

Graphs and Graph Algorithms

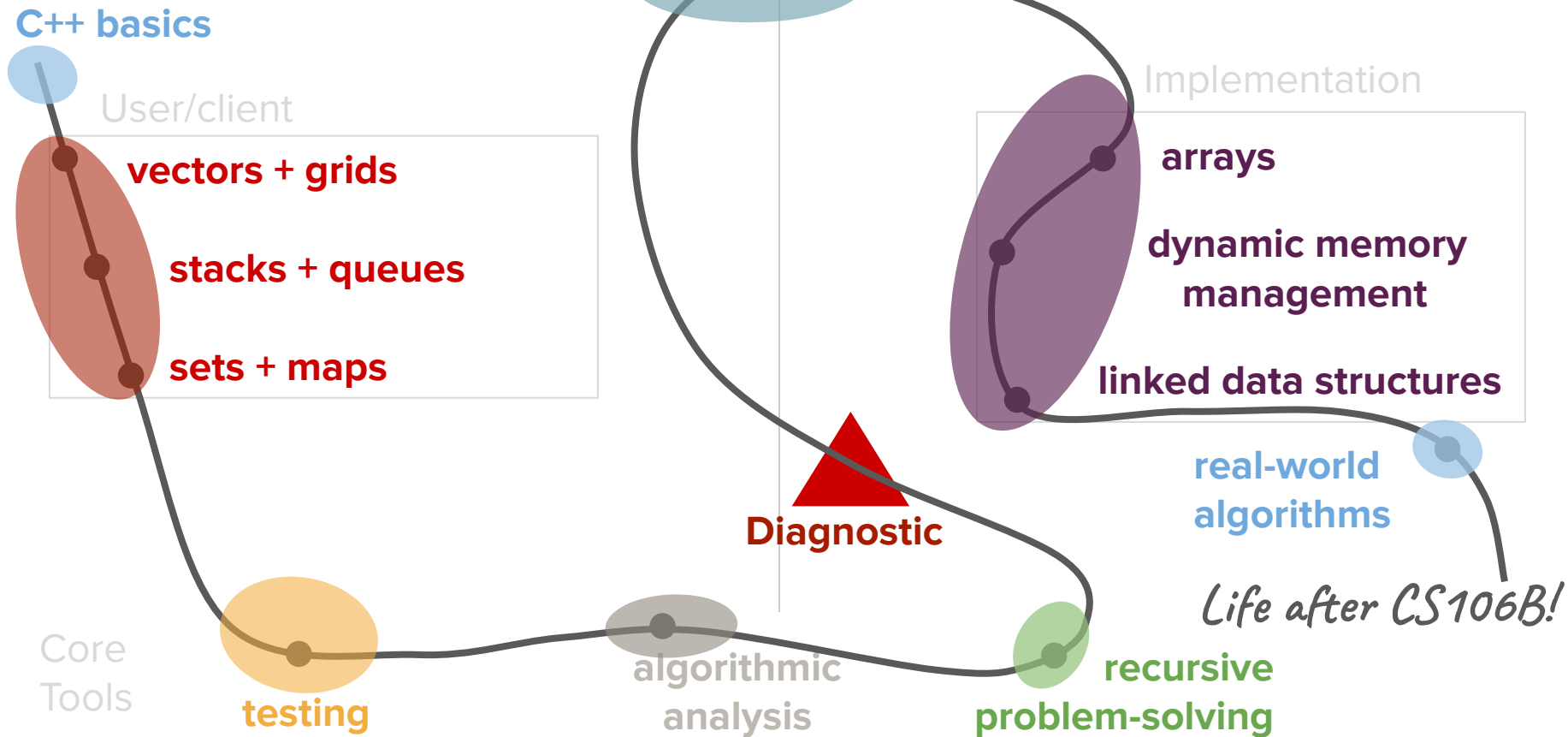
What has been your favorite part of this summer?

(put your answers in the chat)



Roadmap

Roadmap graphic courtesy of Nick Bowman & Kylie Ju



Roadmap

C++ basics

User/client

vectors + grids

stacks + queues

sets + maps

Core
Tools

testing

algorithmic
analysis

**recursive
problem-solving**

Object-Oriented
Programming

Implementation

arrays

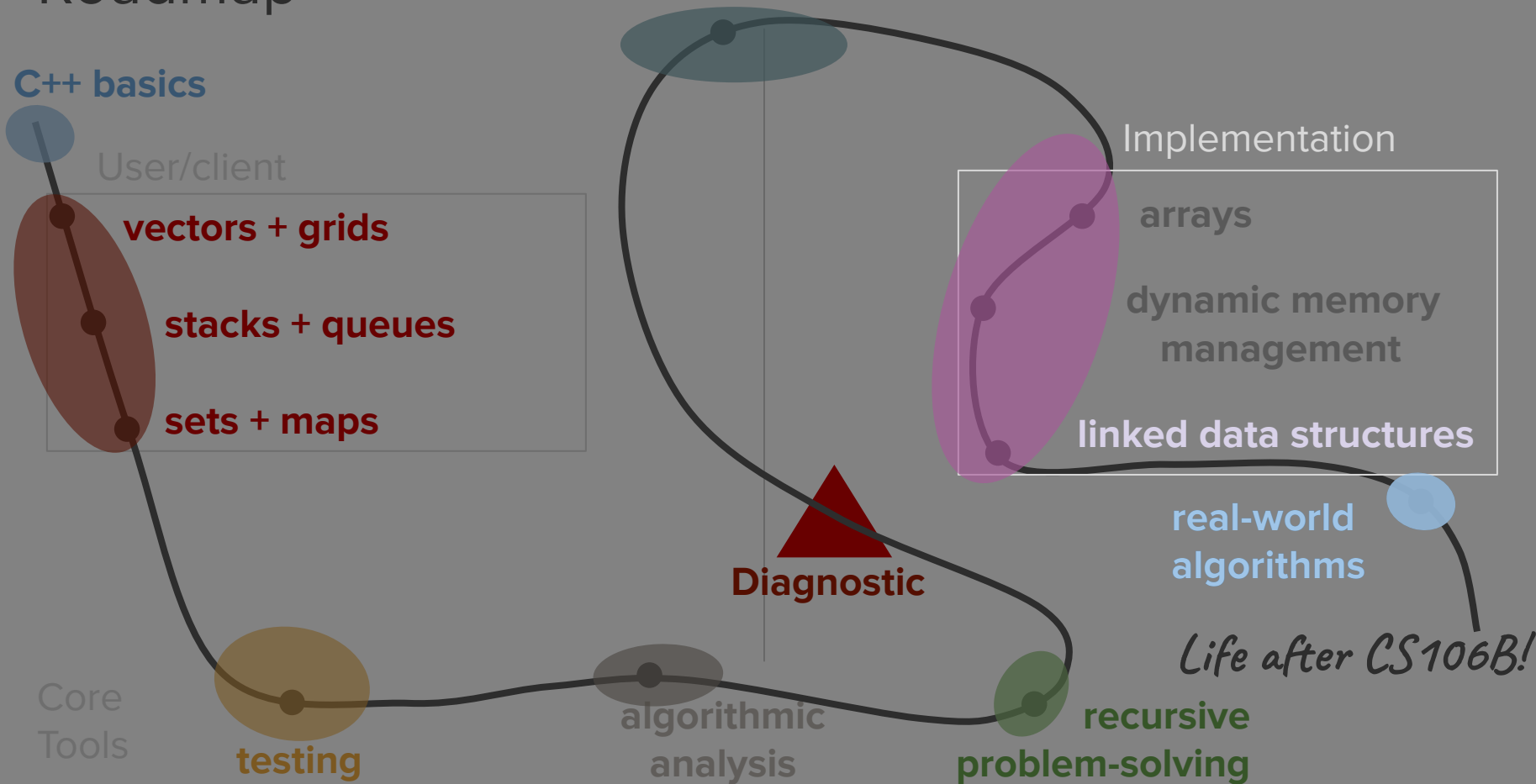
dynamic memory
management

linked data structures

real-world
algorithms

Life after CS106B!

Diagnostic



Today's question

How can we represent and
organize complex systems
of interconnected
components?

Today's topics

1. Use Cases for Graphs
2. Graph Definition and Terminology
3. Graph Algorithms (BFS, Dijkstra and A^*)

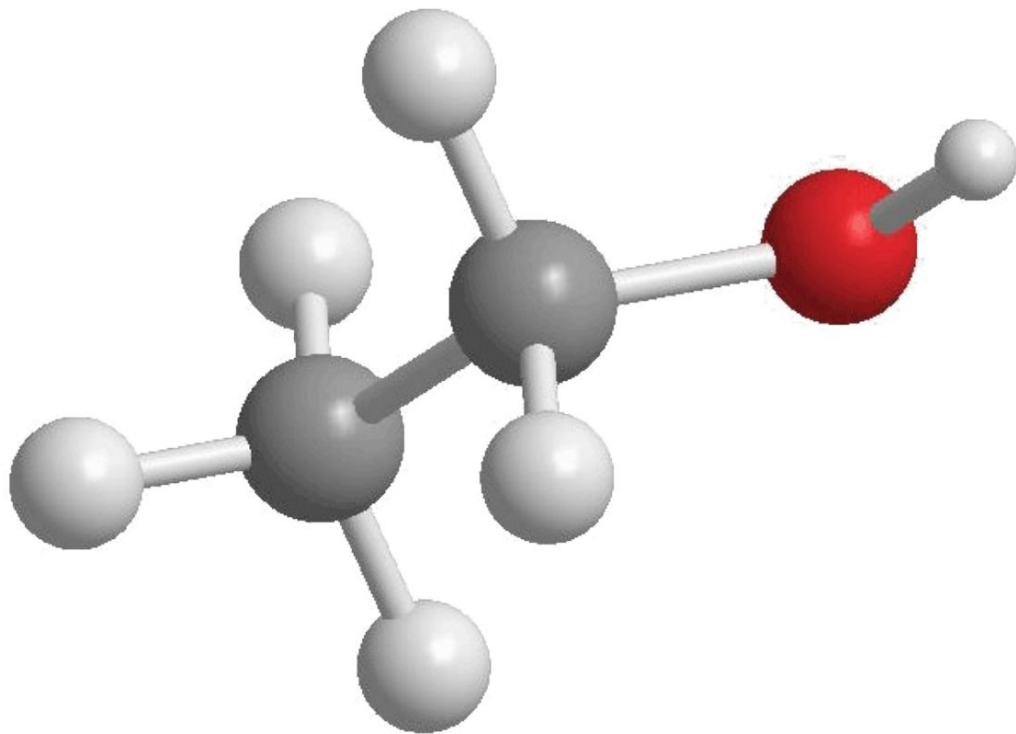
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Graphs

