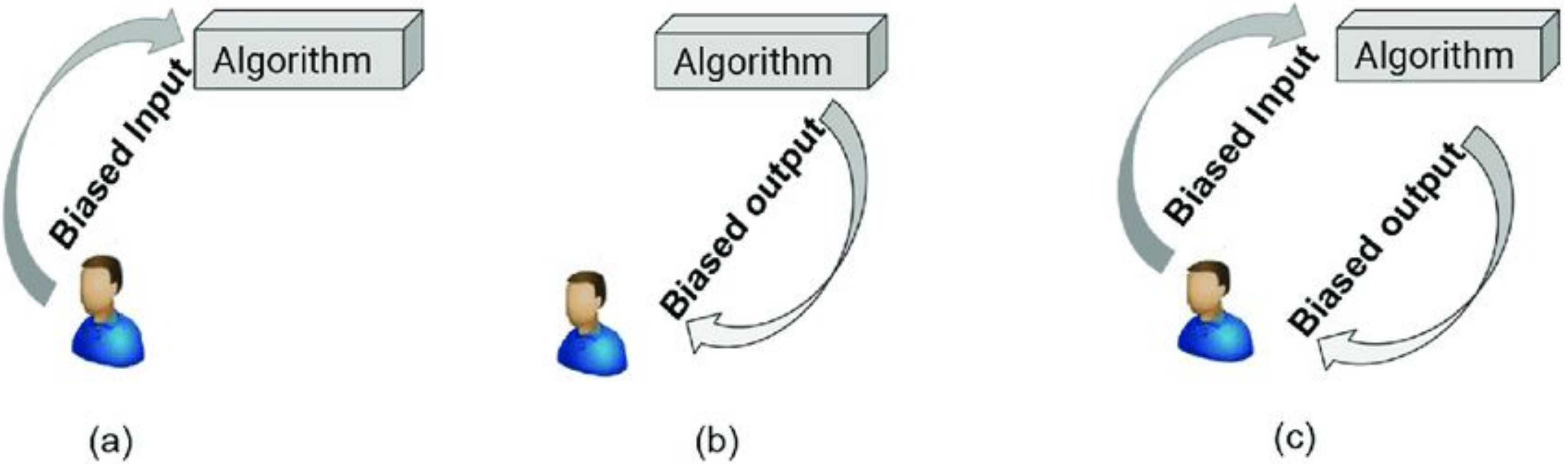
- ### Part 2: Recursive Backtracking

  The second collection of recursive problems pivots towards applying the powerful nature of recursive backtracking to solve some real-world problems. After you've built your solid base of recursive fundamentals in Part 1, you will be well set to apply your newfound skills to solve some practical real-world problems. The material needed to work through the


GET OUT
THERE &
MAKE
SOMETHING!

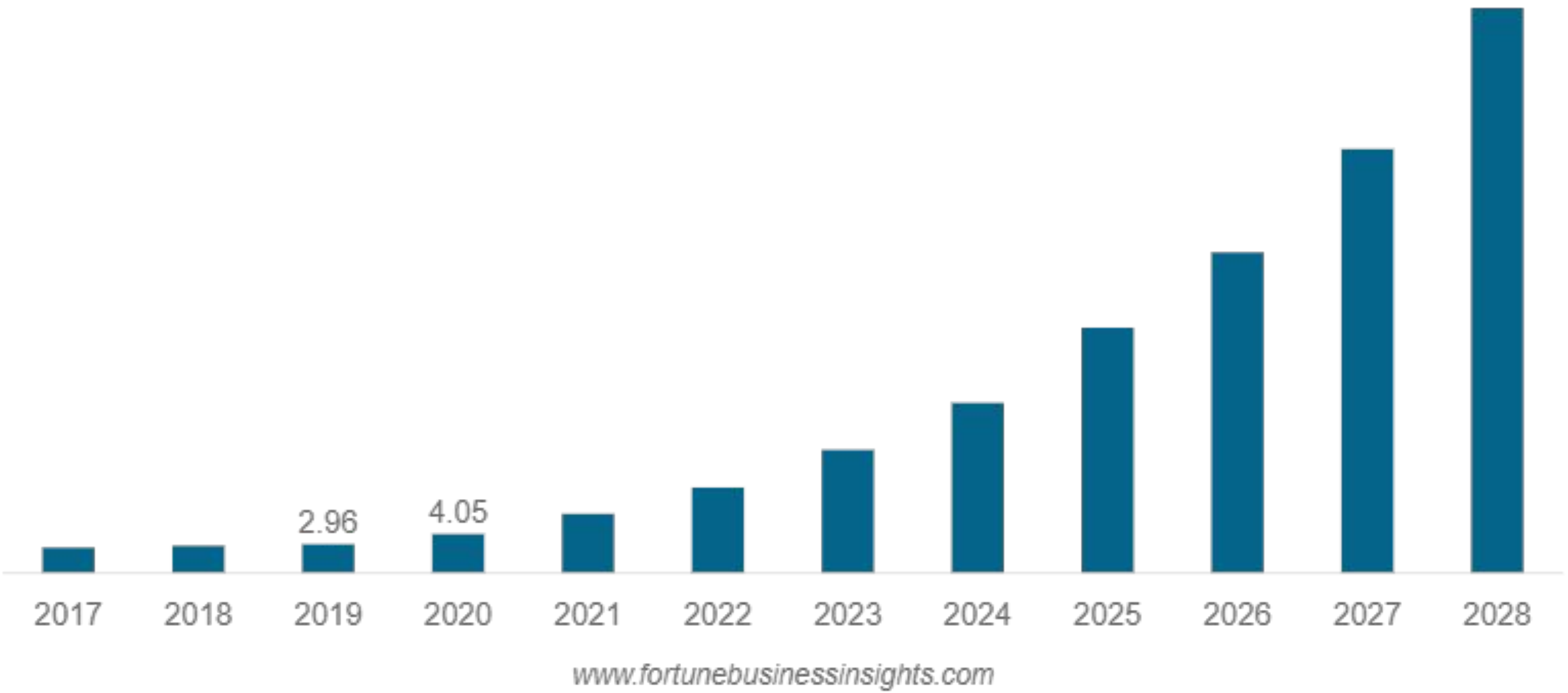
Software Design and Optimization

Algorithmic Bias Poll



Software is ubiquitious

North America Machine Learning Market Size, 2017-2028 (USD Billion)



Error-riddled data sets are warping our sense of how good AI really is

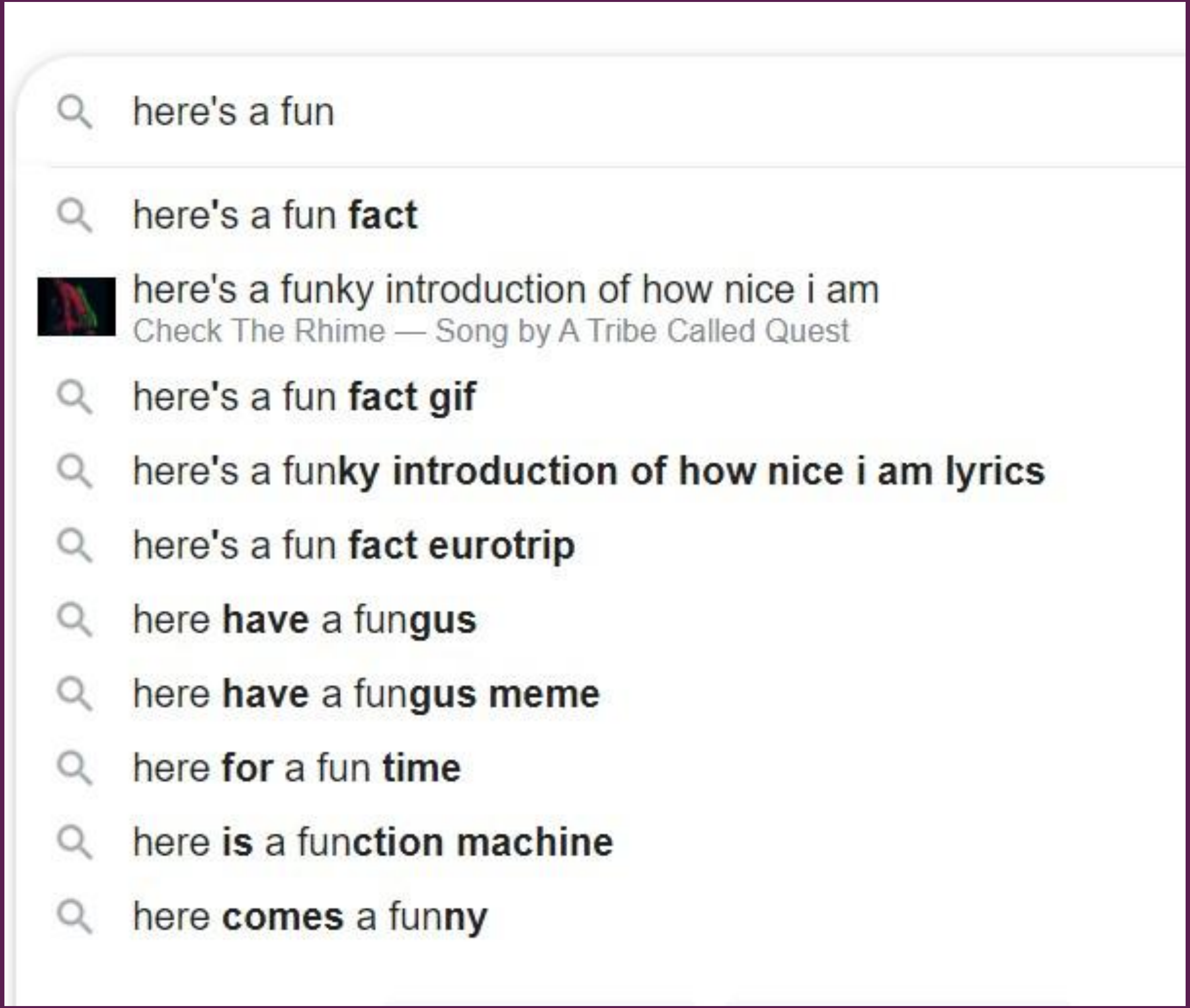
Our understanding of progress in machine learning has been colored by flawed testing data.

by **Karen Hao**

April 1, 2021

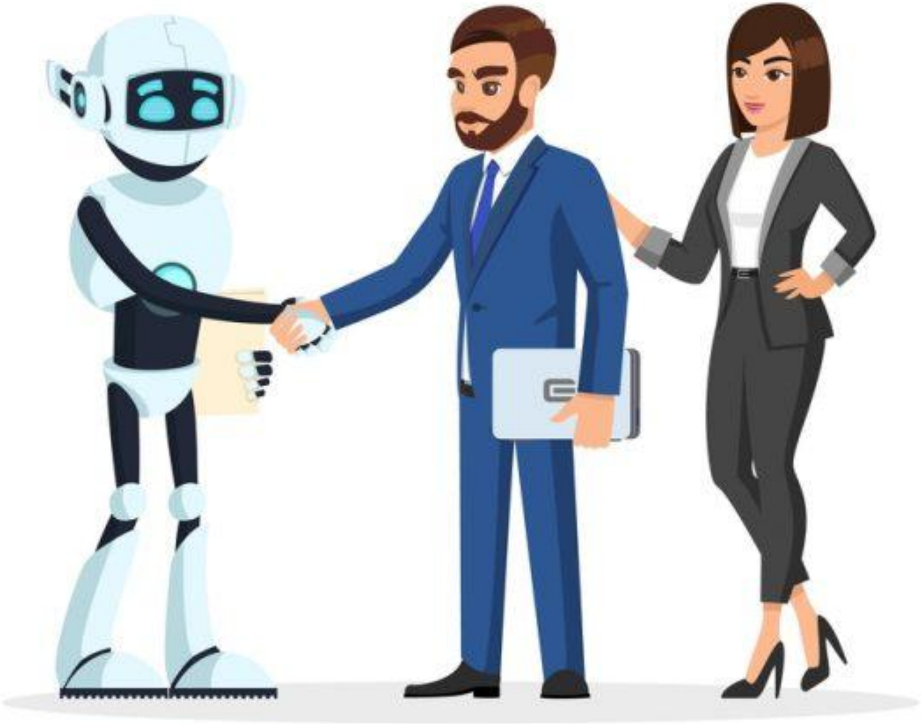
The 10 most cited AI data sets are riddled with label errors, according to a new study out of MIT, and it's distorting our understanding of the field's progress.

