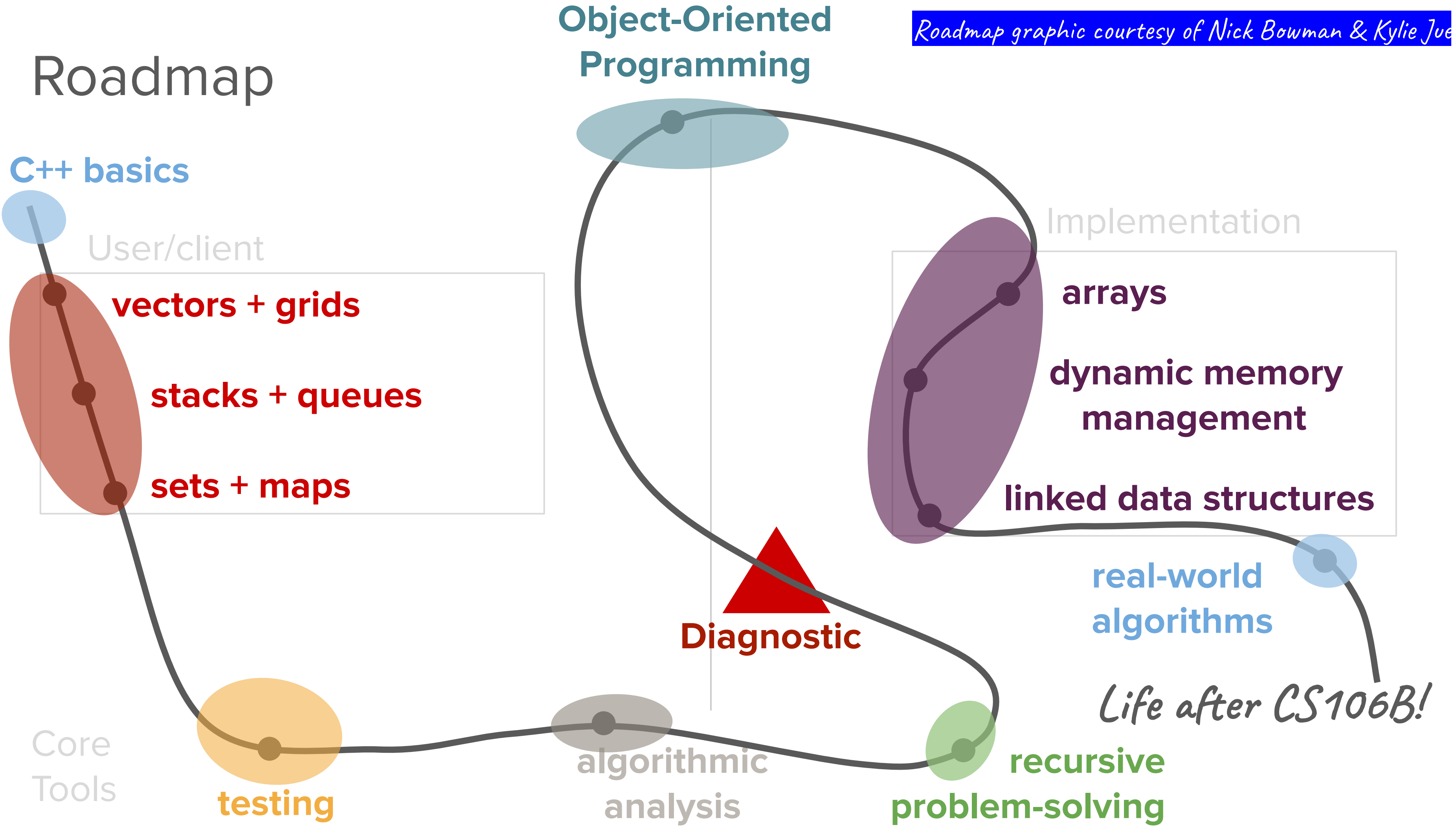
Recursive Fractals

What examples of recursion have you encountered in day-to-day life (not programming-related)?

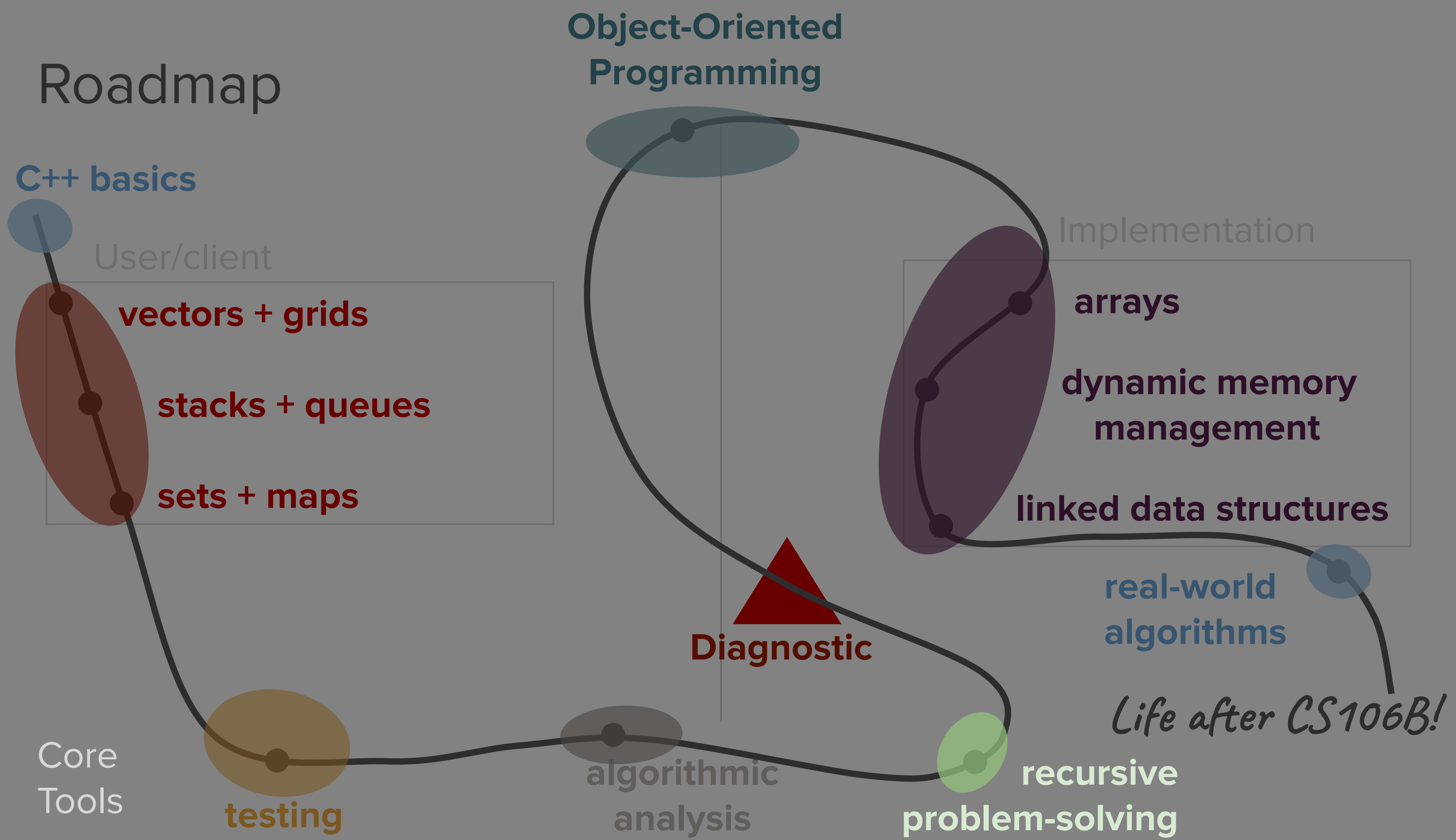
(put your answers the chat)

Roadmap

Roadmap graphic courtesy of Nick Bowman & Kylie Jue



Roadmap


Object-Oriented Programming
C++ basics
User/client
vectors + grids
stacks + queues
sets + maps
Implementation
arrays
dynamic memory management
linked data structures
real-world algorithms
Diagnostic
Life after CS106B!
Core Tools
testing
algorithmic analysis
recursive problem-solving

Today's question

How can we use recursion to make art?

Today's topics

1. Review
2. The Cantor Set
3. The Sierpinski Carpet
4. Revisiting the Towers of Hanoi

Review

Definition

recursion

A problem-solving technique in which tasks are completed by reducing them into repeated, smaller tasks of the same form.

Recursion Review

- Recursion is a problem-solving technique in which tasks are completed by reducing them into **repeated, smaller tasks of the same form**.
    - A recursive operation (function) is defined in terms of itself (i.e. it calls itself).

Recursion Review

- Recursion is a problem-solving technique in which tasks are completed by reducing them into **repeated, smaller tasks of the same form**.

- Recursion has two main parts: the **base case** and the **recursive case**.
    - Base case: Simplest form of the problem that has a direct answer.
    - Recursive case: The step where you break the problem into a smaller, self-similar task.

Recursion Review

- Recursion is a problem-solving technique in which tasks are completed by reducing them into **repeated, smaller tasks of the same form**.

- Recursion has two main parts: the **base case** and the **recursive case**.

- The solution will get built up **as you come back up the call stack**.
    - The base case will define the “base” of the solution you’re building up.
    - Each previous recursive call contributes a little bit to the final solution.
    - The initial call to your recursive function is what will return the completely constructed answer.

Recursion Review

- Recursion is a problem-solving technique in which tasks are completed by reducing them into **repeated, smaller tasks of the same form**.

- Recursion has two main parts: the **base case** and the **recursive case**.

- The solution will get built up **as you come back up the call stack**.

- When solving problems recursively, look for **self-similarity** and think about **what information is getting stored in each stack frame**.

Recursion Review

- Recursion is a problem-solving technique in which tasks are completed by reducing them into **repeated, smaller tasks of the same form**.

- Recursion has two main parts: the **base case** and the **recursive case**.

- The solution will get built up **as you come back up the call stack**.

- When solving problems recursively, look for **self-similarity** and think about **what information is getting stored in each stack frame**.

Example:

`isPalindrome()`

Check out these funny English palindrome sentences that you can read forwards and backwards:

Was it a rat I saw?
A nut for a jar of tuna.
Go dog!
Don't nod!
No lemon, no melon.
Was it a car or a cat I saw?
Oozy rat in a sanitary zoo.
Never odd or even.
Step on no pets.
Mr. Owl ate my metal worm.

Susan’s speedometer in 2012 and 2019

Write a function that returns if a string is a palindrome

A string is a palindrome if it reads the same both forwards and backwards:

- isPalindrome(“level”) ➔ true
- isPalindrome(“racecar”) ➔ true
- isPalindrome(“step on no pets”) ➔ true
- isPalindrome(“high”) ➔ false
- isPalindrome(“hi”) ➔ false
- isPalindrome(“palindrome”) ➔ false
- isPalindrome(“X”) ➔ true
- isPalindrome(“”) ➔ true

Approaching recursive problems

- Look for self-similarity.

- Try out an example and look for patterns.
  - Work through a simple example and then increase the complexity.
  - Think about what information needs to be “stored” at each step in the recursive case (like the current value of **`n`** in each **`factorial`** stack frame).

- Ask yourself:
  - What is the base case? (What is the simplest case?)
  - What is the recursive case? (What pattern of self-similarity do you see?)

Discuss:

What are the base and recursive cases?

(breakout rooms)

isPalindrome()

- Look for self-similarity: **racecar**

isPalindrome()

- Look for self-similarity: **racecar**
  - Look at the first and last letters of “racecar” ➔ both are ‘r’

isPalindrome()

- Look for self-similarity: **racecar**
  - Look at the first and last letters of “racecar” ➔ both are ‘r’
  - Check if “aceca” is a palindrome:

isPalindrome()

- Look for self-similarity: **racecar**
  - Look at the first and last letters of “racecar” ➜ both are ‘r’
  - Check if “aceca” is a palindrome:
    - Look at the first and last letters of “aceca” ➜ both are ‘a’
    - Check if “cec” is a palindrome:

isPalindrome()

- Look for self-similarity: **racecar**
  - Look at the first and last letters of “racecar” ➔ both are ‘r’
  - Check if “aceca” is a palindrome:
    - Look at the first and last letters of “aceca” ➔ both are ‘a’
    - Check if “cec” is a palindrome:
      - Look at the first and last letters of “cec” ➔ both are ‘c’
      - Check if “e” is a palindrome:

isPalindrome()

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  - Look at the first and last letters of “racecar” ➔ both are ‘r’
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      - Look at the first and last letters of “cec” ➔ both are ‘c’
      - Check if “e” is a palindrome:
        - **Base case**: “e” is a palindrome

isPalindrome()

- Look for self-similarity: **racecar**
    - Look at the first and last letters of “racecar” ➔ both are ‘r’
    - Check if “aceca” is a palindrome:
        - Look at the first and last letters of “aceca” ➔ both are ‘a’
        - Check if “cec” is a palindrome:
            - Look at the first and last letters of “cec” ➔ both are ‘c’
            - Check if “e” is a palindrome:
                - **Base case**: “e” is a palindrome

What about the `false` *case?*

isPalindrome()

- Look for self-similarity: **high**

isPalindrome()

- Look for self-similarity: **high**
  - Look at the first and last letters of “high” ➜ both are ‘h’

isPalindrome()

- Look for self-similarity: **high**
  - Look at the first and last letters of “high” ➜ both are ‘h’
  - Check if “ig” is a palindrome:

isPalindrome()

- Look for self-similarity: **high**
  - Look at the first and last letters of “high” ➔ both are ‘h’
  - Check if “ig” is a palindrome:
    - Look at the first and last letters of “ig” ➔ not equal
    - **Base case**: Return `false`

isPalindrome()

- **Base cases**:
  - isPalindrome(“”) ➜ `true`
  - isPalindrome(string of length 1) ➜ `true`
  - If the first and last letters are not equal ➜ `false`

- **Recursive case:** If the first and last letters are equal,
  isPalindrome(string) = isPalindrome(string minus first and last letters)

isPalindrome()

- **Base cases**:
  - isPalindrome(“”) ➜ **`true`**
  - isPalindrome(string of length 1) ➜ **`true`**
  - If the first and last letters are not equal ➜ **`false`**

There can be multiple base (or recursive) cases!