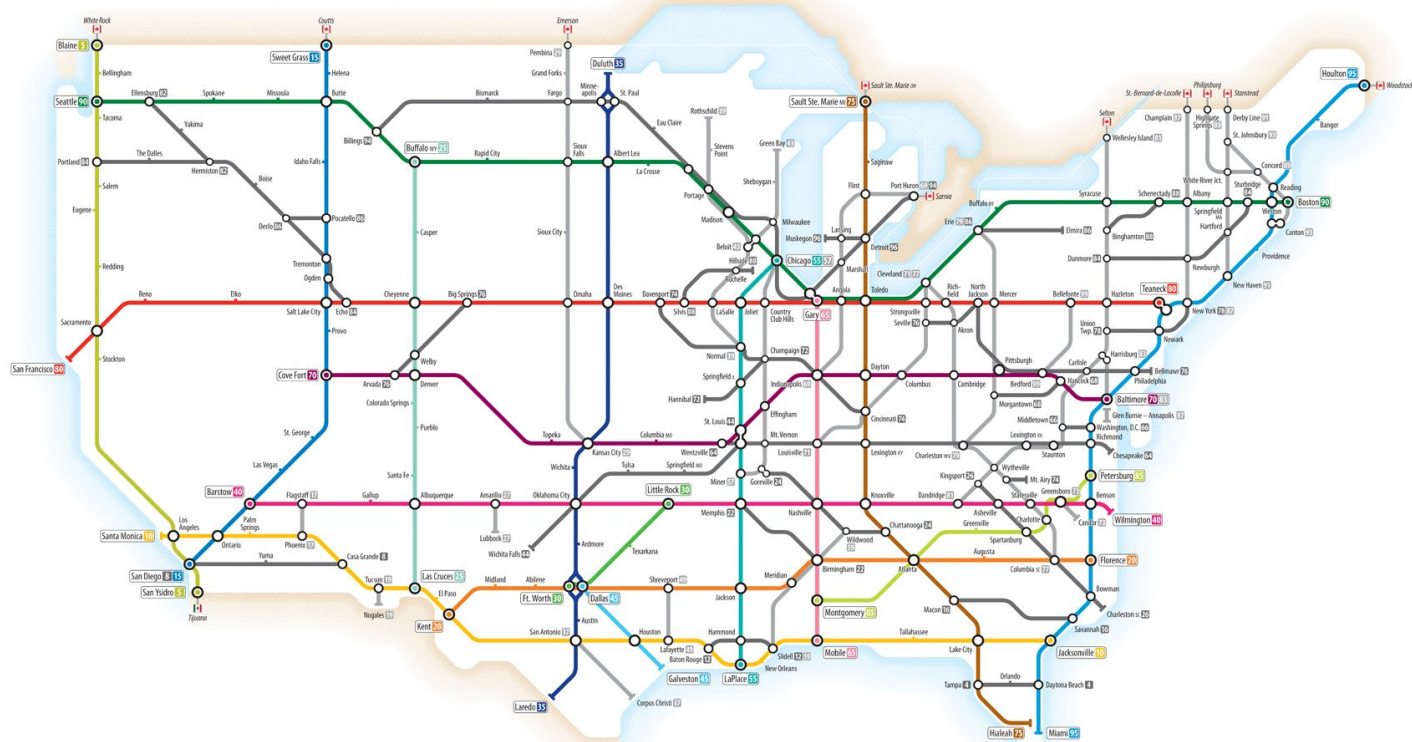
Social Networks



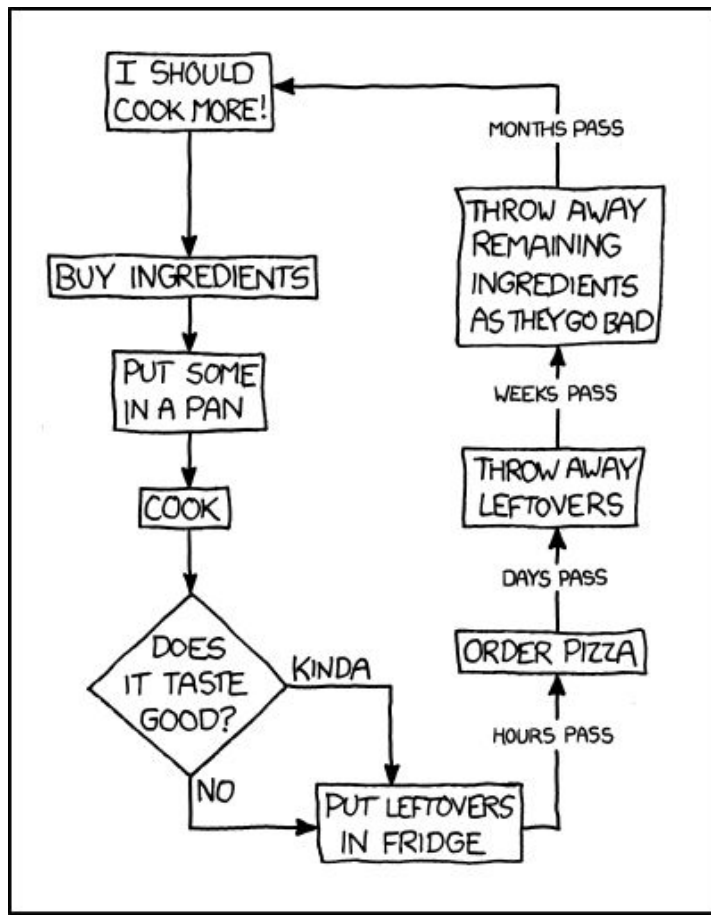
Chemical Bonds



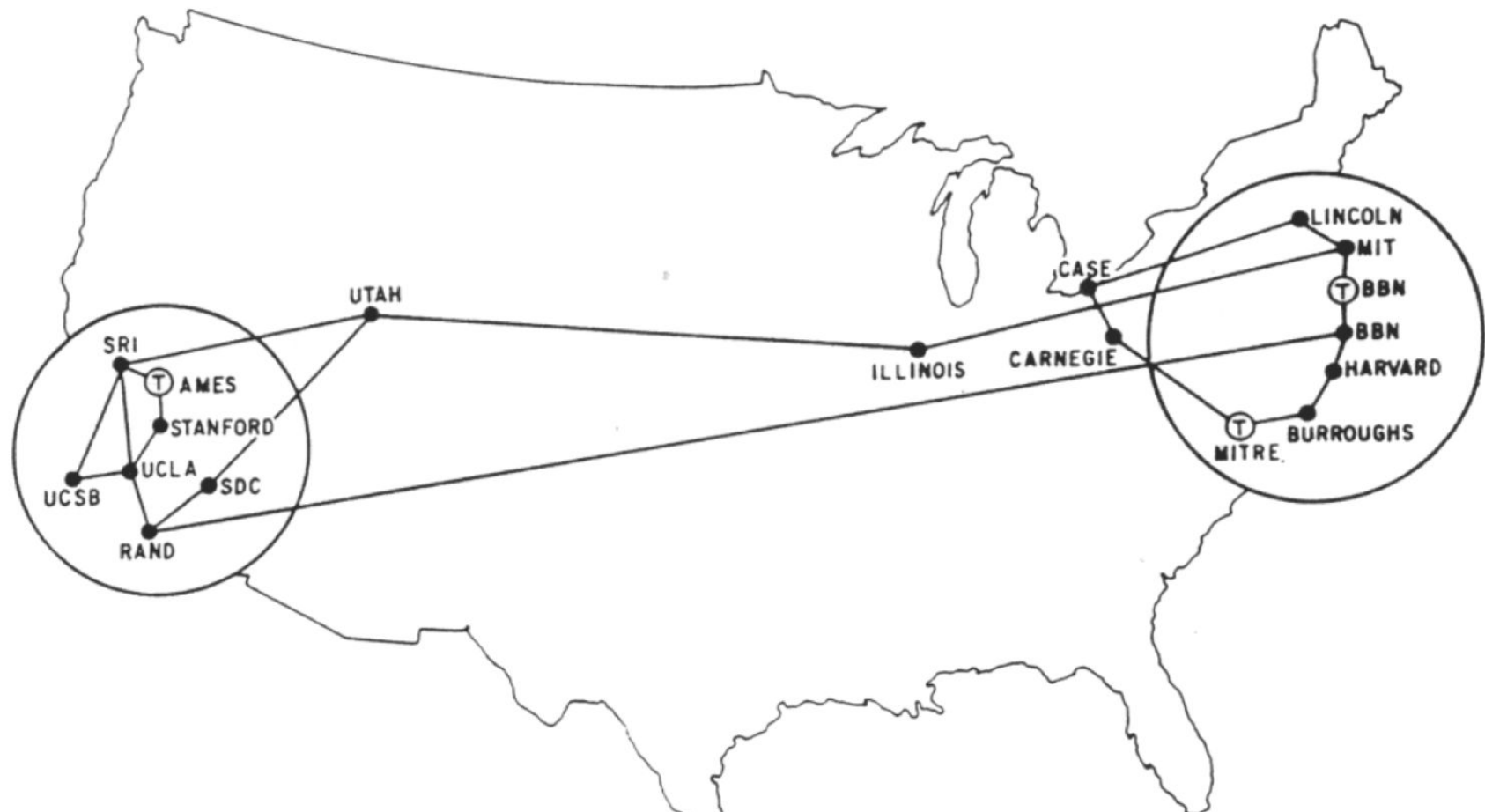
The Interstate Highway System

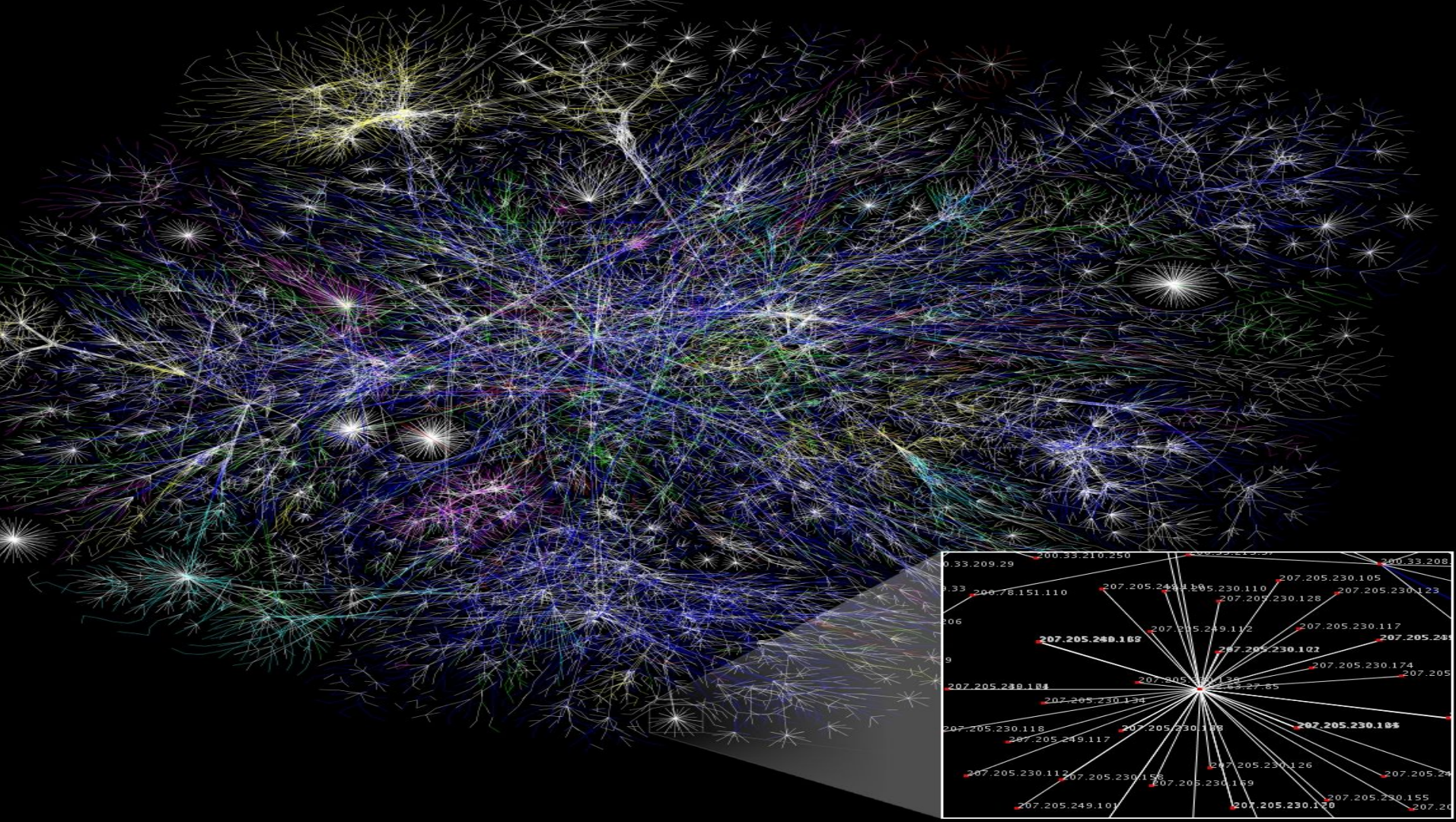


Flowcharts



The Internet!





What is a graph?

Definition

graph

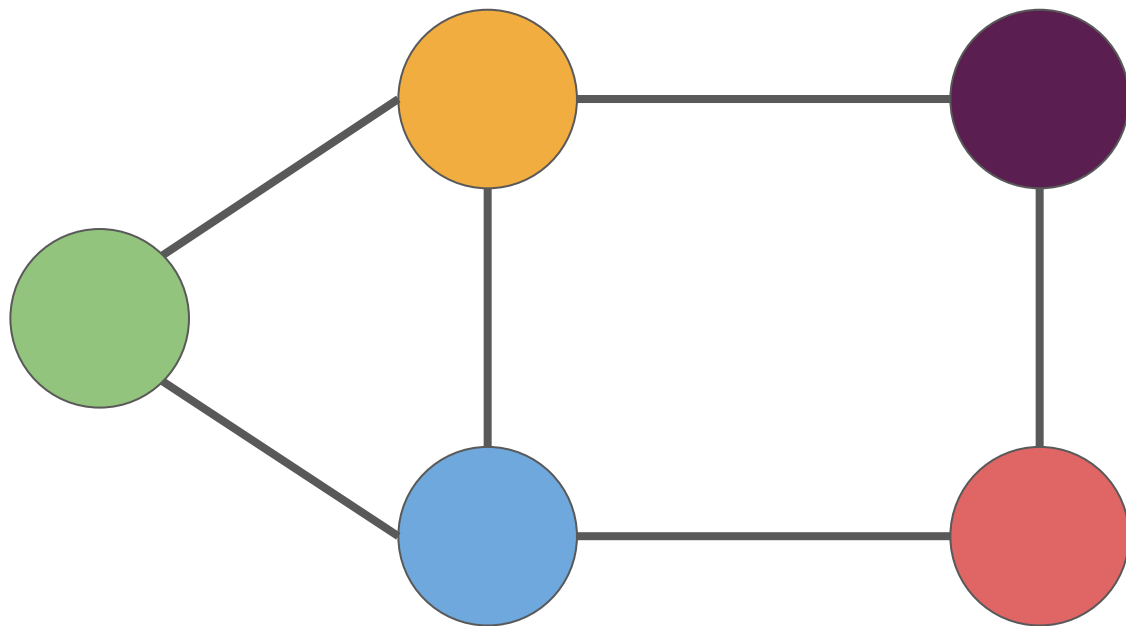