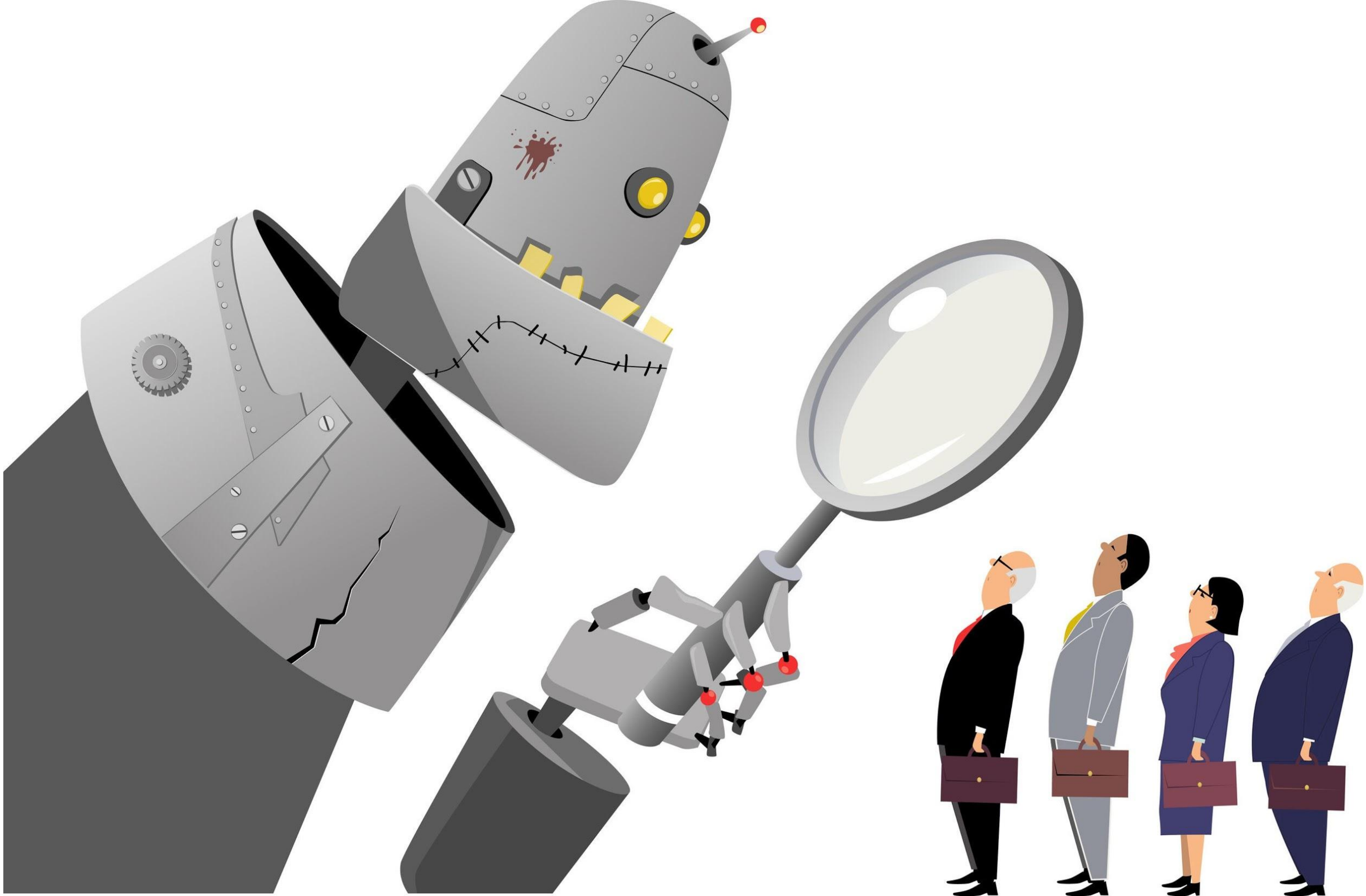
Data backbone: Data sets are the backbone of AI research, but some are more critical than others. There are a core set of them that researchers use to evaluate machine-learning models as a way to track how AI capabilities are advancing over time. One of the best-known is the canonical image-

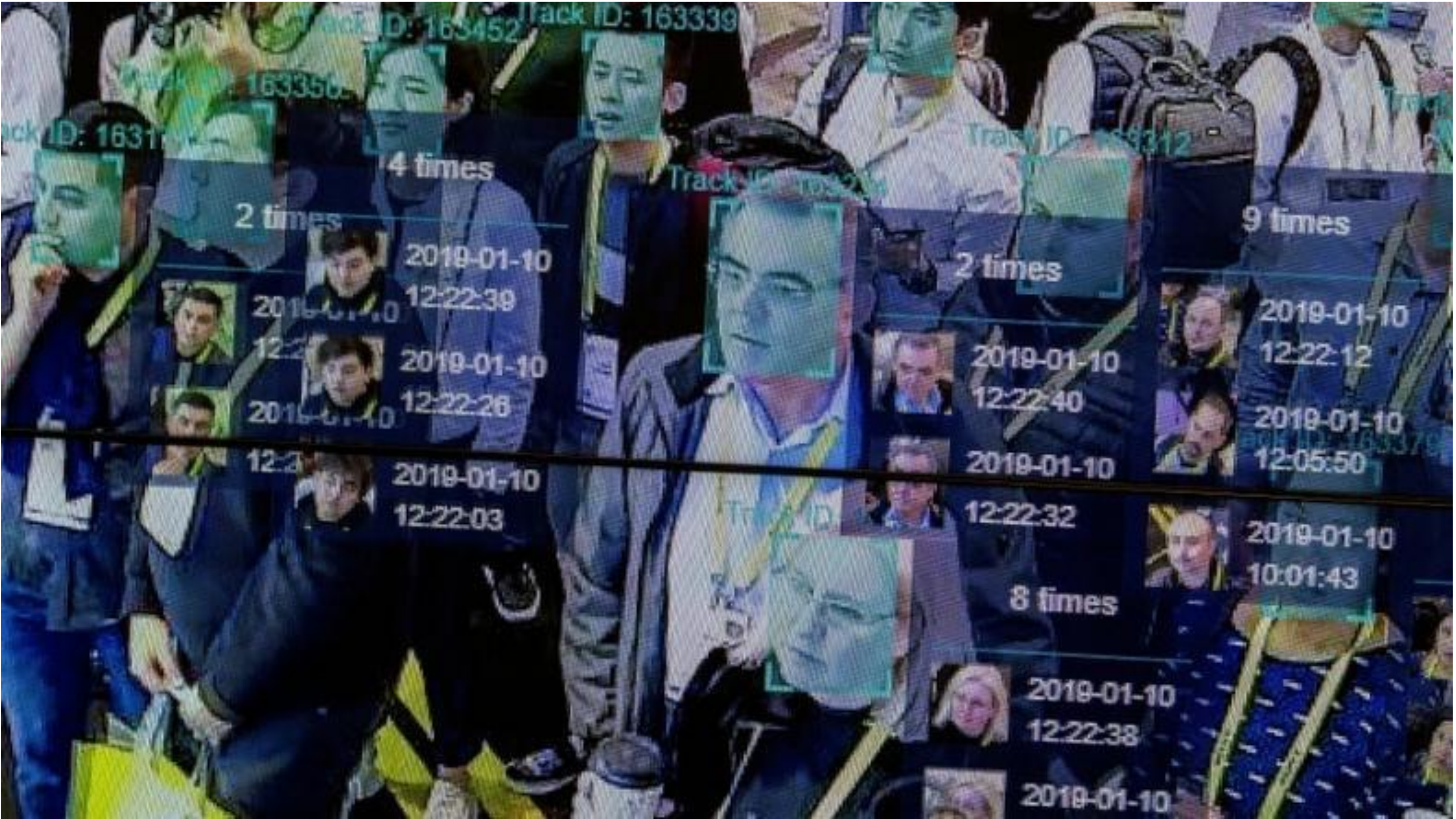

here's a fun
here's a fun fact
here's a funky introduction of how nice i am
Check The Rhime — Song by A Tribe Called Quest
here's a fun fact gif
here's a funky introduction of how nice i am lyrics
here's a fun fact eurotrip
here have a fungus
here have a fungus meme
here for a fun time
here is a function machine
here comes a funny