- **Recursive case:** If the first and last letters are equal, isPalindrome(string) = isPalindrome(string minus first and last letters)

isPalindrome()

```
bool isPalindrome (string s) {
    if (s.length() < 2) {
        return true;
    } else {
      if (s[0] != s[s.length() - 1]) {
          return false;
      }
      return isPalindrome(s.substr(1, s.length() - 2));
    }
}
```

isPalindrome() in action

```
int main() {
    cout << isPalindrome("racecar") << endl;
    return 0;
}
```

isPalindrome() in action

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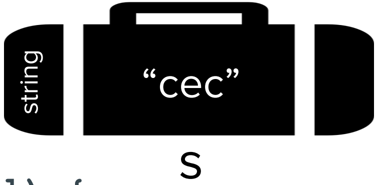

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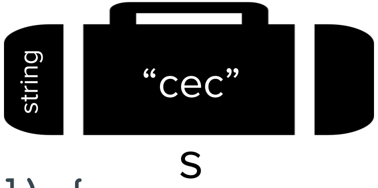

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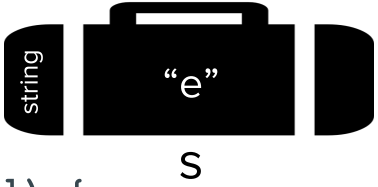

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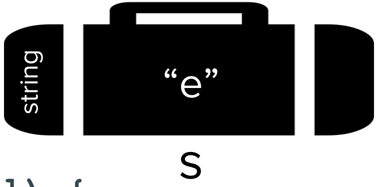

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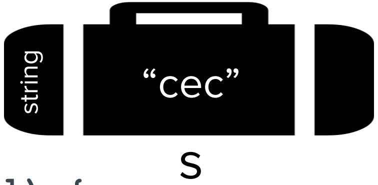

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}
```

true

isPalindrome() in action

