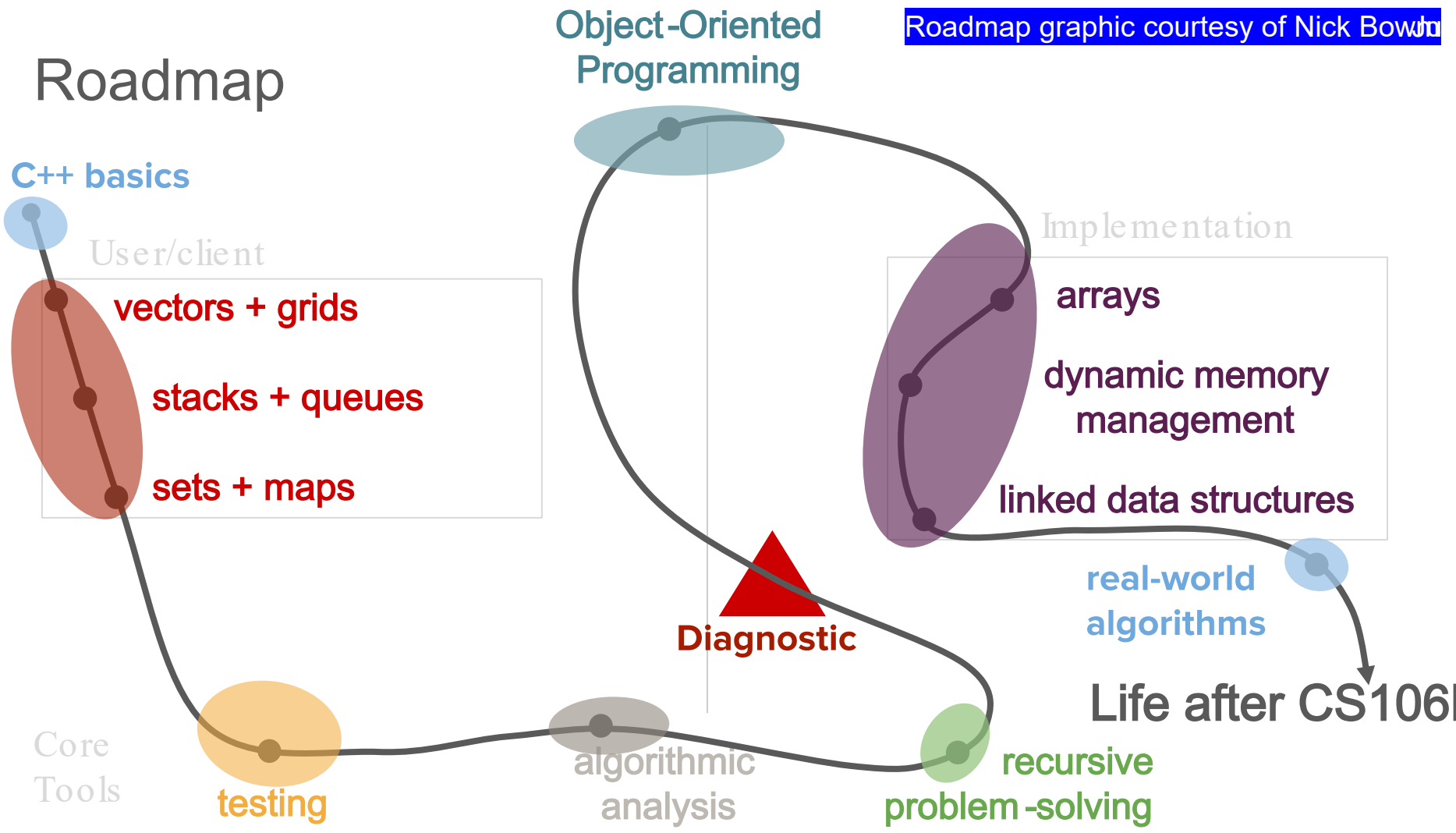

Binary Search Trees
What is your favorite type of tree?
(e.g., oak, redwood, maple)
put your answers in the chat

Roadmap


Object-Oriented Programming
Roadmap graphic courtesy of Nick Bowm
C++ basics
User/client
vectors + grids
stacks + queues
sets + maps
Implementation
arrays
dynamic memory management
linked data structures
real-world algorithms
Diagnostic
Life after CS106
Core Tools
testing
algorithmic analysis
recursive problem-solving

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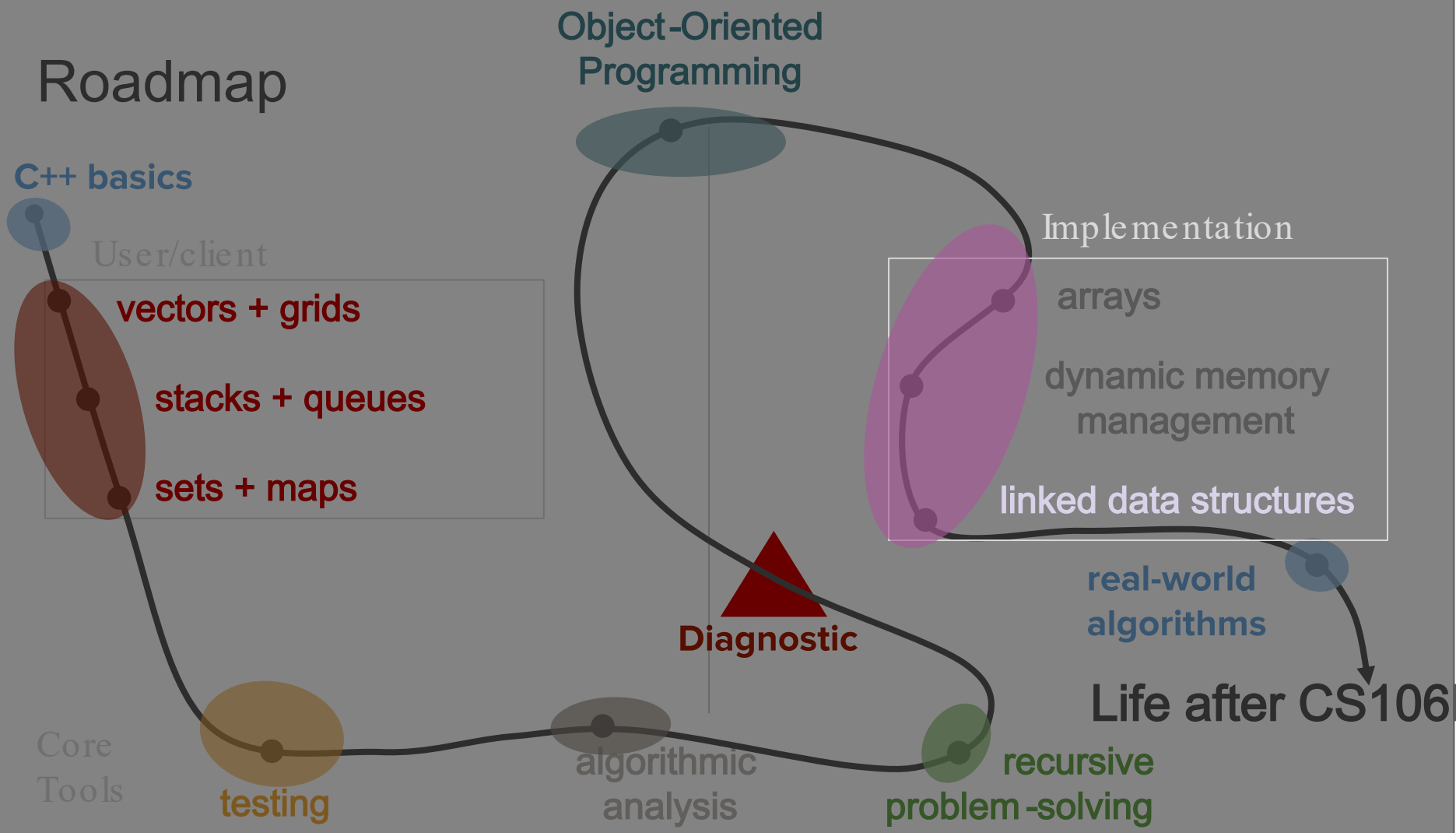
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Today's questions

How can we take advantage of trees to structure and efficiently manipulate data?

Today's topics

1. What is a binary search tree (BST)?

2. Building efficient BSTs

1. Implementing Sets with BSTs

Review

[trees]

Definition

tree

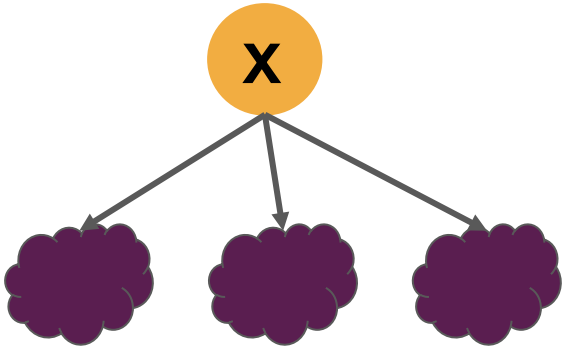
A tree is hierarchical data organization structure composed of a root value linked to zero or more non-empty subtrees.

What is a tree?

A tree is either...

An empty data structure, or...

A single node (parent), with zero or more non-empty subtrees (children)



Tree terminology

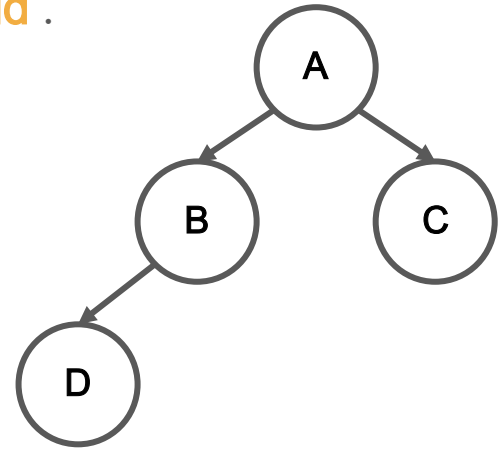
- Types of nodes
  - The **root** node defines the "top" of the tree.
  - Every node has 0 or more **children** nodes descended from it.
  - Nodes with no children are called **leaf nodes**.
  - Every node in a tree has exactly one **parent** node (except for the root node).

- Terminology for quantifying trees
  - A **path** *between two nodes* traverses edges between parents and their children, and **length** *of a path* is the number of edges between the two nodes.
  - The **depth** *of a node* is the length of the path (# of edges) between the root and that node.
  - The **height** *of a tree* is the number of nodes in the longest path through the tree (i.e. the number of **levels** in the tree).

Binary trees

- A **binary tree** is a tree where every node has either 0, 1, or 2 children. No node in a binary tree can have more than 2 children.

- Typically, the two children of a node in a binary tree are referred to as the **left child** and the **right child** .



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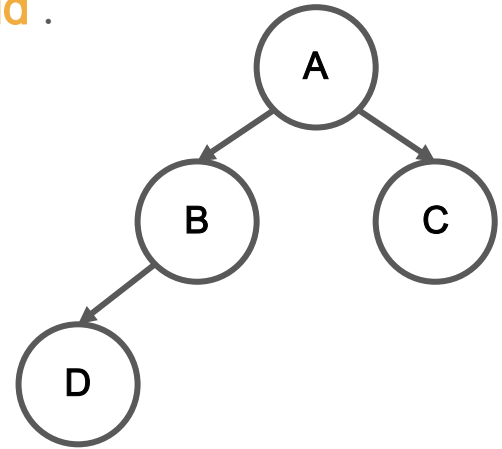


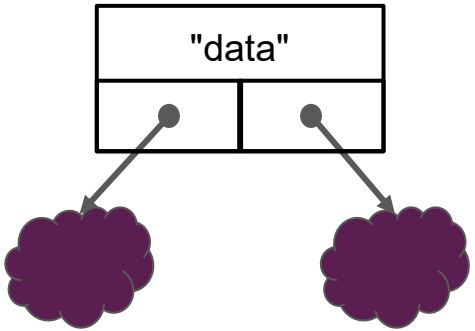

```
struct TreeNode {
    string data;
    TreeNode* left;
    TreeNode* right;
}
```

What is a tree in C++?

A tree is either...

An empty tree represented by **`nullptr`**, or...

A single **`TreeNode`**, with 0, 1, or 2 non-null pointers to other **`TreeNodes`**



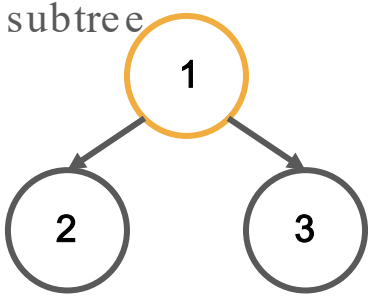
Building a tree

- Building a tree is very similar to the process of building a linked list.
- We create new nodes of the tree by dynamically allocating memory.
- We start by first creating the leaf nodes and then creating their parents.
- We integrate the parents into the tree by rewiring their **`left`** and **`right`** pointers to the already-created children.

Traversing a tree - recursively!

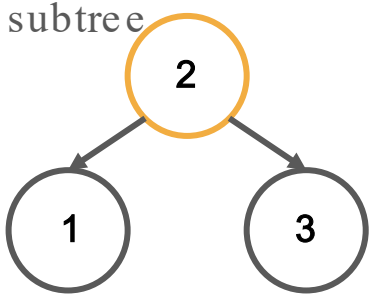
Pre-order

1. "Do something" with the current node
2. Traverse the left subtree
3. Traverse the right subtree



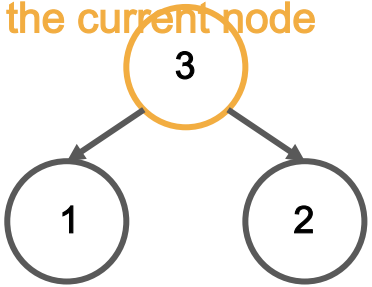
In-order

1. Traverse the left subtree
2. "Do something" with the current node
3. Traverse the right subtree



Post-order

1. Traverse the left subtree
2. Traverse the right subtree
3. "Do something" with the current node



Freeing a tree!

1. What kind of traversal would you use?

2. Where does the delete call go?

Let’s code it!

freeTree()

Key Idea: The distance from each element (node) in a tree to the top of the tree (the root) is small, even if there are many elements.

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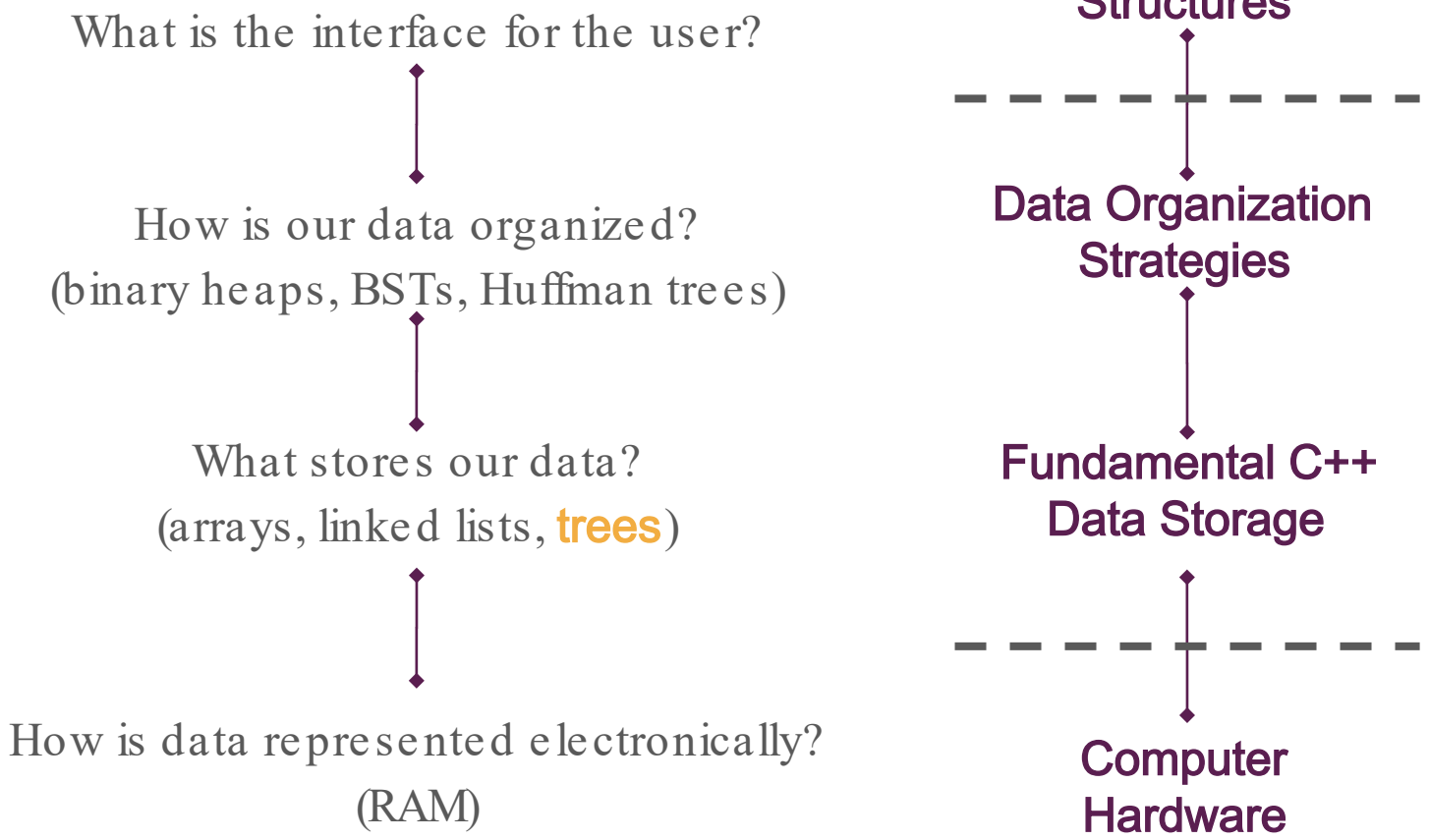
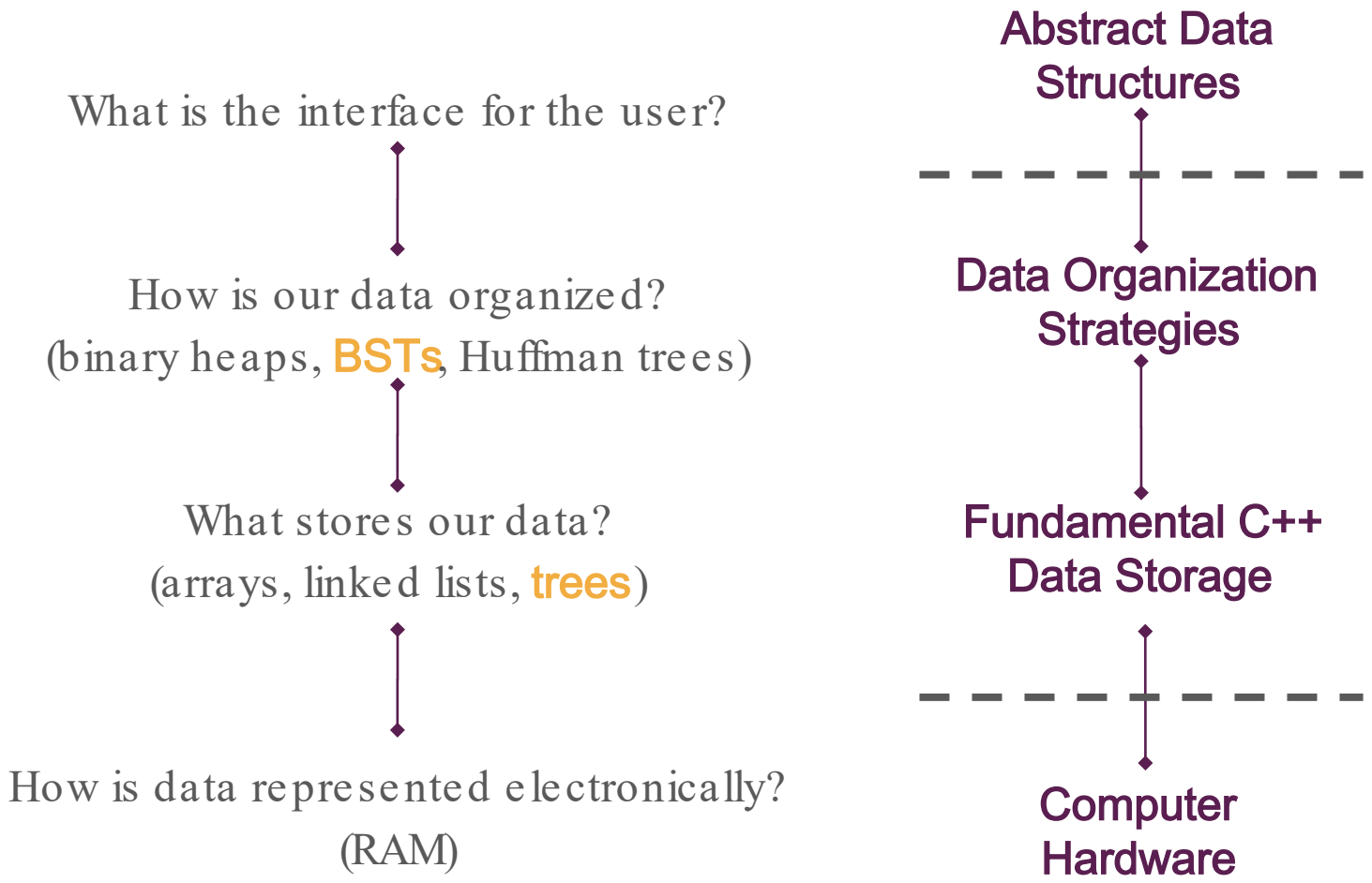
How can we take advantage of trees to structure and manipulate data?

Revisiting our levels of abstraction...