WE'RE HIRING

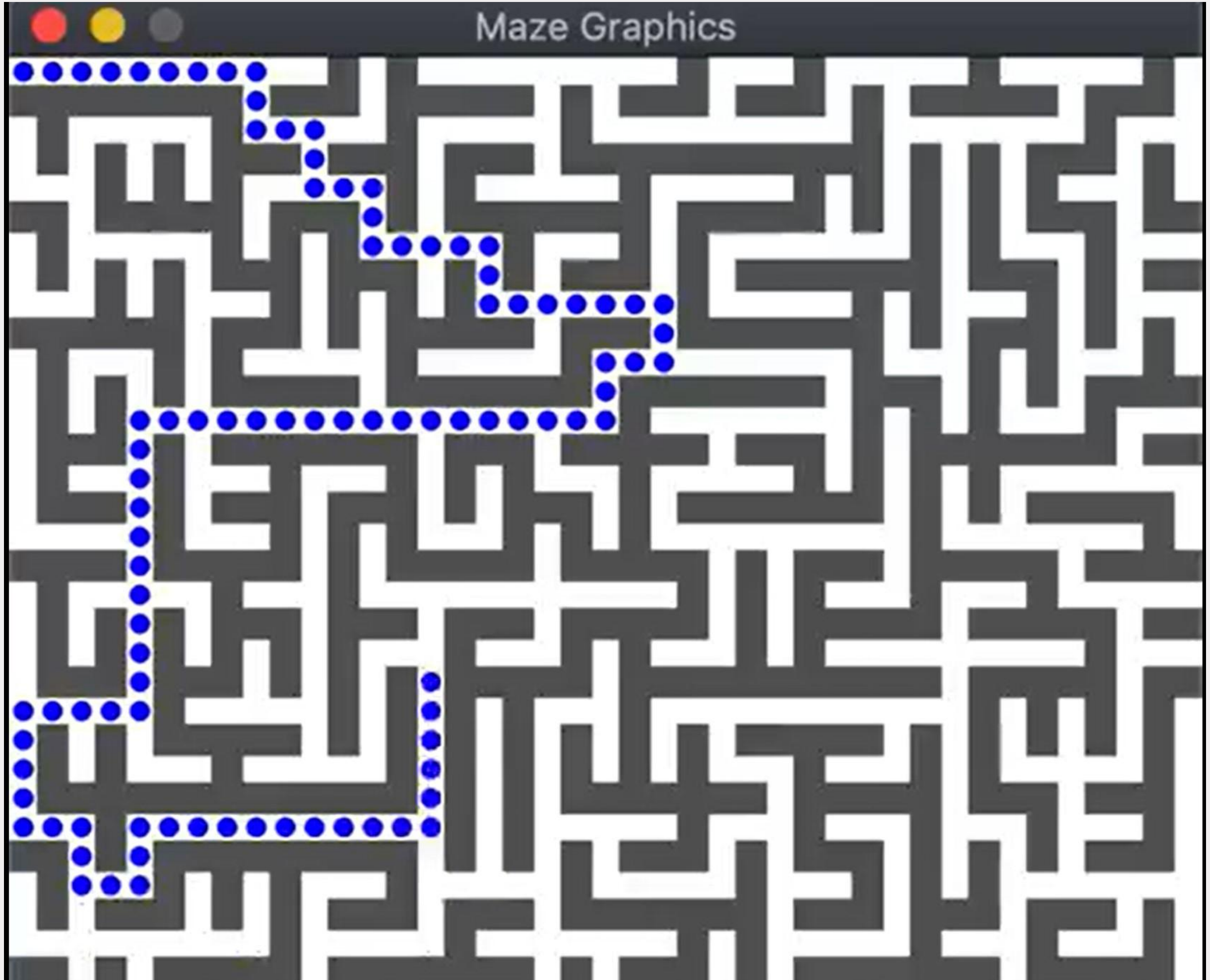
READ MORE







Track ID: 163339
4 times
2 times
2019-01-10
12:22:39
2019-01-10
12:22:26
2019-01-10
12:22:03
2 times
2019-01-10
12:22:40
2019-01-10
12:22:32
9 times
2019-01-10
12:22:12
2019-01-10
12:05:50
2019-01-10
10:01:43
8 times
2019-01-10
12:22:38


Maze Graphics

Shortest-Path Algorithms

- Find the "shortest" path from point A to point B
- "Shortest" in time, distance, cost, …
- Numerous applications
  - Map navigation
  - Flight itineraries
  - Circuit wiring
  - Network routing

With soaring demand for social services, field-tested technology can help governments scale up

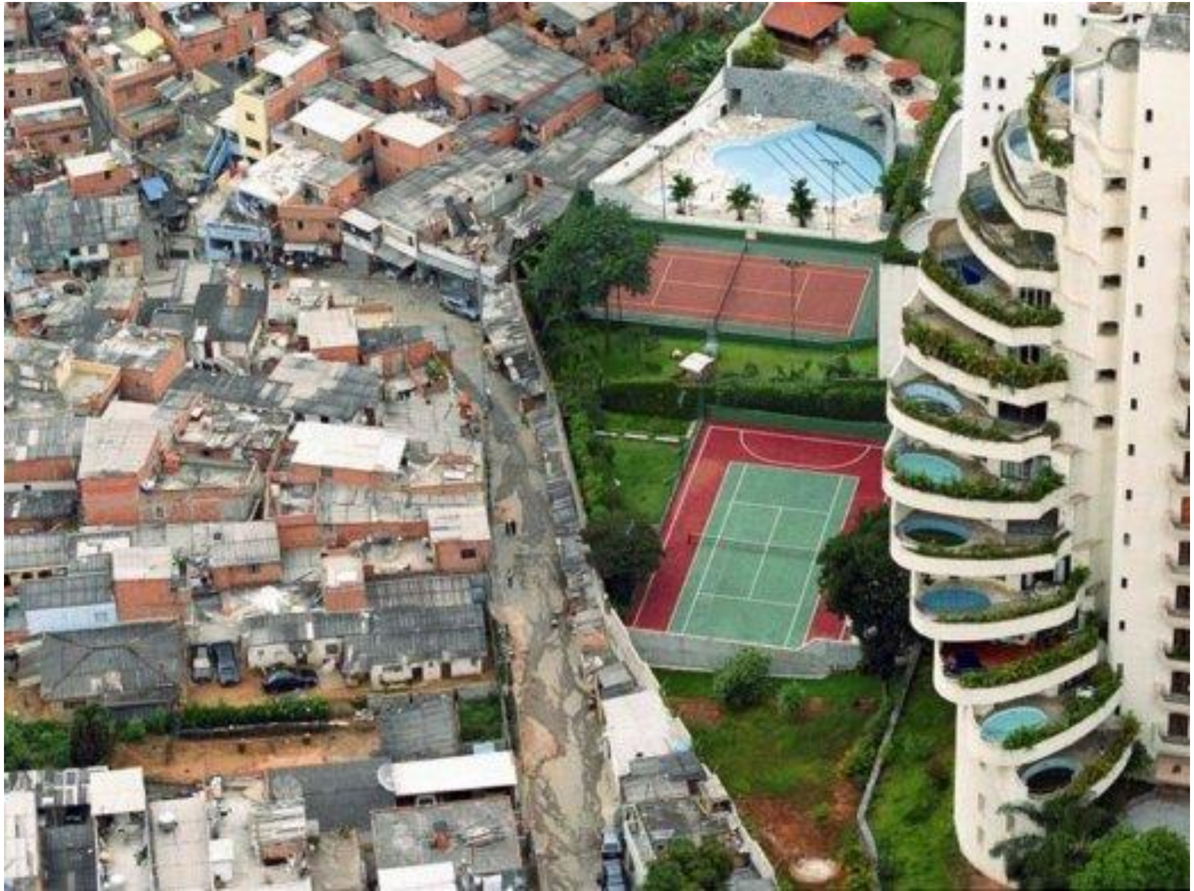
By Emily O'Neill | 2 minute read | May 19, 2020


Design
Justice

Who might be impacted by the software?

Who should be at the table helping to design the software?

Gaining Perspective



Breakout Room Discussion #1

There are two discussion prompts:

- Does the background of the programmer **matter**?

- **How** can we involve those impacted in development of software?


HELP

Moral Relativity

Who decides the target audience?

Who needs the app or software the most?

If there are unintended consequences, are these consequences fairly distributed among groups of people?

How do we define fairness?

If we need funds to develop our software,
who is able to buy it and does the cost to develop it
inherently make it inequitable?

Should the government play a role in regulating this?


MAN ACCUSED OF USING SNAPCHAT TO SELL DRUGS
TEWKSBURY
DOWNLOAD THE NBC10 BOSTON APP
11:16 75°
WORCESTER AREA
NOW
10
BOSTON

Software Usage Labels?

MATERIAL SAFETY DATA SHEET

NOVUS PLASTIC POLISH

NOVUS Inc.
10425 Hampshire Avenue South
Minneapolis, MN 55438

Medical Emergencies (800) 228-5635 x 334
Transportation Emergencies (800) 424-9300
(Outside U.S. call (703) 527-3887 collect)
Business phone # (612) 944-8000

SECTION 1 - PRODUCT IDENTIFICATION AND USE

PRODUCT NAME(S): NOVUS Plastic Polish #2 (P/N 7030, 7032, 7033, & 7072)

NOVUS I.D. NUMBER: 7011

CHEMICAL NAME: NA

PRODUCT USE: Clean and restore plastic surfaces.

SECTION 2 - HAZARDOUS INGREDIENTS

INGREDIENT	% BY WEIGHT	CAS #	EXPOSURE LIMITS	LD50 (SPECIES & ROUTE)	LC50 (SPECIES)
Odorless Mineral Spirits	7-13	68551-17-7	400 ppm (5)	NE	NE
Silica, Amorphous, Diatomaceous Earth	3-7	61790-53-2	.05 mg/kg (2)	NE	NE
Morpholine	1-5	110-91-8	20 ppm (1,2) 30 ppm (3,4)	Rat: 1050 mg/kg Rabbit, skin: 500 mgkg	Rat: 12,000 ppm (8 hr.)
Oleic Acid	1-5	112-80-1	NE	Rat: 20 ml/kg	NE

NOTES: (1) ACGIH TLV (TWA) (2) - OSHA PEL (TWA) (3) - ACGIH STEL (4) - OSHA STEL (5) - MFR/SUPPLIER TLV
LD 50 VALUES ARE VIA ORAL ROUTE UNLESS OTHERWISE INDICATED
** TOXIC CHEMICAL REPORTABLE PER SECTION 313, TITLE III SARA AND 40 CFR372

SECTION 3 - PHYSICAL DATA

PHYSICAL STATE: Liquid | *APPEARANCE:* Tan opaque liquid | *ODOR:* Slight

VAPOR PRESSURE (mmHg): <75 | *% VOLATILE:* 76 | *VAPOR DENSIT (AIR=1):*

EVAPORATION RATE (BuAc=1): < 1 | *BOILING POINT (deg C):* 80 | *FREEZING POIN (deg C):*

PH: NE | *DENSITY (gr/ml):* 1.01 | *SOLUBILITY IN WATER:*

SECTION 4 - FIRE AND EXPLOSION DATA

FLAMMABILITY YES ☐ NO ☒ *IF YES, UNDER WHICH CONDITIONS:* Combustible - Keep away from heat, open flame, and other sou

EXTINGUISHING MEDIA: Carbon Dioxide, Dry Chemical, or Foam.

SPECIAL FIRE FIGHTING PROCEDURES: Use water spray or fog to cool fire exposed containers.

FLASH POINT (PMCC) deg F: 199 | *UPPER FLAMMABILITY LIMIT: (% BY VOLUME)* NE | *LOWER FLAMM (% BY VOLUME*

AUTOIGNITION TEMPERATURE (deg C): NE | *HAZARDOUS COMBUSTION PRODUCTS:* Carbon Dioxide and C

EXPLOSION DATA *SENSITIVITY TO IMPACT:* NE *SENSITIVITY TO STATIC DISCHARGE:*

SECTION 5 - REACTIVITY DATA

CHEMICAL STABILITY: YES ☒ NO ☐ *IF NO, UNDER WHICH CONDITIONS:*

INCOMPATIBILITY WITH OTHER SUBSTANCES: YES ☒ NO ☐ *IF SO, WHICH ONES?* Strong oxidizers.

REACTIVITY, AND UNDER WHICH CONDITIONS? Product is not considered highly reactive.

HAZARDOUS DECOMPOSITION PRODUCTS: Carbon Dioxide and Carbon Monoxide.