```
int main() {
    cout << isPalindrome("racecar") << endl;
    return 0;
}
```

Prints **true**!

Announcements

Announcements

- Assignment 1 grades will be released on Paperless by the end of the day today.

- Assignment 2 is due **tonight** at 11:59pm PDT.

- Assignment 3 will be released by the end of the day on Thursday.
  - YEAH for A3 will be **7/8 at 11am PT.** Info for the session will be posted on Ed.

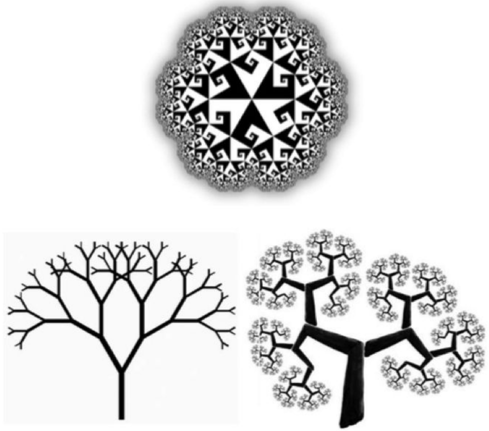
- Make sure to check out our posts on Ed – there's important info there!

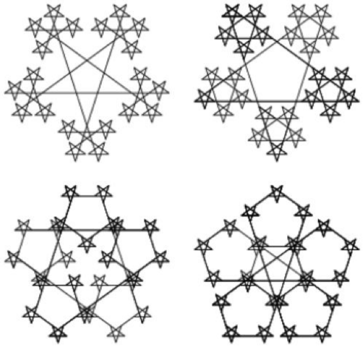
Self-Similarity

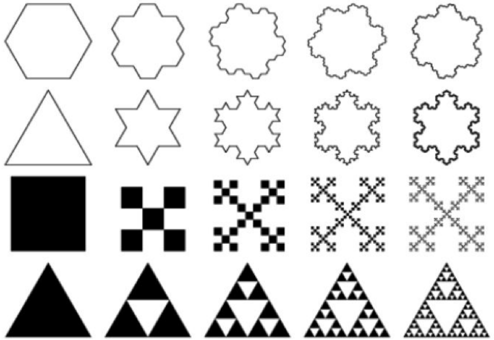
Fractals

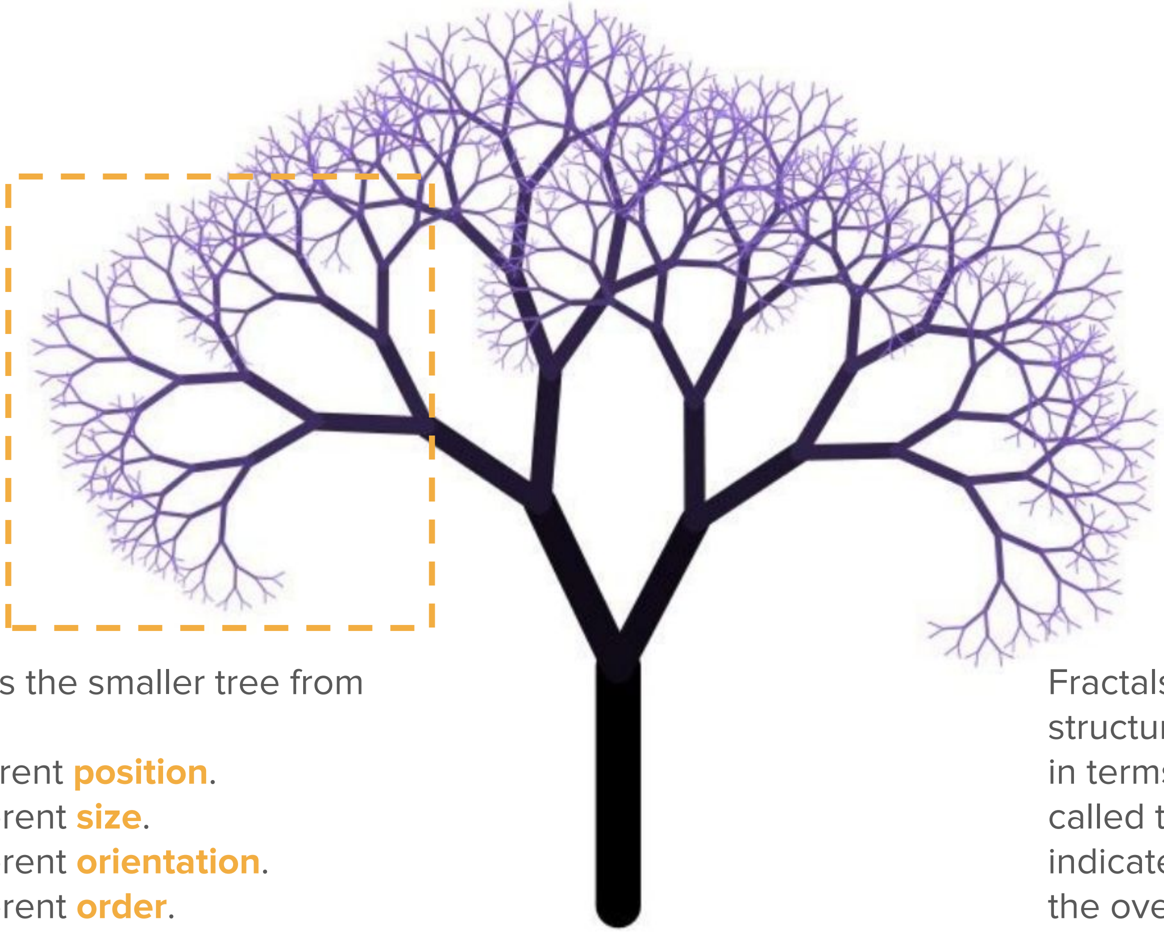
- A **fractal** is any repeated, graphical pattern.

- A fractal is composed of **repeated instances of the same shape or pattern**, arranged in a structured way.









What differentiates the smaller tree from the bigger one?

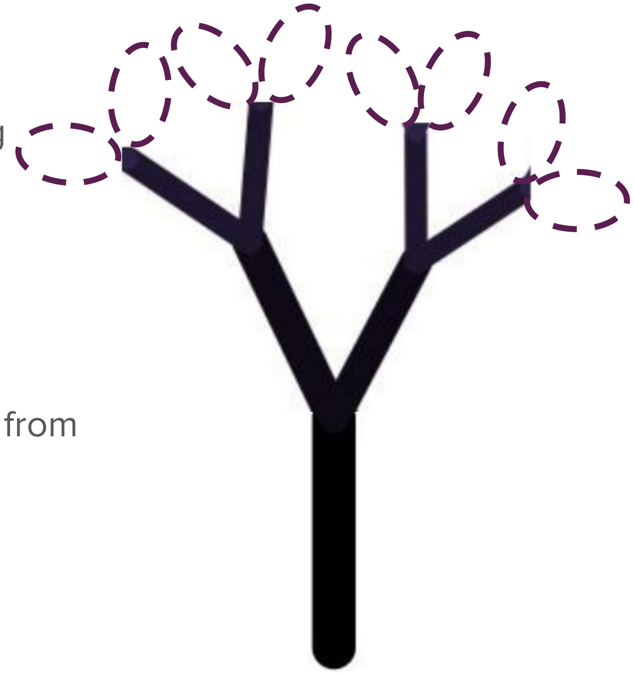
1. It's at a different **position**.
2. It has a different **size**.
3. It has a different **orientation**.
4. It has a different **order**.

Fractals and self-similar structures are often defined in terms of some parameter called the **order**, which indicates the complexity of the overall structure.

An order-3 tree

An order-0 tree is nothing at all.

An order-`n` tree is a line with two smaller order-`(n-1)` trees starting at the end of that line.



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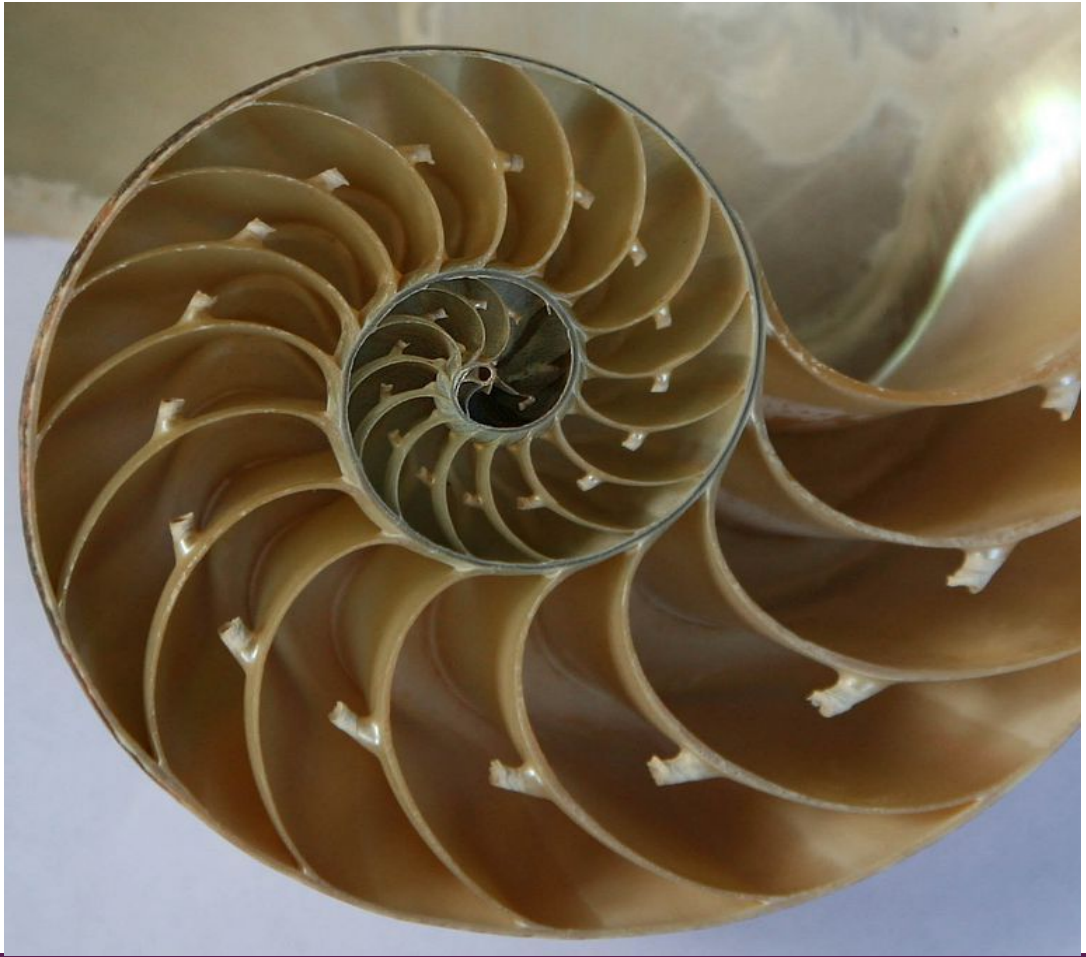


How can we use recursion to make art?

C++ Stanford graphics library

Graphics in CS106B

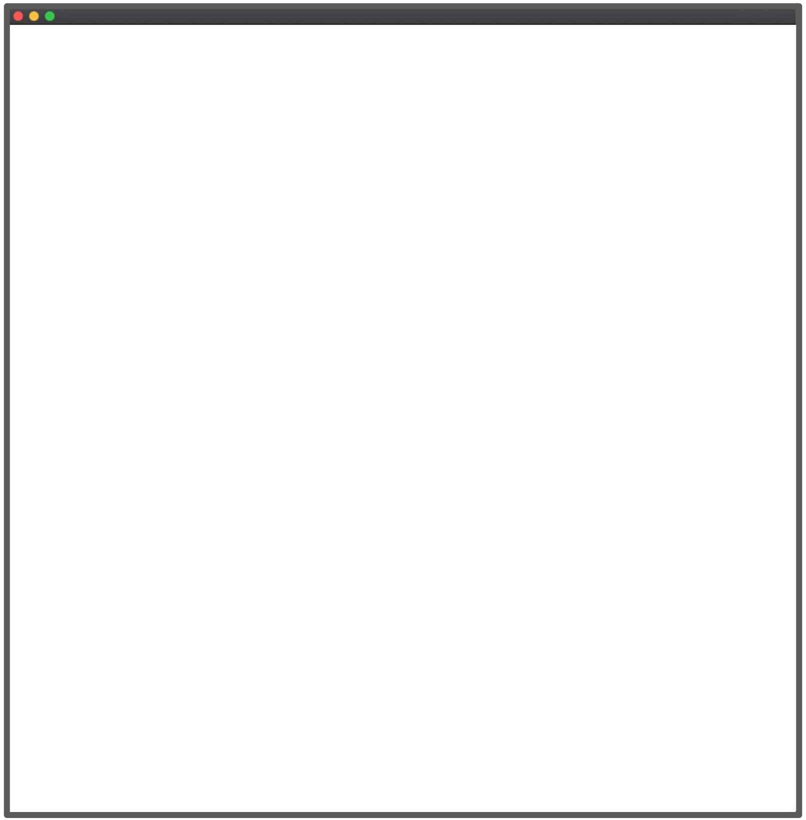
- Creating graphical programs is not one of our main focuses in this class, but a brief crash course in working with graphical programs is necessary to be able to code up some fractals of our own.