Levels of abstraction



Levels of abstraction



ADT Big-O Matrix

- Vectors
  - `.size() - O(1)`
  - `.add() - O(1)`
  - `v[i] - O(1)`
  - `.insert() - O(n)`
  - `.remove() - O(n)`
  - `.clear() - O(n)`
  - `traversal - O(n)`
- Grids
  - `.numRows()/.numCols() - O(1)`
  - `g[i][j] - O(1)`
  - `.inBounds() - O(1)`
  - `traversal - O(n^2)`
- Queues
  - `.size() - O(1)`
  - `.peek() - O(1)`
  - `.enqueue() - O(1)`
  - `.dequeue() - O(1)`
  - `.isEmpty() - O(1)`
  - `traversal - O(n)`
- Stacks
  - `.size() - O(1)`
  - `.peek() - O(1)`
  - `.push() - O(1)`
  - `.pop() - O(1)`
  - `.isEmpty() - O(1)`
  - `traversal - O(n)`
- Sets
  - `.size() - O(1)`
  - `.isEmpty() - O(1)`
  - `.add() - O(log(n))`
  - `.remove() - O(log(n))`
  - `.contains() - O(log(n))`
  - `traversal - O(n)`
- Maps
  - `.size() - O(1)`
  - `.isEmpty() - O(1)`
  - `m[key] - O(log(n))`
  - `.contains() - O(log(n))`
  - `traversal - O(n)`

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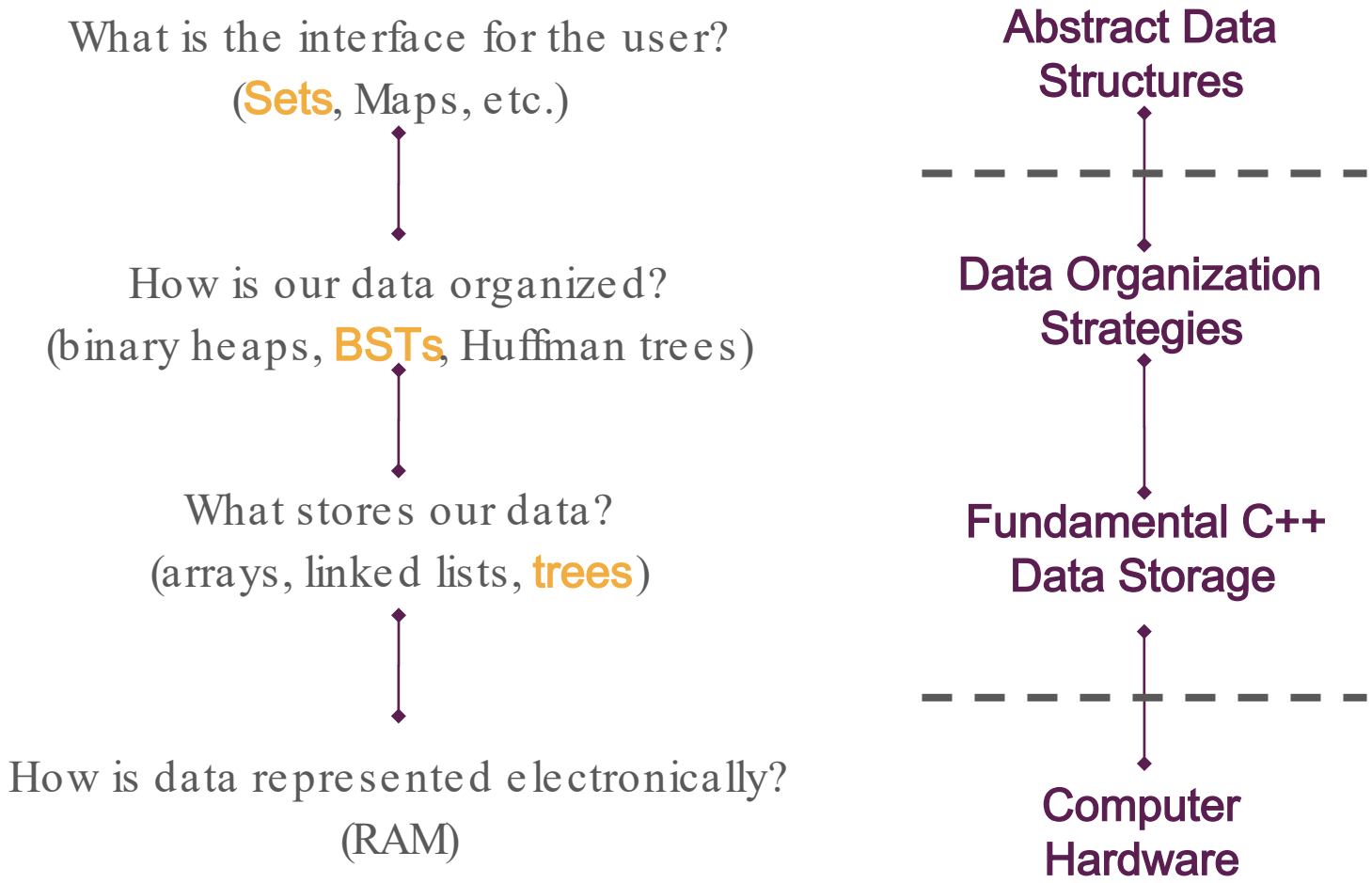
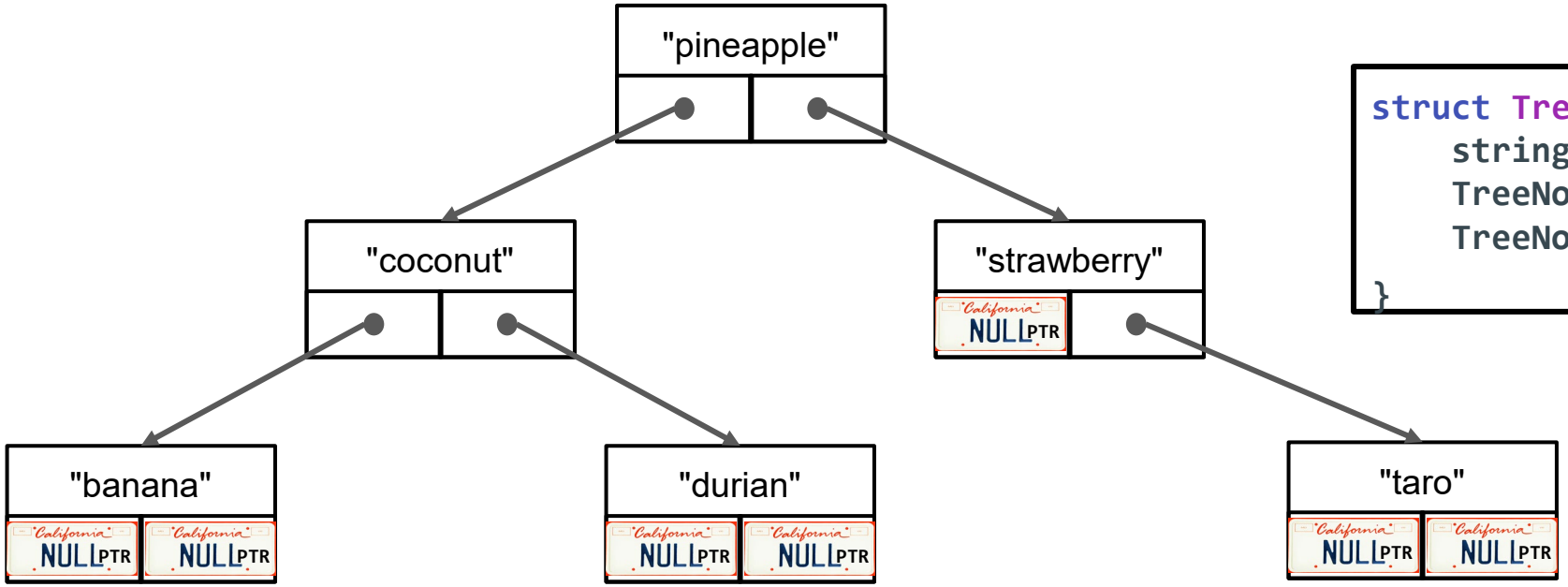
Levels of abstraction



What is a binary search tree (BST)?

Building Trees Programmatically

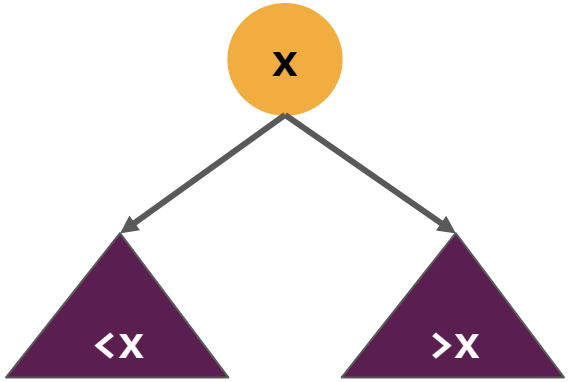



```
struct TreeNode {
    string data;
    TreeNode* left;
    TreeNode* right;
}
```

A binary search tree is either...

an empty data structure represented by nullptr or...

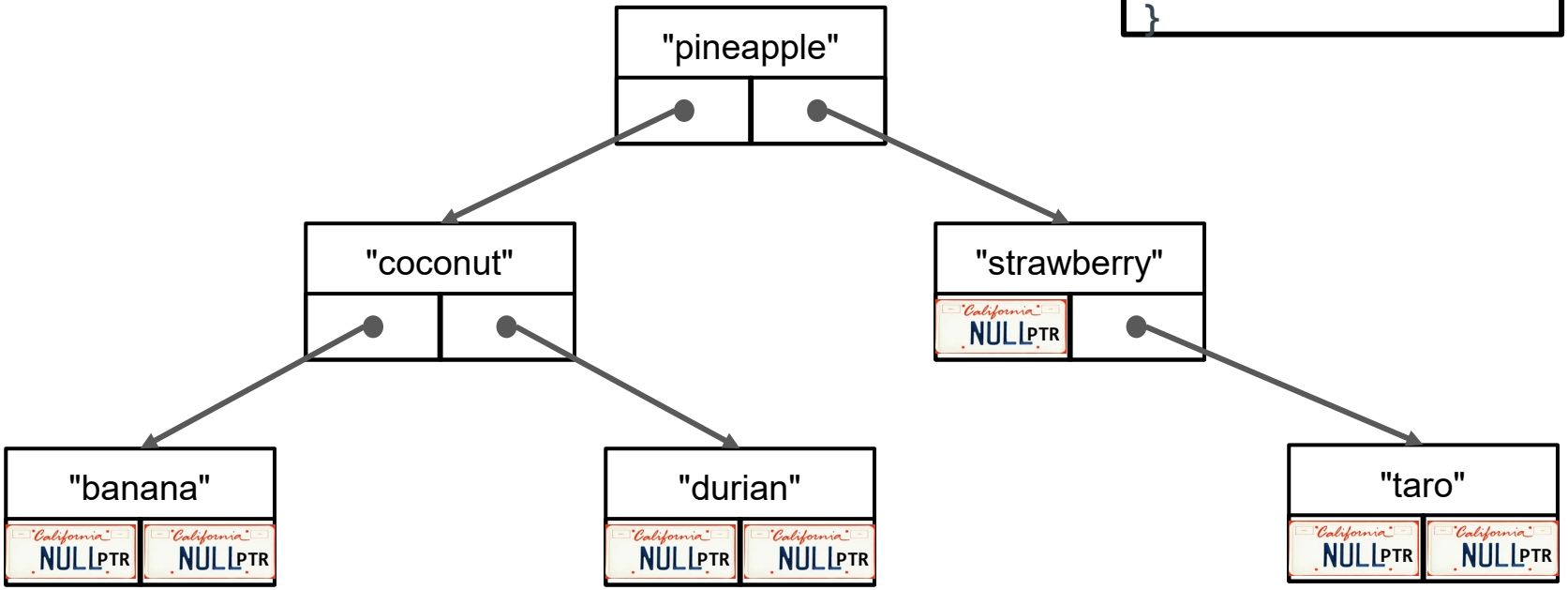
a single node, whose left subtree is a BST of smaller values than **x**…



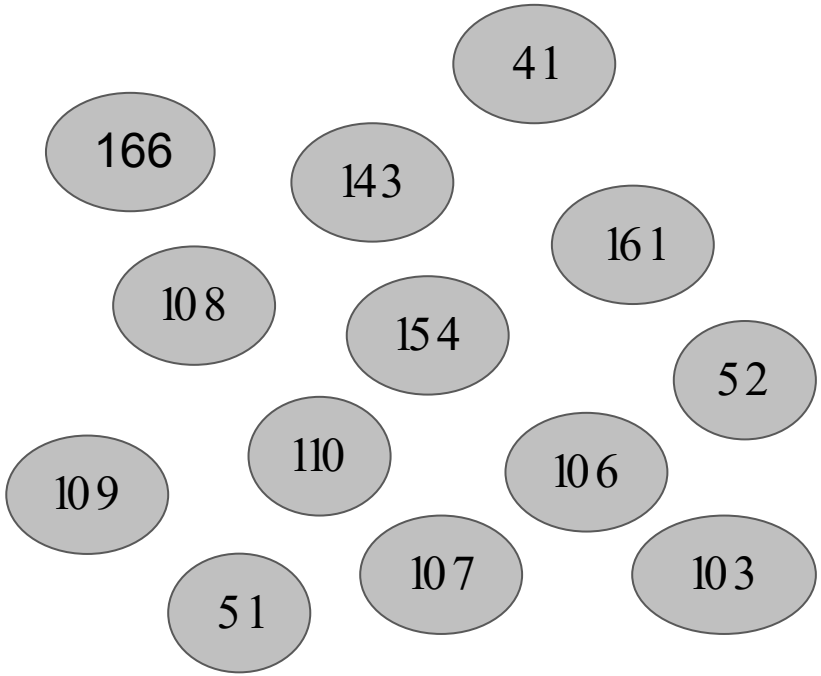
and whose right subtree is a BST of larger values than **x**.

Building Trees Programmatically

```
struct TreeNode {
    string data;
    TreeNode* left;
    TreeNode* right;
}
```



Building a BST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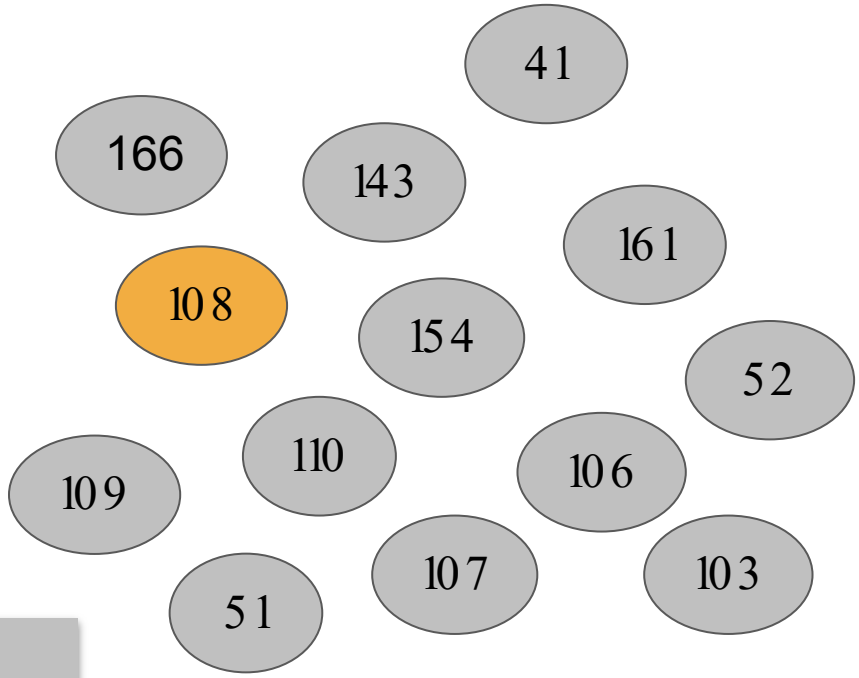
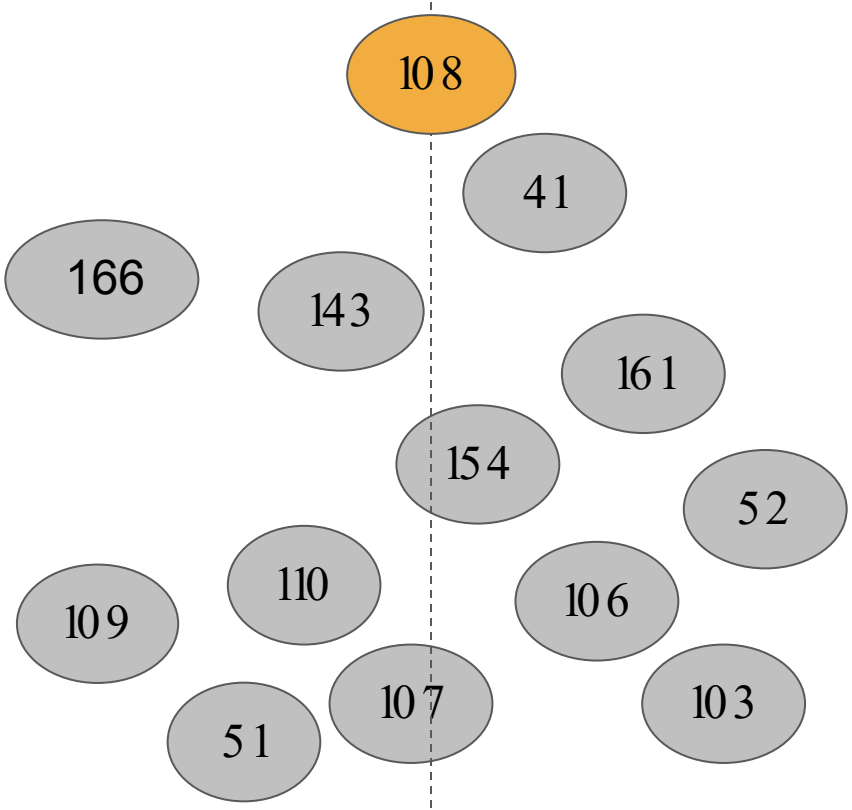


Pick the median element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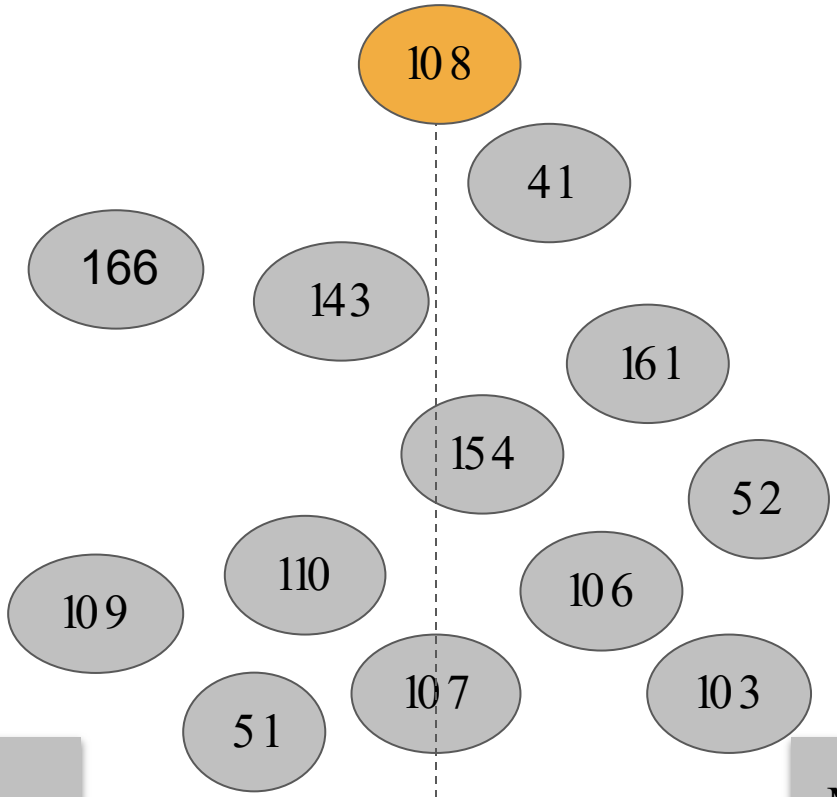
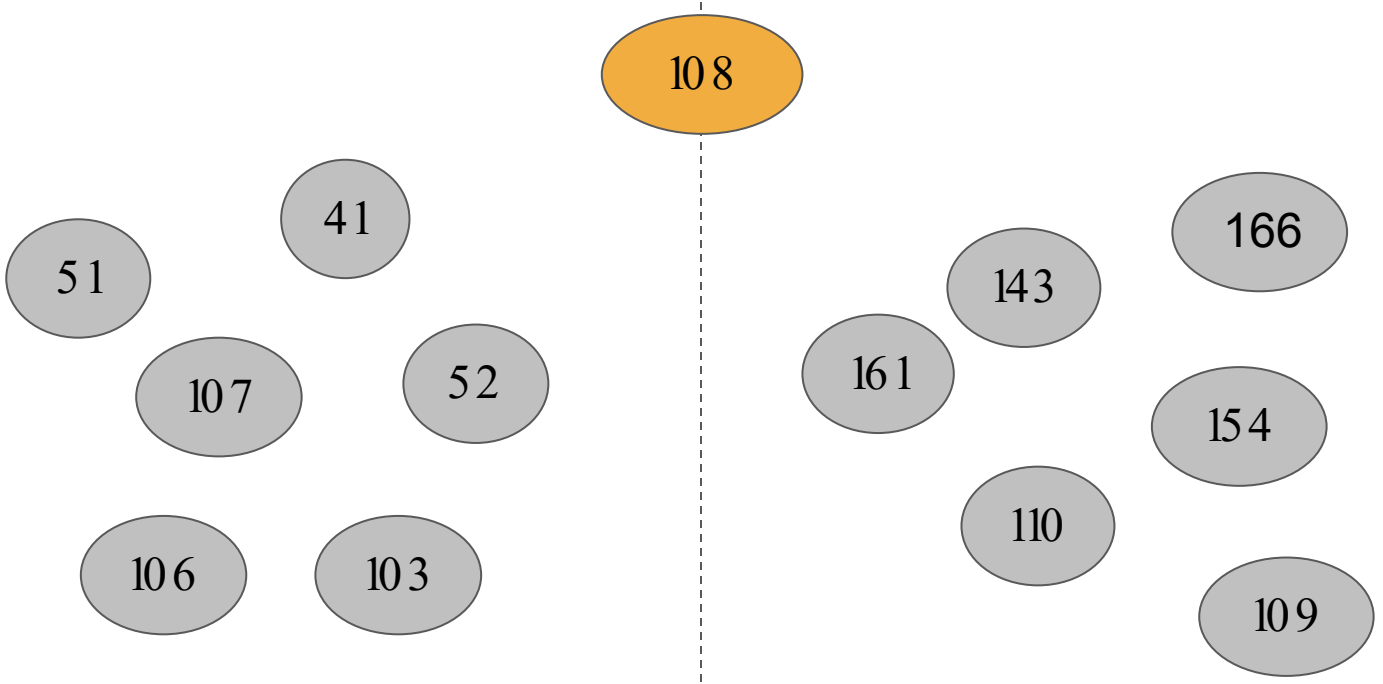