H **WARNINGS**

Take This Medication With Food.

This Drug May Impair The Ability To Drive Or Operate Machinery. Use Care Until You Become Familiar With Its Effects.

Do Not Take Other Medicines Without Checking With Your Doctor Or Pharmacist.

A **Miss Member**
1111 Meadow Drive, Anywhere, RI 00000

B DATE: 01/01/2016

C Metformin 500mg
IC Glucophage 500mg

D **TAKE 1 TABLET BY MOUTH UP TO 2 TIMES DAILY**

E **RX 1234567-09**

F QTY: 60
2 Refills
03/01/2016

G **Your Pharmacy**
1111 State Road, Rhode Island 00000

Rx

Nutrition Facts

Serving Size 1/2 cup (about 82g)
Servings Per Container 8

Amount Per Serving

Calories 200 Calories from Fat 130

	% Daily Value*
Total Fat 14g	**22%**
Saturated Fat 9g	**45%**
Trans Fat 0g	
Cholesterol 55mg	**18%**
Sodium 40mg	**2%**
Total Carbohydrate 17g	**6%**
Dietary Fiber 1g	**4%**
Sugars 14g	
Protein 3g	

Vitamin A 10% • Vitamin C 0%
Calcium 10% • Iron 6%

*Percent Daily Values are based on a 2,000 calorie diet. Your daily values may be higher or lower depending on your calorie needs:

		Calories:	2,000	2,500
Total Fat		Less than	65g	80g
Saturated Fat		Less than	20g	25g
Cholesterol		Less than	300mg	300 mg
Sodium		Less than	2,400mg	2,400mg
Total Carbohydrate			300g	375g
Dietary Fiber			25g	30g

Calories per gram:
Fat 9 • Carbohydrate 4 • Protein 4

Breakout Room Discussion #2

Would software usage labels be **helpful**?

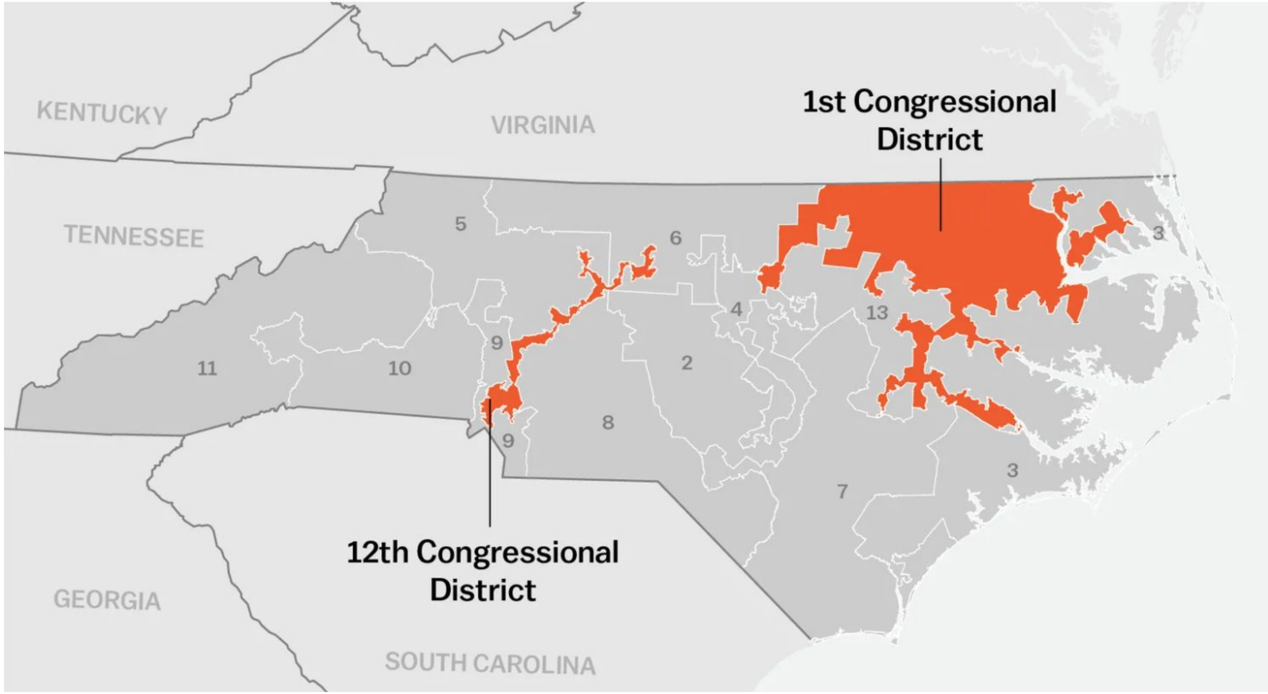
What would the labels say?

Announcements

Announcements

- The mid-quarter diagnostic is coming up at the end of this week.
  - You will have a 72-hour period of time from Friday to Sunday to complete the diagnostic.
  - The diagnostic is designed to take about an hour and a half to complete, but you can have up to 3 hours to work on it if you so choose.
  - The diagnostic will be administered digitally using Gradescope.
  - A practice diagnostic and many additional review materials have been posted on the diagnostic page.
- Assignment 3 is due on **Thursday, July 15 at 11:59pm.**

Gerrymandering & Algorithmic Thinking



The Law of the Relational Database

If the only tool you have is a relational database,
everything looks like a table.

Today's question

What problems *should* we solve with recursive backtracking?

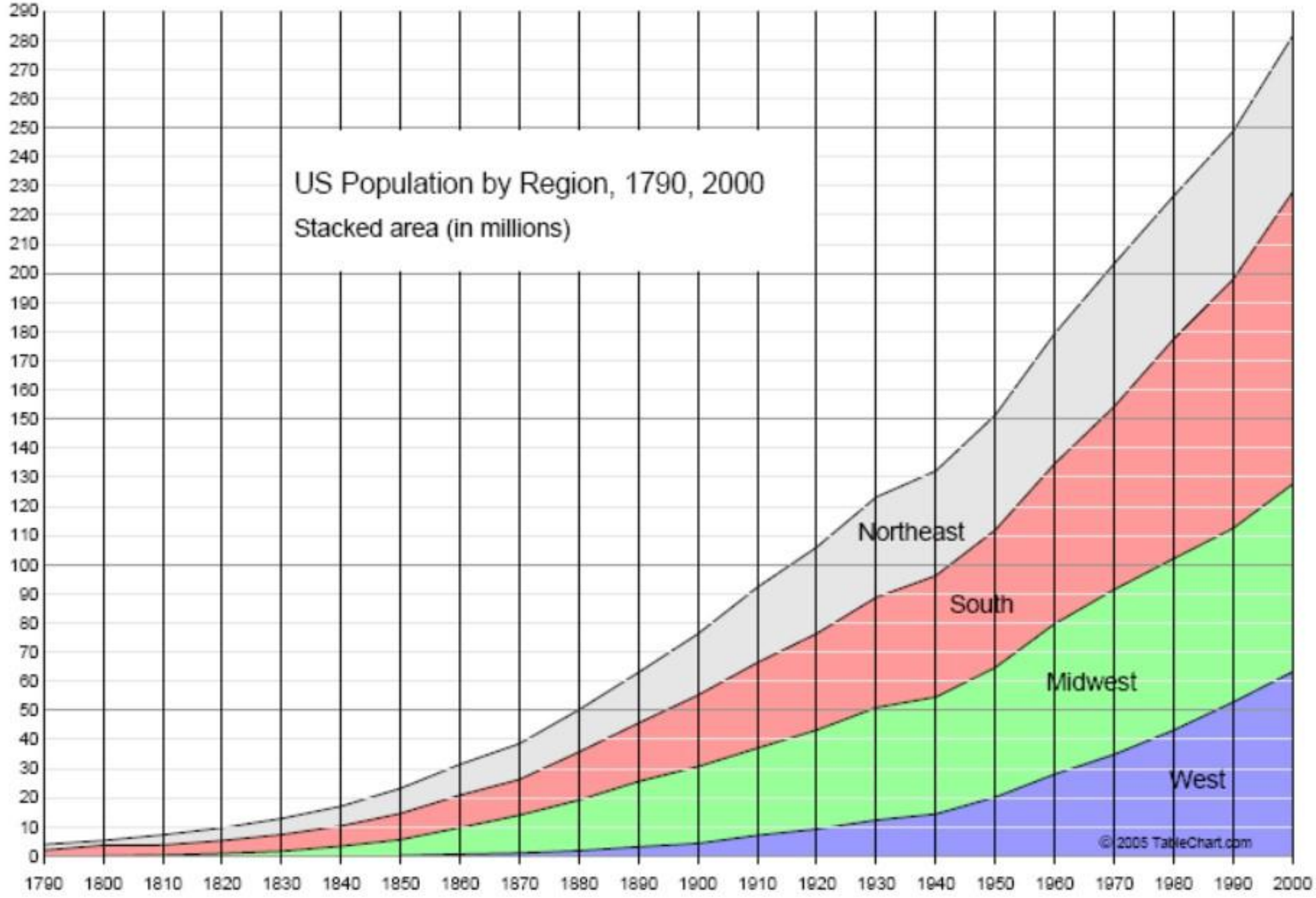
History of Voter Suppression

15th Amendment to the US Constitution (1870)

The "right of citizens of the United States to vote shall not be denied ***or abridged*** *by the United States or by any state on account of race, color, or previous condition of servitude."*

Addresses ***racial gerrymandering***, which abridges the right to vote on account of race.


HIGHLAND COLONY
BAPTIST CHURCH
VOTE
VOTE
VOTE
VOTE
VOTE


US Population by Region, 1790, 2000
Stacked area (in millions)
Northeast
South
Midwest
West
© 2005 TableChart.com
290
280
270
260
250
240
230
220
210
200
190
180
170
160
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0
1790 1800 1810 1820 1830 1840 1850 1860 1870 1880 1890 1900 1910 1920 1930 1940 1950 1960 1970 1980 1990 2000