A structured way to represent relationships between different entities.

Our first graph!

- A structured way to represent relationships between different entities.

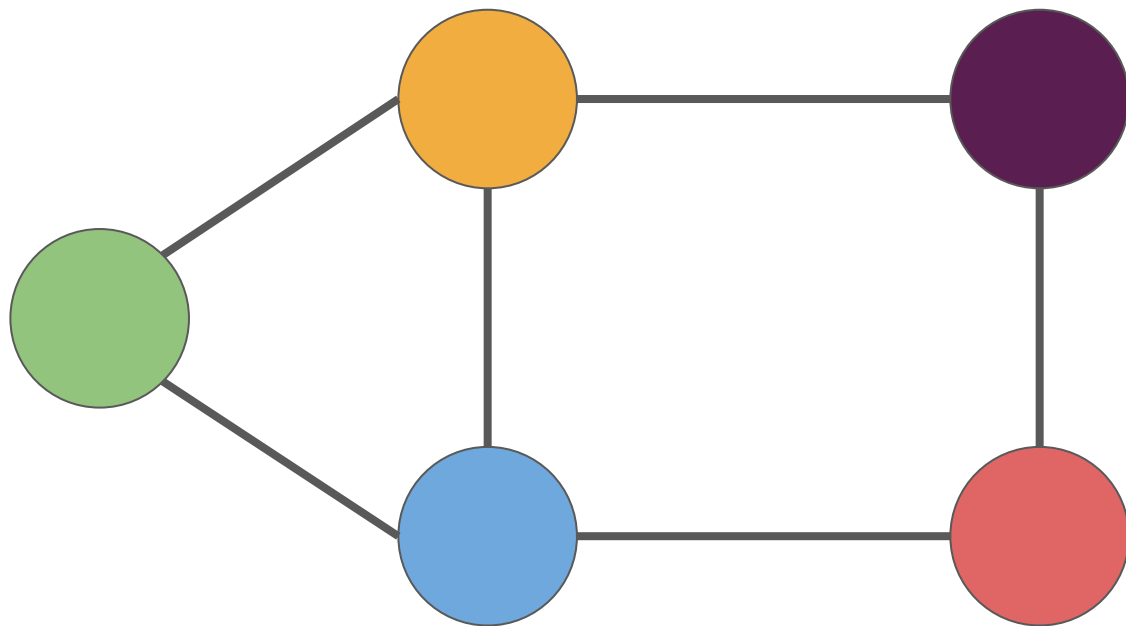
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Our first graph!

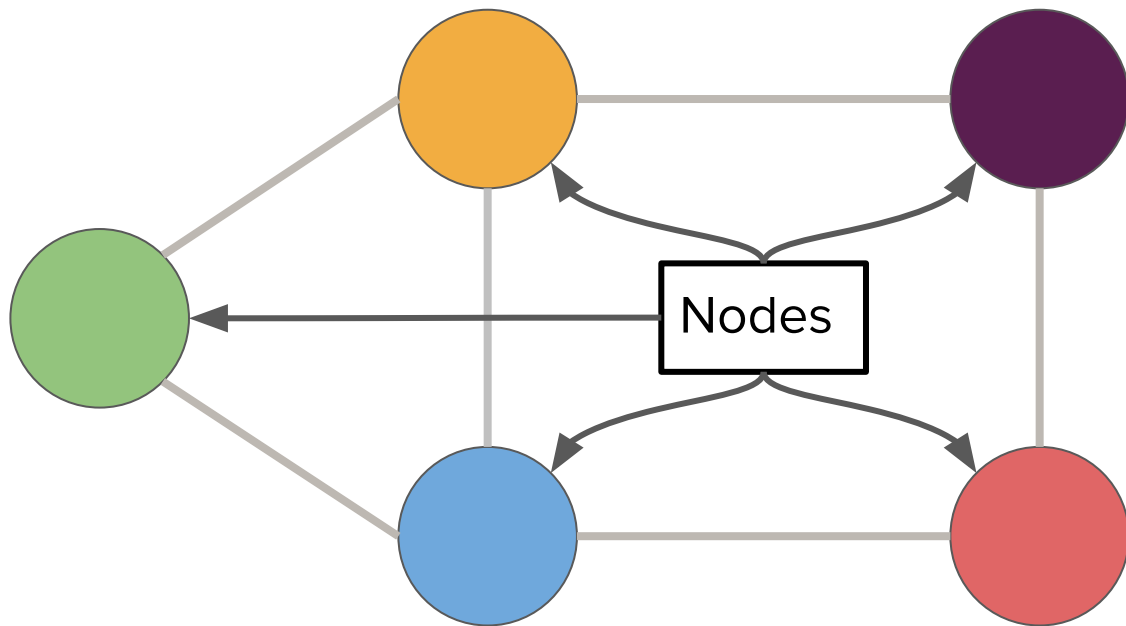
- A structured way to represent relationships between different entities.