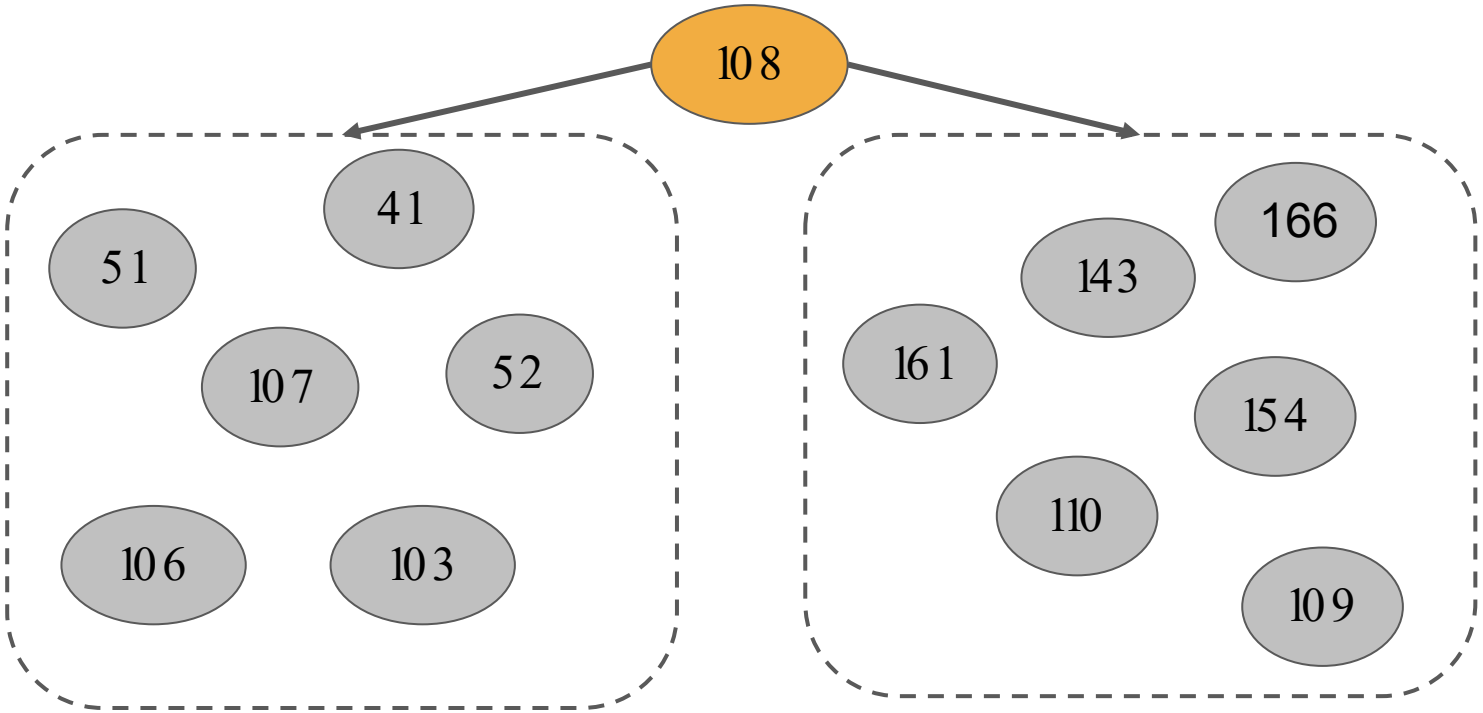
Move elements less than 108 to this side

Move elements greater than 108 to this side



Move elements less than 108 to this side

Move elements greater than 108 to this side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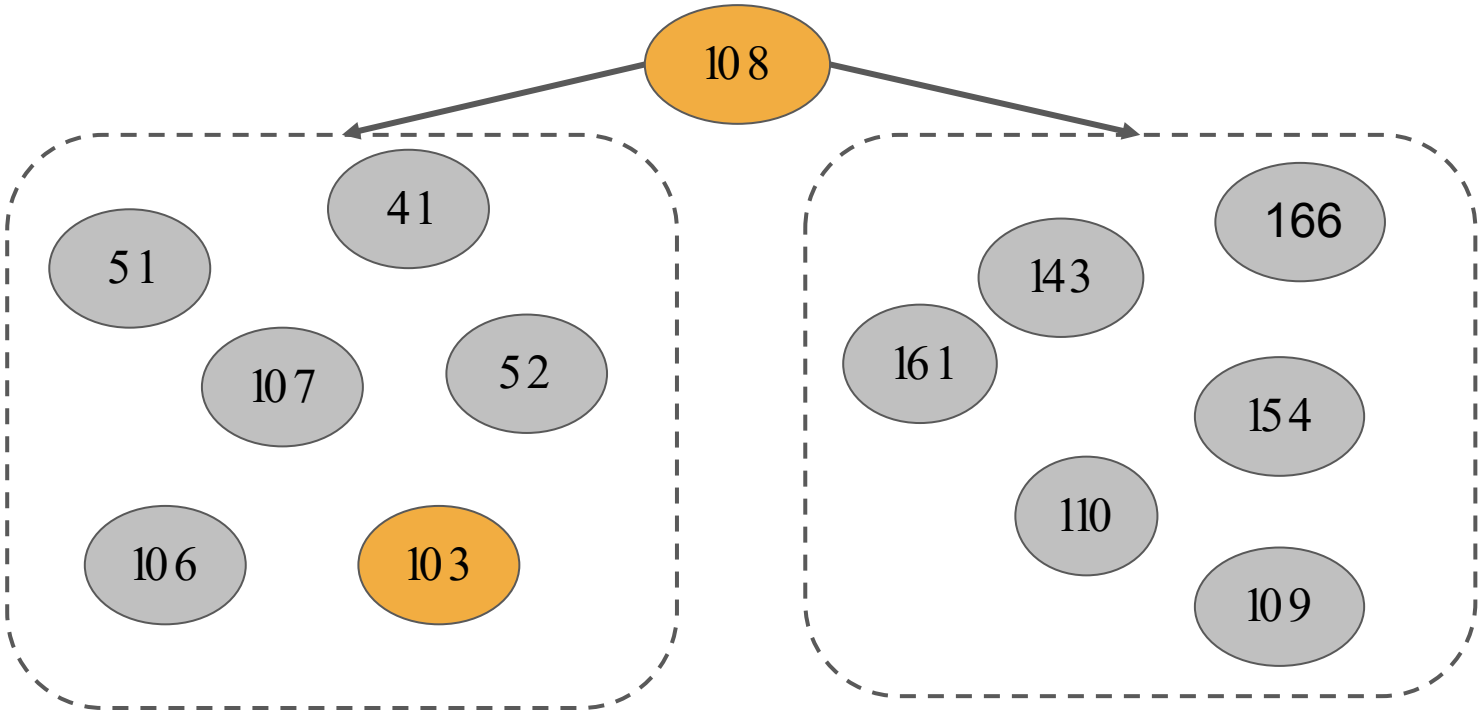
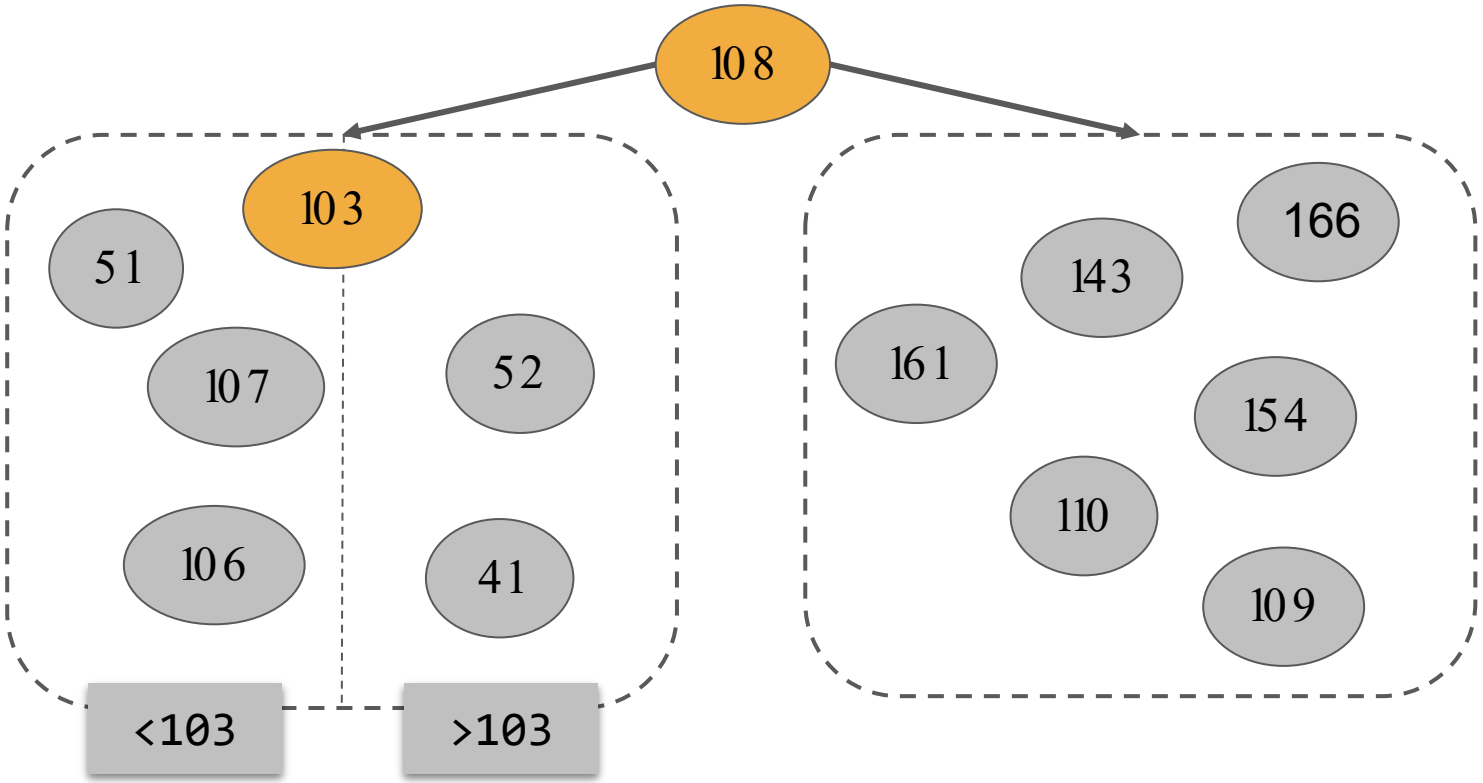


Pick the median element of the left side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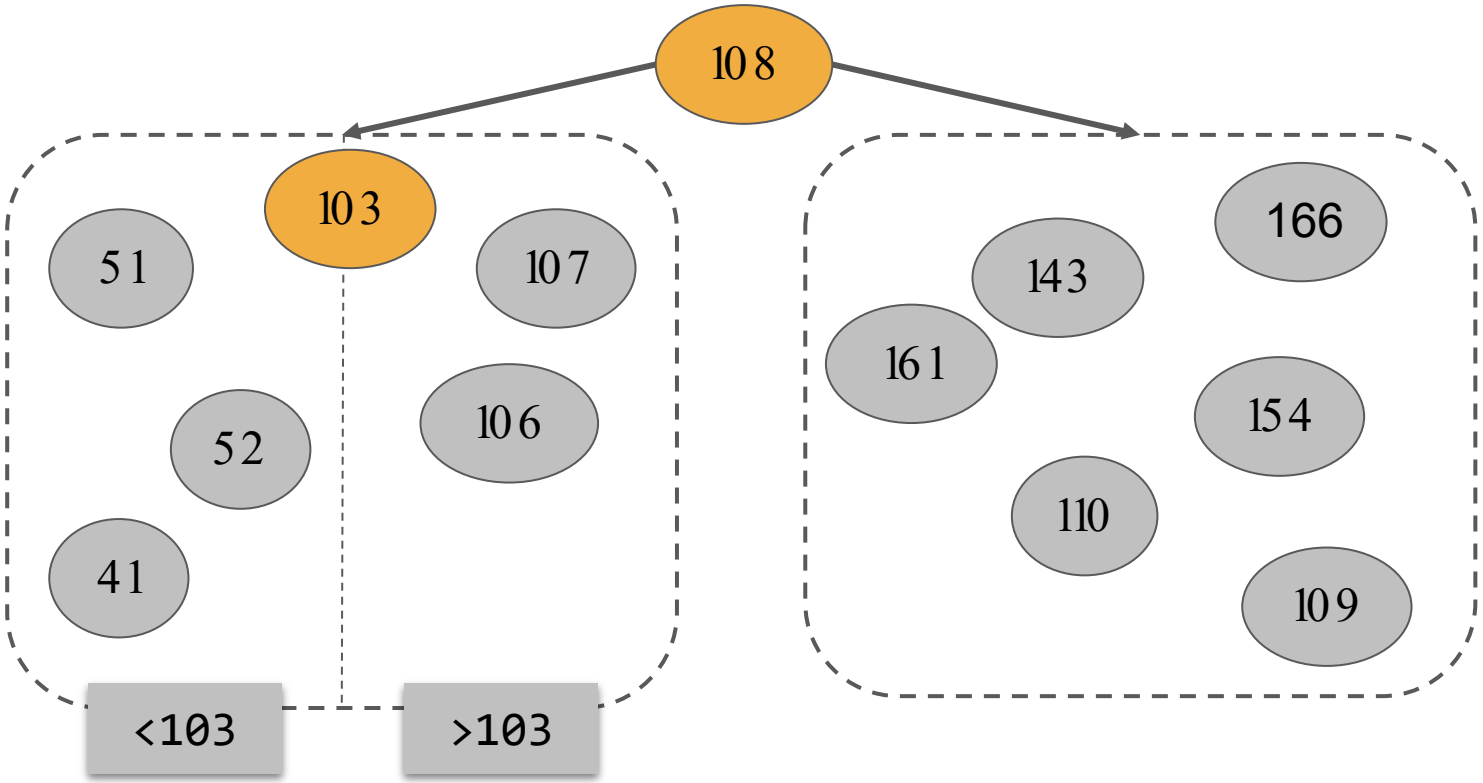
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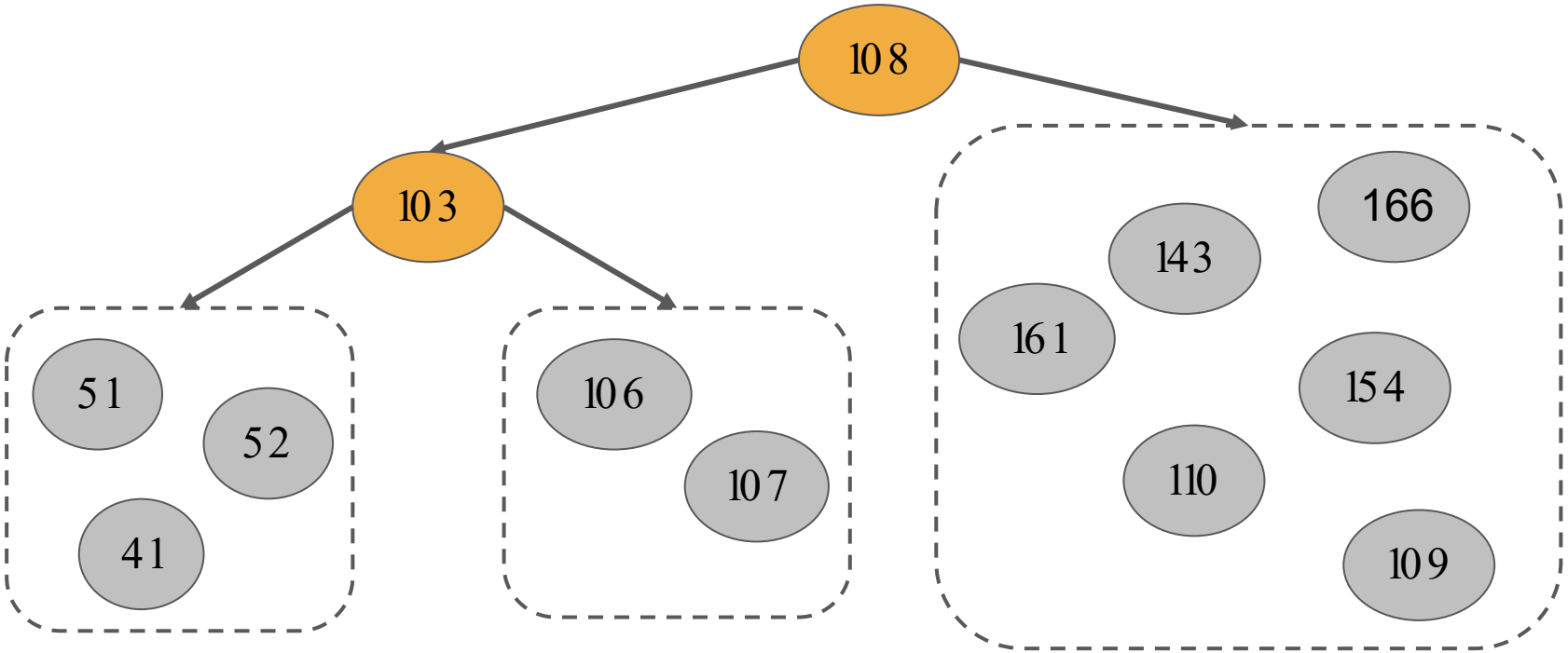
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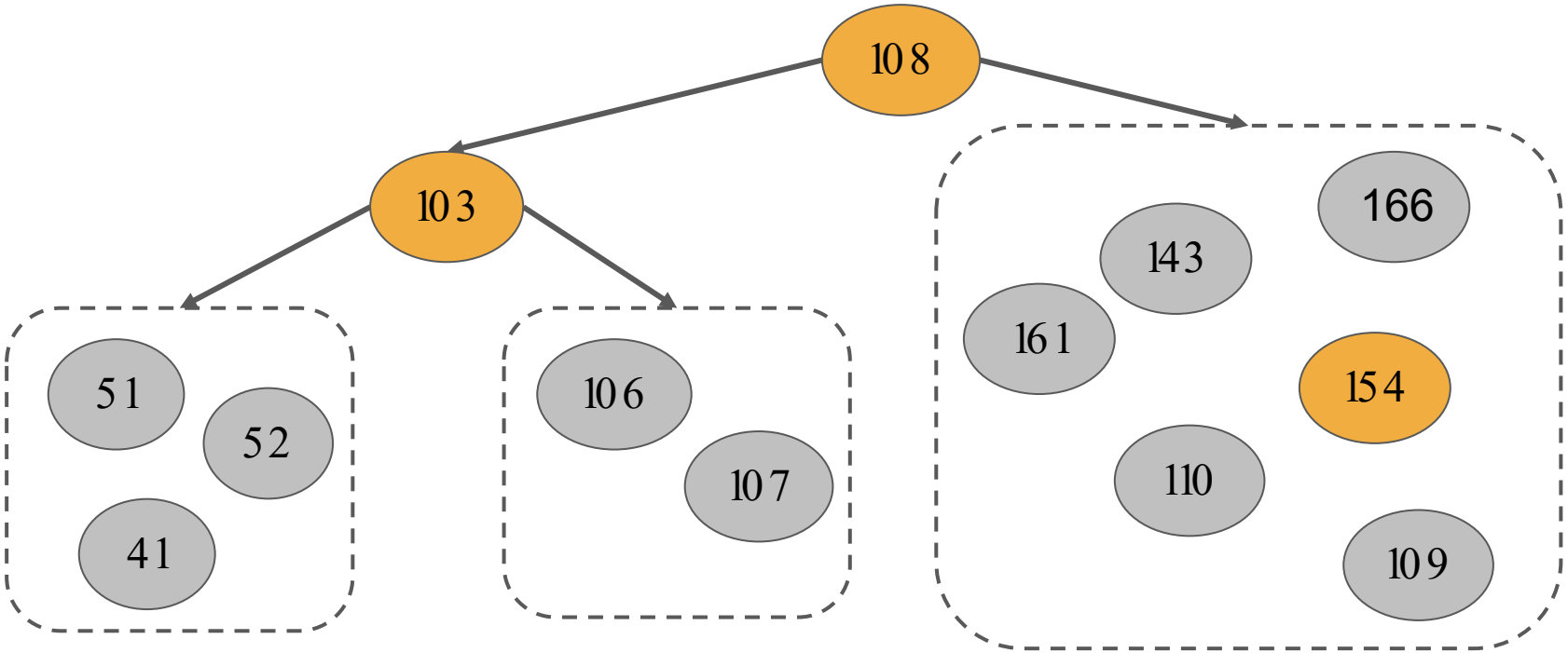
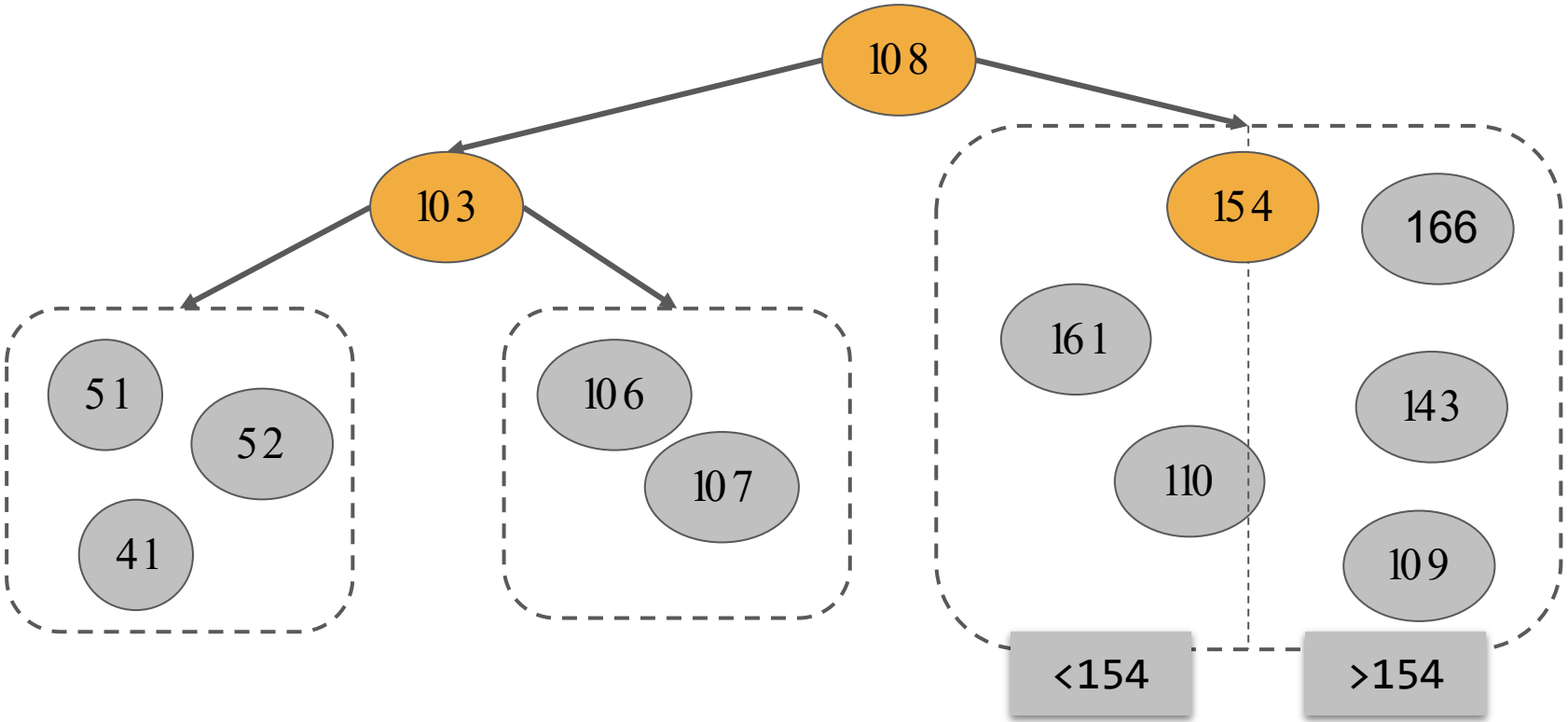