- The Stanford C++ libraries provide extensive capabilities to create custom graphical programs. The full documentation of these capabilities can be found in the official documentation.

- We will abstract away almost all of the complexity for you via provided helper functions.
    - There are two main classes/components of the library you need to know: **`GWindow`** and **`GPoint`**

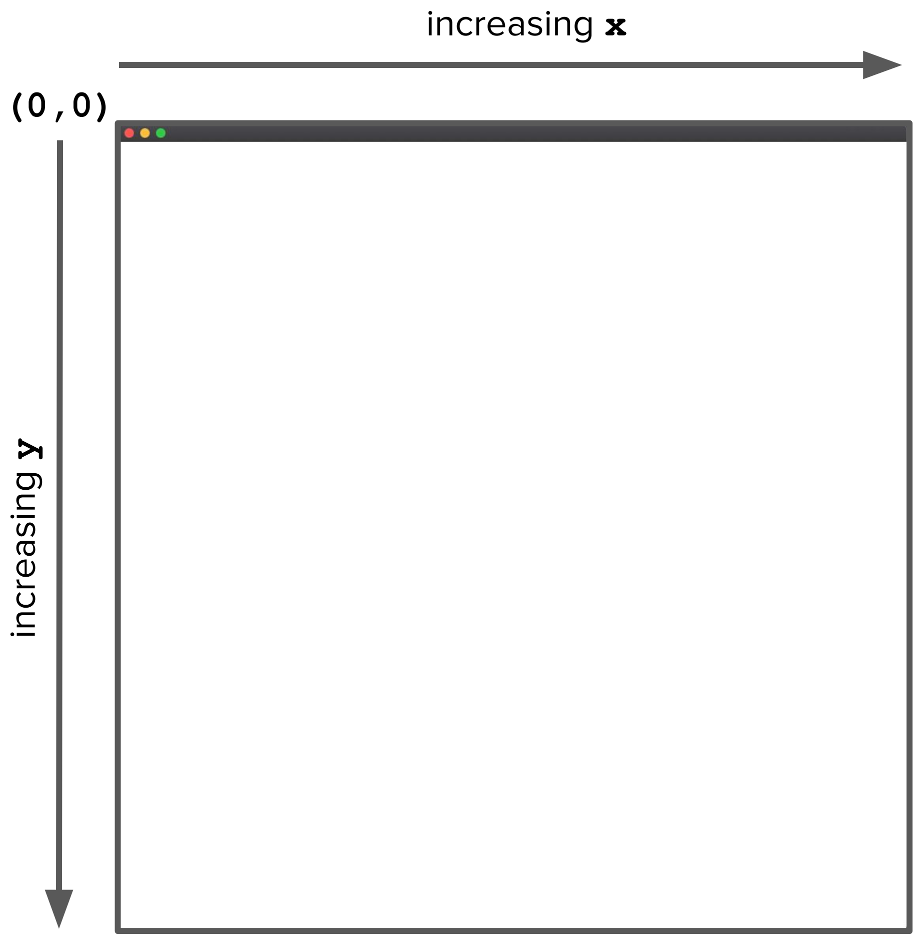
GWindow

- A **`GWindow`** is an abstraction for the graphical window upon which we will do all of our drawing.



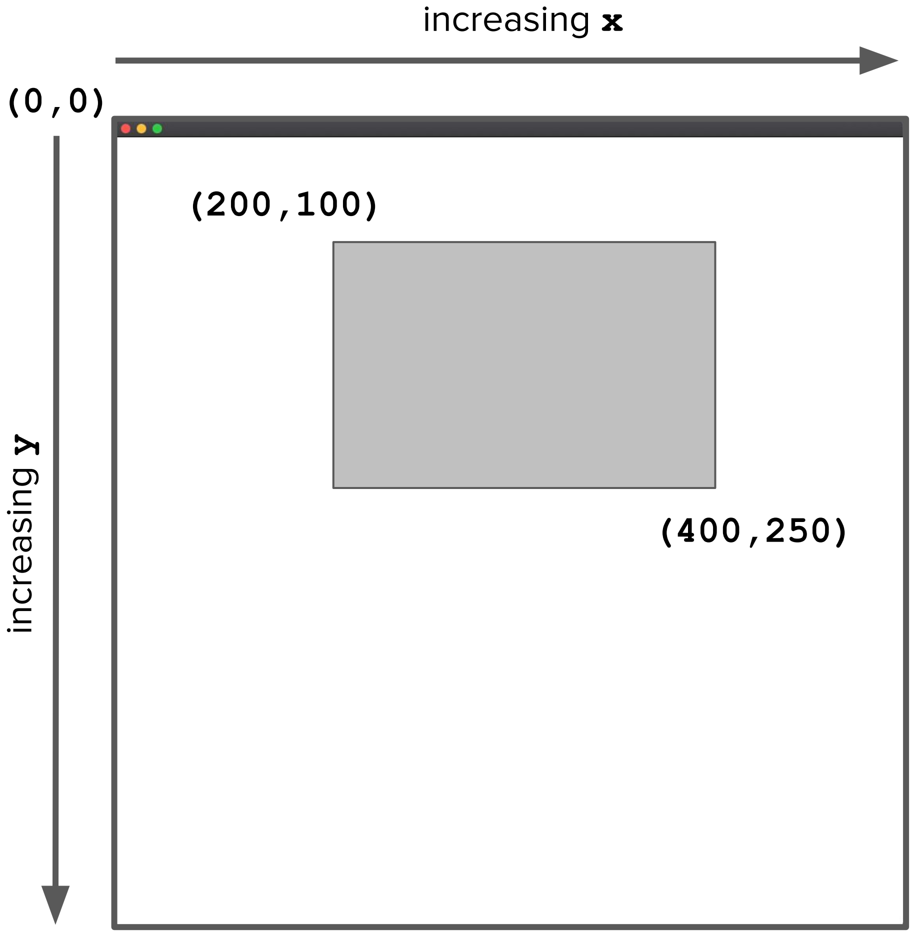
GWindow

- A **GWindow** is an abstraction for the graphical window upon which we will do all of our drawing.
- The window defines a coordinate system of x-y values
  - The top left corner is **(0, 0)**
  - The bottom right corner is **(windowWidth, windowHeight)**



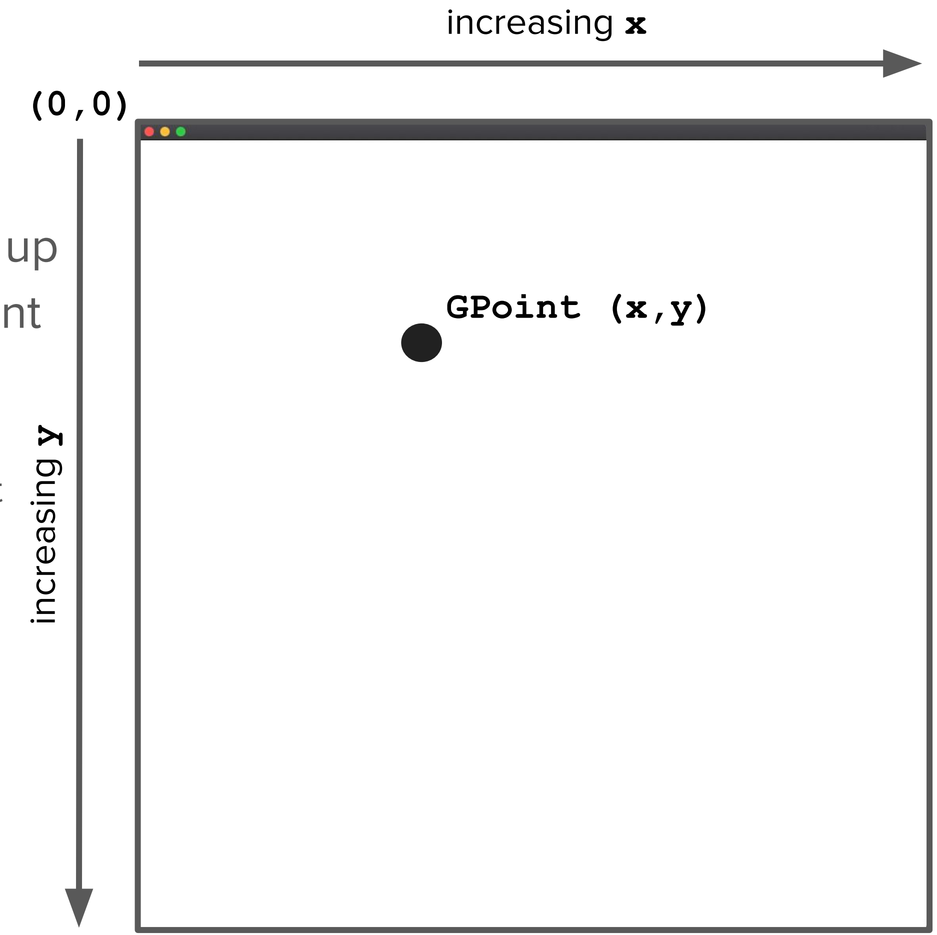
GWindow

- A **GWindow** is an abstraction for the graphical window upon which we will do all of our drawing.
- The window defines a coordinate system of x-y values
    - The top left corner is **(0, 0)**
    - The bottom right corner is **(windowWidth, windowHeight)**
- All lines and shapes drawn on the window are defined by their **(x,y)** coordinates



GPoint

- A **GPoint** is a handy way to bundle up the x-y coordinates for a specific point in the window.
  - Very similar in functionality to the **GridLocation** struct we learned about before!

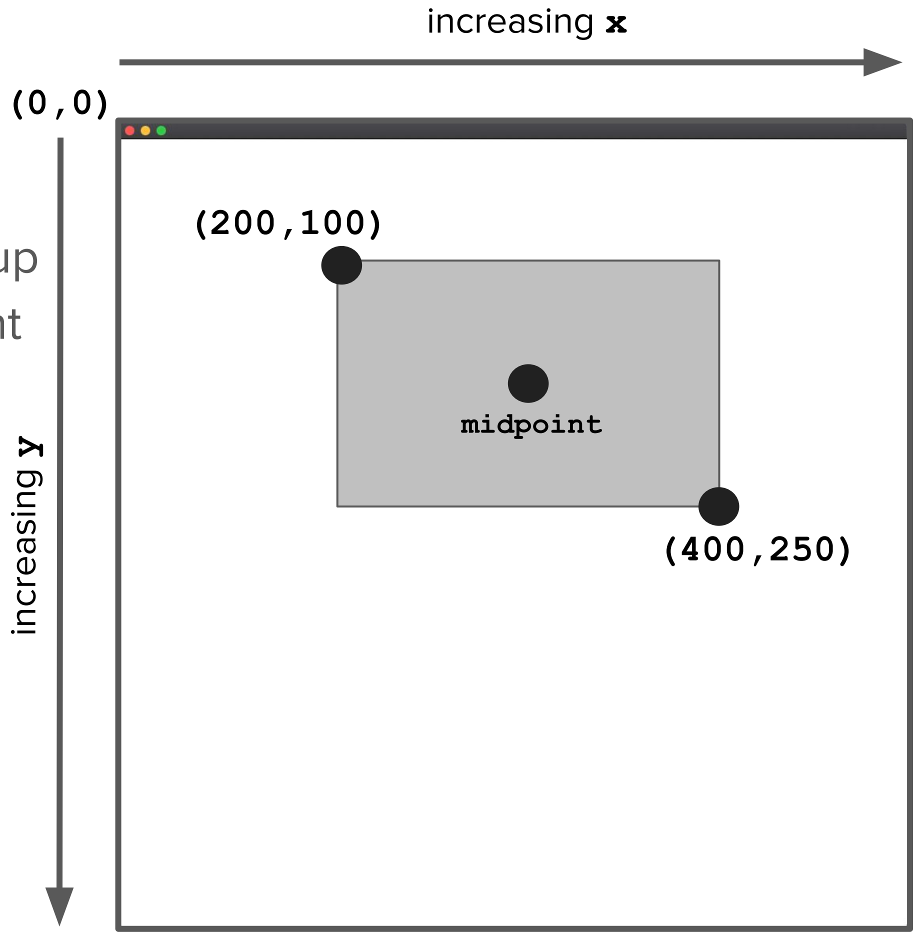


GPoint

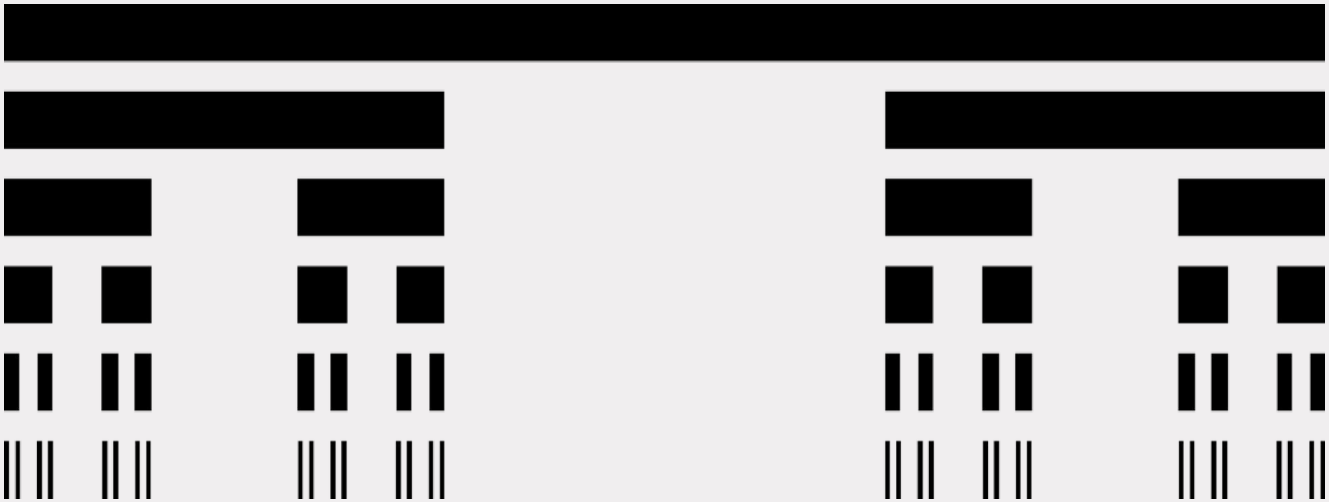
- A **GPoint** is a handy way to bundle up the x-y coordinates for a specific point in the window.
  - Very similar in functionality to the **GridLocation** struct we learned about before!