Redistricting for House Elections

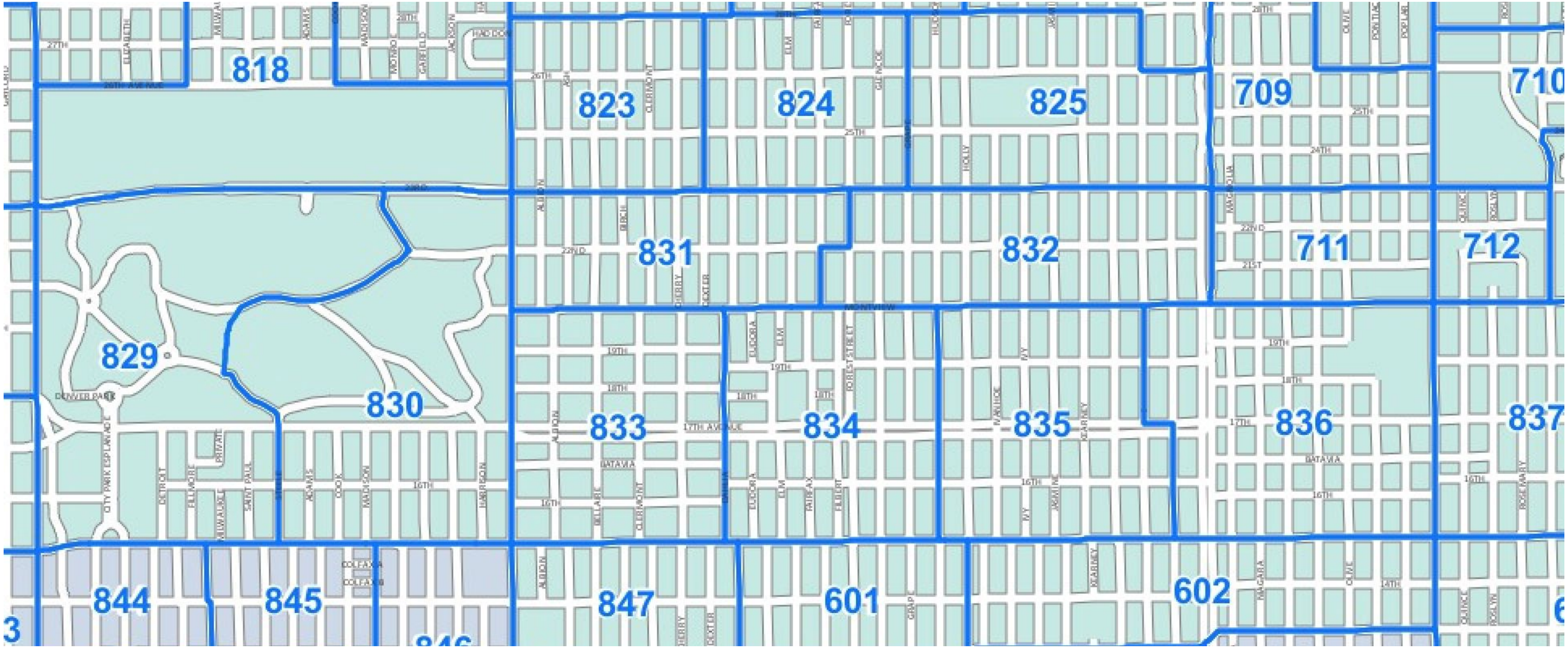
435 Districts of (roughly) equal population. Each elects a Representative

... but populations shift over time.

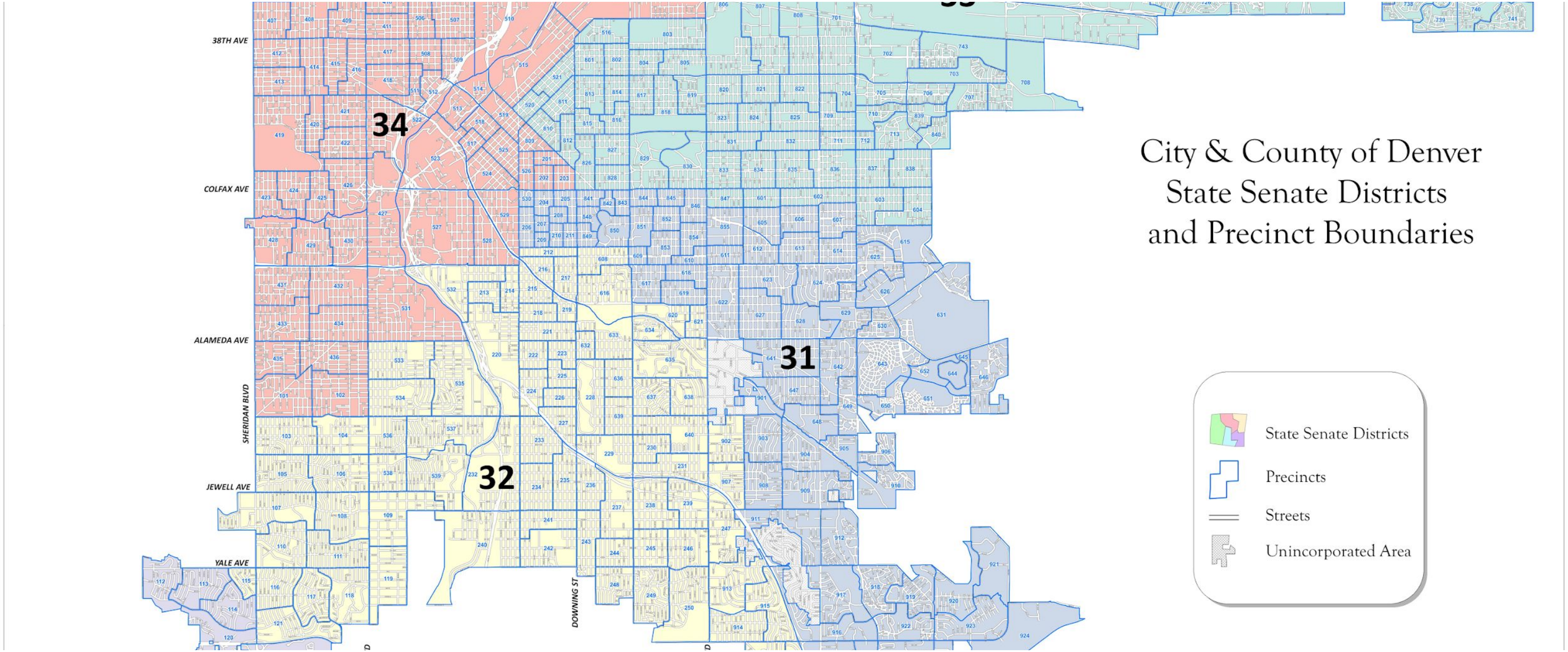
US Census measures population every 10 years

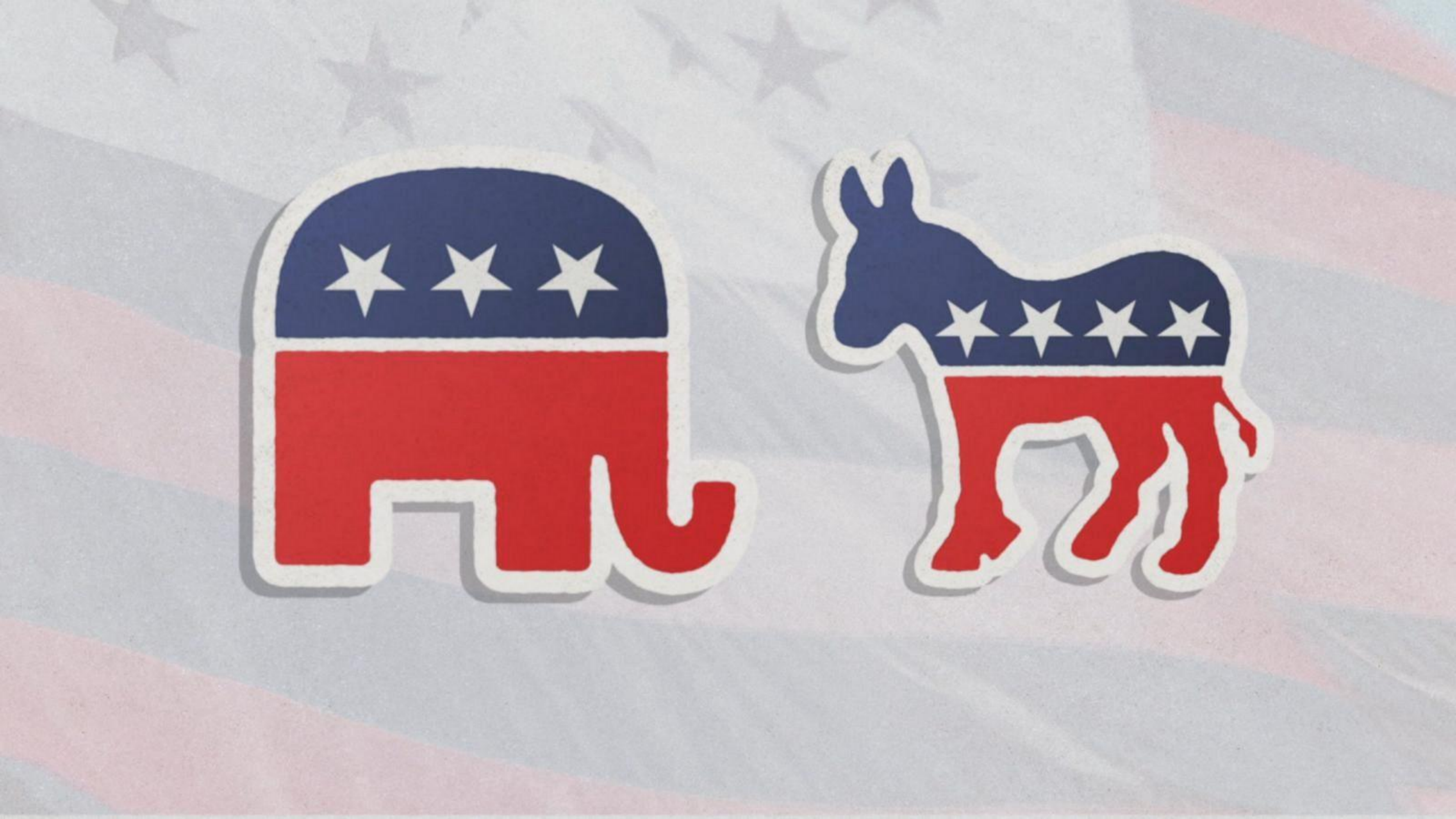
... so redistrict based on the new Census.

Precincts: tiny boxes



Districts: bigger ... boxes?





In order to find out, we need to understand the ...

HISTORY OF GERRYMANDERING

“The Gerry Mander” (1812)

The Computational Problem of Redistricting

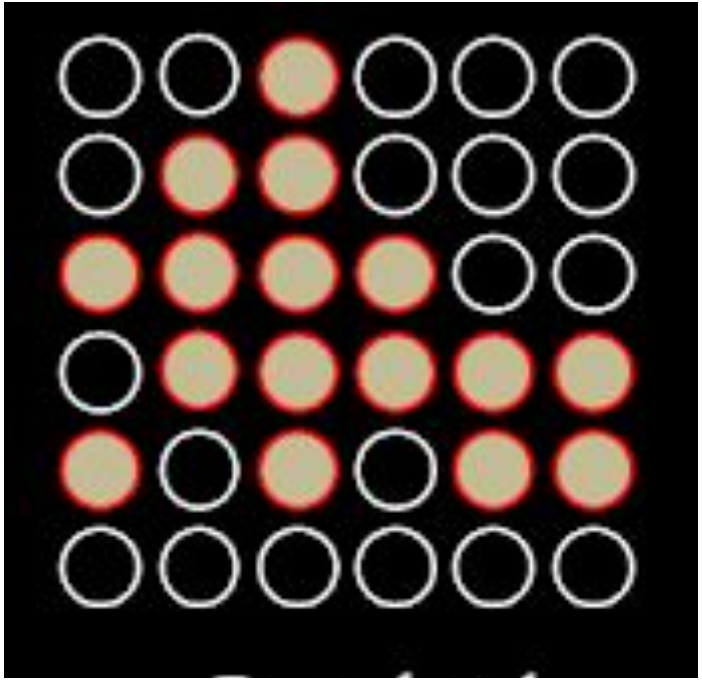
1. count residents (Census)
2. apportion Congressional Representatives to the states
3. partition the states by sorting precincts into districts

The Math (combinatorics) Problem of Redistricting

If a state has n residents (with attributes) and k Representatives, how to form k groups of approximately size n out of census blocks k in accordance with various rules and values.