Pick the median element of the right side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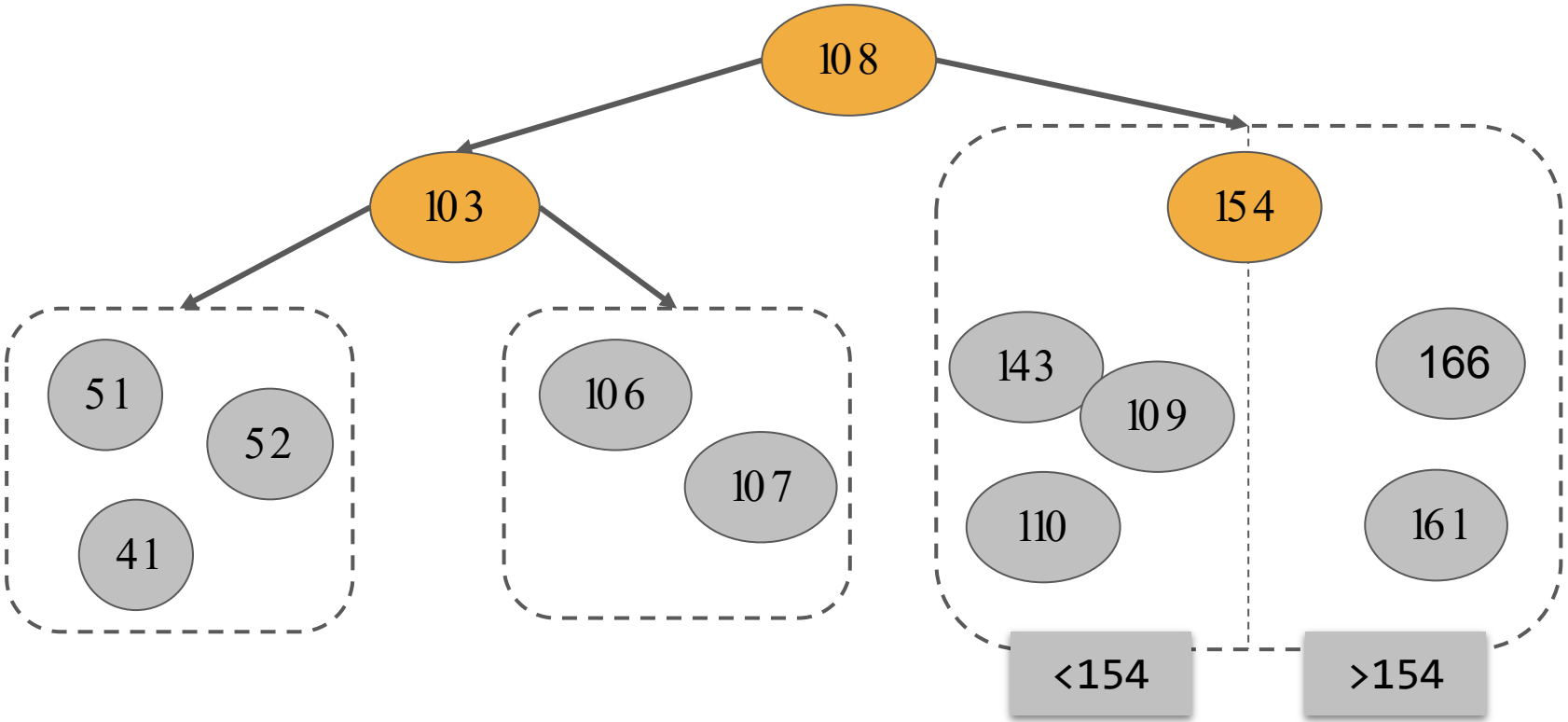
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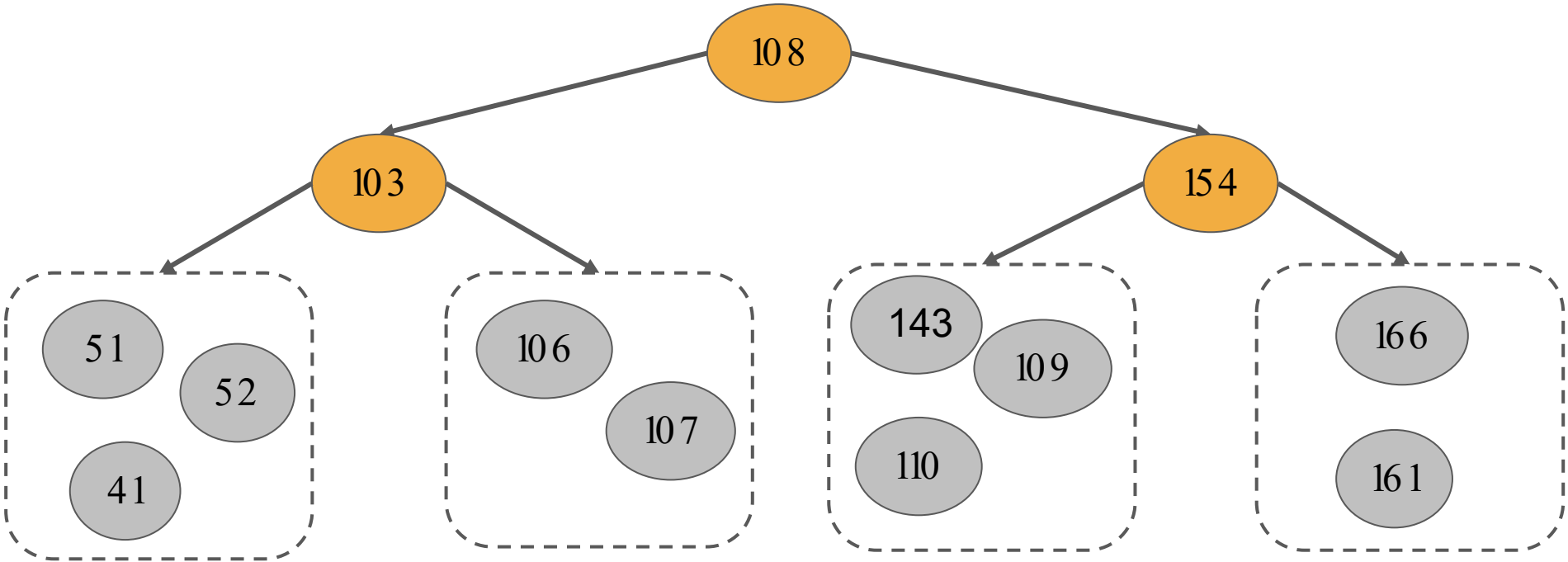
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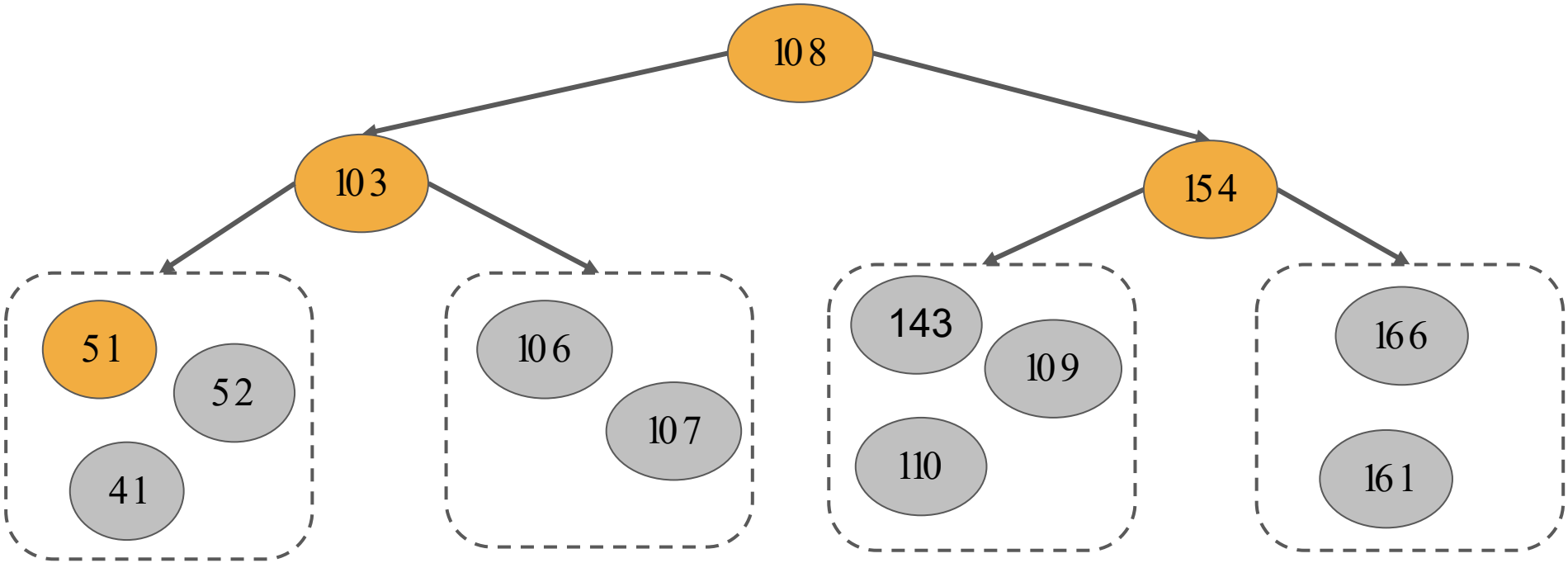
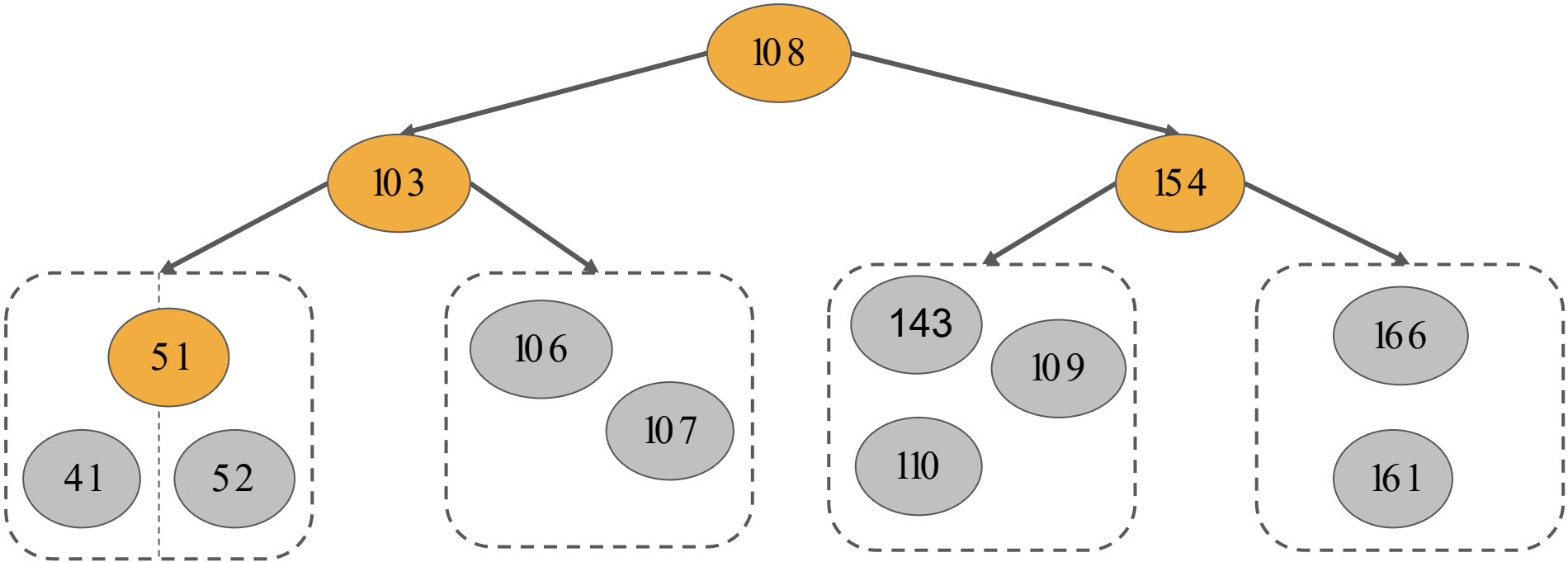


Pick the median element of the left side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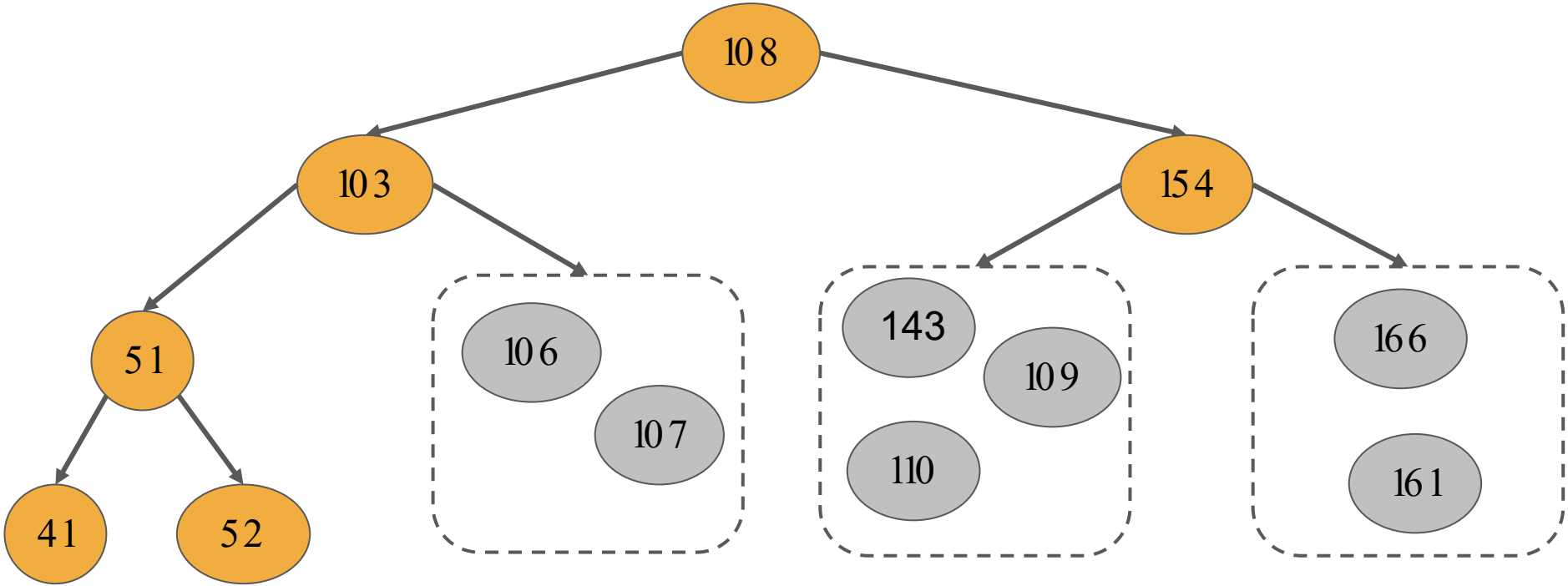
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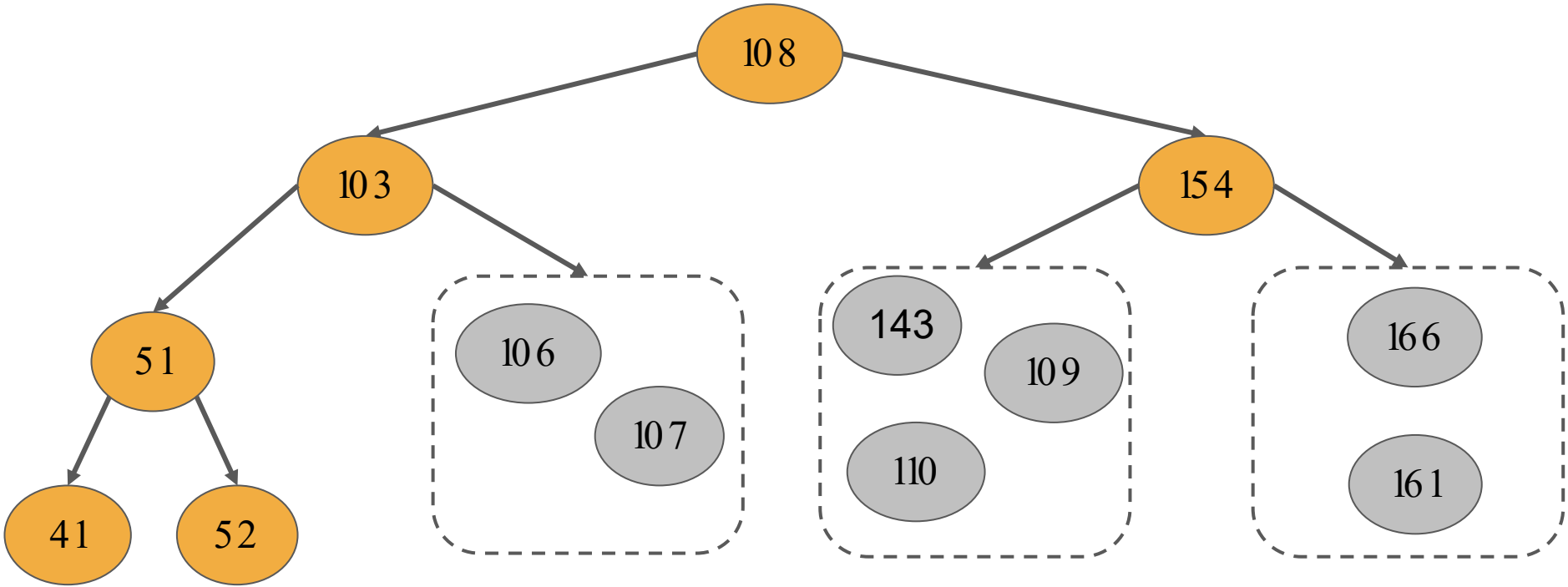
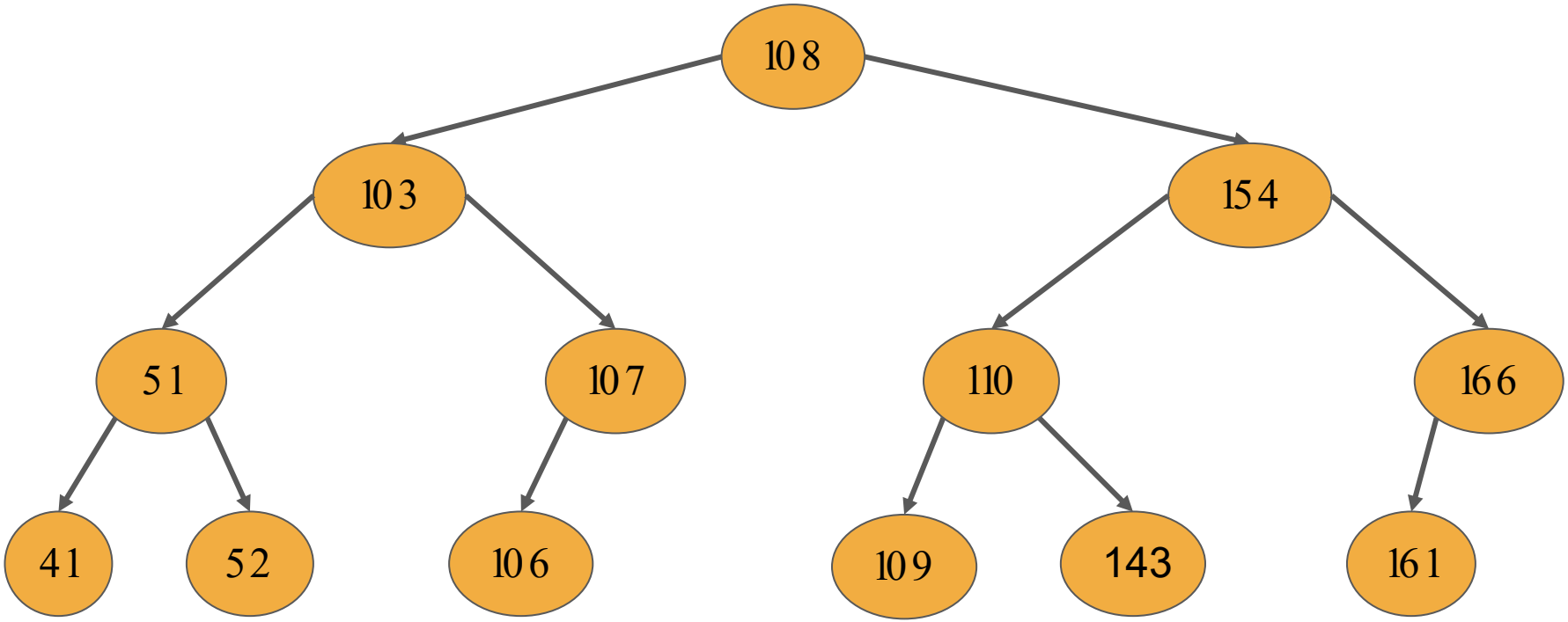