```
GPoint topLeft(200, 100);
GPoint bottomRight(400, 250);
drawFilledRect(topLeft, bottomRight);

GPoint midpoint = {
(topLeft.x + bottomRight.x)/ 2,
(topLeft.y + bottomRight.y)/ 2 };
```



Cantor Set example



Cantor Set

- The Cantor fractal is a set of lines where there is one main line, and below that there are two other lines: each ⅓ of the width of the original line, with one on the left and one on the right (with a ⅓ separation of whitespace between them)
- Below each of the other lines is an identical situation: two ⅓ lines.
- This repeats until the lines are no longer visible.
- The factors to differentiate the fractal components:
  size, **position**, **orientation**, and **order**

An order-0 Cantor Set

An order-1 Cantor Set

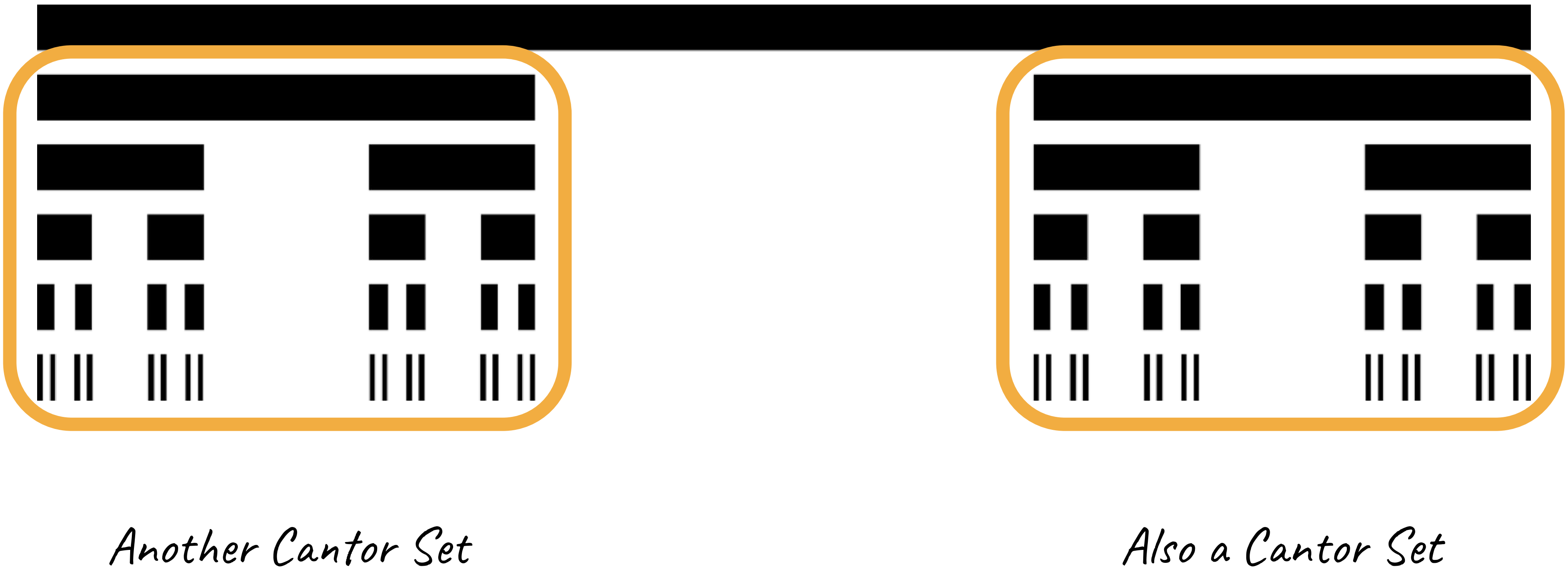
An order-2 Cantor Set

An order-6 Cantor Set

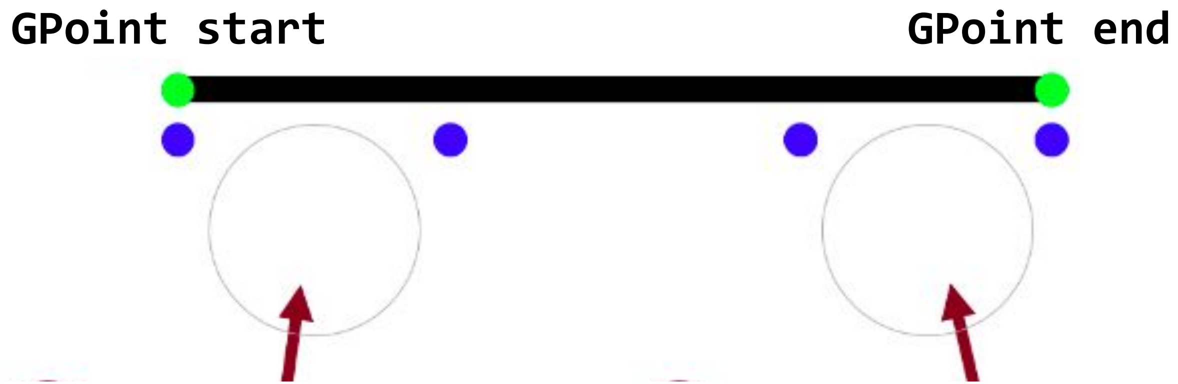
An order-6 Cantor Set

Another Cantor Set

An order-6 Cantor Set

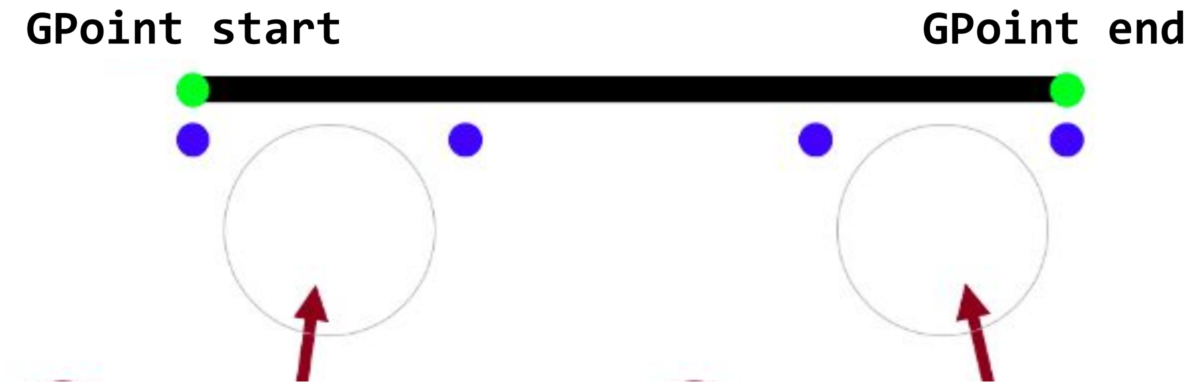


How to draw an order-n Cantor Set



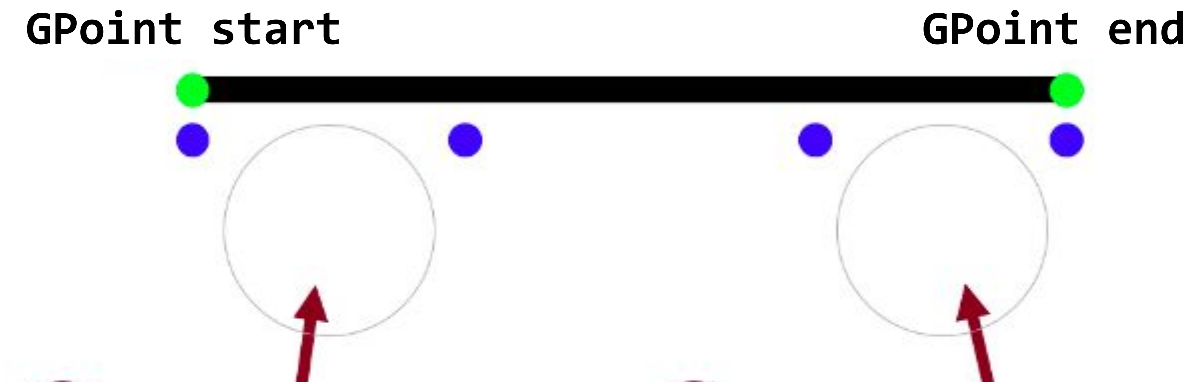
How to draw an order-n Cantor Set

1. Draw a line from **start** to **end**.



How to draw an order-n Cantor Set

1. Draw a line from **`start`** to **`end`**.



2. Underneath the left third, draw a Cantor Set of order-(**`n - 1`**).

How to draw an order-n Cantor Set

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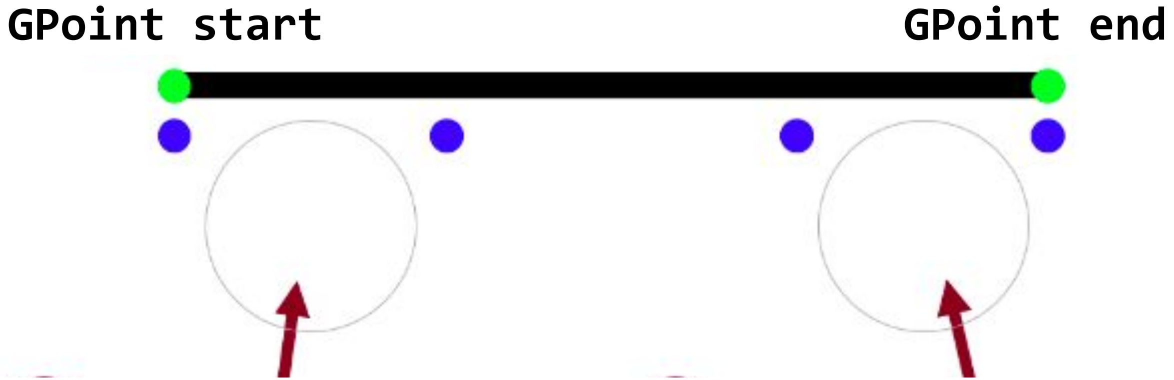


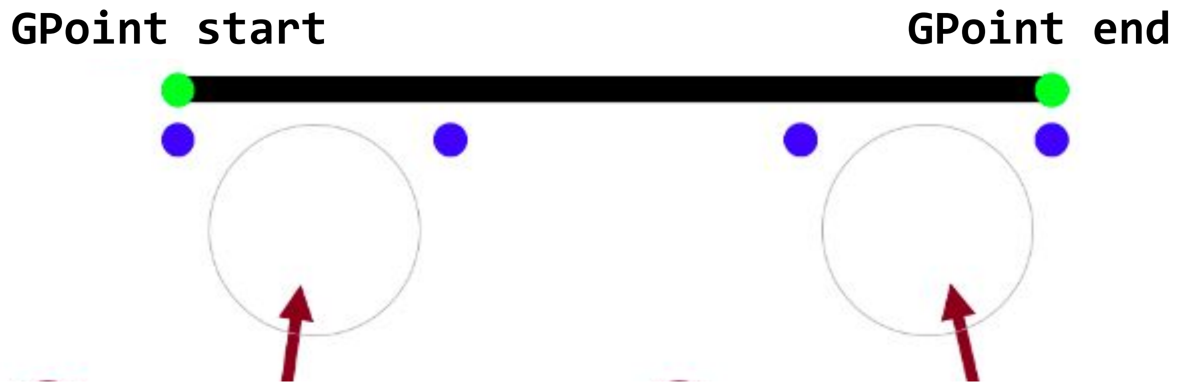
2. Underneath the left third, draw a Cantor Set of order-(**`n - 1`**).
3. Underneath the right third, draw a Cantor Set of order-(**`n - 1`**).

How to draw an order-n Cantor Set

Base case:

`order == 0`

1. Draw a line from **`start`** to **`end`**.

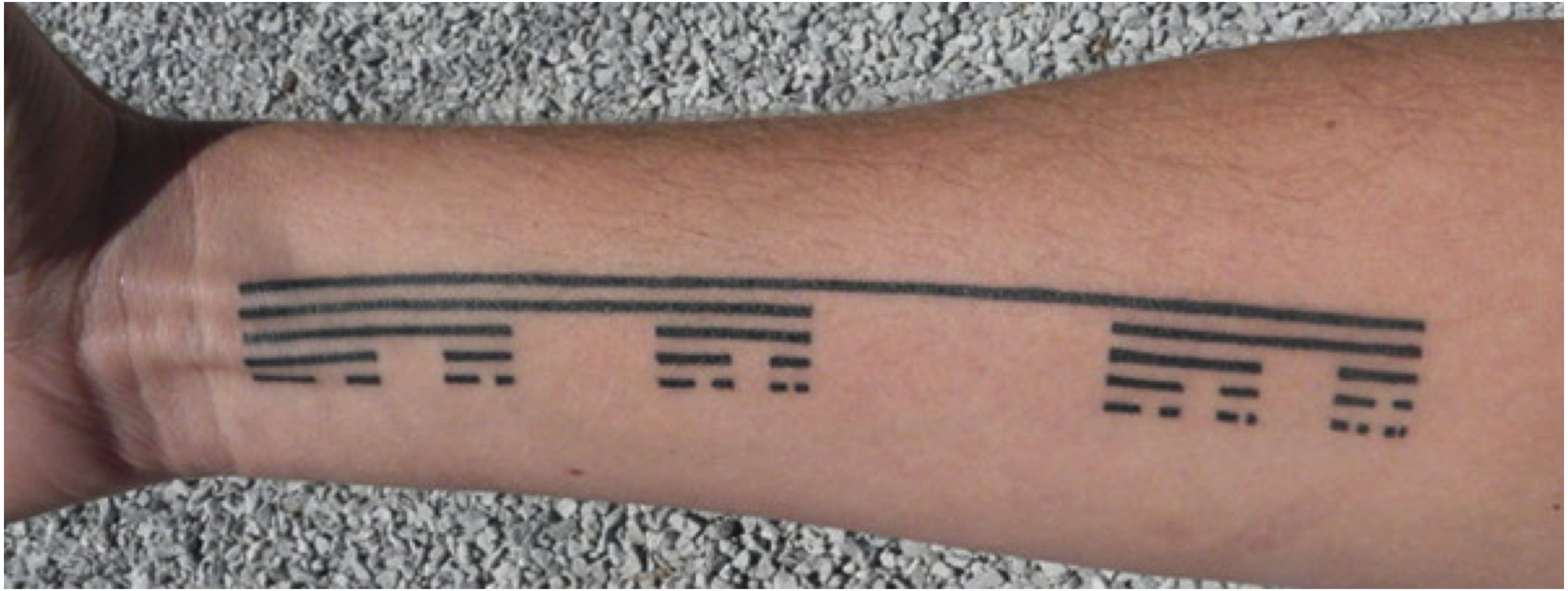


2. Underneath the left third, draw a Cantor Set of order-(**`n - 1`**).
3. Underneath the right third, draw a Cantor Set of order-(**`n - 1`**).

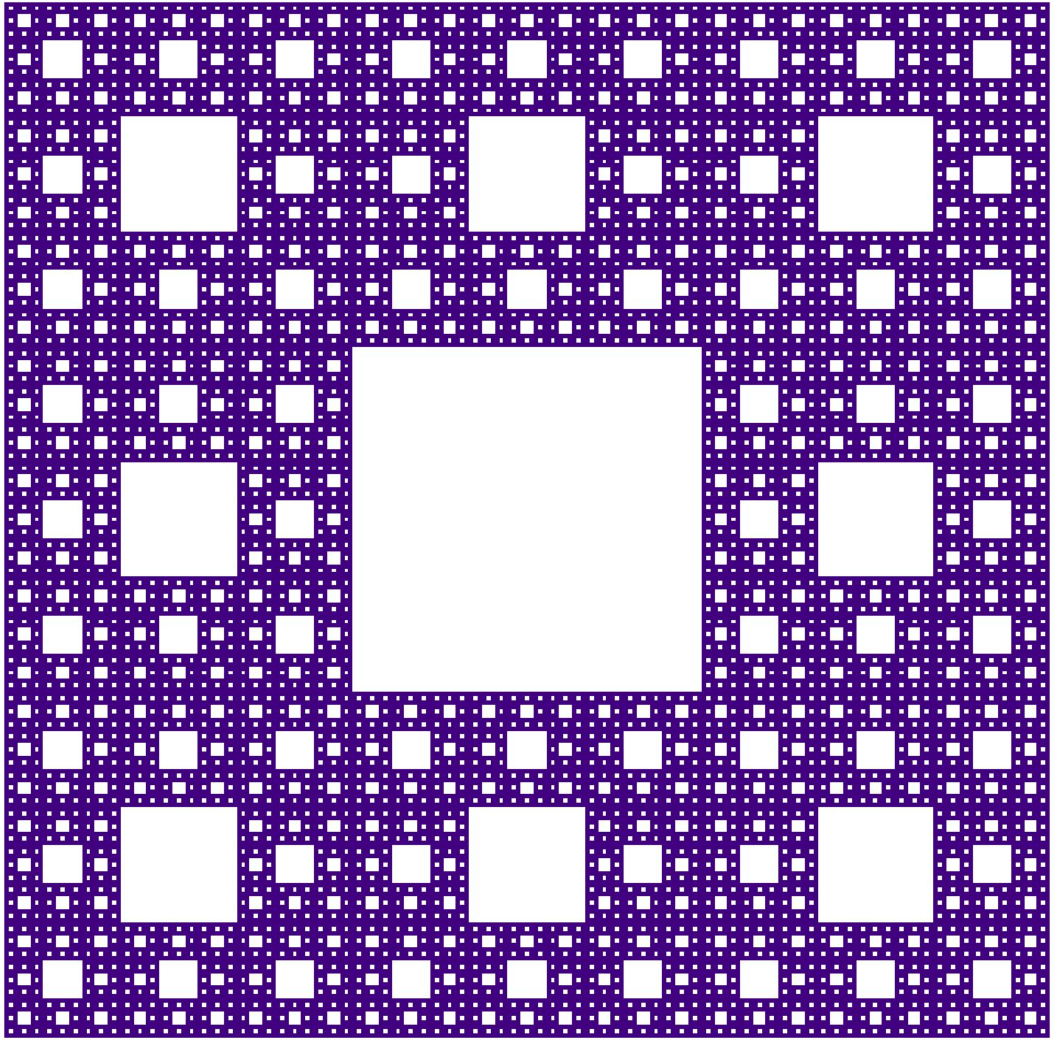