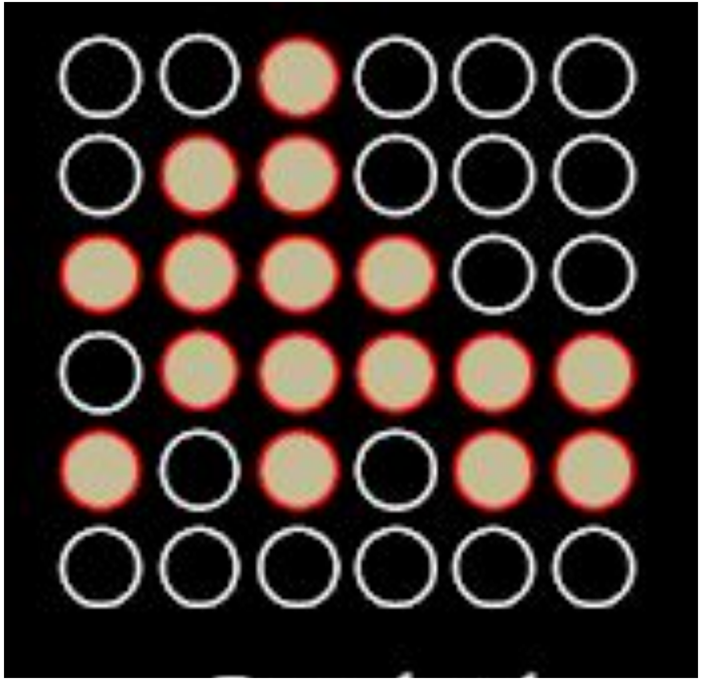
Ways Voting Rights can be Restricted

- There are four districts, each with 9 people.
- 36 people total
- 20 are in the majority group, 16 are in the minority group



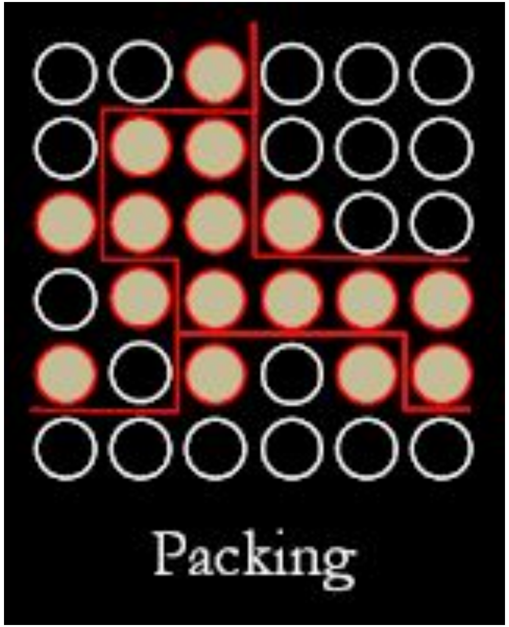
Ways Voting Rights can be Restricted

- If the minority and majority groups in are politically polarized and tend to prefer different parties, we would expect 2 candidates from each party to be elected.

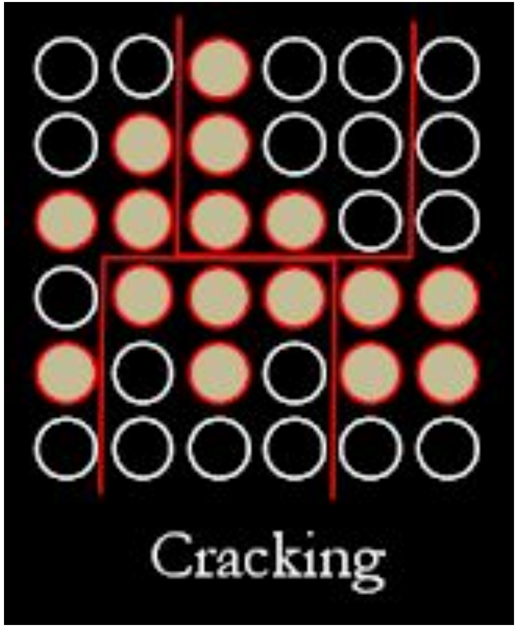


Racially Polarized Voting

only 1 minority-preferred candidate elected



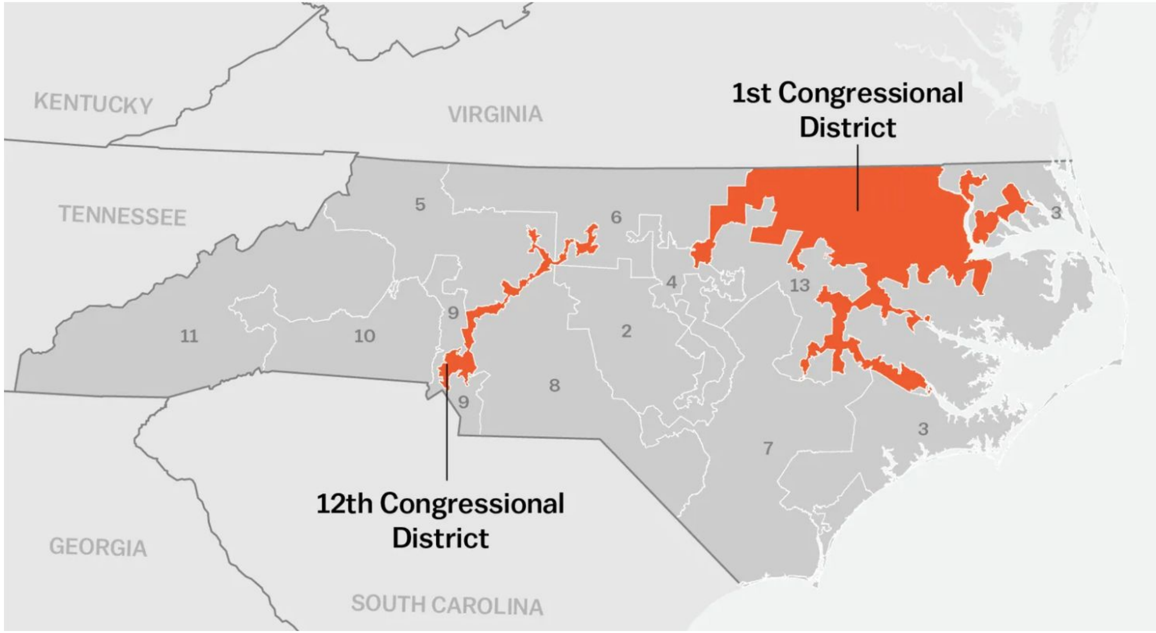
0 minority-preferred candidates elected



1965: Voting Rights Act

“The Civil Rights Division has the responsibility for enforcement of provisions of the Voting Rights Act that seek to ensure that redistricting plans do not discriminate on the basis of **race, color, or membership in a protected language minority group**.”

“[The Civil Rights Act] prohibits not only election-related practices and procedures that are intended to be racially discriminatory, but also those that are shown to have a racially discriminatory result.”



Racial Redistricting: Example of Packing a District

Republican lawmakers in the state, after the 2010 census, had redrawn the map to add more black voters into Districts 1 and 12. The Supreme Court concluded 5-3 that North Carolina violated the Equal Protections Clause of the 14th Amendment by separating voters in different districts on the basis of race without “sufficient justification” for doing so.

Partisan Gerrymandering defined

This is redistricting for the purpose of gaining or preserving an advantage for one political party at the cost of equitable political representation for voters.

Why has Gerrymandering Gotten Worse?

Veteran redistricter: “Give the chairman of a state redistricting committee a powerful enough computer and neighborhood-block-level Census data, so that he suddenly discovers he can draw really weird and aggressive districts—and he will.”

Software + big data making a problem worse?

- New Software: Maptitude, RedAppl, and autoBound
- New Data: block-by-block census data

What Problem-Solving Technique or Algorithm Should We Use?

Recursion to the Rescue!

- Exhaustive algorithm -- generate all the possible solutions, constrained by our rules and ideally matched to our values.

- Recursive backtracking (choose-explore-unchoose) to find all the potential districting maps

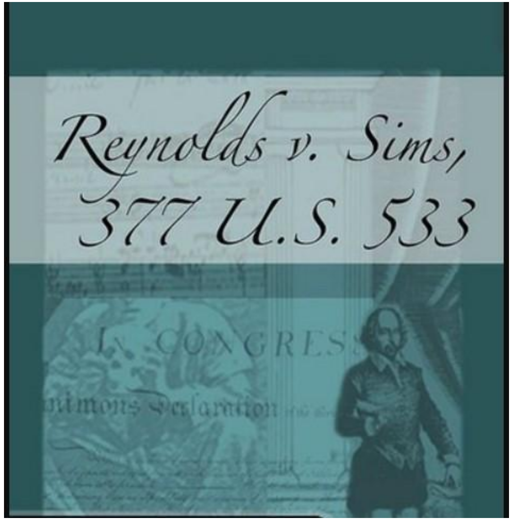
- Optimize: identify the worst (or best!) maps

CONSTRAINTS AND PRINCIPLES

What are our metrics?

How should we redistrict responsibly?

Principle 1: One person, one vote



"… as nearly as is practicable **one man's vote** in a Congressional election is **worth as much as another's**."

One Person, One Vote: The Efficiency Gap Metric

- Any vote cast for the losing party is a wasted vote for that party

- Any vote cast for the winning party that was more than the simple majority needed to win is also a wasted vote for that party

District	D Votes	R Votes	D Wasted Votes	R Wasted Votes	Net Wasted Votes
1	75	25	24	25	1 R
2	60	40	9	40	31 R
3	43	57	43	6	37 D
4	48	52	48	1	47 D
5	49	51	49	0	49 D
Total	275	225	173	72	101 D

One Person One Vote: The Efficiency Gap Metric

The circled votes on the top are “wasted” in that they didn’t influence the outcome in that district.

6 Congressional Districts

Lime Party wins 4 Congressional seats.
Teal Party wins 2 seats.