A graph consists of a set of **nodes** connected by **edges**.

Our first graph!

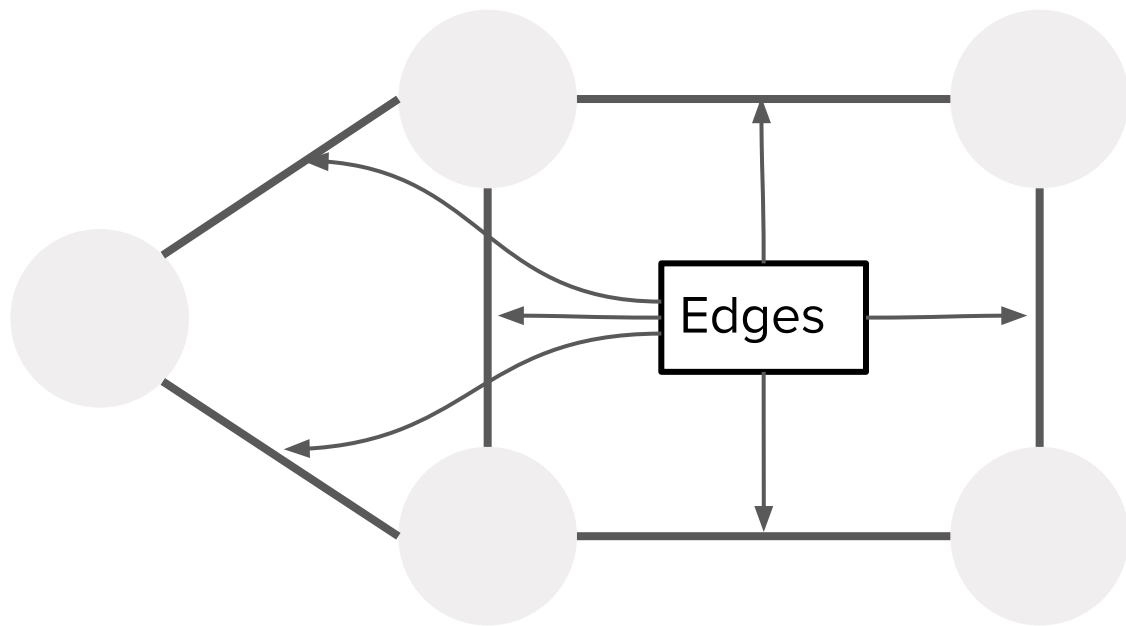
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Our first graph!

- A structured way to represent **relationships** between different entities.



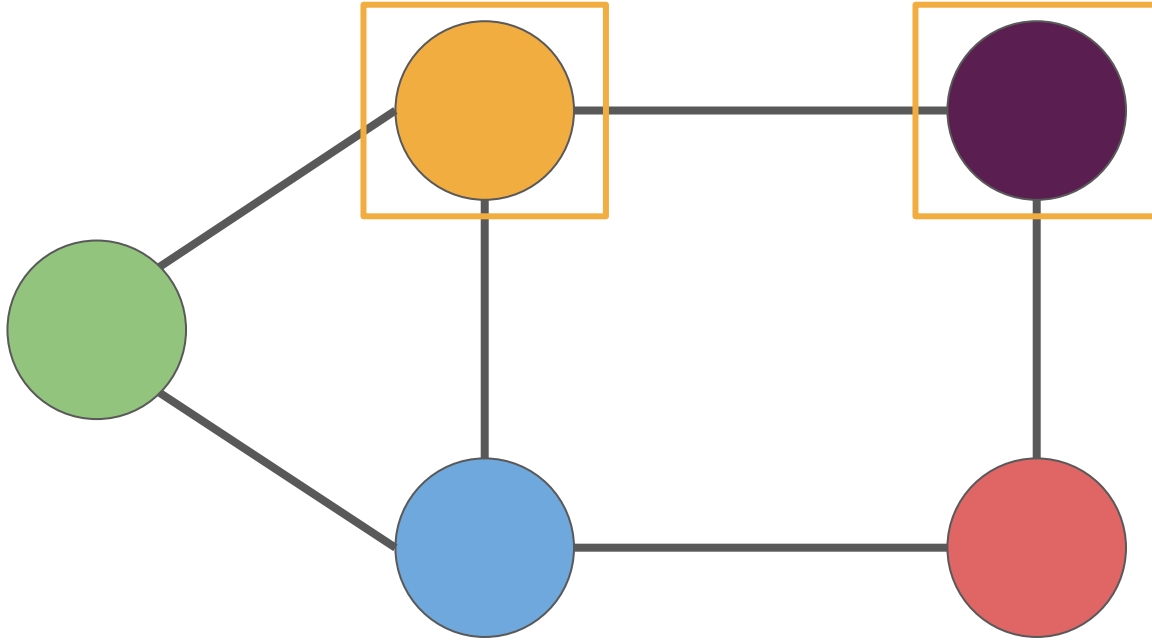
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Graph Terminology

Graph Terminology

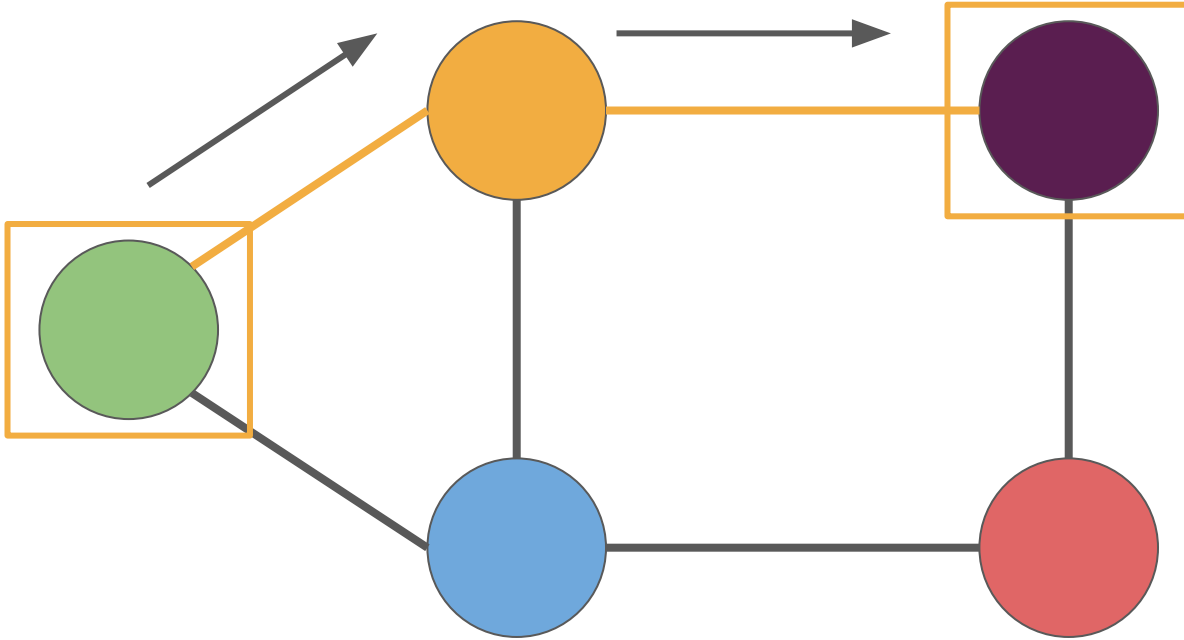
- There are lots of different terms used when talking about graphs and their properties. Let's explore some of them!

Graph Terminology



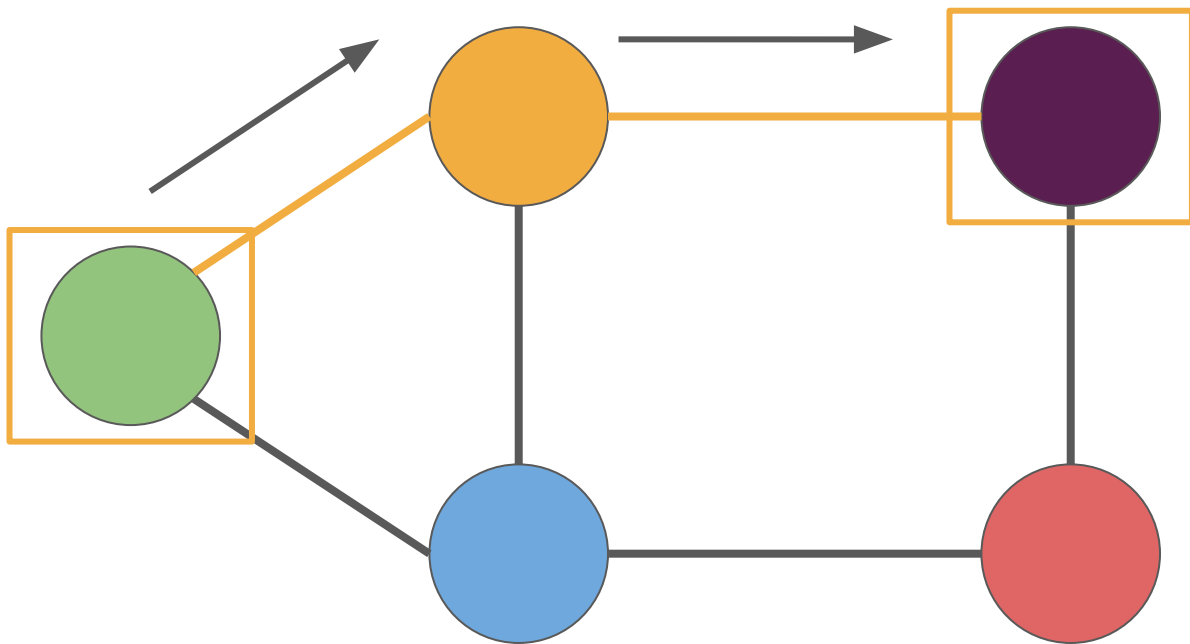
Two nodes are **neighbors** if they are directly connected by an edge.