Keep repeating this process for all the subtrees!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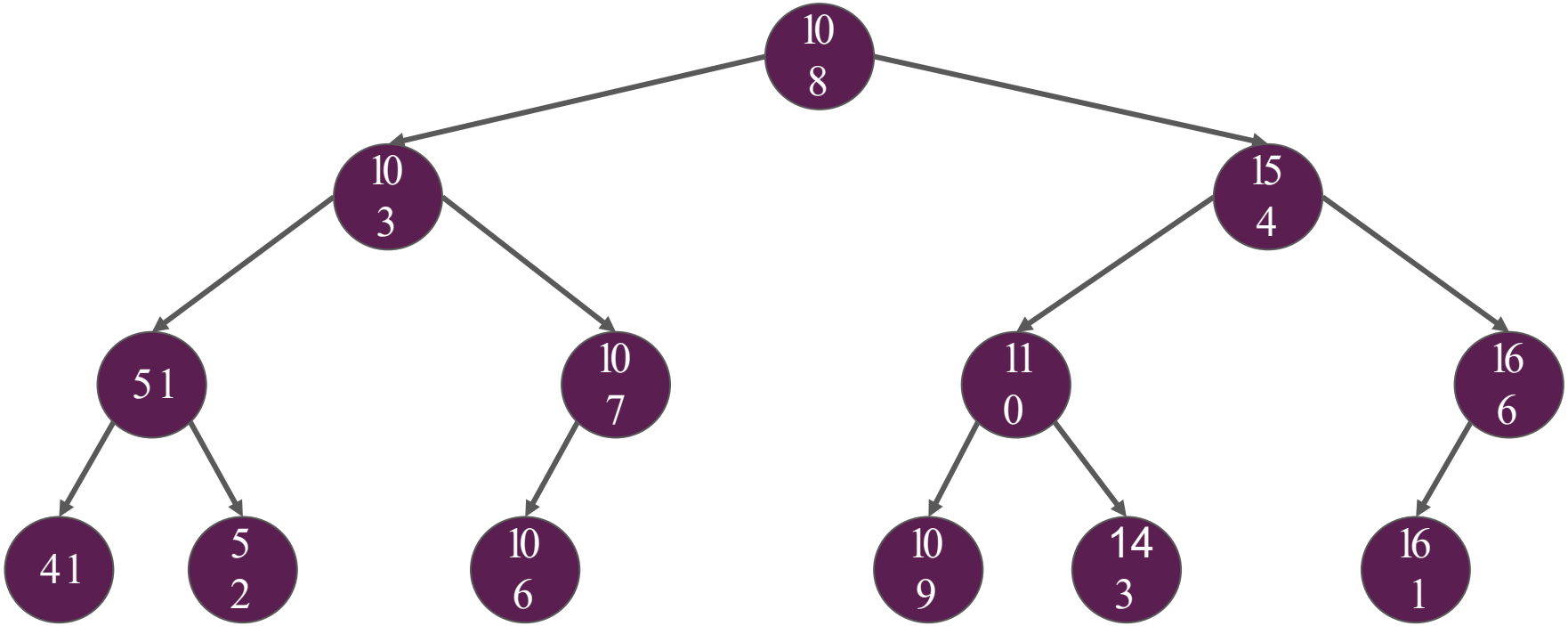
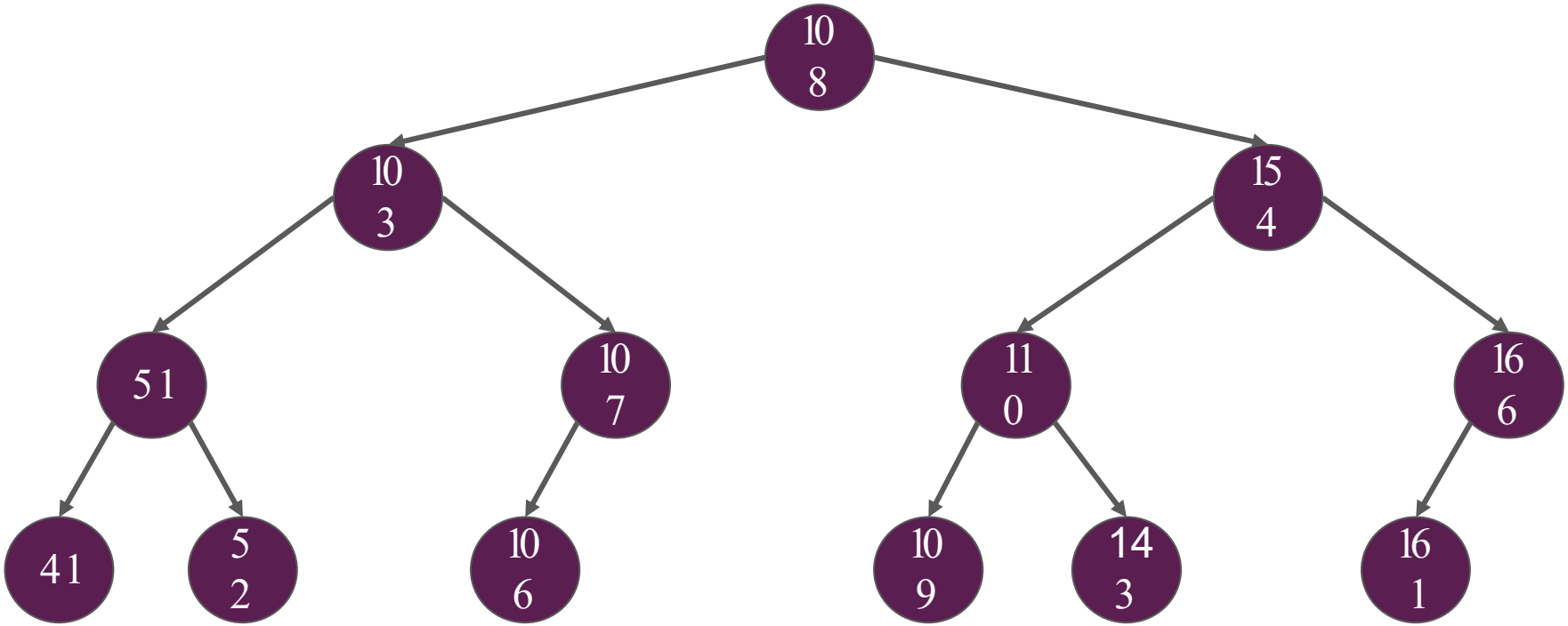
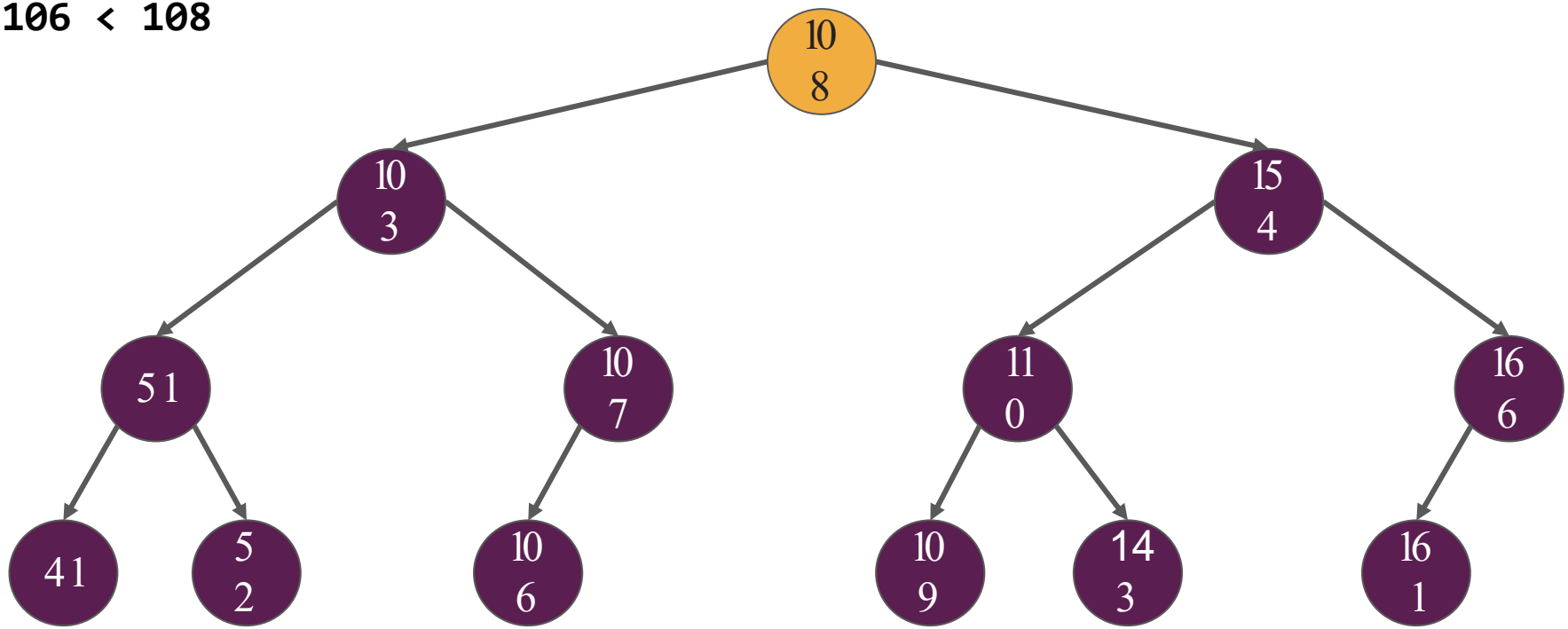


There are 13 nodes in the tree, but the path to each node is short (**~O(log 13)**)!



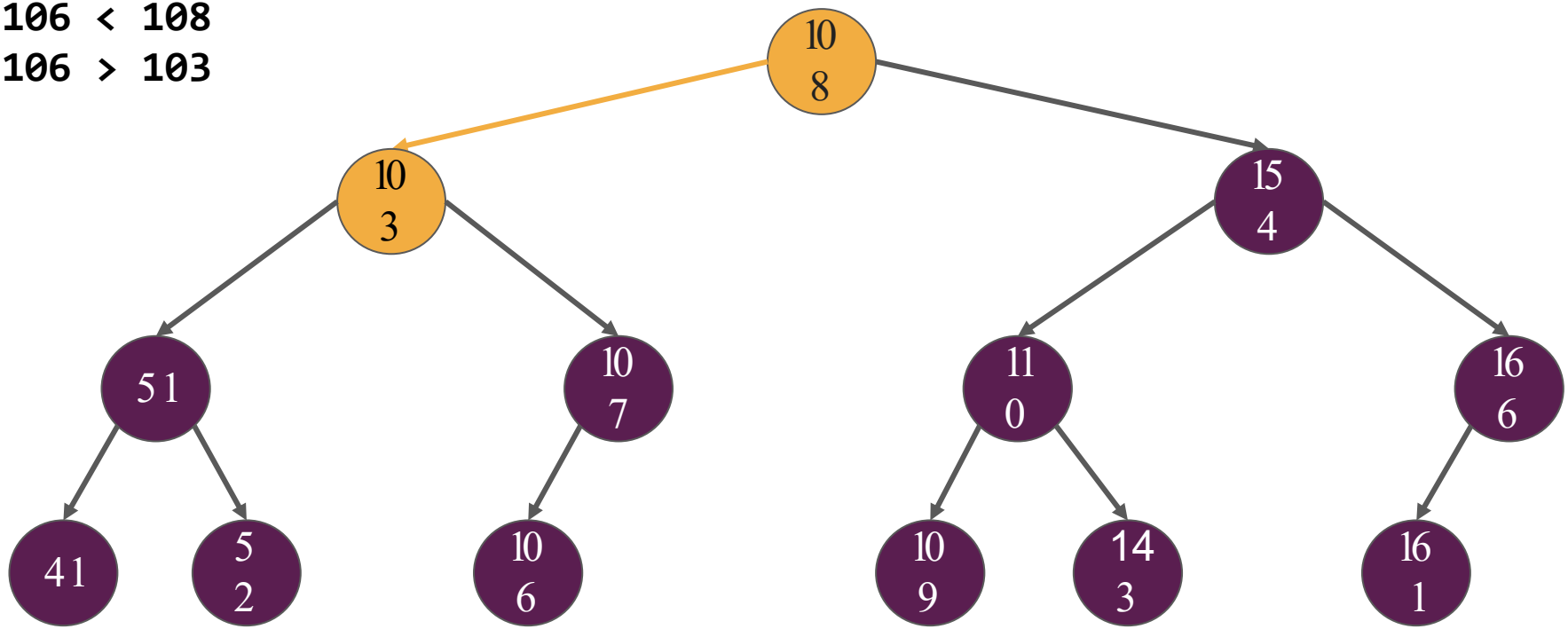
How could we check if **106** is in this tree?

106 < 108



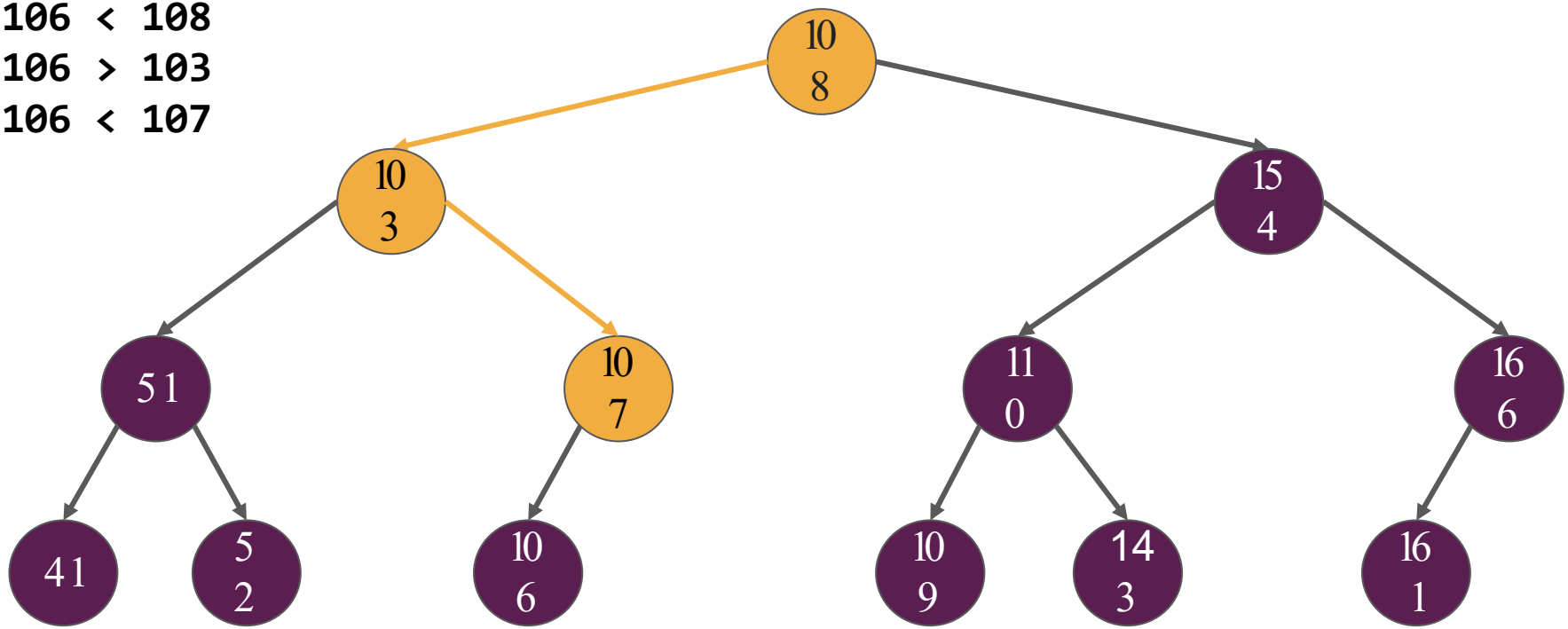
How could we check if **106** is in this tree?

```
106 < 108
106 > 103
```



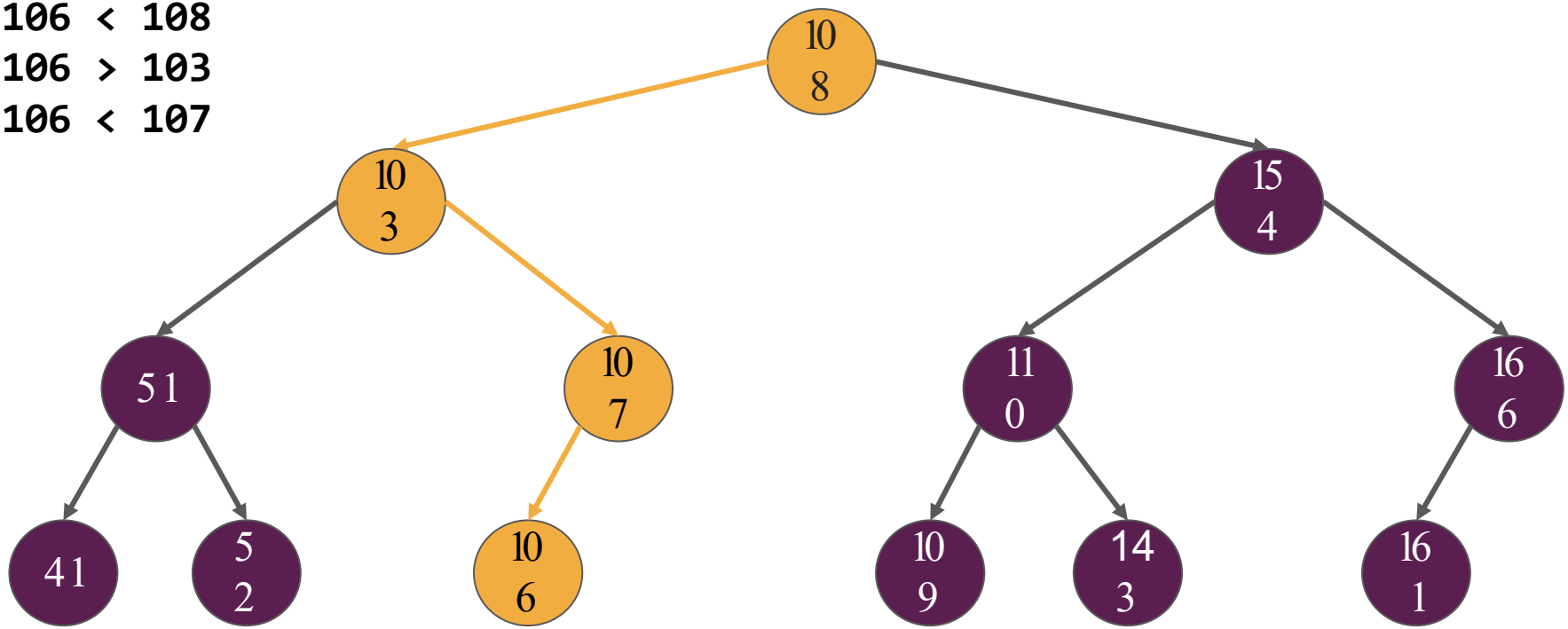
How could we check if **106** is in this tree?

```
106 < 108
106 > 103
106 < 107
```

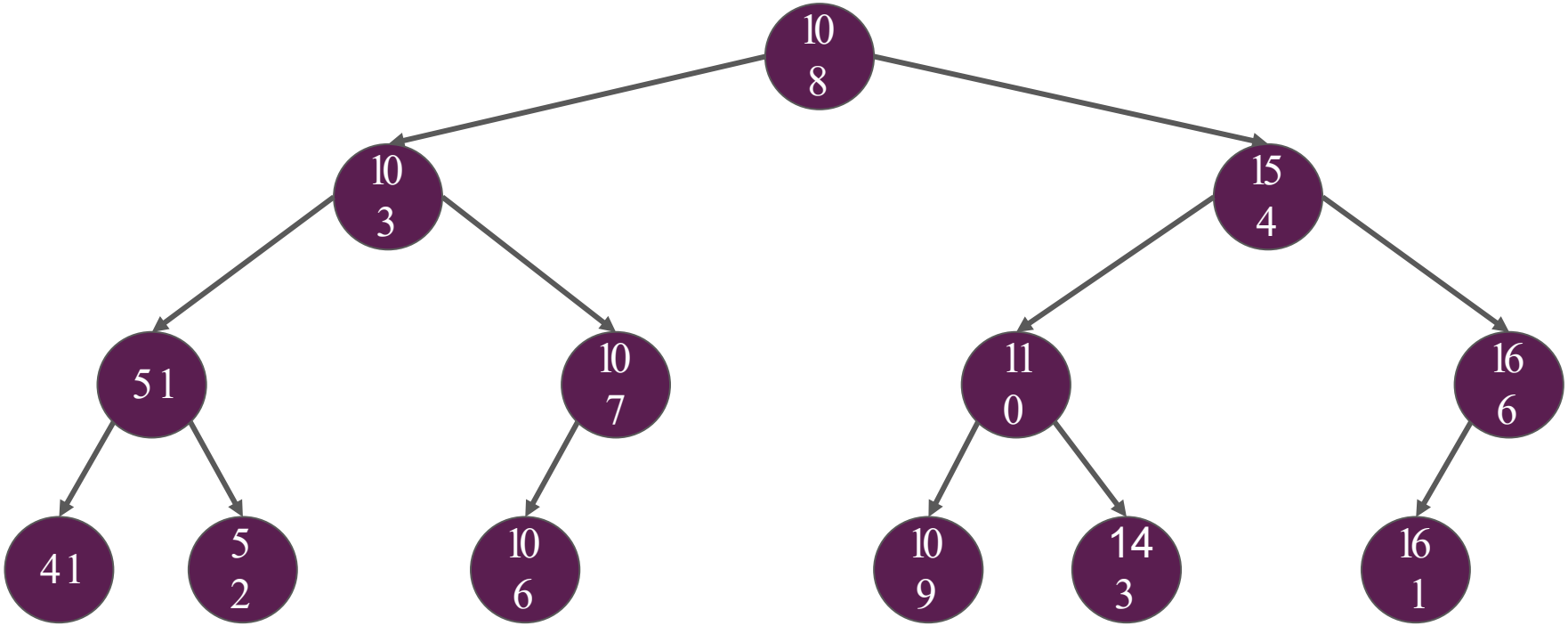


How could we check if **106** is in this tree?

```
106 < 108
106 > 103
106 < 107
```



We found **106** so we're done!