CALCULATING THE EFFICIENCY GAP

Efficiency Gap (EG) =

$$\frac{\text{abs(wasted green votes - wasted teal votes)}}{\text{total votes cast}}$$

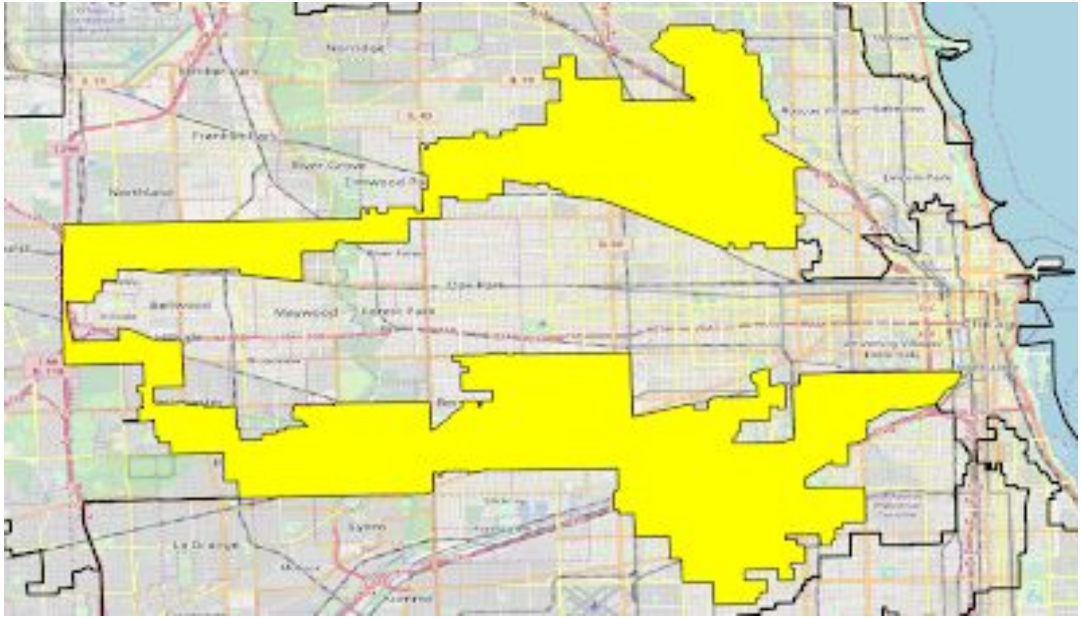
Principle 2: Communities of Interest

Interpretation A:
a community of interest is a community that shares an identity group

Interpretation B:
a community of interest is “compact” or geographically contiguous

Community of Interest: IL-04: “The Earmuffs”

- Follows a predominantly Latine community
- Created in the 1990s; elected the first Latine representative to Congress from the Midwest.



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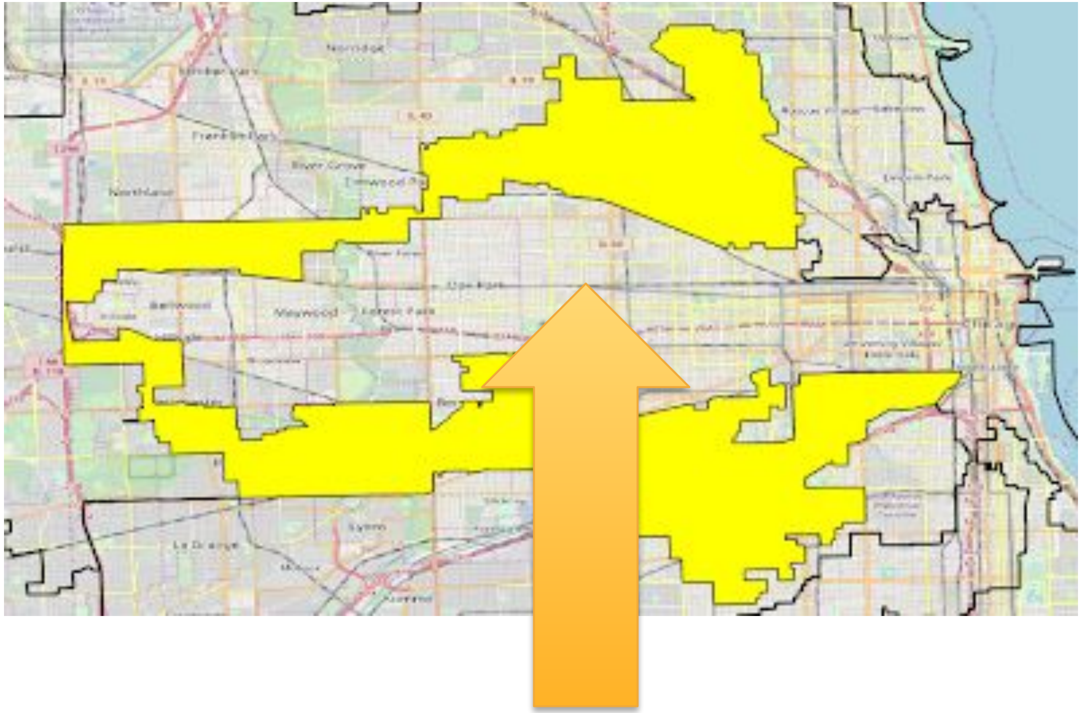
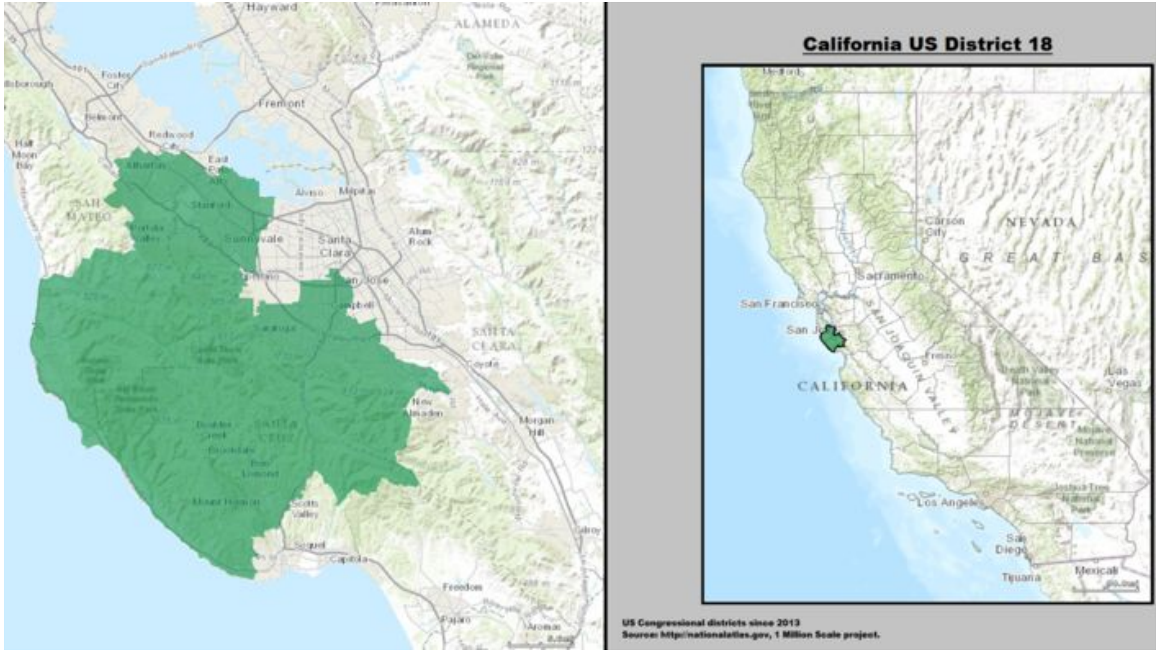


Surrounds IL-07, a predominantly African-American district

Interpretation B: Compactness

- Geography also matters for political representation
- People who live in the same physical area often have interests in common just in virtue of their location

What are the Interests of Santa Clara County & CA District 18?



- Silicon Valley
- Housing & Land Use Policy
- Transportation

Silicon Valley: A Different Story

- Semiconductors & microprocessor manufacturing (silicon!) in the 1950s-1990s by Fairchild, Hewlett-Packard, Intel, Apple, Atari, Xerox, etc.
- Hardware is mostly gone, but left behind are 23 Superfund sites contaminated with toxic chemicals
- Highest density of Superfund sites in the country
- Groundwater is safe, but plumes of toxic gas can escape, including at Google's Quad Campus in 2012-2013
- **What do we in Santa Clara Country have in common?**
  A special desire to clean up our Superfund sites!

THE ROLE OF COMPUTER SCIENTISTS

What should we prioritize in redistricting?
And what is a legitimate way to choose?

PRINCIPLE 1: ONE PERSON, ONE VOTE

PRINCIPLE 2: COMMUNITIES OF INTEREST

Who Should Control Redistricting?

- Biggest problem: US is one of the few countries in which politicians have control over redistricting.

- In most other countries, all redistricting is done by an independent commission.

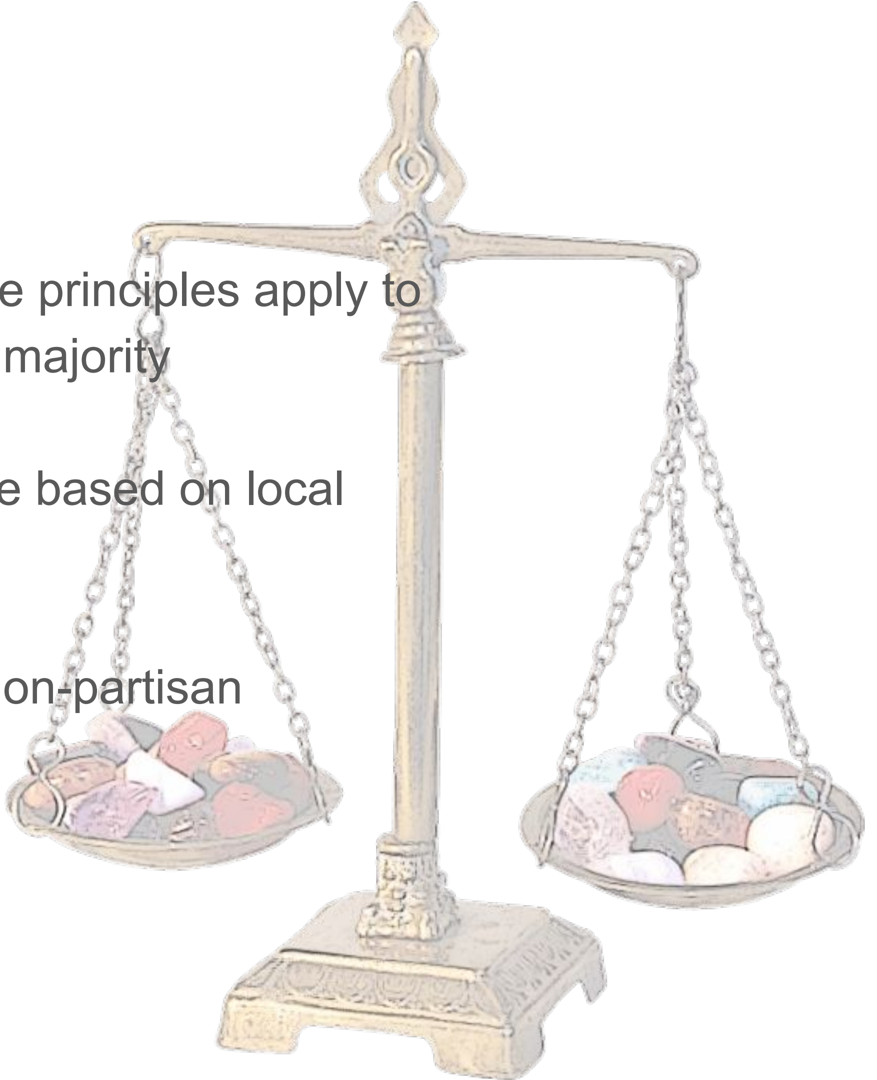
- **Politicians districting based on their own interests is unfair on either principle.**