Graph Terminology



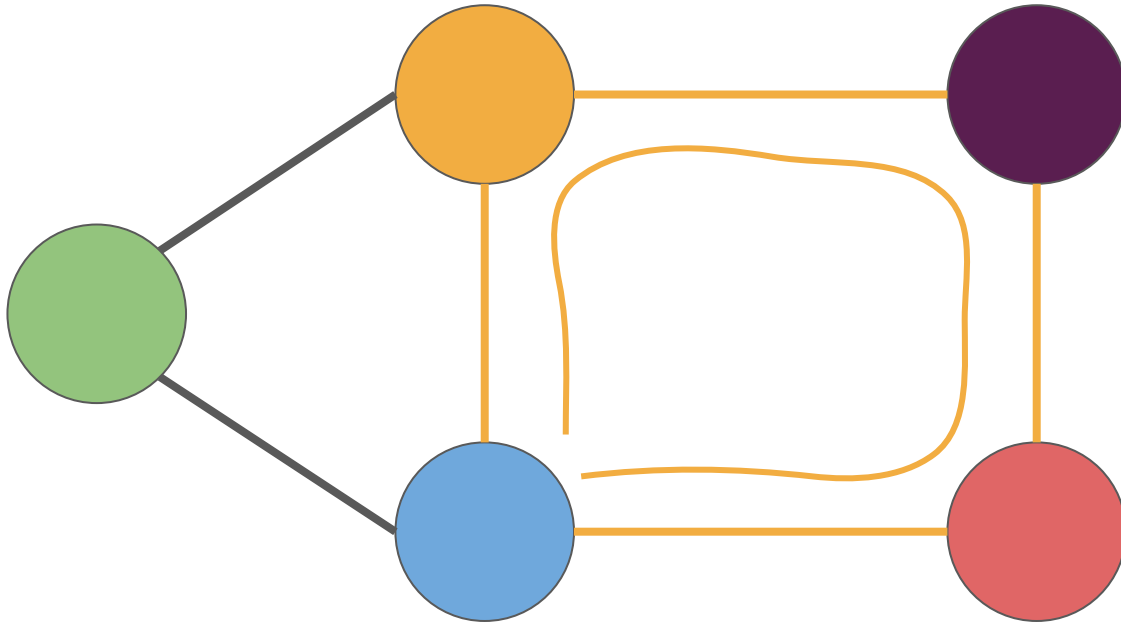
A **path** between two nodes is defined be a sequence of edges that can be followed to traverse between the two nodes.

Graph Terminology



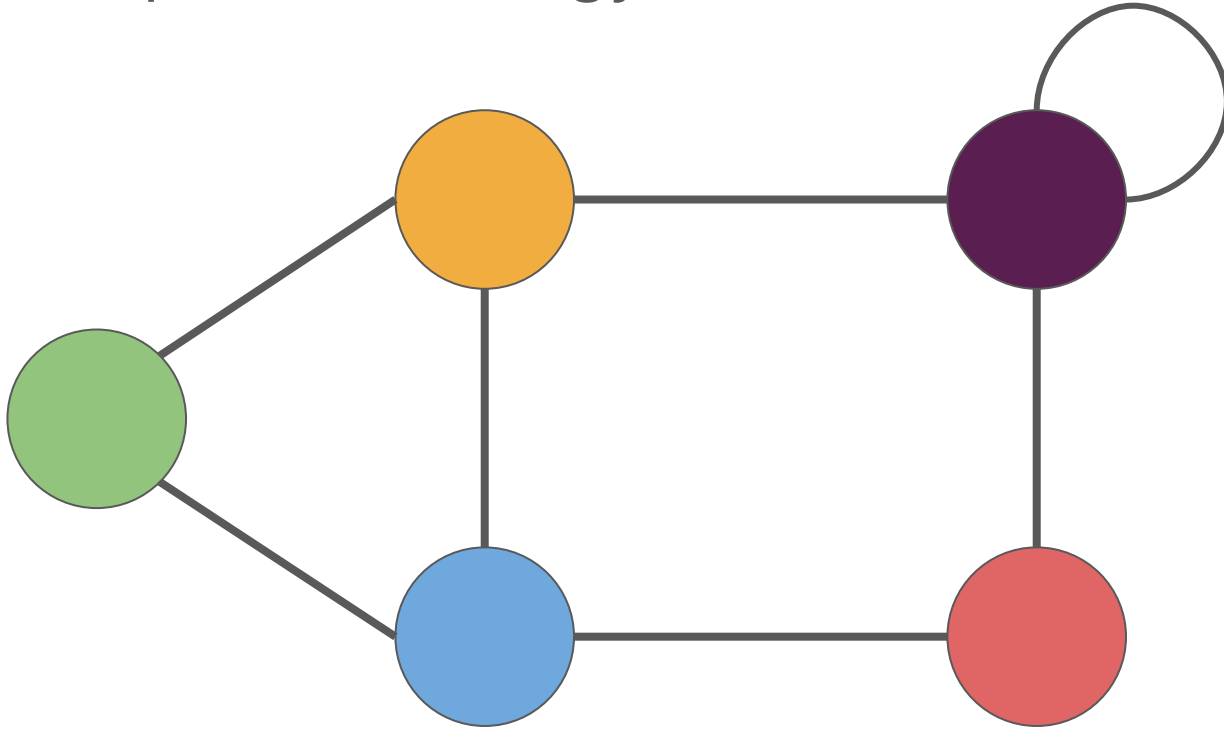
The **length** of a path is the number of edges that make up the path. This path has length 2.

Graph Terminology

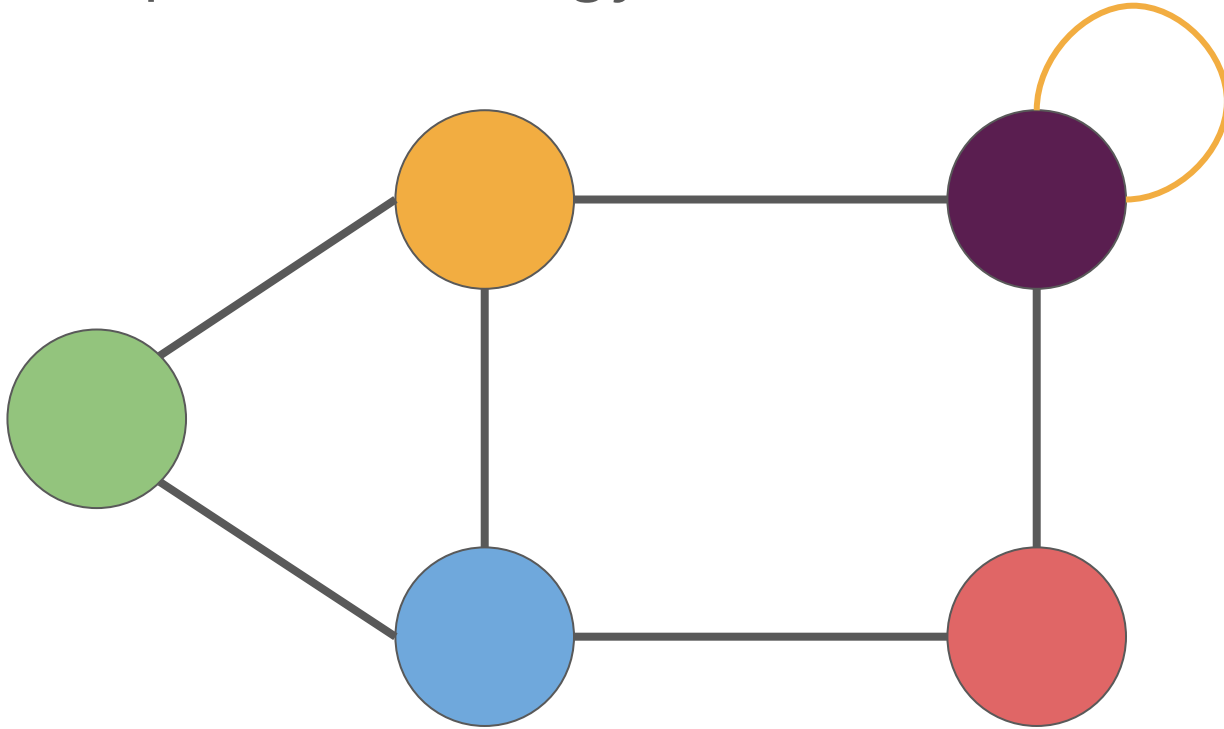


A **cycle** is a path that begins and ends at the same node.