108
103
154
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166
41
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106
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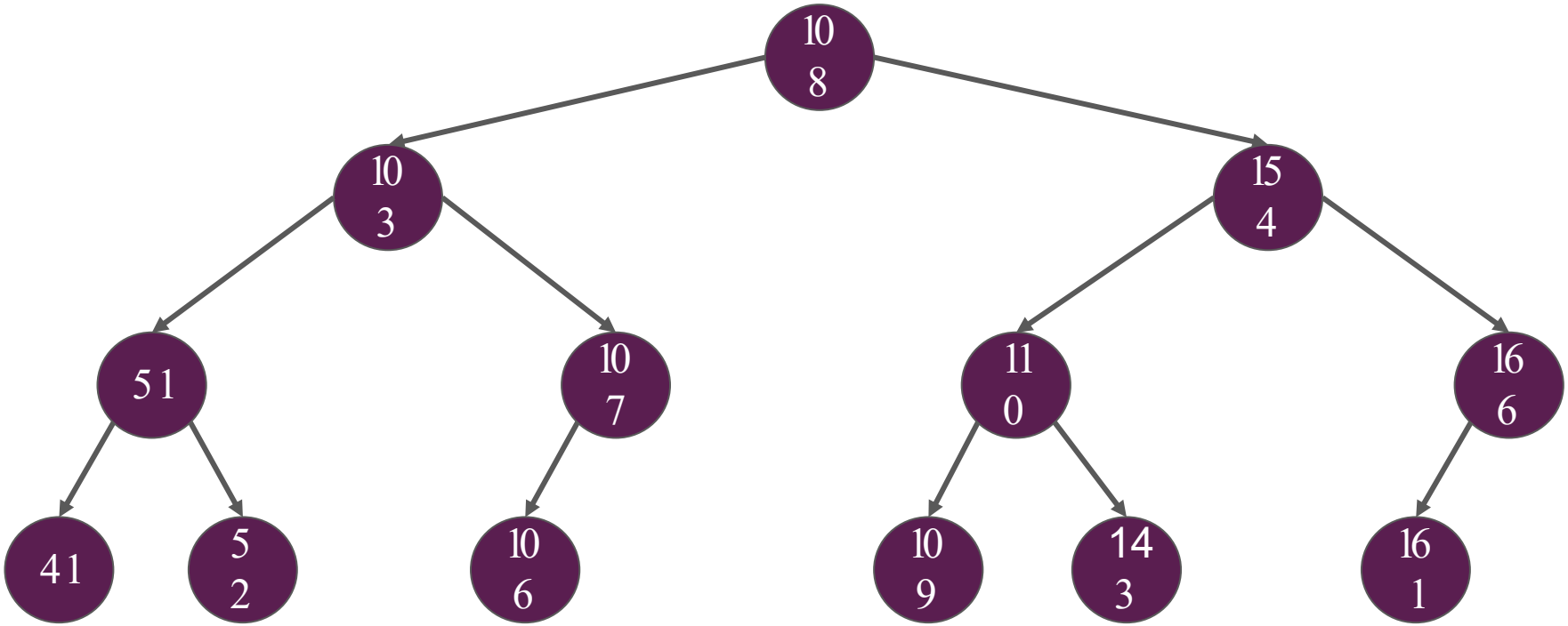
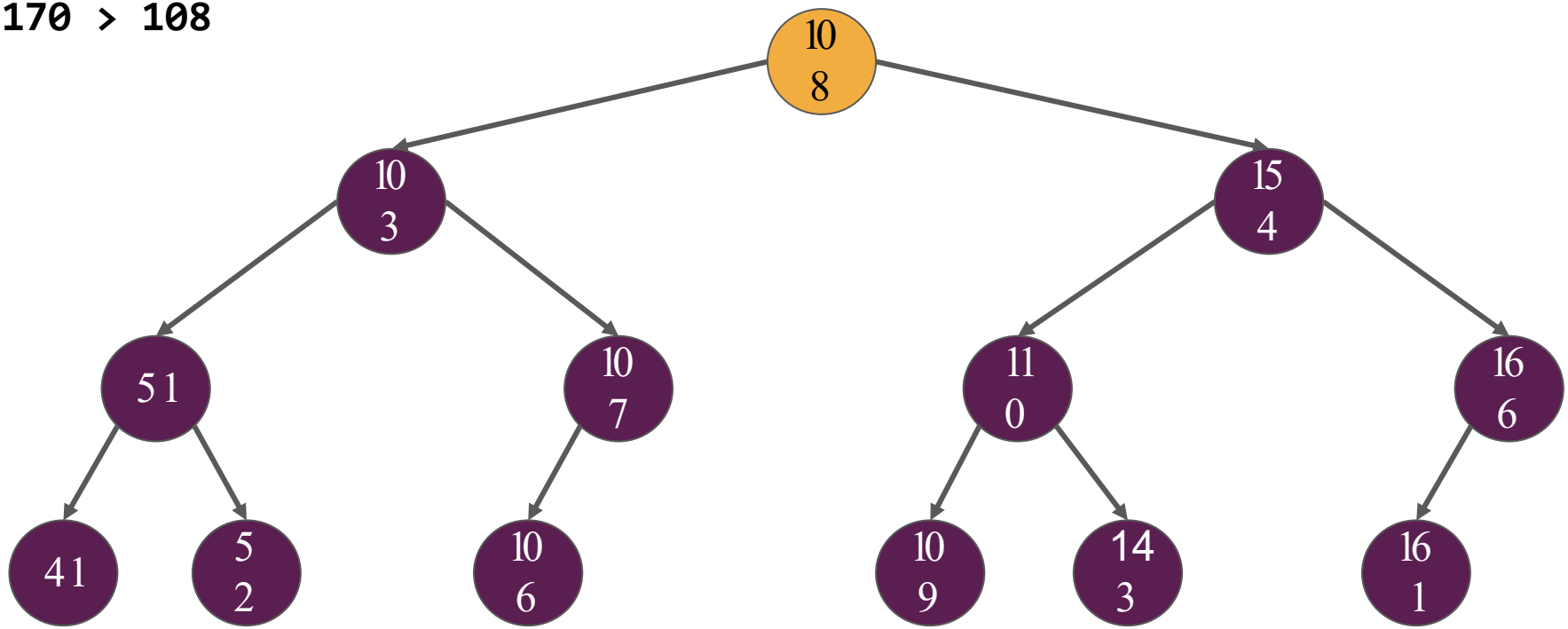
143
161



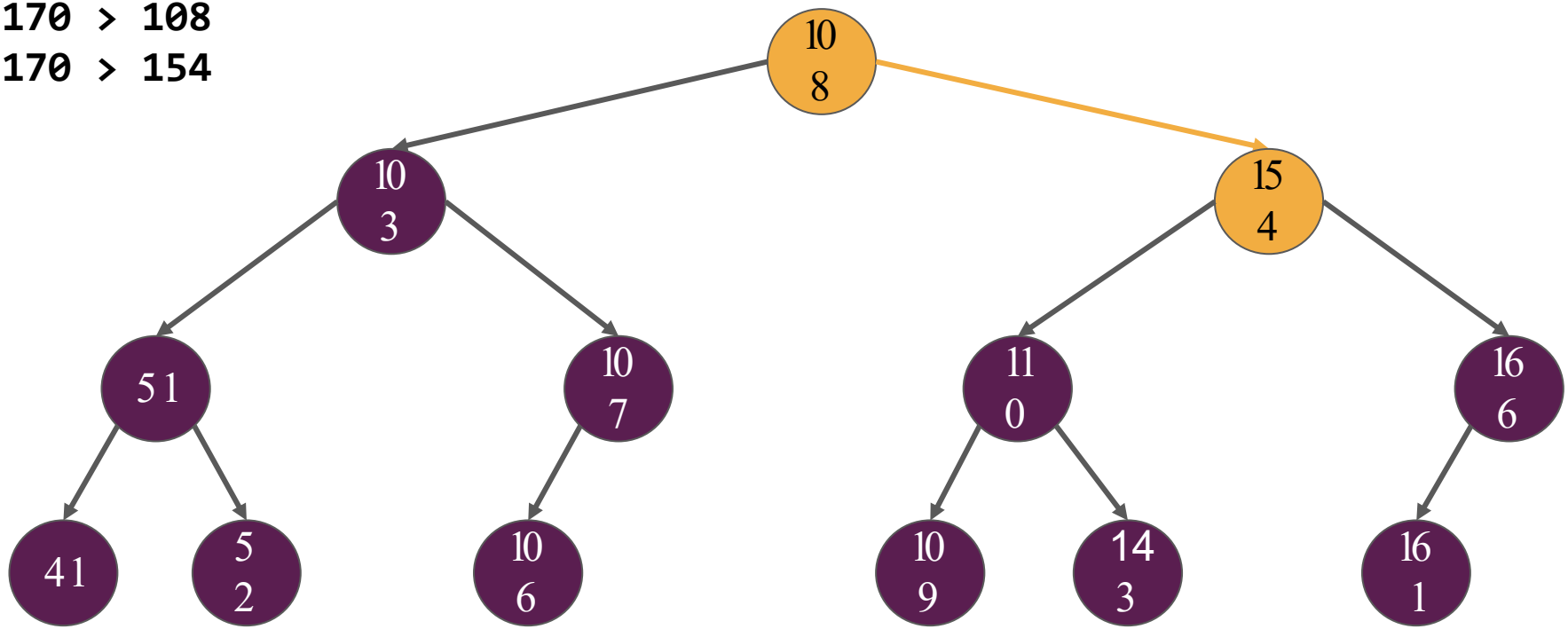
How could we check if **`170`** is in this tree?

170 > 108



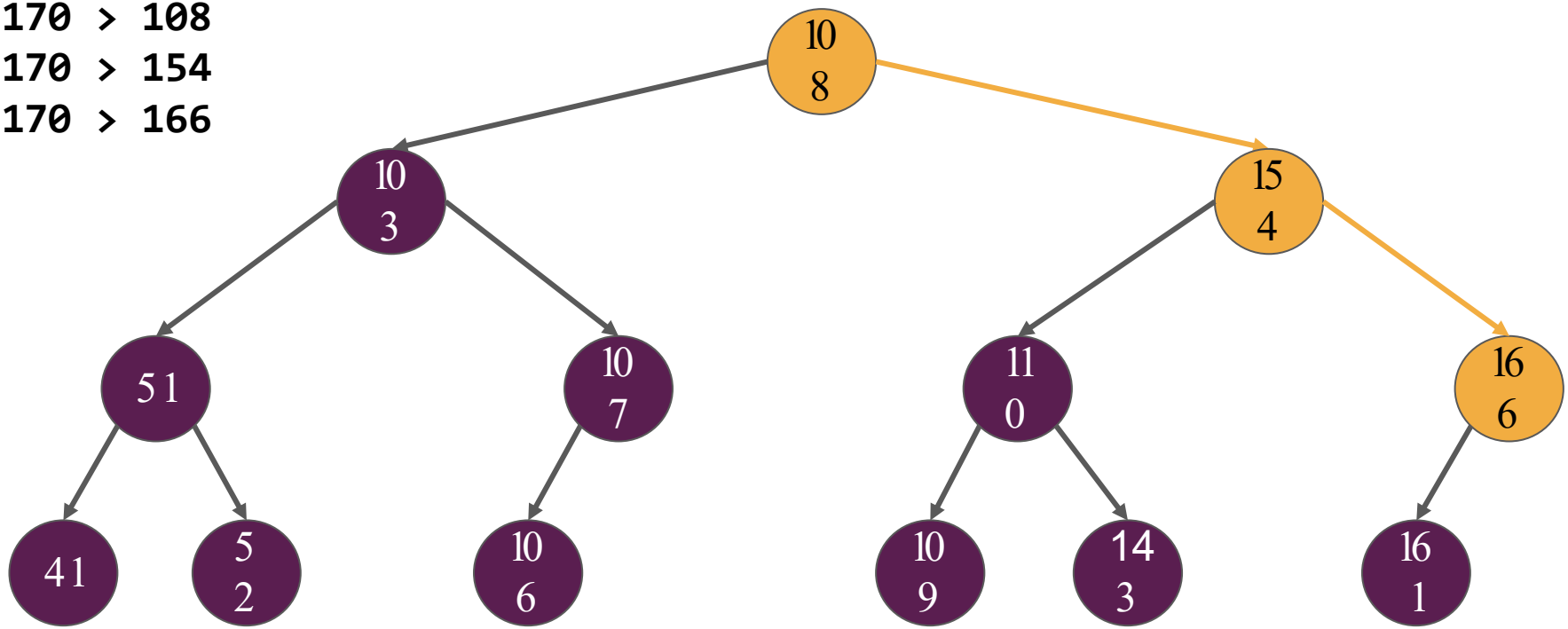
How could we check if **170** is in this tree?

170 > 108
170 > 154



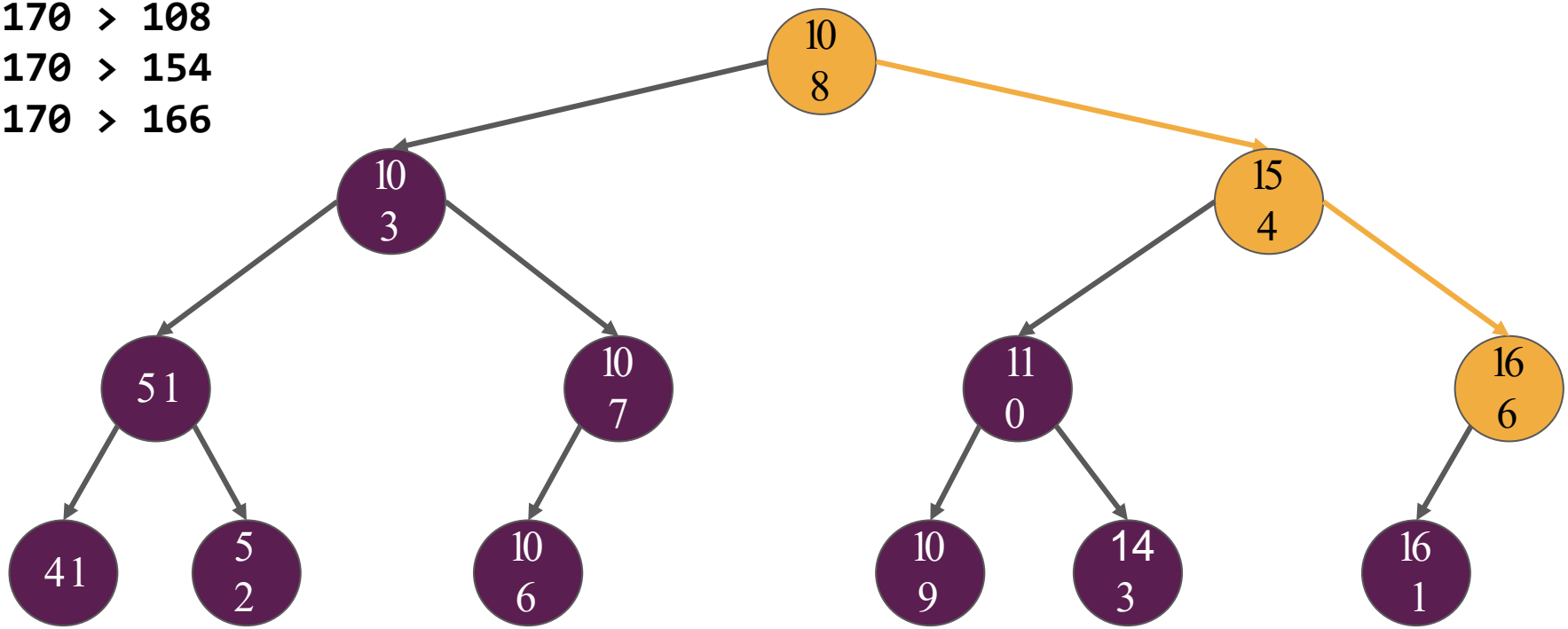
How could we check if **170** is in this tree?

```
170 > 108
170 > 154
170 > 166
```



How could we check if **170** is in this tree?

```
170 > 108
170 > 154
170 > 166
```



Right child is **`nullptr`** so we're done!

Building a BST

- An optimal BST is built by repeatedly choosing the median element as the root node of a given subtree and then separating elements into groups less than and greater than that median.

Building a BST

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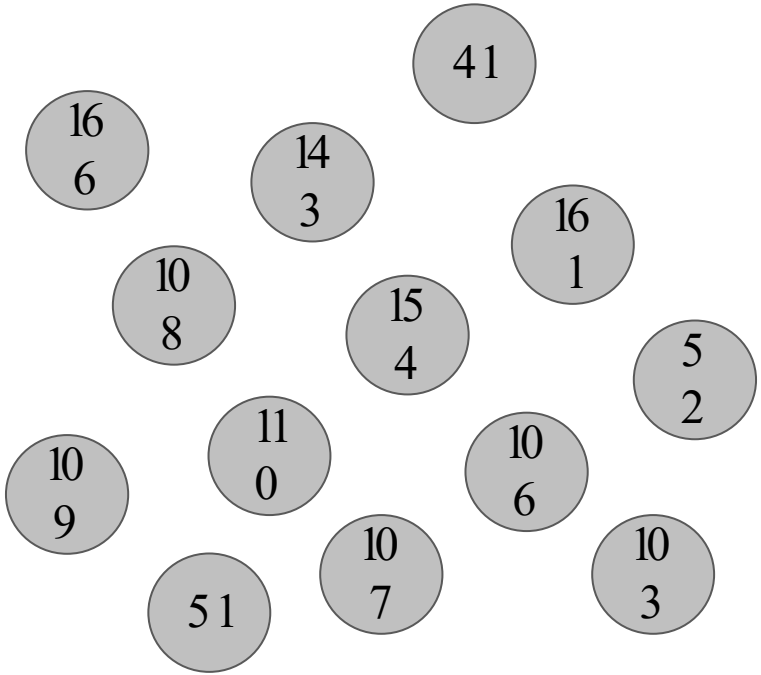
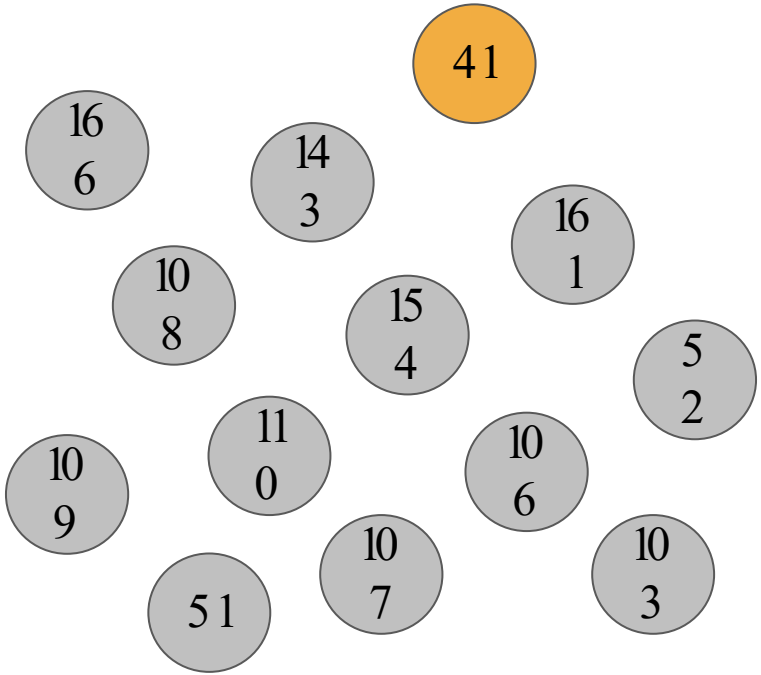
What does “optimal” mean?