Consistency for Fairness

- However we balance these two, the same principles apply to every district, regardless of who is in the majority

- Redistricting principles should not change based on local circumstances (or politician interests)

- Easier to implement consistency with a non-partisan independent commission



Community Control for Justice

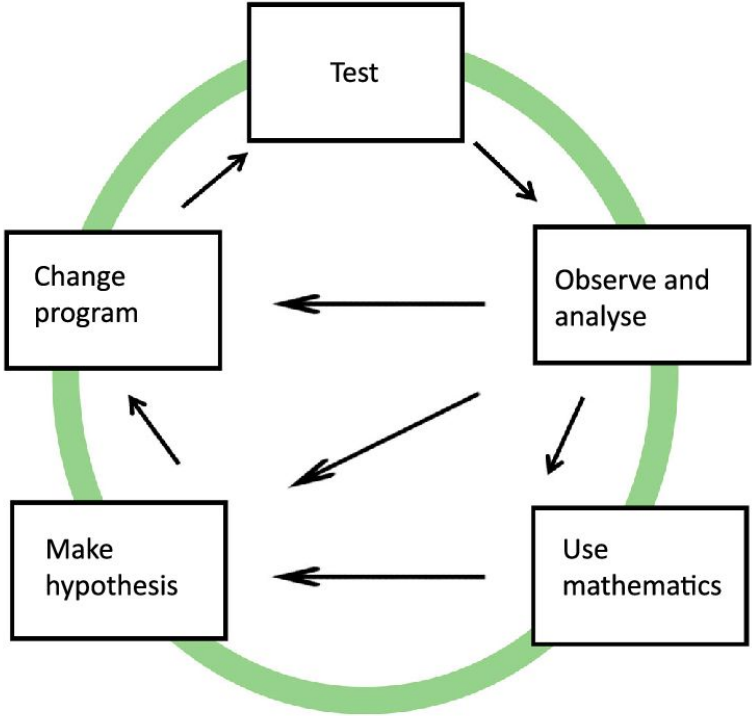
- Allow local communities to directly vote on or otherwise choose the balance of principles which should guide districting (and thus their own representation).

- Have an independent non-partisan commission implement the principles in drawing up a map.

Iterating and Revisiting our Solutions

- Sometimes our first algorithmic solution does not work as expected

- The Voting Rights Act (VRA) attempts to indicate the fair middle ground between packing and cracking, but the interpretation of the VRA has been closer to packing



Ethics of Care

- Few social problems can be solved exactly once by an algorithm
- Any algorithmic solution should be revisited often as society changes and we understand its implications better.
- Caring for an algorithm, and for the community that relies on it, can mean updating the data ... or changing the algorithm and implementation.
- Creating “living” algorithms and machine learning models that can grow and change over time is a huge focus in data science and AI

Roles you will be ready to take on after CS106B

- Make better systems yourself!

- Using your CS106B knowledge to advocate for communities affected by unfair practices that rely on algorithmic decision-making

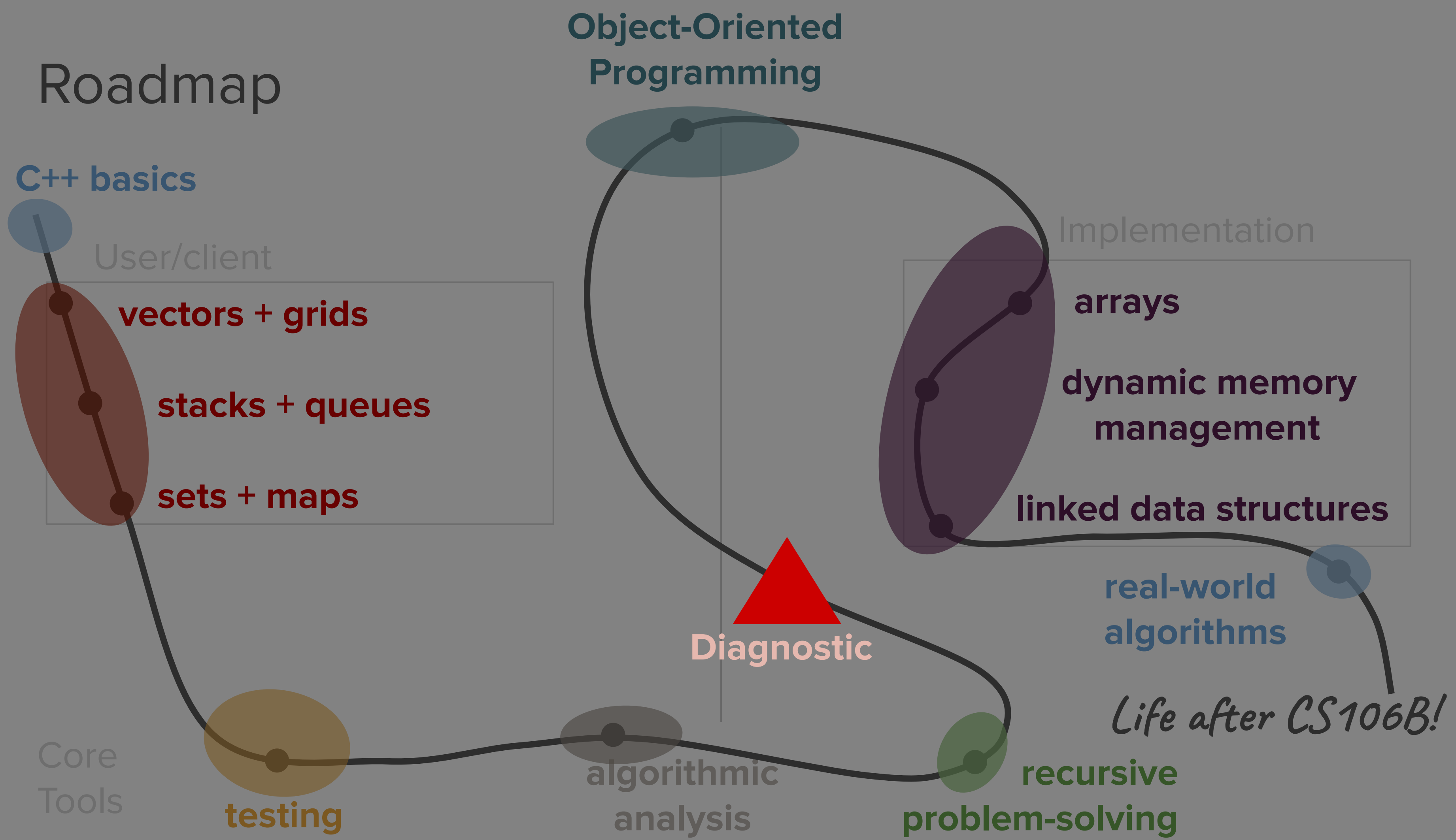
- Formulating problems carefully based on knowledge of the history of the problem and understanding what people affected by the problem may see as a solution

What problems *should* we solve with recursive backtracking?

Congratulations on making it this far!

What’s next?

Roadmap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vectors + grids
stacks + queues
sets + maps
Implementation
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Core
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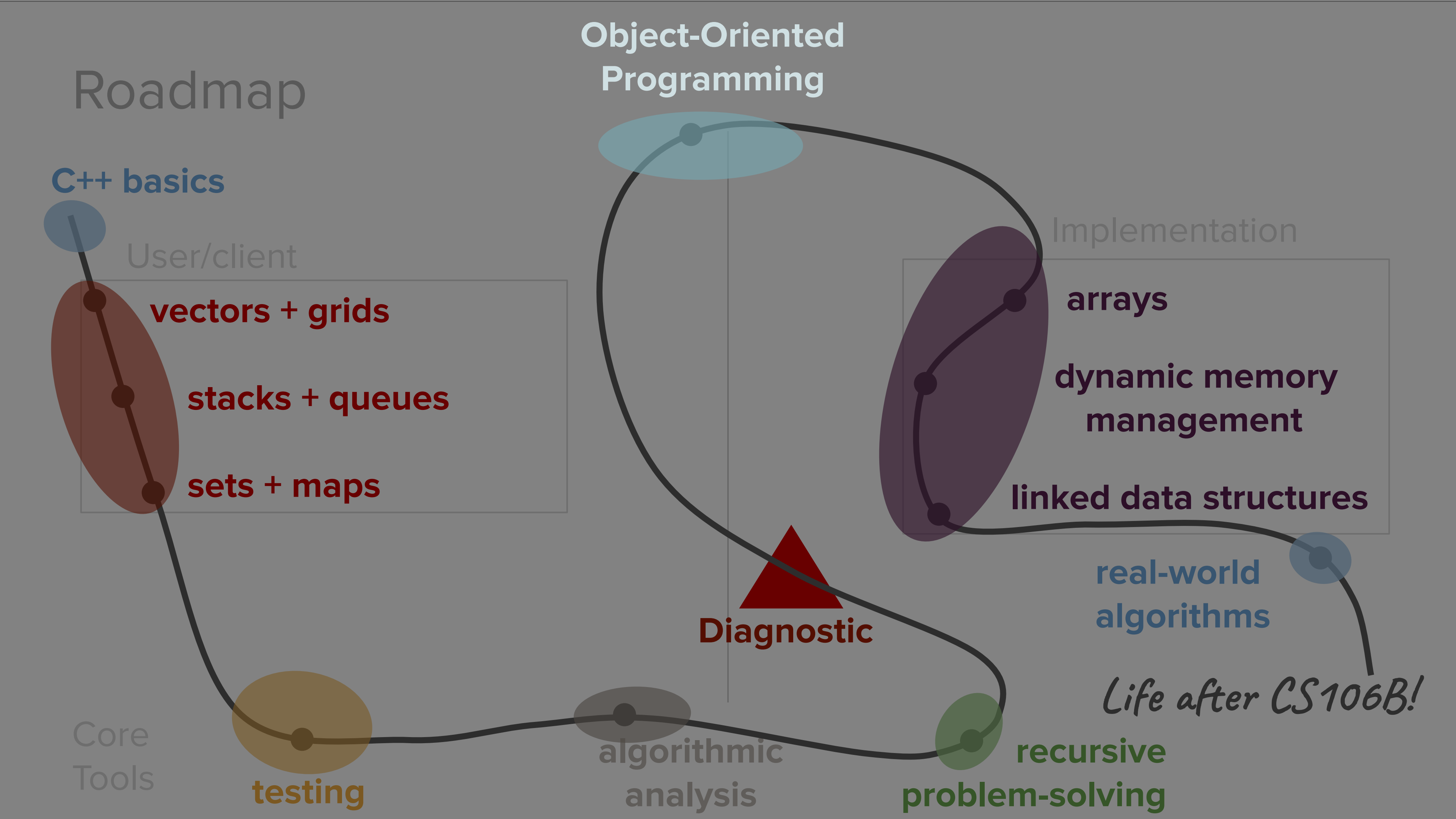
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Classes and Object-Oriented Programming