Graph Terminology

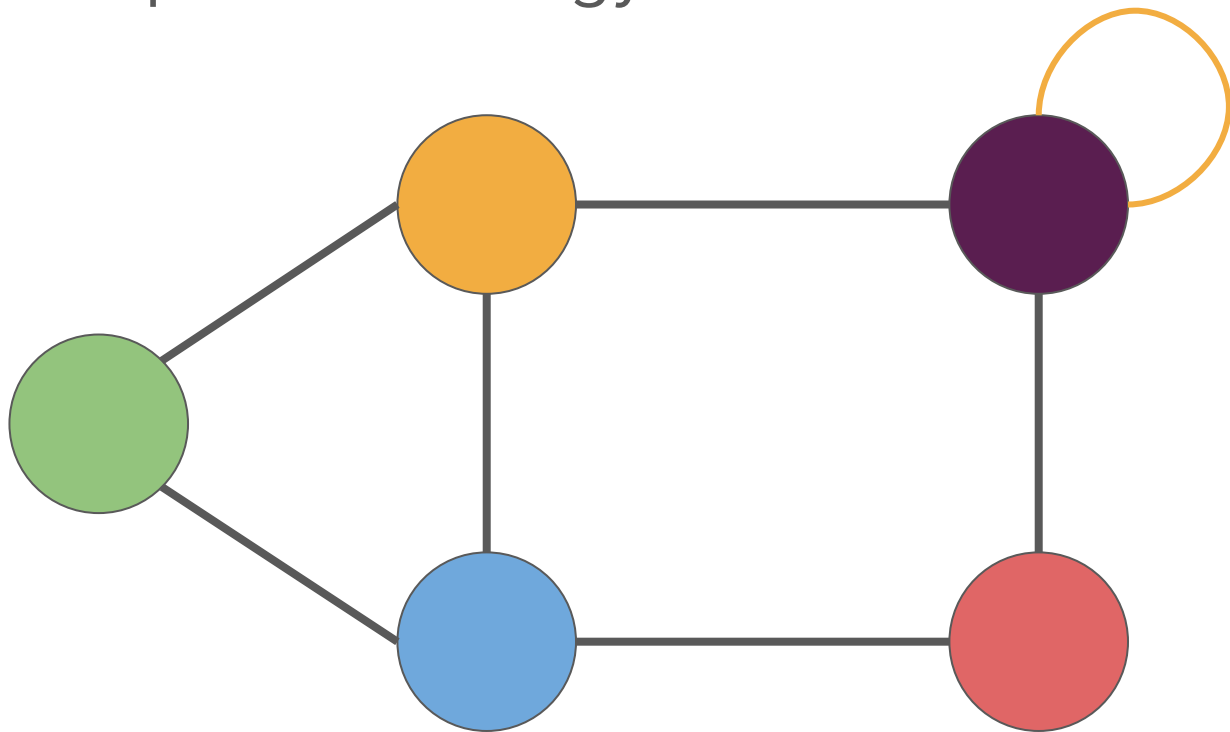


Graph Terminology



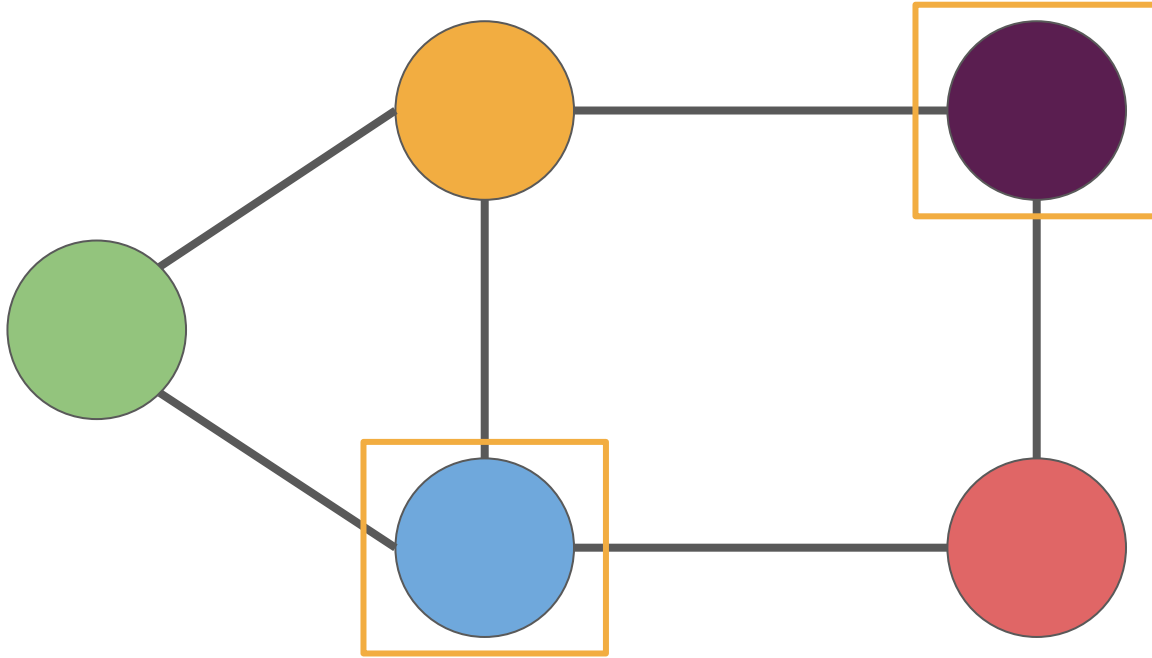
Are we allowed to have edges that look like this?

Graph Terminology



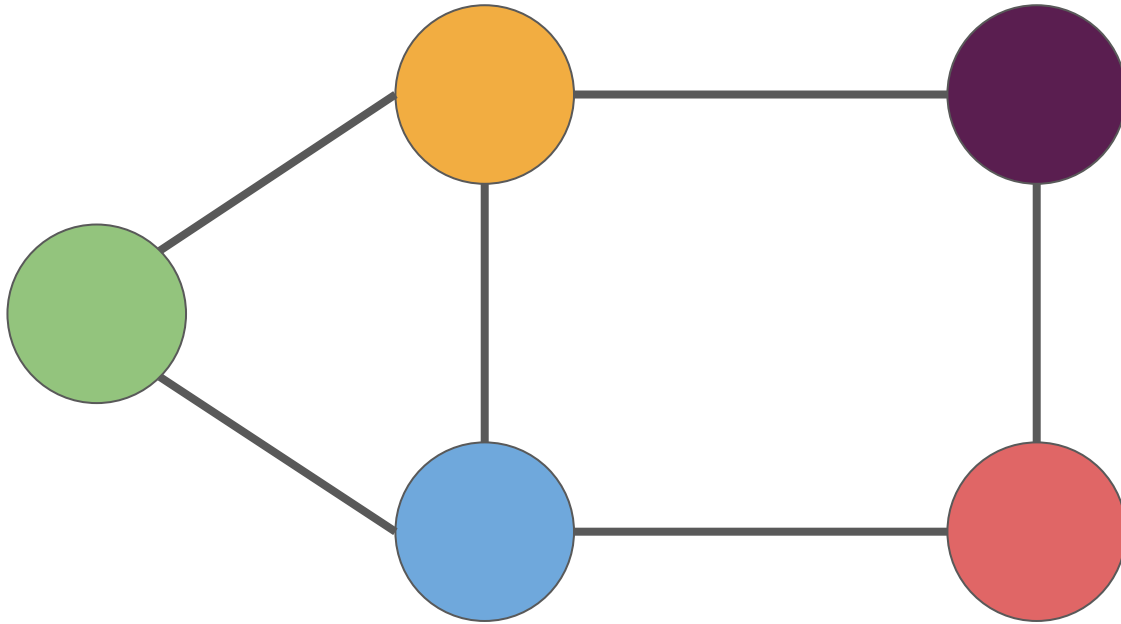
A **loop** is an edge directly from a node back to itself. Some graphs allow loops and some graphs don't!

Graph Terminology



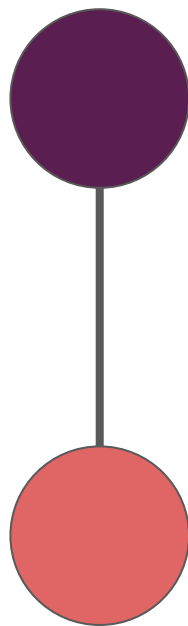
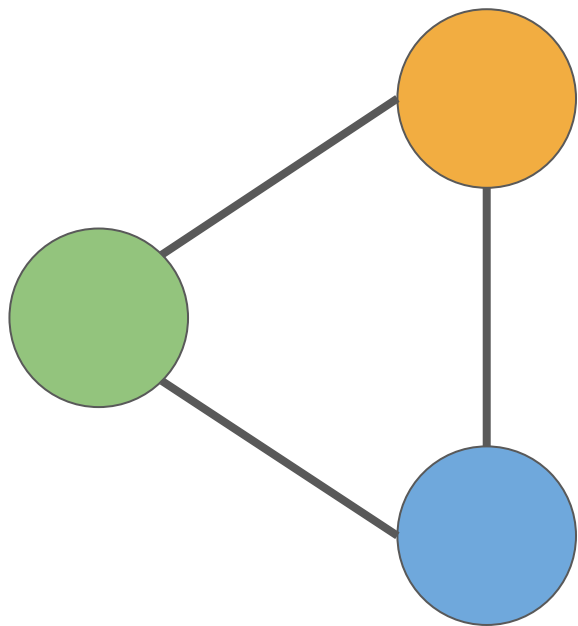
A node is **reachable** from another node if a path exists between the two nodes.

Graph Terminology



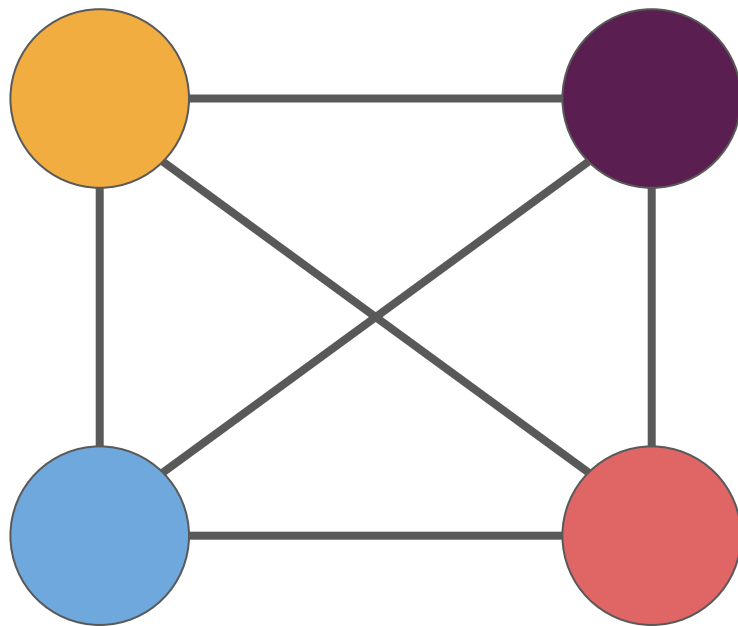
A graph is **connected** if all nodes are reachable from all other nodes. This graph is connected!

Graph Terminology



A graph is **connected** if all nodes are reachable from all other nodes. This graph is ***not*** connected!

Graph Terminology



A graph is **complete** if every node has an edge connecting it to every other node!

Graph Terminology Summary

- Graph structures
 - Two nodes are **neighbors** if they are directly connected by an edge.
 - A **path** between two nodes is a sequence of edges connecting them. The **length** of a path is defined by the number of edges in the path.
 - A **cycle** is a path that starts and ends at the same node.
 - A **loop** is an edge that connects a node to itself.
- Graph properties
 - A node is **reachable** from another node if a path between the two nodes in the graph exists.
 - A graph is **connected** if all nodes are reachable from all other nodes.
 - A graph is **complete** if edges exist between all pairs of nodes in the graph.