Cantor Set demo

[Qt Creator]

Real-world application of the Cantor Set

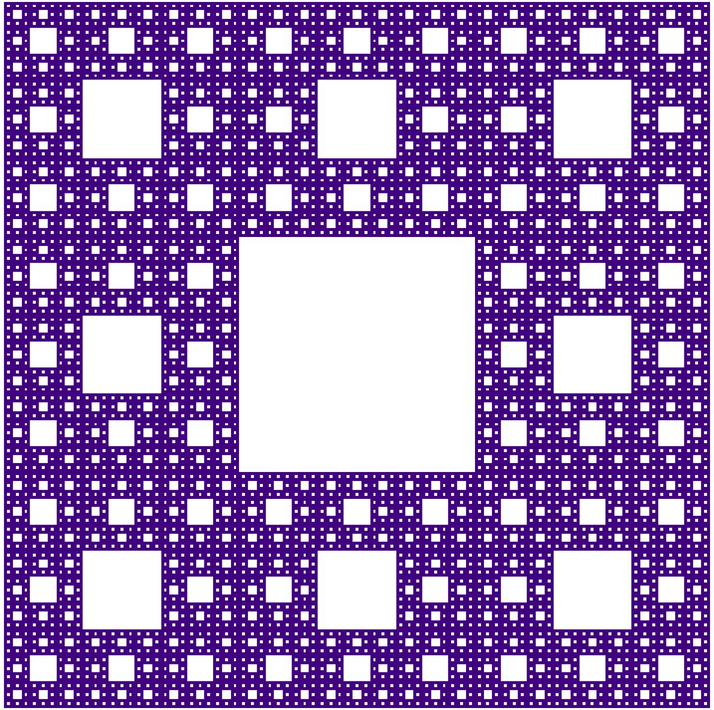


Sierpinski Carpet example

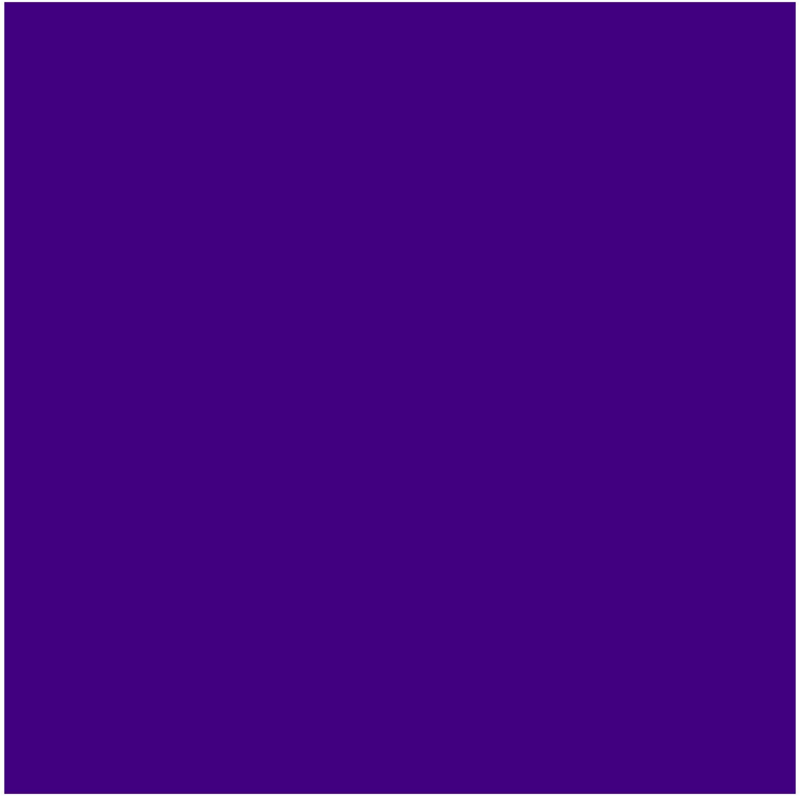


Sierpinski Carpet

- First described by Wacław Sierpiński in 1916
- A generalization of the Cantor Set to two dimensions!
- Defined by the subdivision of a shape (a square in this case) into smaller copies of itself.
  - The same pattern applied to a triangle yields a Sierpinski triangle, which you will code up on the next assignment.

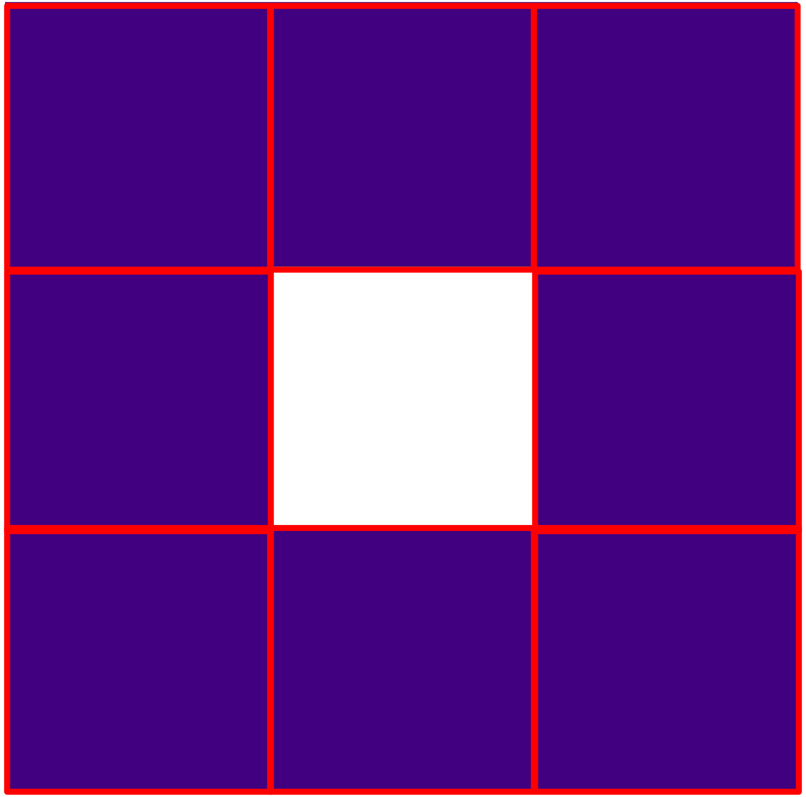


An order-0 Sierpinski Carpet



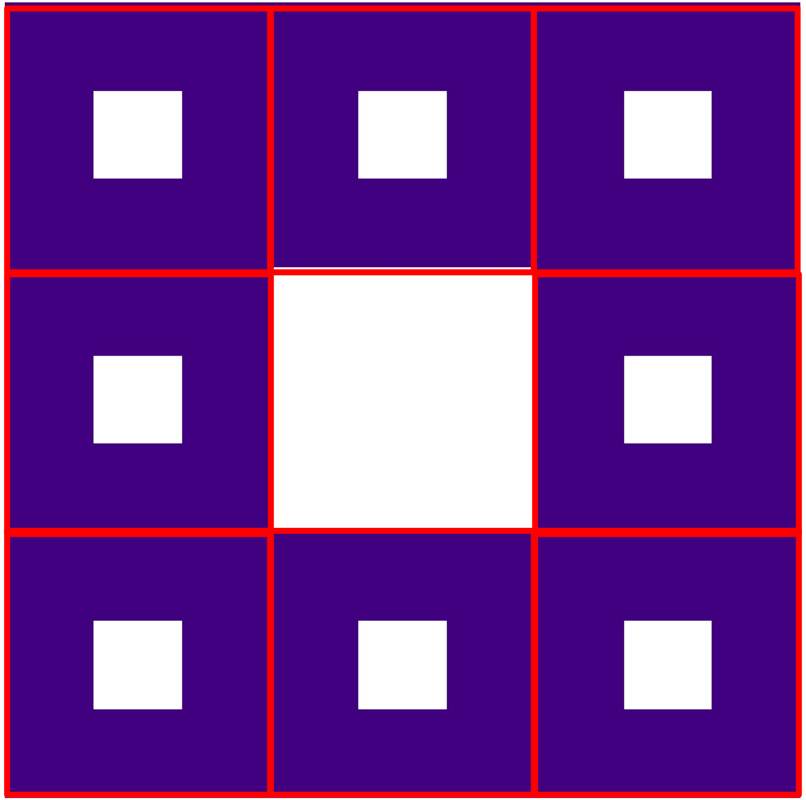
An order-1 Sierpinski Carpet

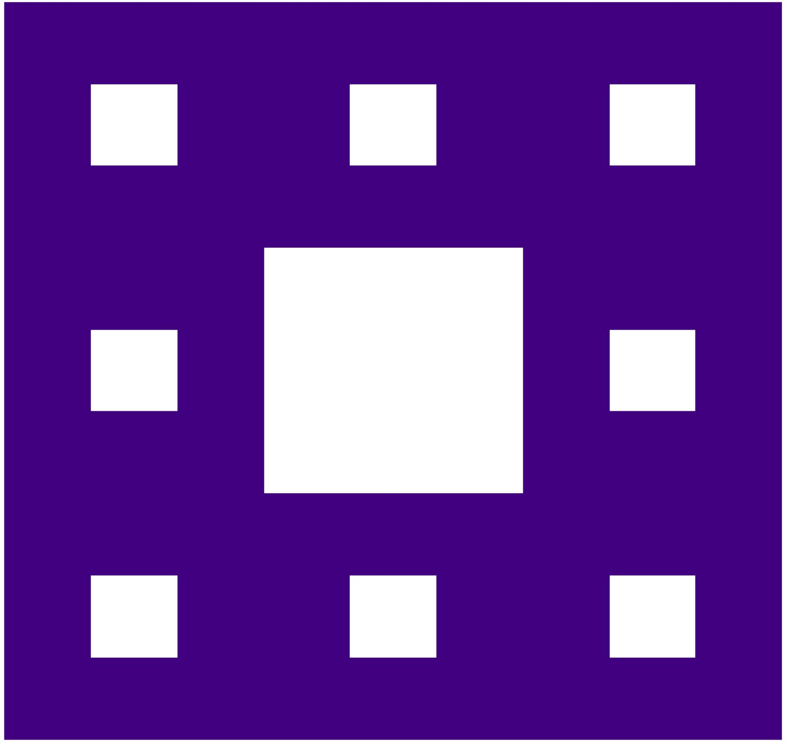
An order-1 carpet is subdivided into eight order-0 carpets arranged in this grid pattern



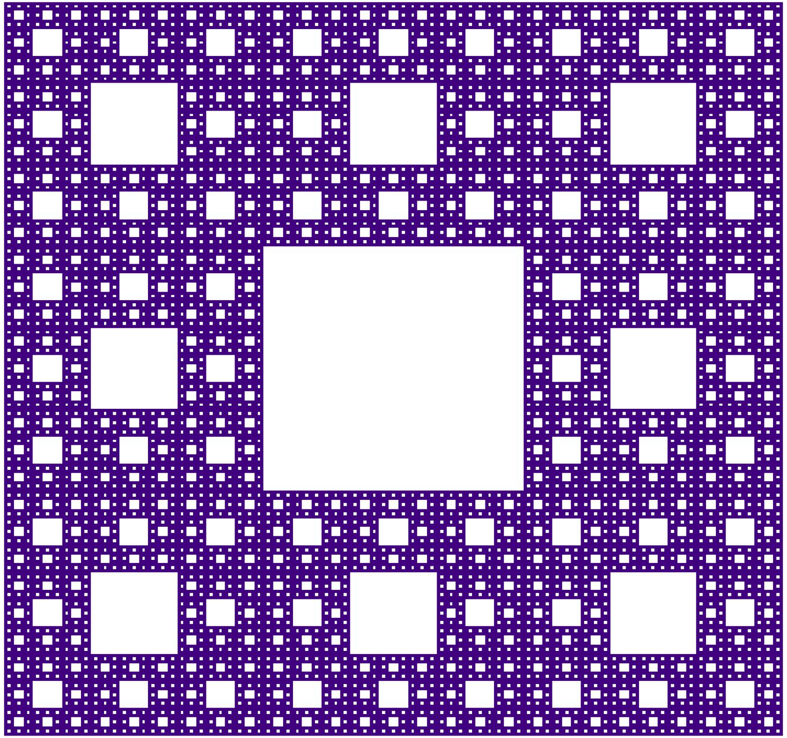
What are the base and recursive cases that define an order 2 Sierpinski Carpet fractal?

An order-2 Sierpinski Carpet





Order 2 Sierpinski carpet



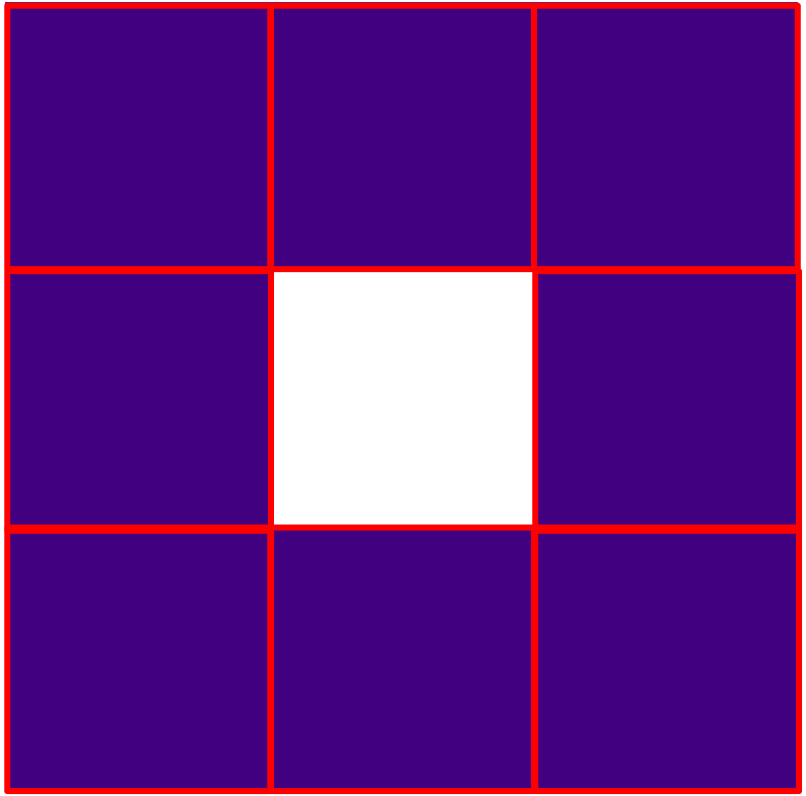
Order 5 Sierpinski carpet

Sierpinski Carpet Formalized

- Base Case (order-0)
  - Draw a filled square at the appropriate location

Sierpinski Carpet Formalized

- Base Case (order-0)
  - Draw a filled square at the appropriate location
- Recursive Case (order-n, n ≠ 0)
  - Draw 8 order n-1 Sierpinski carpets, arranged in a 3x3 grid, omitting the center location



Sierpinski Carpet Formalized

- Base Case (order-0)
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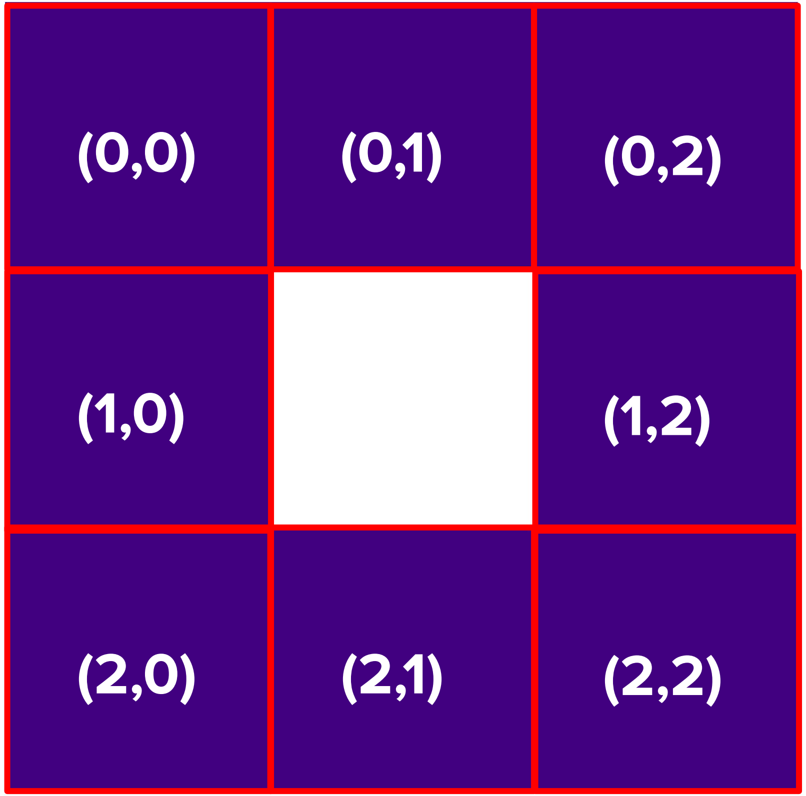
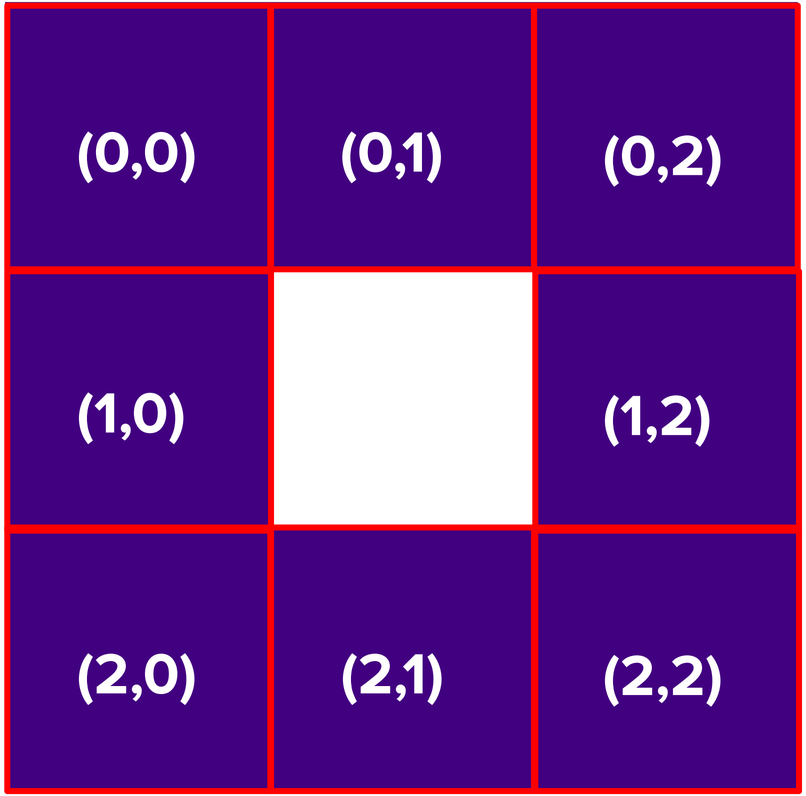


Sierpinski Carpet Formalized

- Base Case (order-0)
  - Draw a filled square at the appropriate location
- Recursive Case (order-n, n ≠ 0)
  - Draw 8 order n-1 Sierpinski carpets, arranged in a 3x3 grid, omitting the center location
    - i.e. Draw an n-1 fractal at (0,0), draw an n-1 fractal at (0,1), draw an n-1 fractal at (0,2)...



Sierpinski Carpet Pseudocode (Take 1)