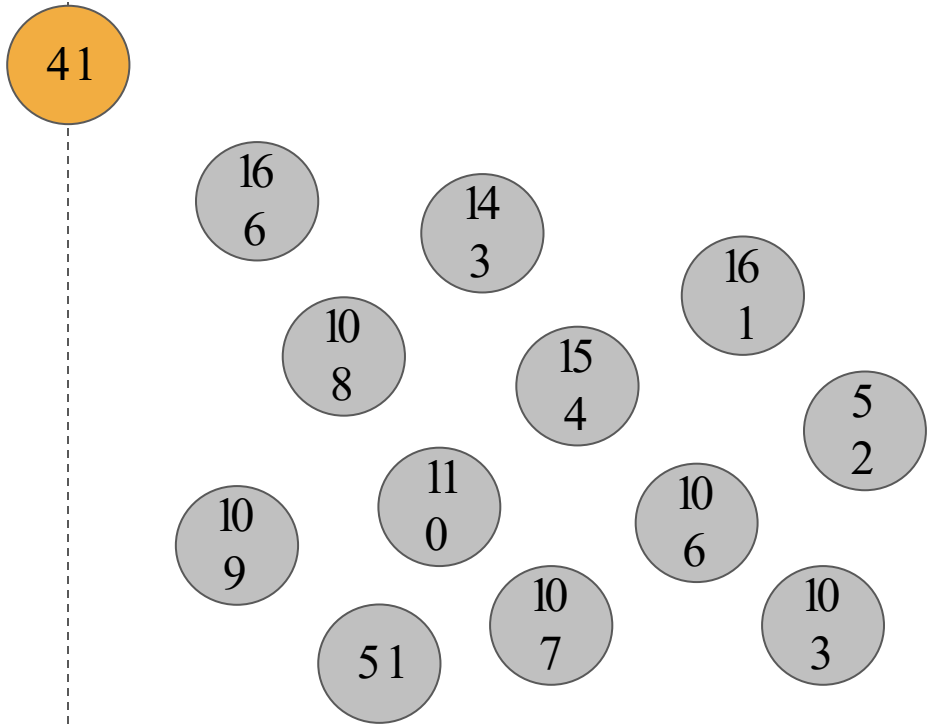
What if we didn’t choose the median?



Let’s choose the smallest element instead...



Let’s choose the smallest element instead...



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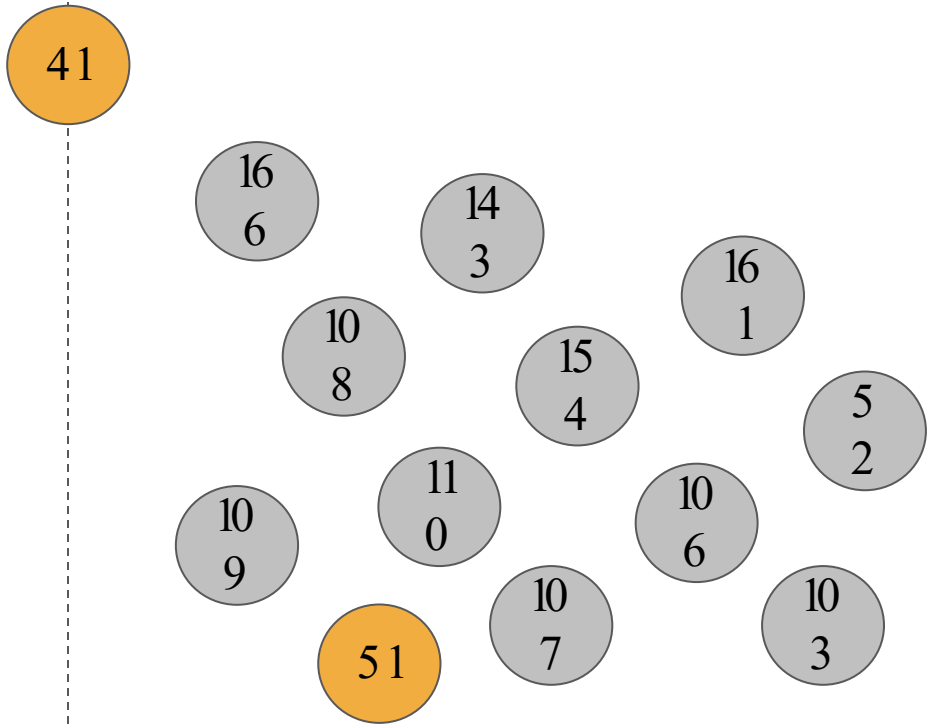

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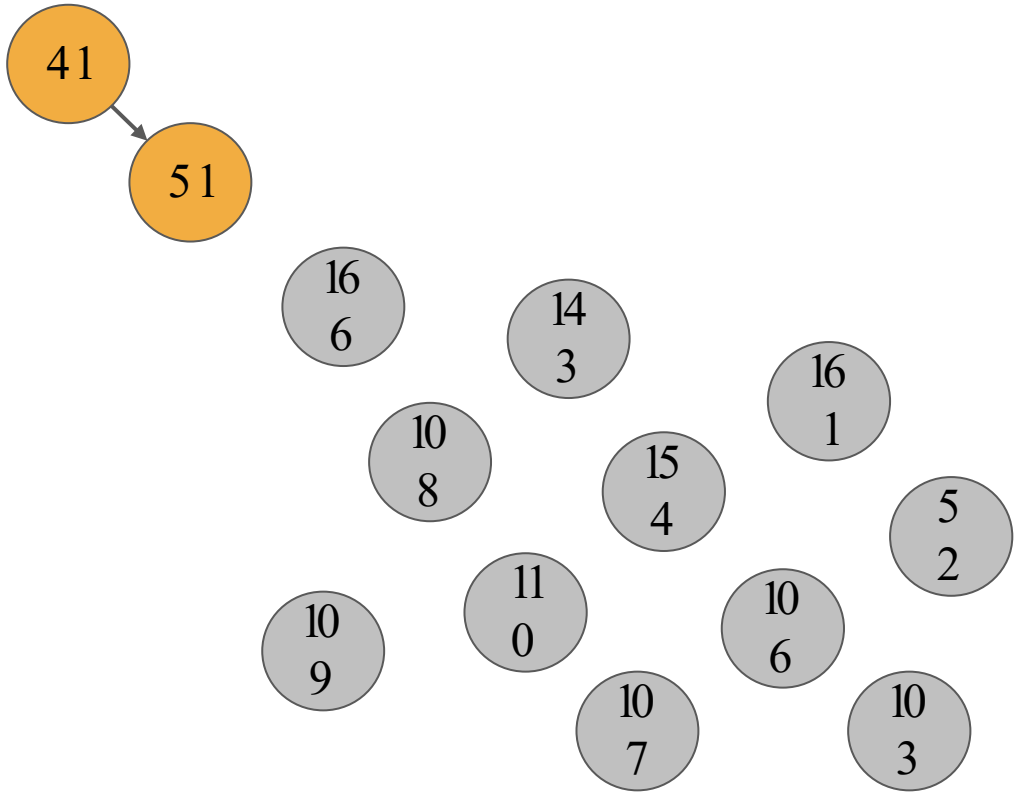

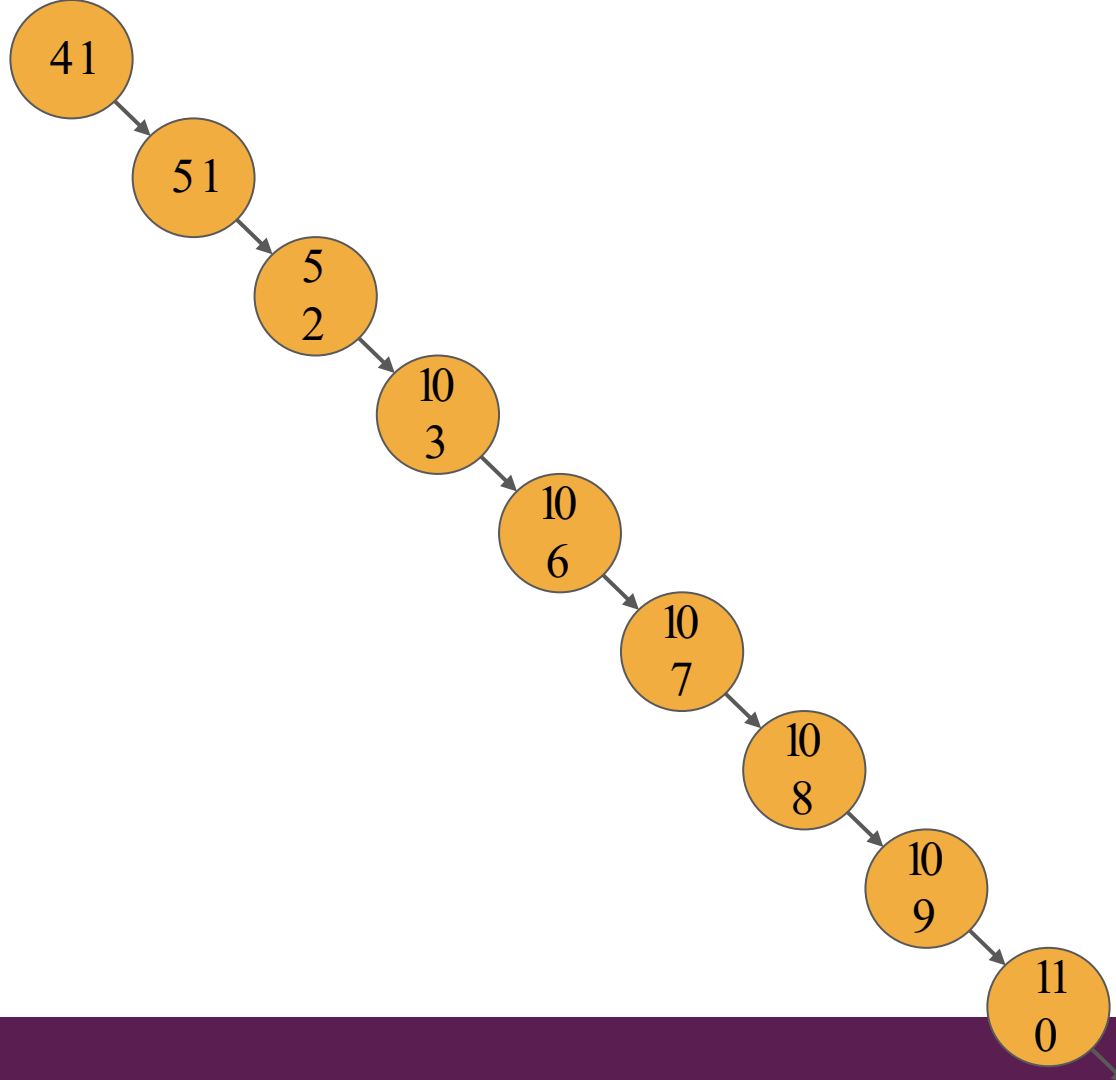
41
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106
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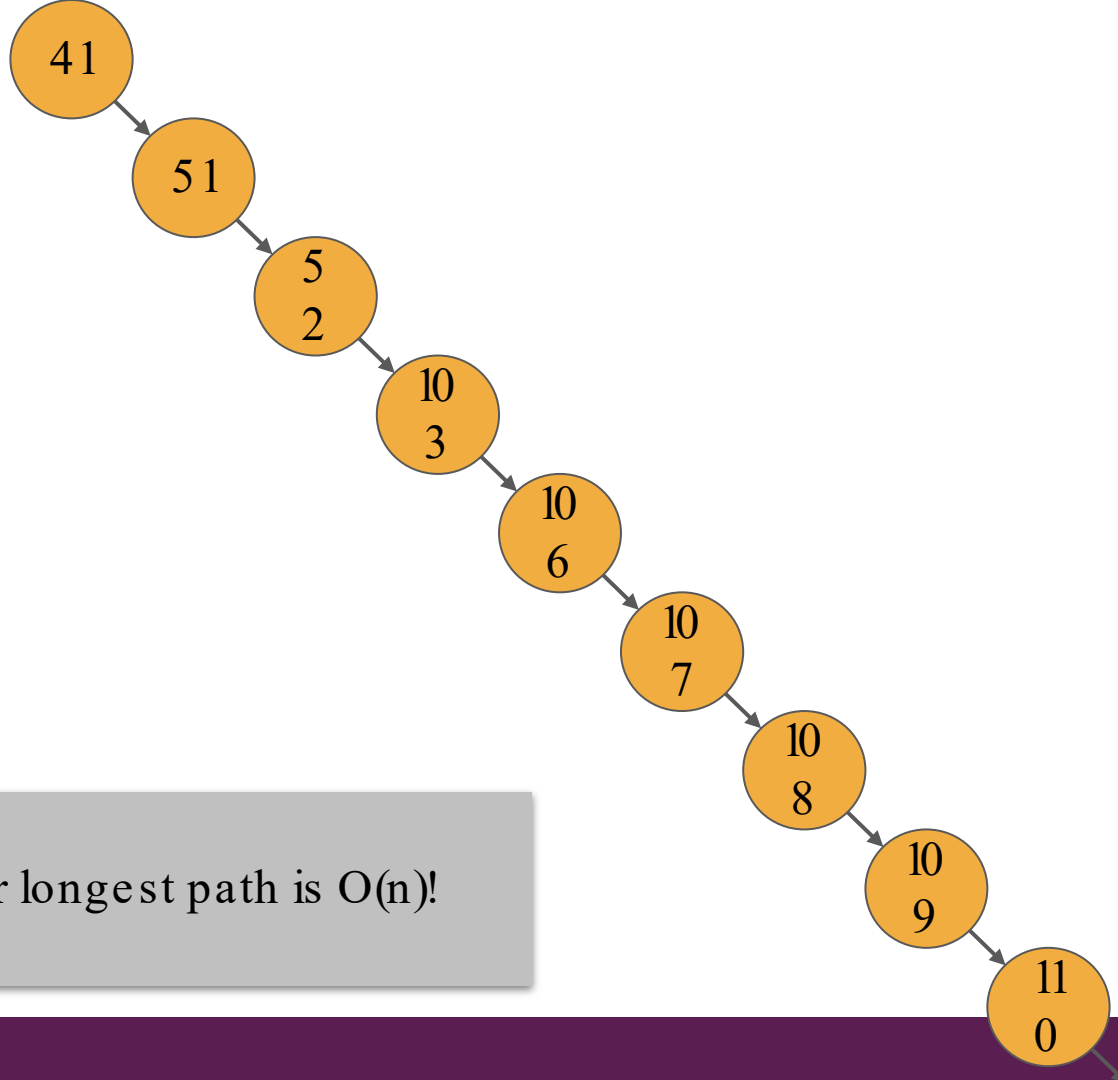
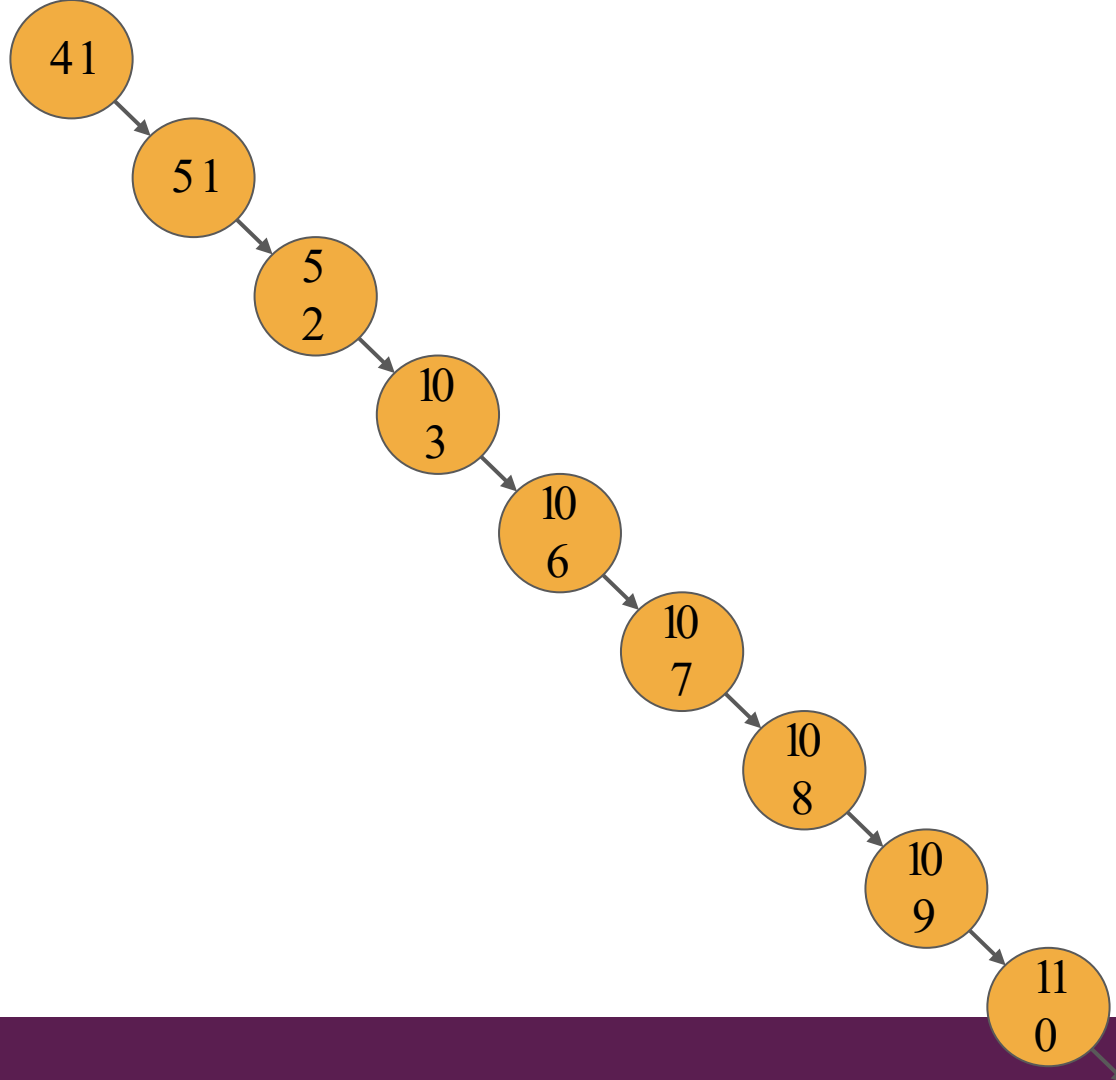
110



Now our longest path is O(n)!

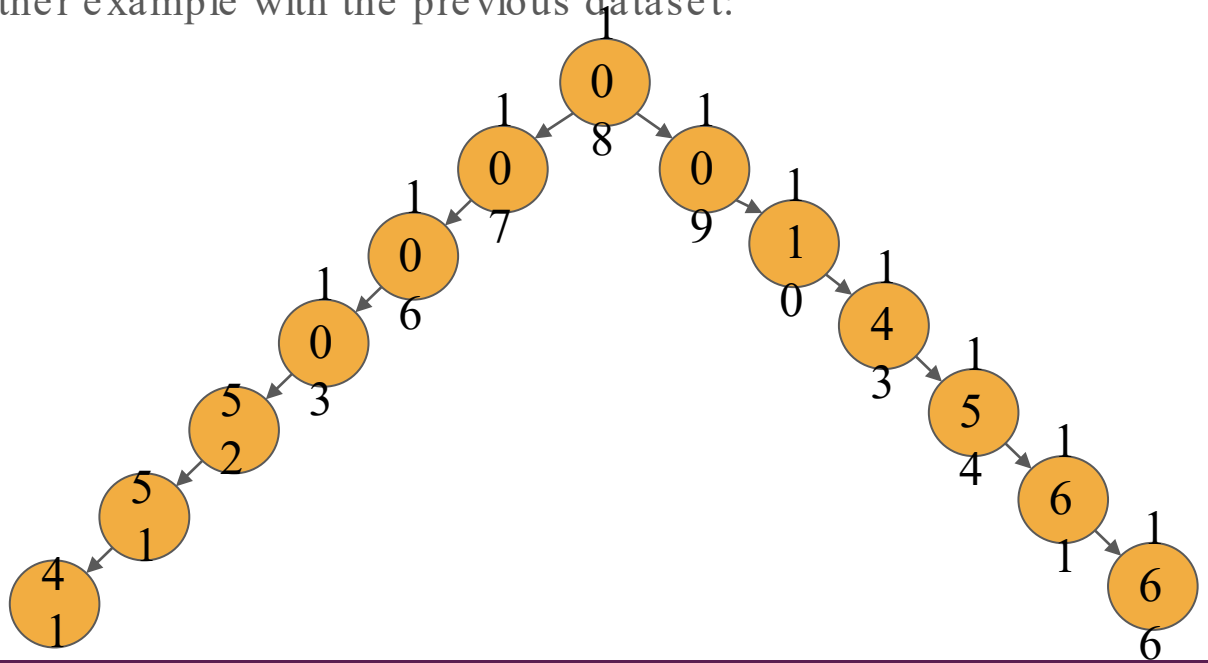
Takeaways

- There are multiple valid BSTs for the same set of data.


41
51
52
103
106
107
108
109
110

Takeaways

- There are multiple valid BSTs for the same set of data.
  - Another example with the previous dataset:



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- A binary search tree is balanced if its height is **`O(log n)`**, where **`n`** is the number of nodes in the tree (i.e. left/right subtrees don't differ in height by more than 1).
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  - **Theorem**: If you start with an empty tree and add in random values, then with high probability the tree is balanced. →take an algorithmic analysis class to learn why!

Takeaways

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  - Lookup, insertion, and deletion with balanced BSTs all operate in **`O(log n)`** runtime.
  - **Theorem**: If you start with an empty tree and add in random values, then with high probability the tree is balanced. →take CS161 to learn why!
  - A **self-balancing** BST reshapes itself on insertions and deletions to stay balanced (how to do this is beyond the scope of this class).

Announcements

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- Assignment 5 is due **tonight at 11:59pm PDT**.
- Assignment 6 will be released by the end of the day tomorrow and will be due on **Wednesday, August 11 at 11:59pm PDT**This is a hard deadline – there is **no grace period and no submissions will be accepted after this time** .
- The End-quarter Assessment will take place over 3 days from **Friday, August 13 to Sunday, August 15.**

Implementing Sets with BSTs

We’re going to implement a Set using a BST!

- Our Set will only store strings as its data type.

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- Our Set will only store strings as its data type.

```
struct TreeNode {
   std::string data;
   TreeNode* left;
   TreeNode* right;

   // default constructor does not initialize
   TreeNode() {}
   // 3-arg constructor sets fields from arguments
   TreeNode(std::string d, TreeNode* l, TreeNode* r) {
       data = d;
       left = l;
       right = r;
   }
};
```

We’re going to implement a Set using a BST!

- Our Set will only store strings as its data type
- We have a header file that will include a public interface already defined.

`OurSet` Public Interface

```
class OurSet {
public:
    OurSet();   // constructor
   ~OurSet();   // destructor

   bool contains(string value);
   void add(string value);
   void remove(string value);
   void clear();
   int size();
   bool isEmpty();
   void printSetContents();

private:
    /* To be defined soon! */
};
```

We’re going to implement a Set using a BST!