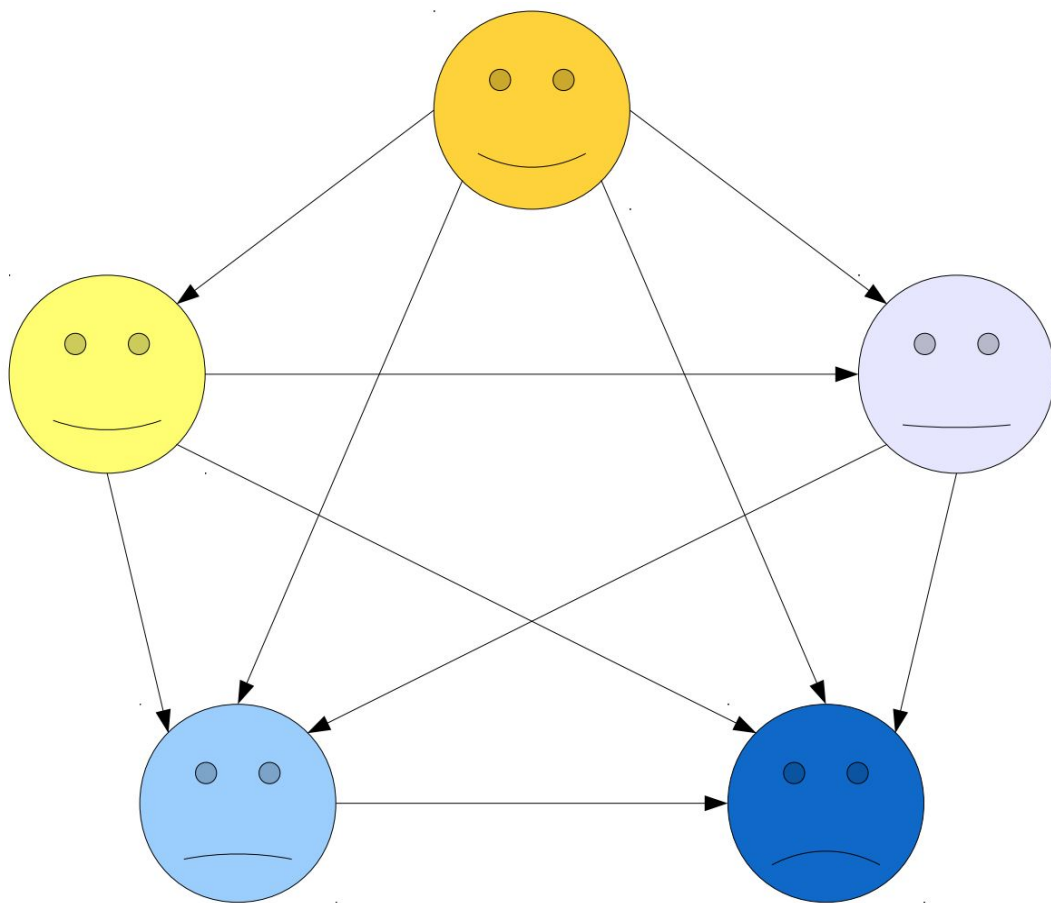
Types of graphs

Different types of graphs

- Some graphs are **directed**. These represent situations where relationships are unidirectional (an action/verb that explicitly implies only one direction).

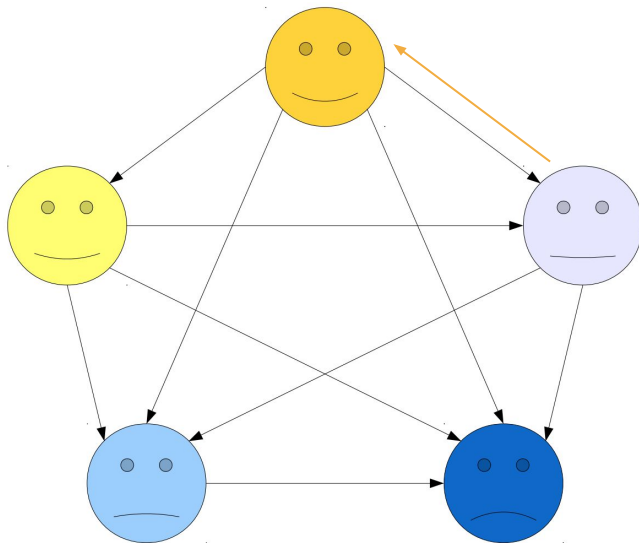
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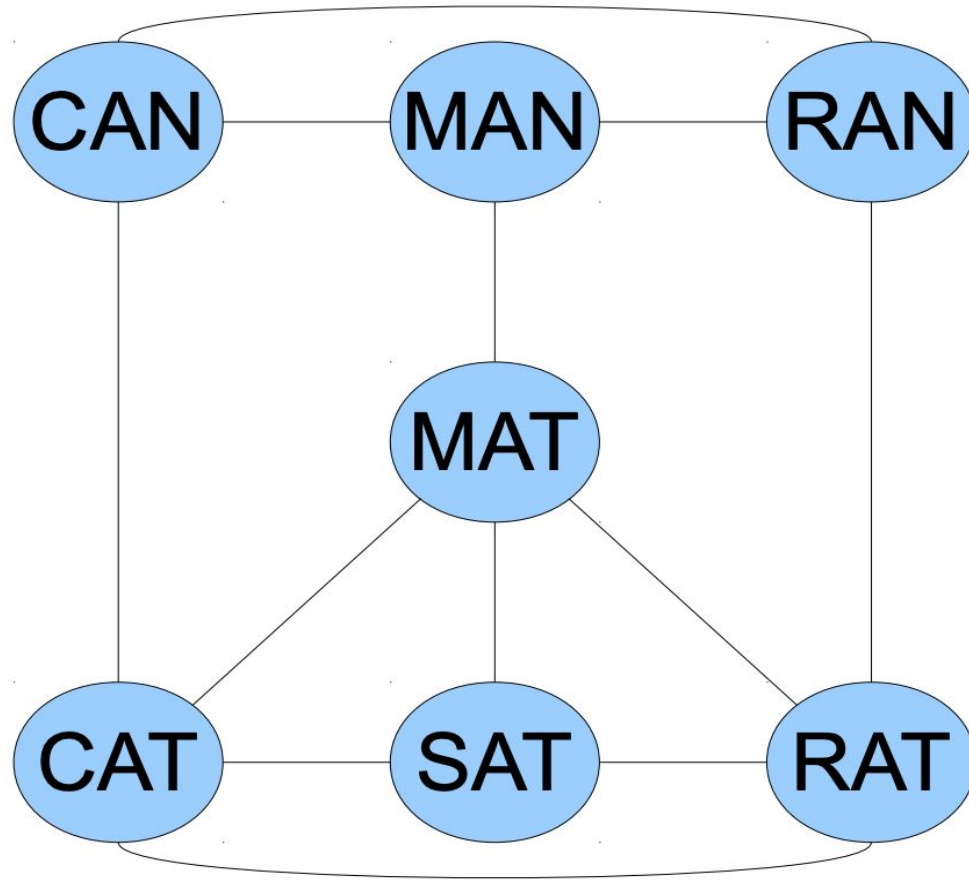
Note: It is possible for a relationship in a directed graph to go both ways between two nodes, but it would need to be explicitly stated.

Different types of graphs

- Some graphs are **undirected**. These represent situations where relationships are bidirectional (the action/verb inherently applies to both entities).

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 - Ex: I am related to my brother, and he is related to me. The relationship applies to both of us.



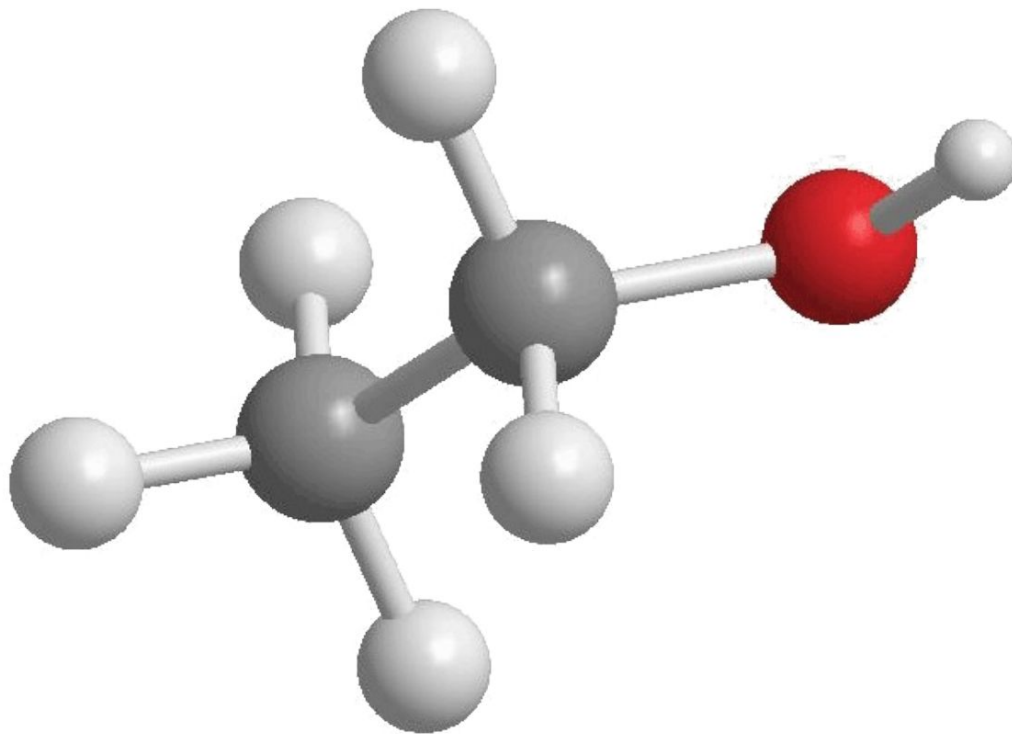
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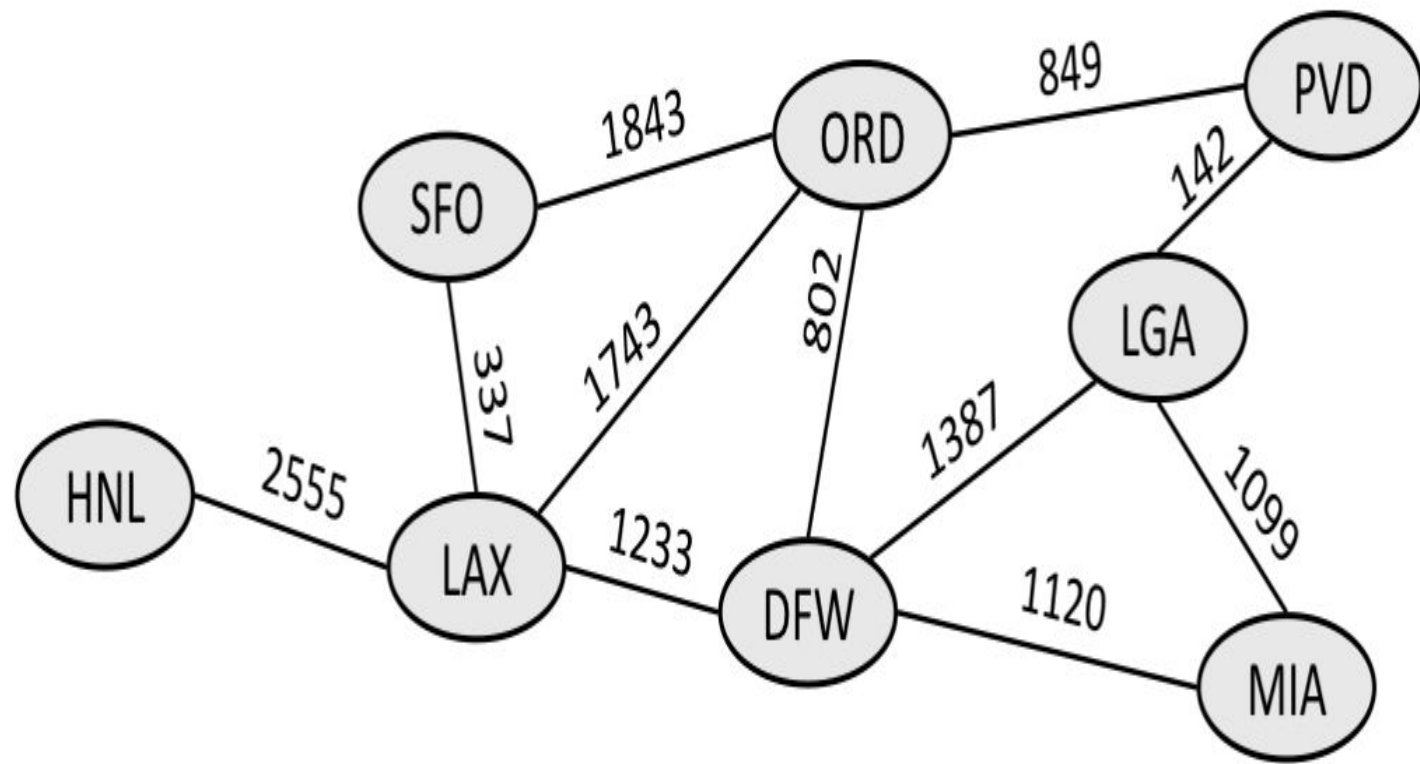
- Some graphs are **weighted**. These represent situations where not all relationships between entities are equal.