```
drawSierpinskiCarpet (x, y, order):

  if (order == 0)
     drawFilledSquare(x, y, BASE_SIZE)
```

Sierpinski Carpet Pseudocode (Take 1)

```
drawSierpinskiCarpet (x, y, order):

  if (order == 0)
     drawFilledSquare(x, y, BASE_SIZE)
  else
     drawSierpinskiCarpet(newX(x, y, 0, 0), newY(x, y, 0, 0), order -1)
     drawSierpinskiCarpet(newX(x, y, 0, 1), newY(x, y, 0, 1), order -1)
     drawSierpinskiCarpet(newX(x, y, 0, 2), newY(x, y, 0, 2), order -1)
     drawSierpinskiCarpet(newX(x, y, 1, 0), newY(x, y, 1, 0), order -1)
     drawSierpinskiCarpet(newX(x, y, 1, 2), newY(x, y, 1, 2), order -1)
     drawSierpinskiCarpet(newX(x, y, 2, 0), newY(x, y, 2, 0), order -1)
     drawSierpinskiCarpet(newX(x, y, 2, 1), newY(x, y, 2, 1), order -1)
     drawSierpinskiCarpet(newX(x, y, 2, 2), newY(x, y, 2, 2), order -1)
```

Sierpinski Carpet Pseudocode (Take 1)

```
drawSierpinskiCarpet (x, y, order):

  if (order == 0)
     drawFilledSquar
  else
     drawSierpinskiC                                    , order -1)
     drawSierpinskiC                                    , order -1)
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     drawSierpinskiC                                    , order -1)
     drawSierpinskiC                                    , order -1)
     drawSierpinskiC                                    , order -1)
     drawSierpinskiCarpet(newX(x, y, 2, 1), newY(x, y, 2, 1), order -1)
     drawSierpinskiCarpet(newX(x, y, 2, 2), newY(x, y, 2, 2), order -1)
```

This isn't very pretty, can we do better?

Sierpinski Carpet Pseudocode (Take 2)