- Our Set will only store strings as its data type
- We have a header file that will include a public interface already defined.
- As we write the Set methods, think about how their runtimes would change for a balanced vs. an unbalanced BST.
  - Note: Actual sets are self-balancing, but we won’t go into the details of how to implement that!

How do we design `OurSet`?

We must answer the following three questions:

1. Member functions: *What public interface should* **`OurSet`** *support? What functions might a client want to call?*

2. Member variables: *What private information will we need to store in order to keep track of the data stored in* **`OurSet`***?*

3. Constructor: *How are the member variables initialized when a new instance of* **`OurSet`** *is created?*

`OurSet` Public Interface

```
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```

Let’s code it!

constructor and destructor

Let’s code it!

size(), isEmpty(), `clear()`

Let’s code it!

printSetContents ()

`OurSet` Public Interface

```
class OurSet {
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```
106 < 108
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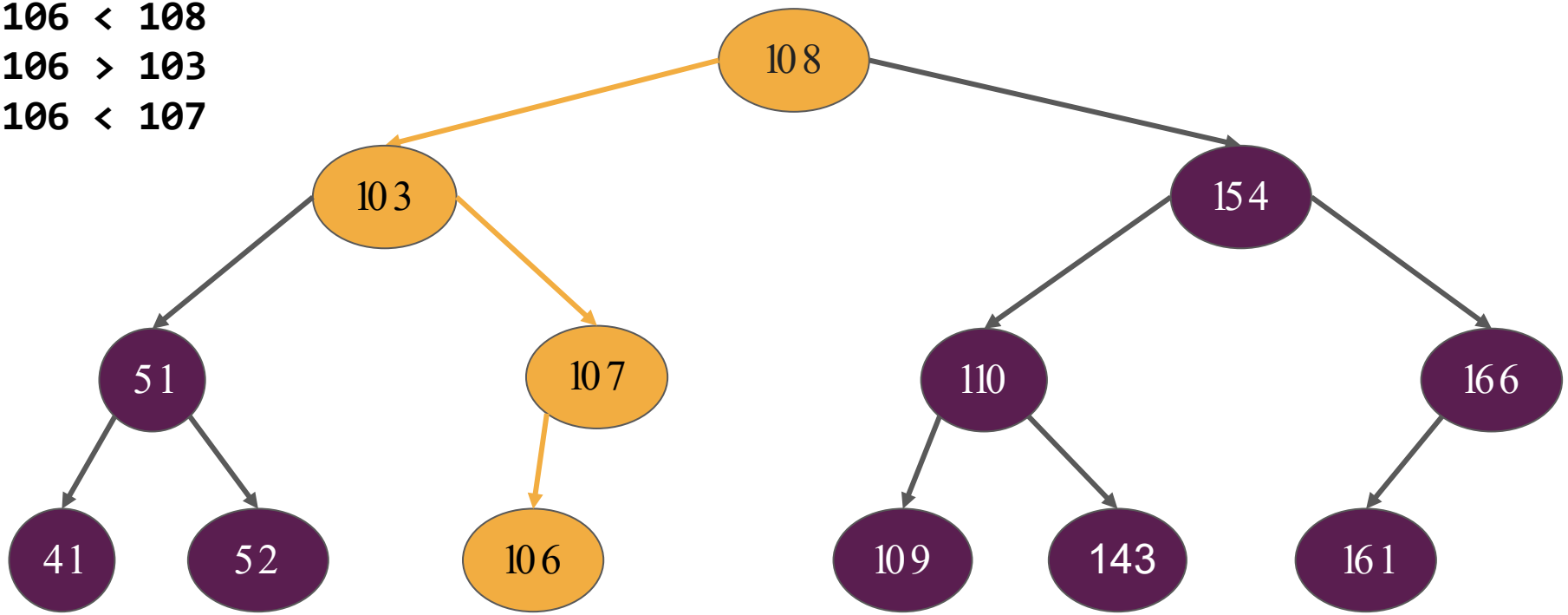
```

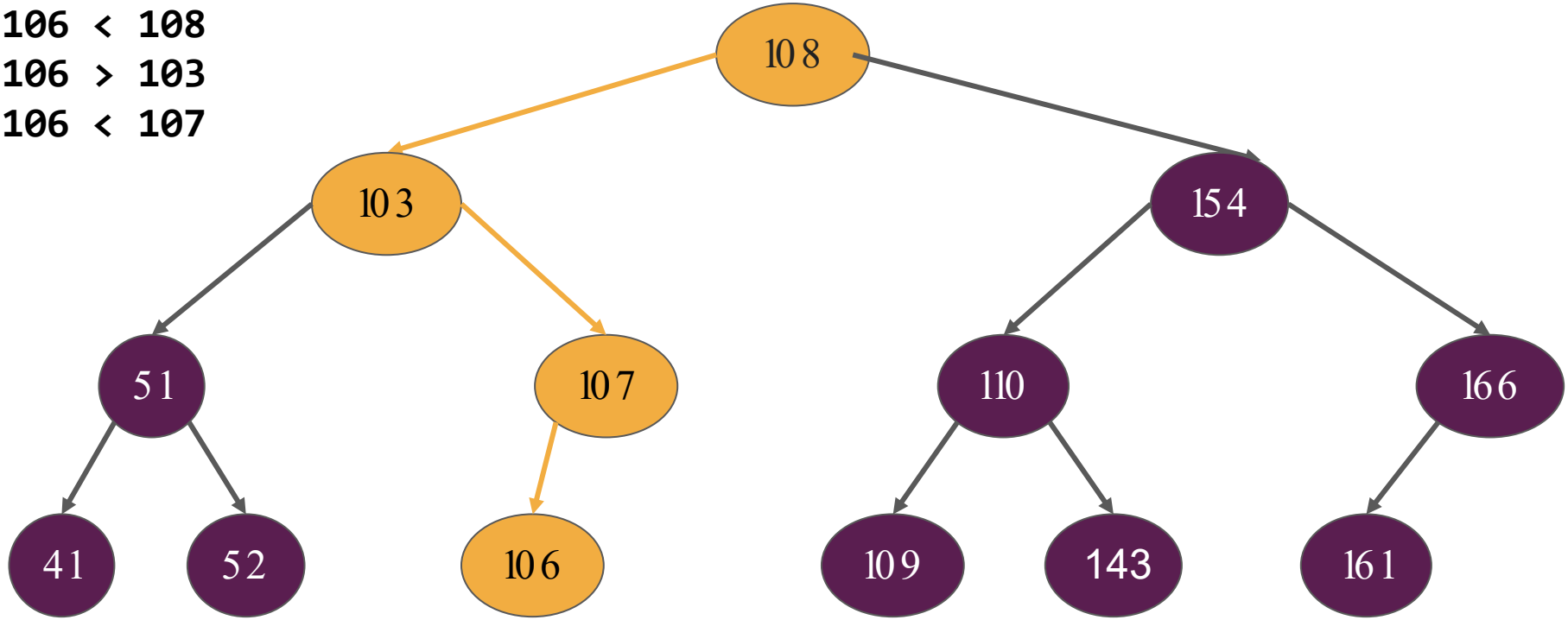


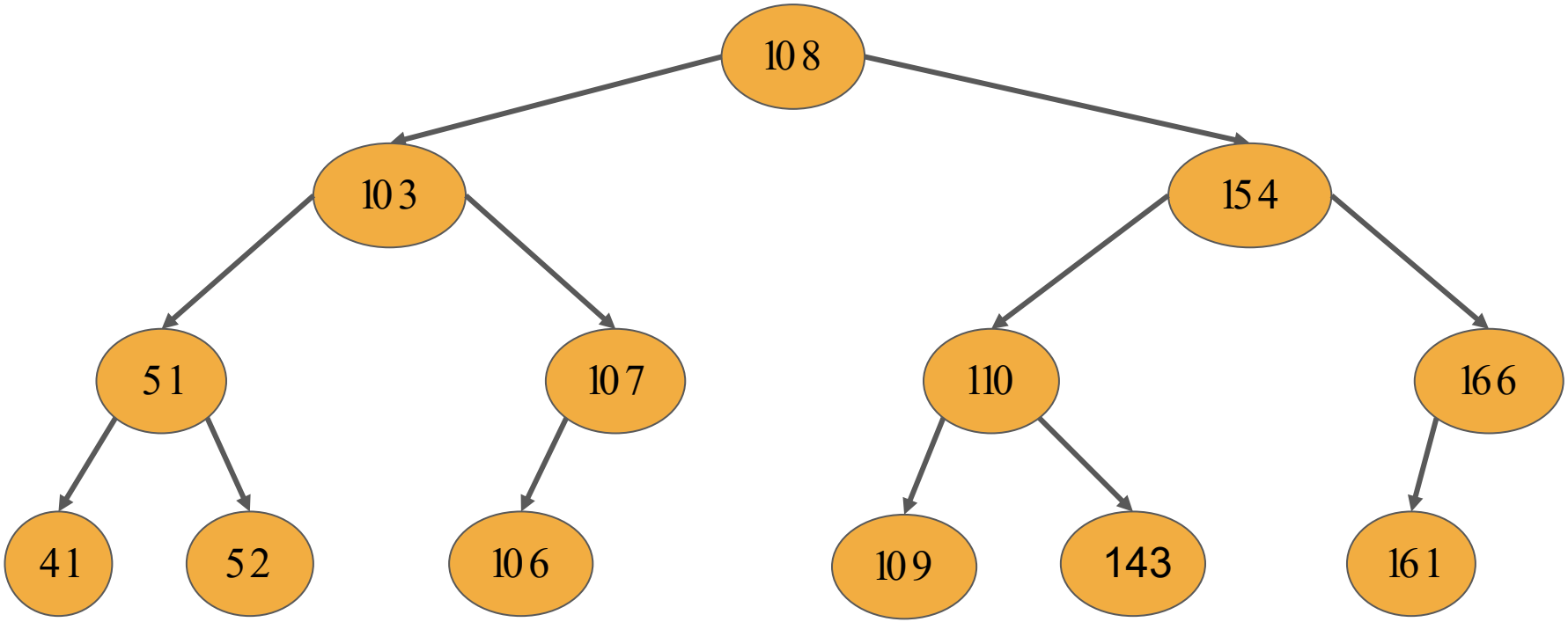
We found **106** so we're done!

Let’s code it!

`contains()`

```
106 < 108
106 > 103
106 < 107
```




108
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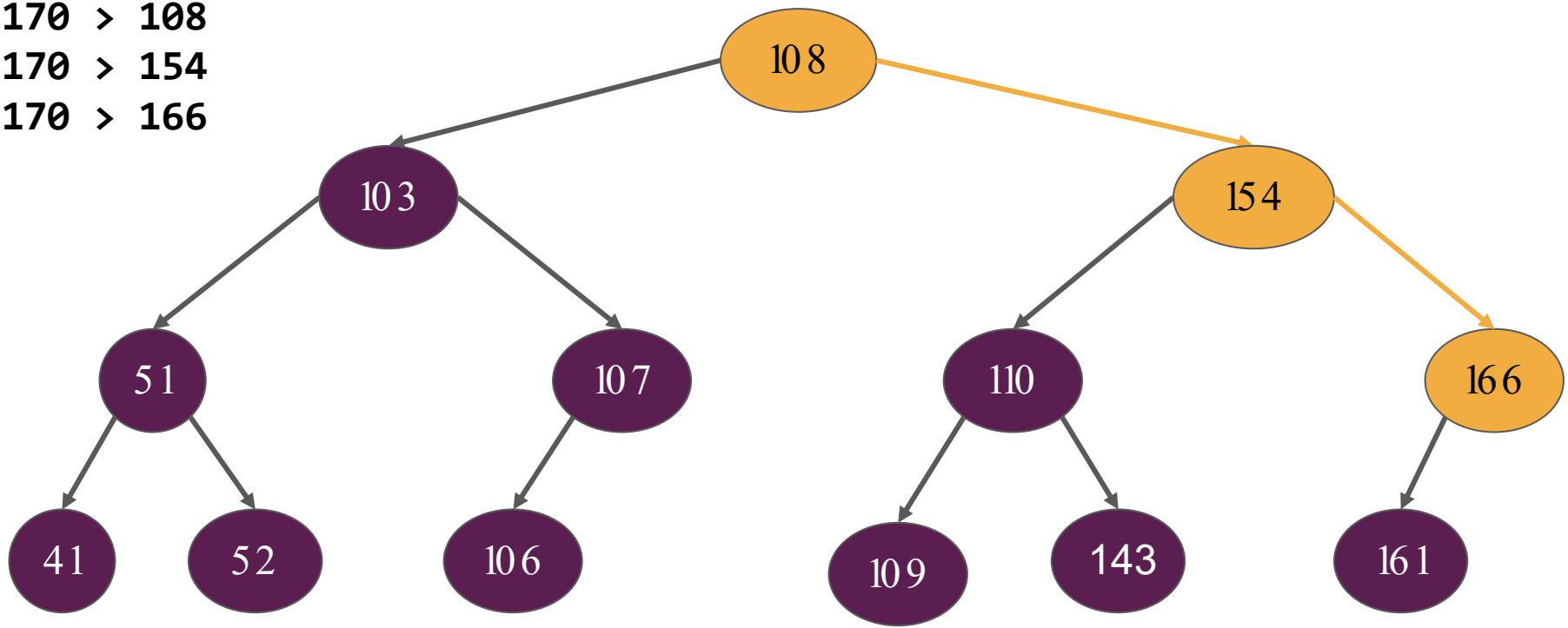
Let’s code it!

`add()`

```
170 > 108
170 > 154
170 > 166
```



Right child is **`nullptr`**

`OurSet` summary

- Our tree utility functions (**`inorderPrint`**, **`freeTree`**) showed up as private member functions/helpers!
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  - In-order traversal prints our elements in the correctly sorted order!

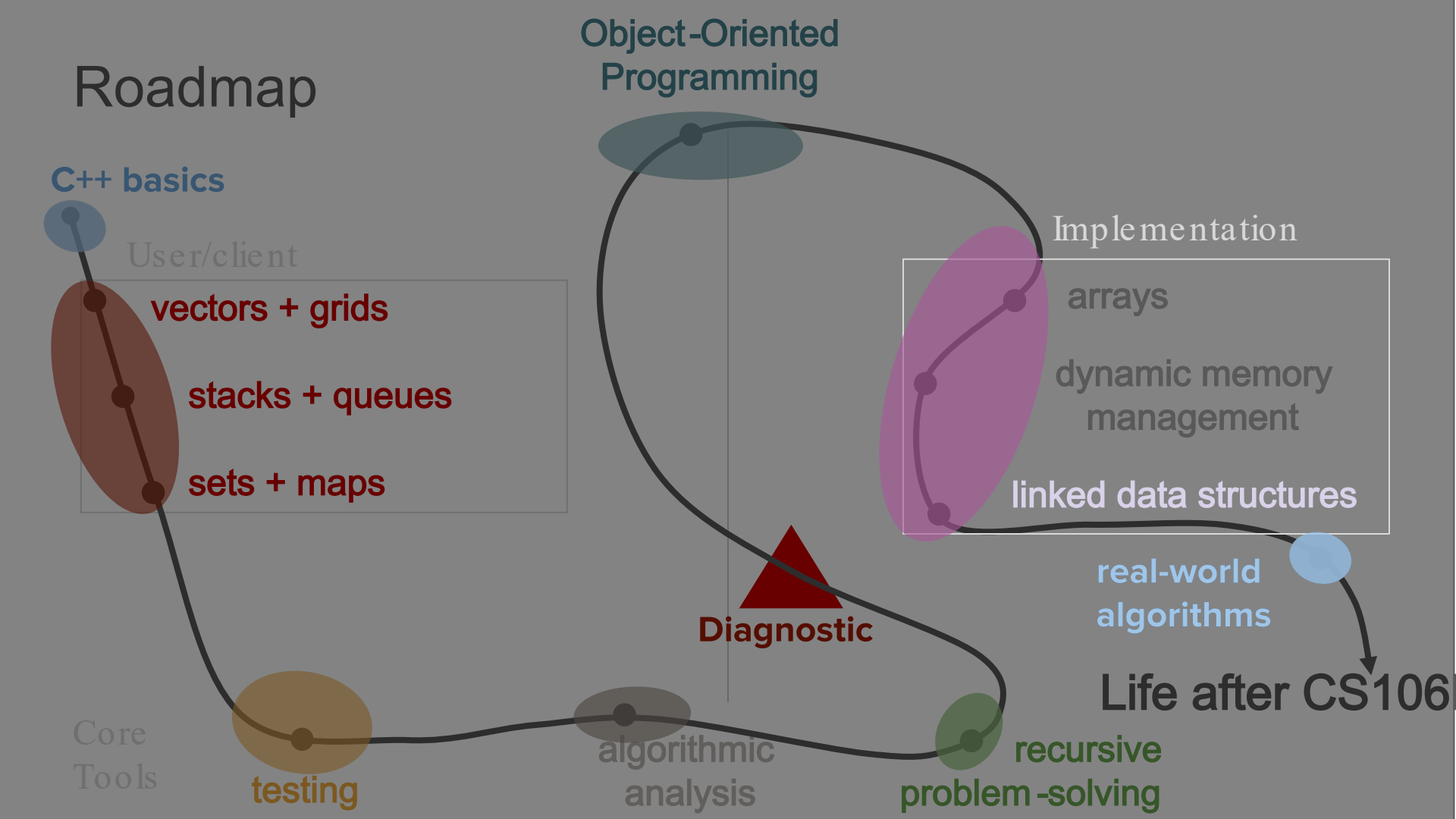
- Using a BST allowed us to take advantage of recursion to traverse our data and get an O(log n) runtime for our methods.

- Rewiring trees can be complicated!
  - Make sure to consider when nodes need to be passed by reference.
  - Check out the remove method after class if you're interested in seeing an example of tree rewiring (you won't be required to do anything this complex with tree rewiring).

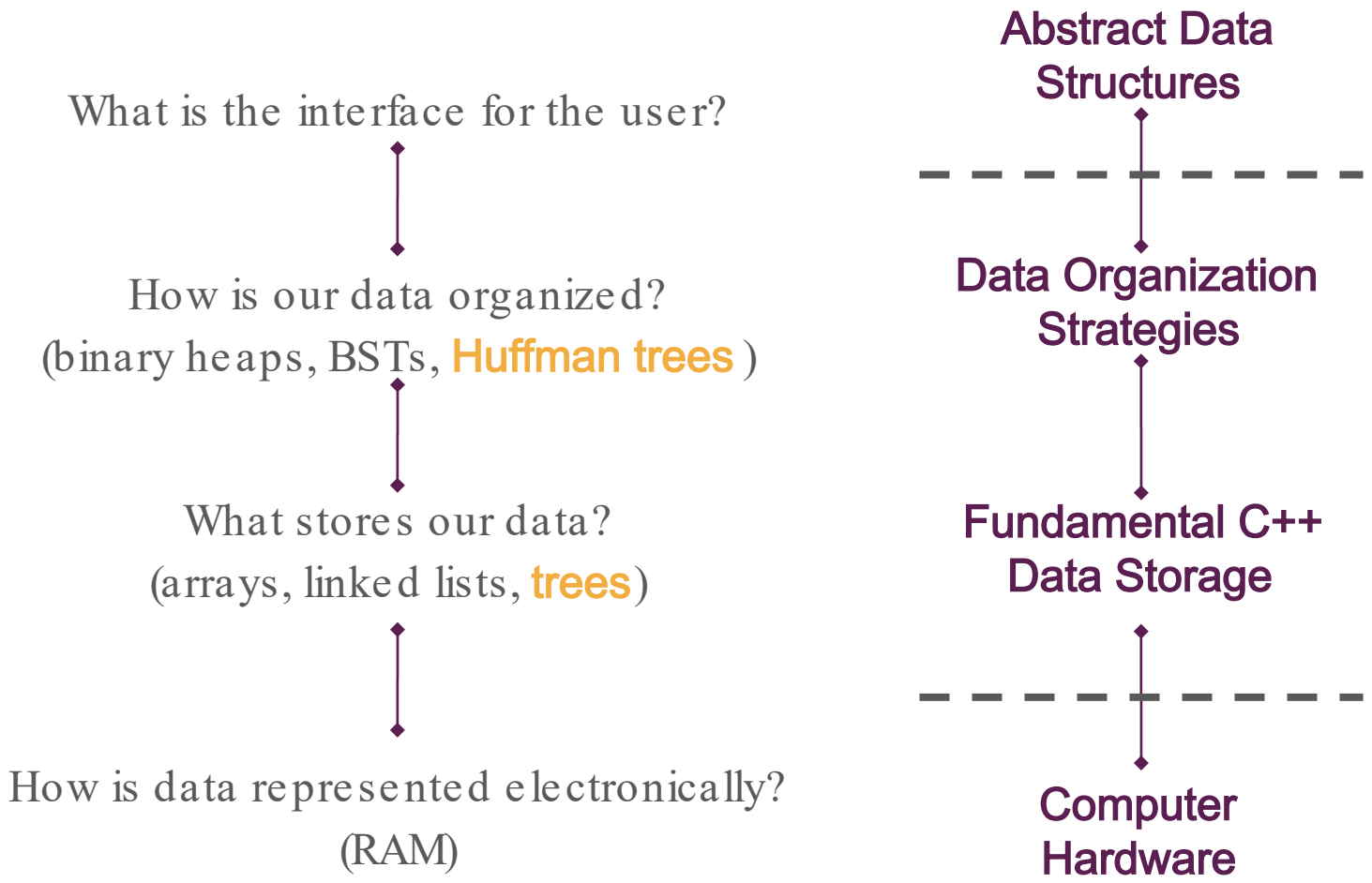
You can play around with a simulation of BSTs to see traversal, lookup, insertion, and deletion, and try balanced and unbalanced trees at this website:

https://visualgo.net/bn/bst

What’s next?


Roadmap
Object-Oriented Programming
C++ basics
User/client
vectors + grids
stacks + queues
sets + maps
Implementation
arrays
dynamic memory management
linked data structures
real-world algorithms
Diagnostic
Life after CS106
Core Tools
testing
algorithmic analysis
recursive problem-solving

Levels of abstraction



Huffman coding