```
drawSierpinskiCarpet (x, y, order):
    if (order == 0)
        drawFilledSquare(x, y, BASE_SIZE)
    else
        for row = 0 to row = 2:
            for col = 0 to col = 2:
                if (col != 1 || row != 1):
                    x_i = newX(x, y, row, col)
                    y_i = newY(x ,y, row, col)
                    drawSierpinskiCarpet(x_i, y_i, order - 1)
```

Iteration + Recursion

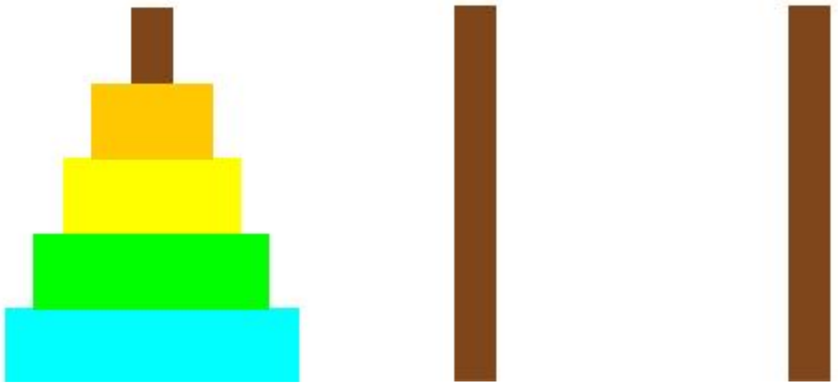
- It’s completely reasonable to mix iteration and recursion in the same function.
- Here, we’re firing off 8 recursive calls, and the easiest way to do that is with a double for loop.
- Recursion doesn’t mean “the absence of iteration.” It just means “solving a problem by solving smaller copies of that same problem.”
- Iteration and recursion can be very powerful in combination!

Sierpinski Carpet

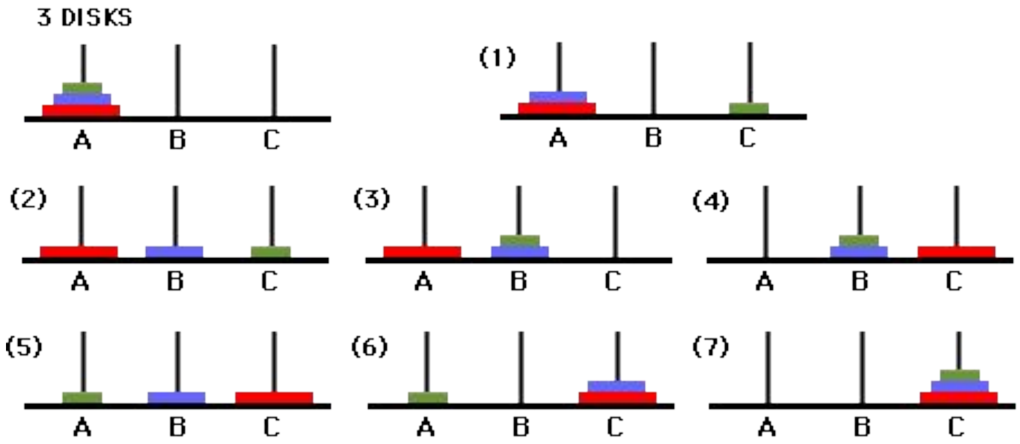
demo

Towers of Hanoi

- How to solve the problem as you increase the number of disks.
- How to define this problem recursively?



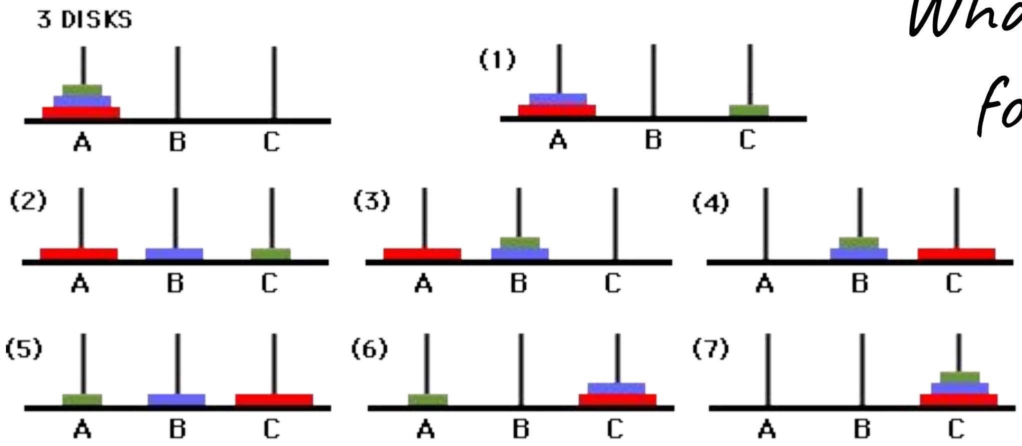
Pseudocode for 3 disks



(1) Move disk 1 to destination
(2) Move disk 2 to auxiliary
(3) Move disk 1 to auxiliary
(4) Move disk 3 to destination
(5) Move disk 1 to source
(6) Move disk 2 to destination
(7) Move disk 1 to destination

Pseudocode for 3 disks

What if we add a fourth disk?



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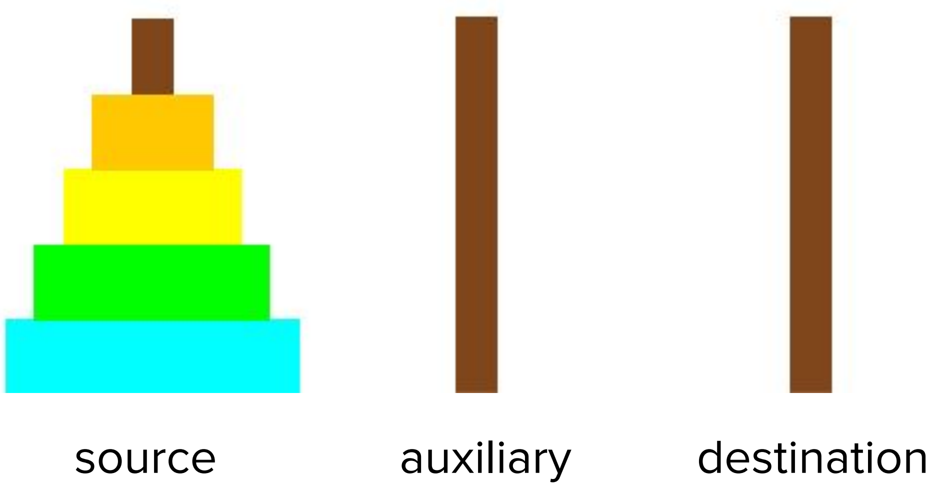
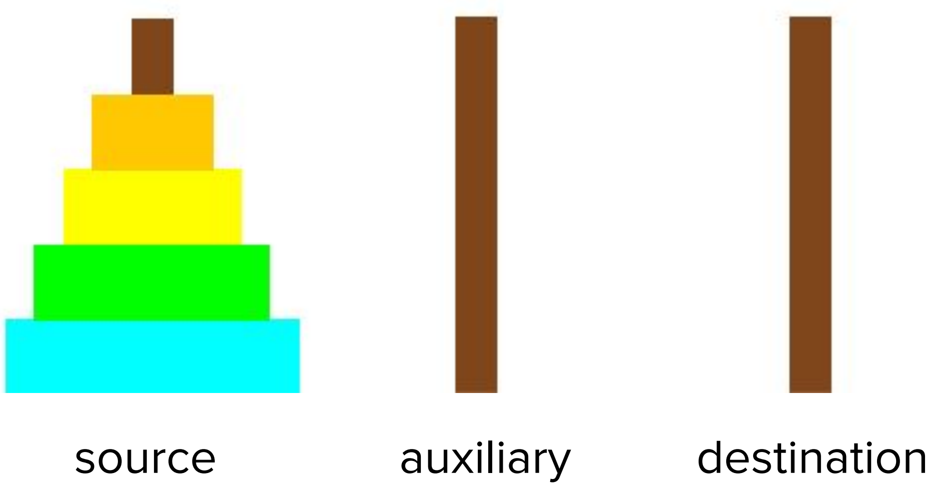
Towers of Hanoi with 4 disks



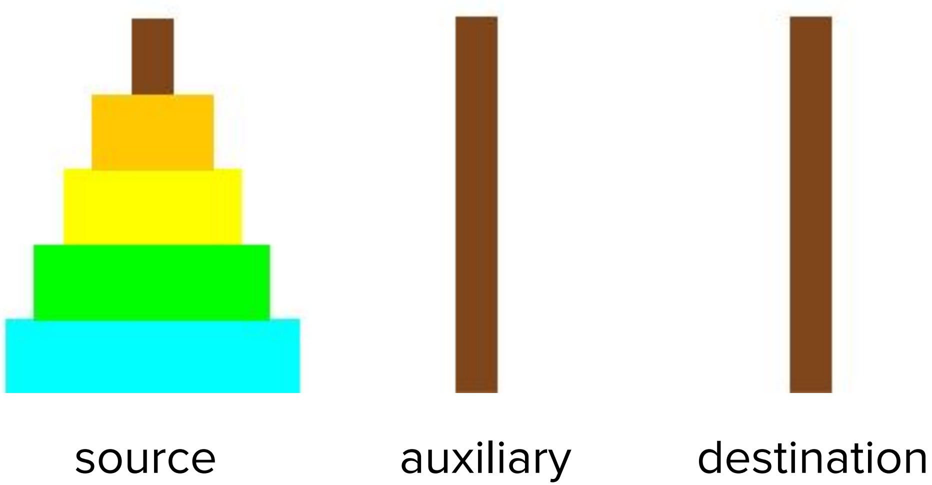
Towers of Hanoi with 4 disks

- We want to first move the biggest disk over to the destination peg.



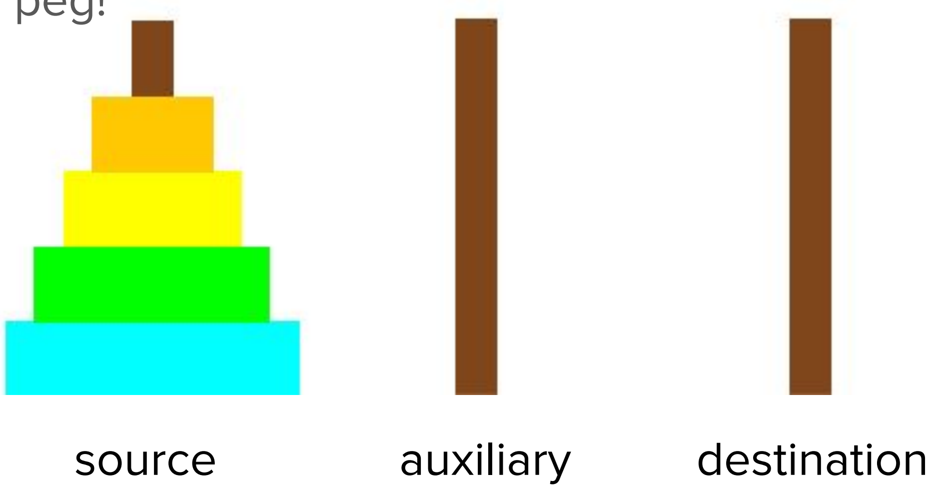
Towers of Hanoi with 4 disks

- We want to first move the biggest disk over to the destination peg.
  - We need to get the top three disks out of the way.



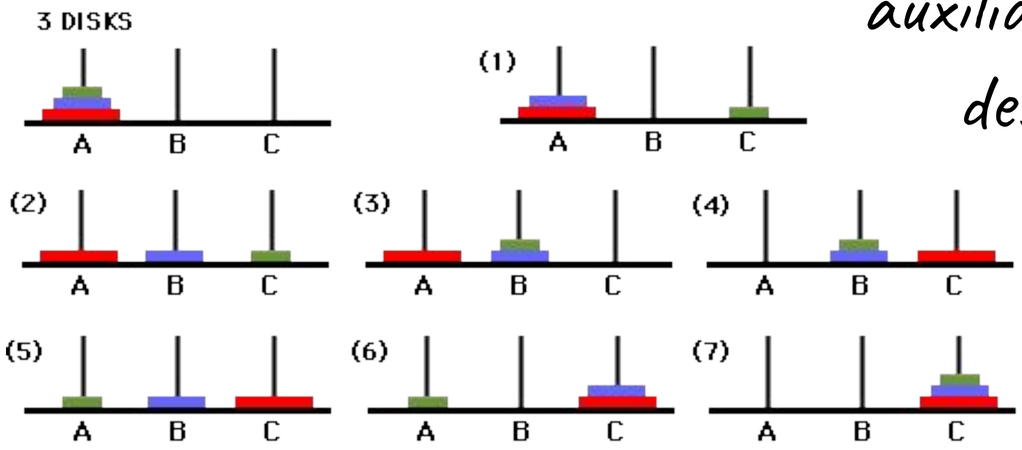
Towers of Hanoi with 4 disks

- We want to first move the biggest disk over to the destination peg.
  - We need to get the top three disks out of the way.
  - We already have an algorithm for moving three disks from a source peg to a destination peg!



Pseudocode for 3 disks

Idea: Move disks to auxiliary instead of destination!



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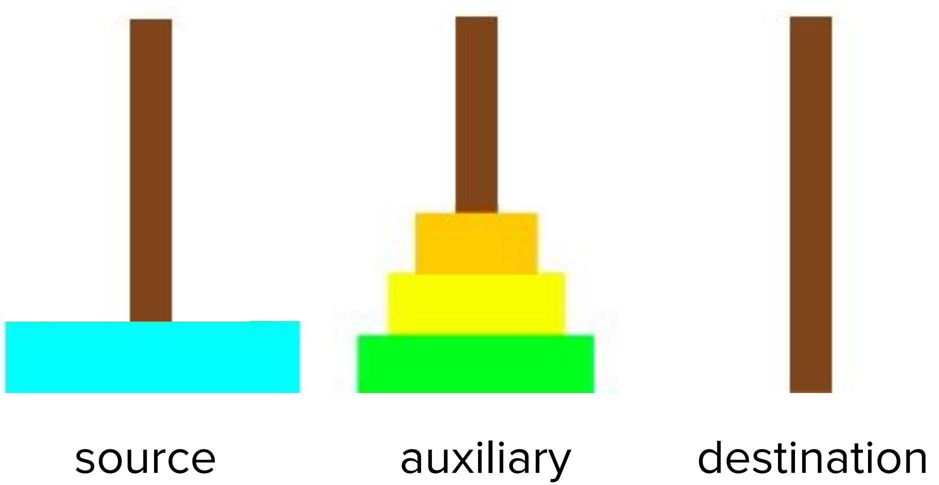
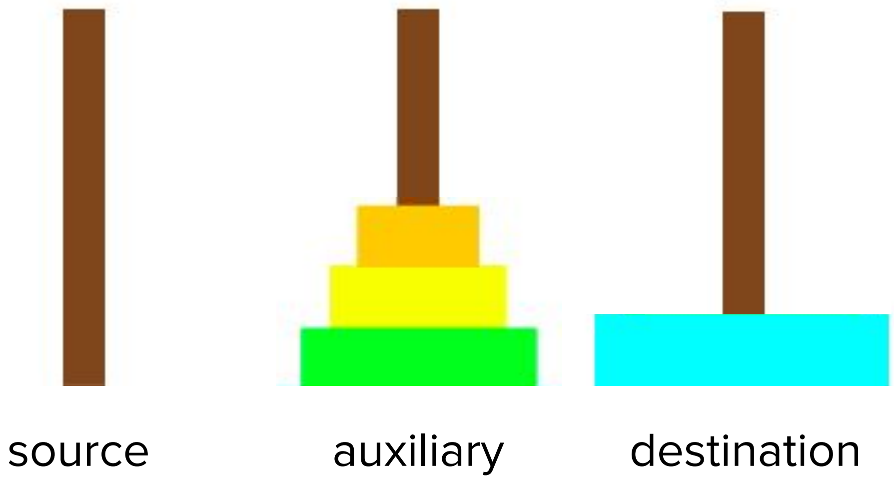
Towers of Hanoi with 4 disks

- We want to first move the biggest disk over to the destination peg.



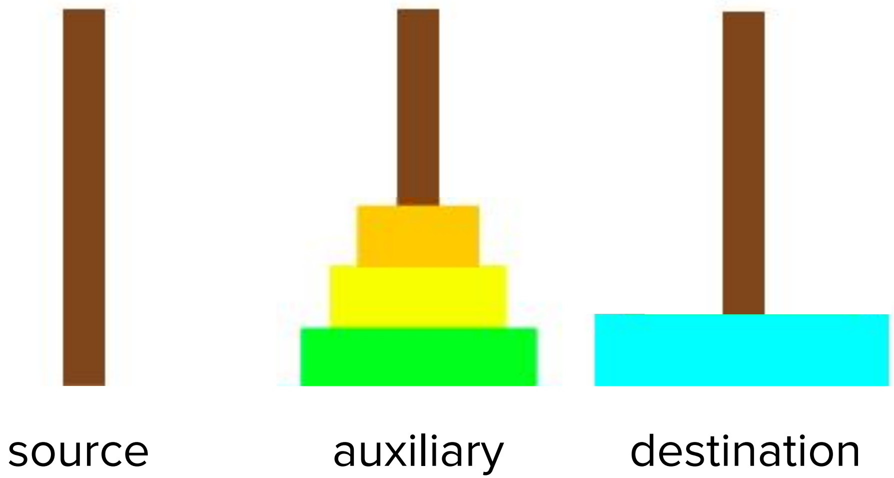
Towers of Hanoi with 4 disks

- We want to first move the biggest disk over to the destination peg.



Towers of Hanoi with 4 disks

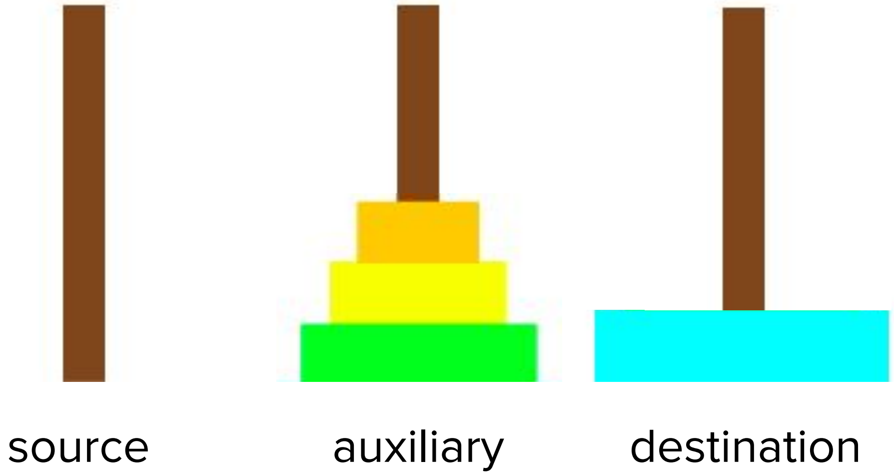
- We want to first move the biggest disk over to the destination peg.
- Now we need to move the stack of three from auxiliary to destination.



Towers of Hanoi with 4 disks

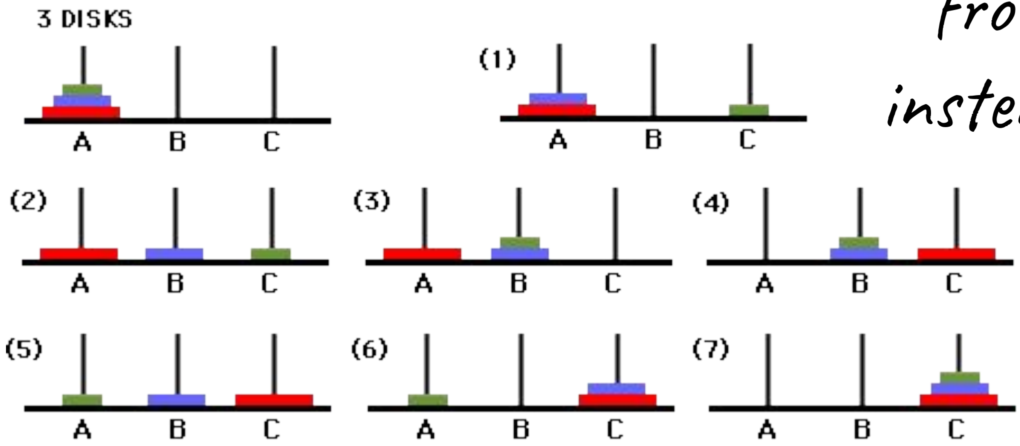
- We want to first move the biggest disk over to the destination peg.
- Now we need to move the stack of three from auxiliary to destination.

Use our existing 3-disk algorithm!



Pseudocode for 3 disks

Idea: Move disks from auxiliary instead of source!



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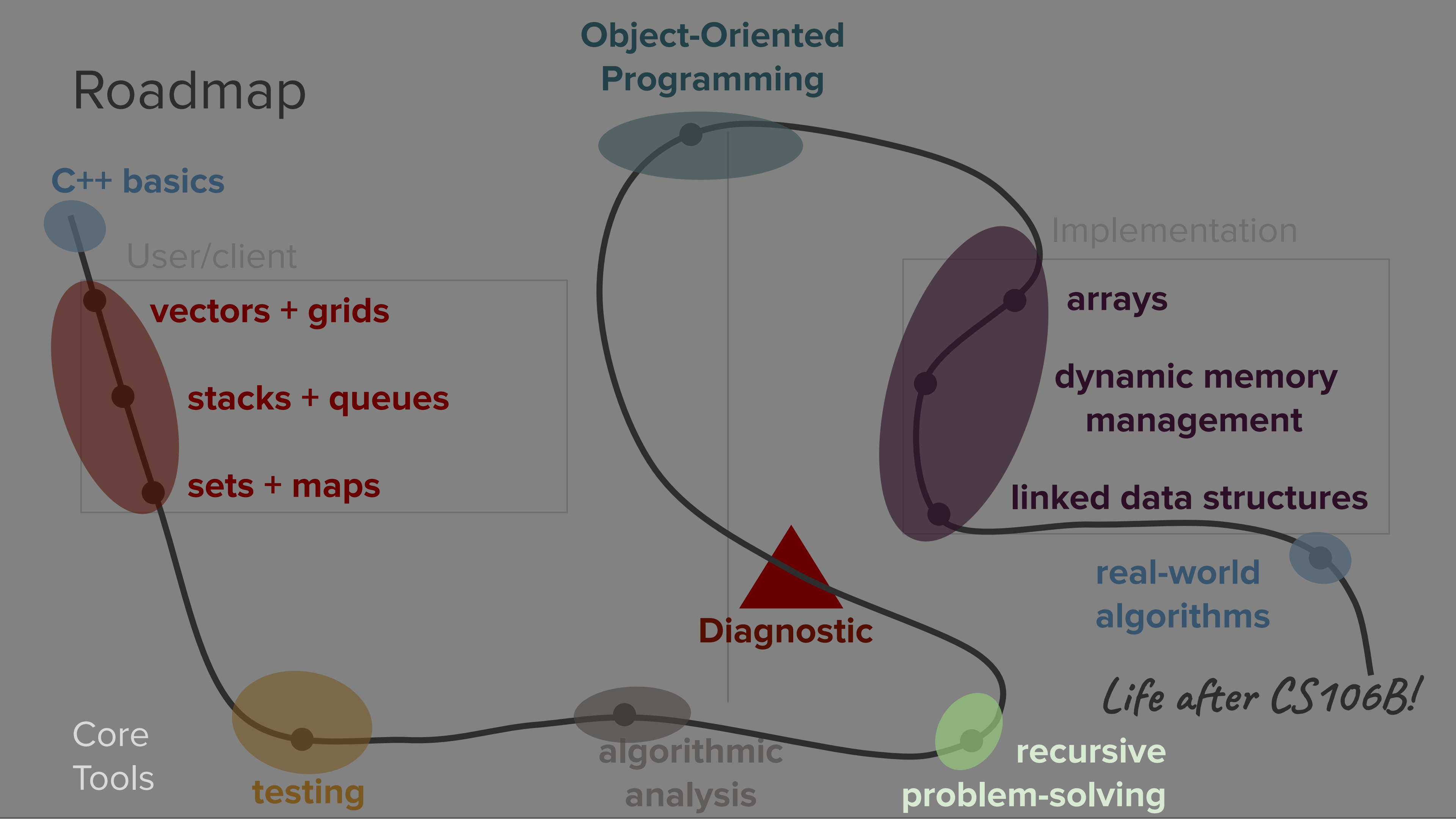
How could we define the Towers of Hanoi solution recursively?

Towers of Hanoi solution

[live coding]

What’s next?


Roadmap
Object-Oriented Programming
C++ basics
User/client
vectors + grids
stacks + queues
sets + maps
Implementation
arrays
dynamic memory management
linked data structures
real-world algorithms
Diagnostic
Life after CS106B!
Core Tools
testing
algorithmic analysis
recursive problem-solving

Advanced Recursion Examples