Different types of graphs

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 - Ex: The different bonds between atoms in a single molecule all have different bond energies and strengths.

Chemical Bonds



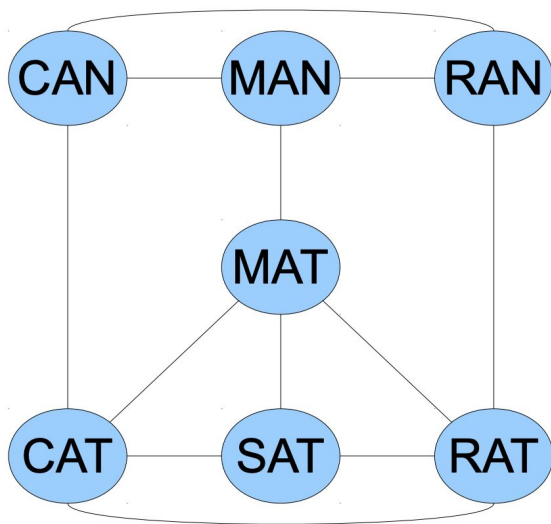


Different types of graphs

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- Some graphs are **unweighted**. These represent situations where all relationships between entities have equal importance.
 - Ex: All connected words in a word ladder are one letter apart from one another.



Types of Graphs Summary

- **Directed:** Unidirectional relationships between nodes, represented with a pointed arrow.
- **Undirected:** Bidirectional relationships between nodes, represented with an arrow-less line.
- **Weighted:** Each edge is assigned a numerical "weight" representing its relative significance/strength.
- **Unweighted:** Each edge has equal significance, no labels assigned.

Revisiting Graph Examples

Revisiting Graph Examples: Social Network

Properties

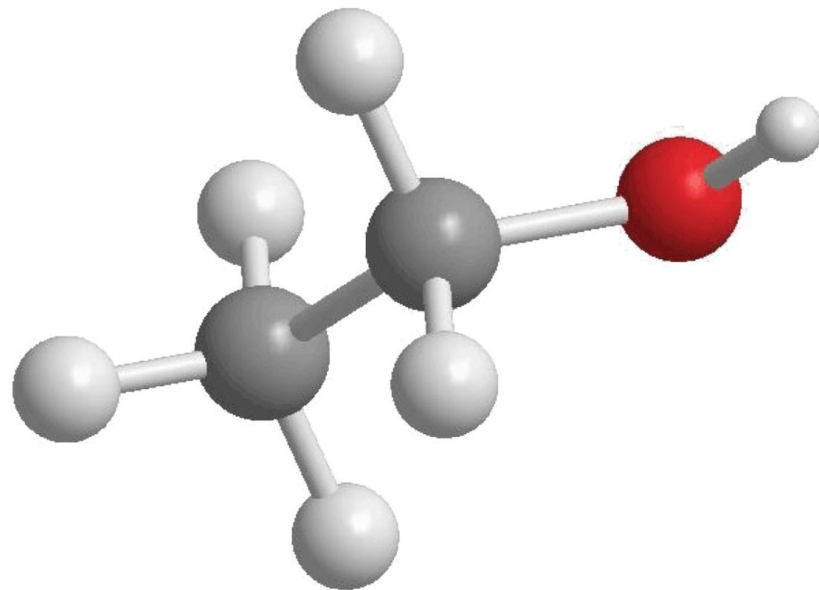
- Nodes: People
- Edges: "Friendship" or "Following"
- Undirected (Facebook) or Directed (Instagram)
- Unweighted



Revisiting Graph Examples: Chemical Bonds

Properties

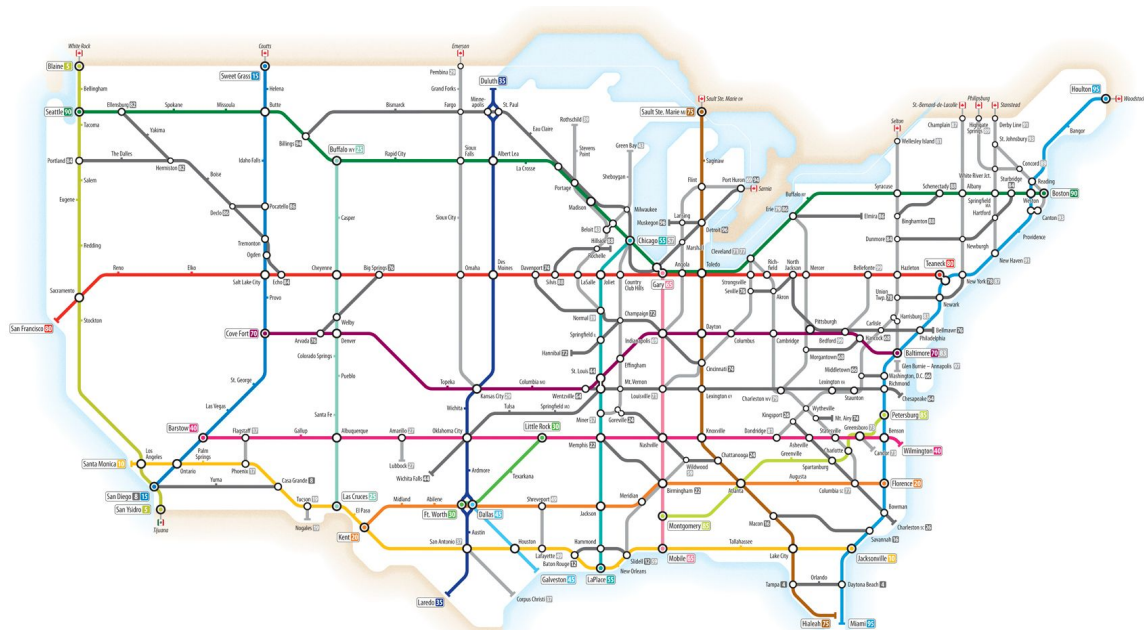
- Nodes: Atoms
- Edges: Bonds
(covalent or ionic)
- Undirected
- Weighted



Revisiting Graph Examples: Interstate Highways

Properties

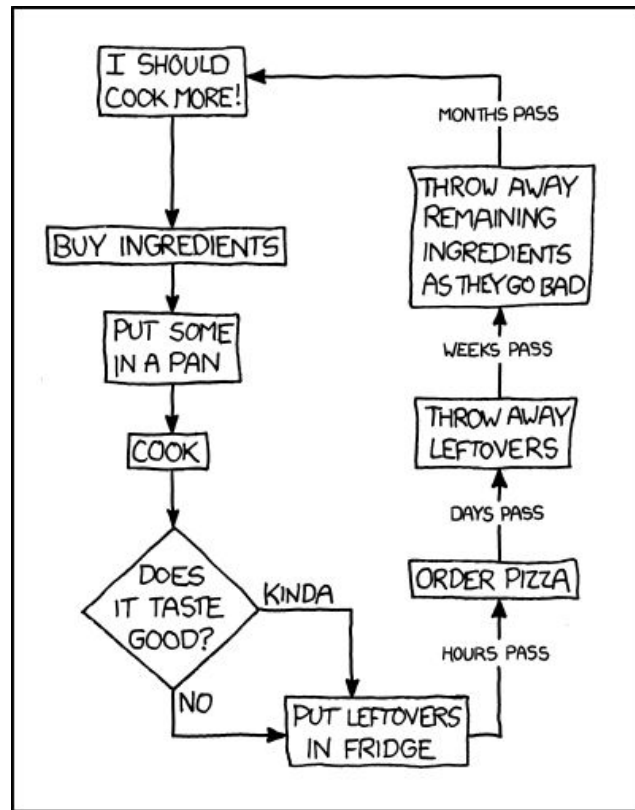
- Nodes: Cities
- Edges: Highways/roads
- Undirected
- Weighted



Revisiting Graph Examples: Flowcharts

Properties

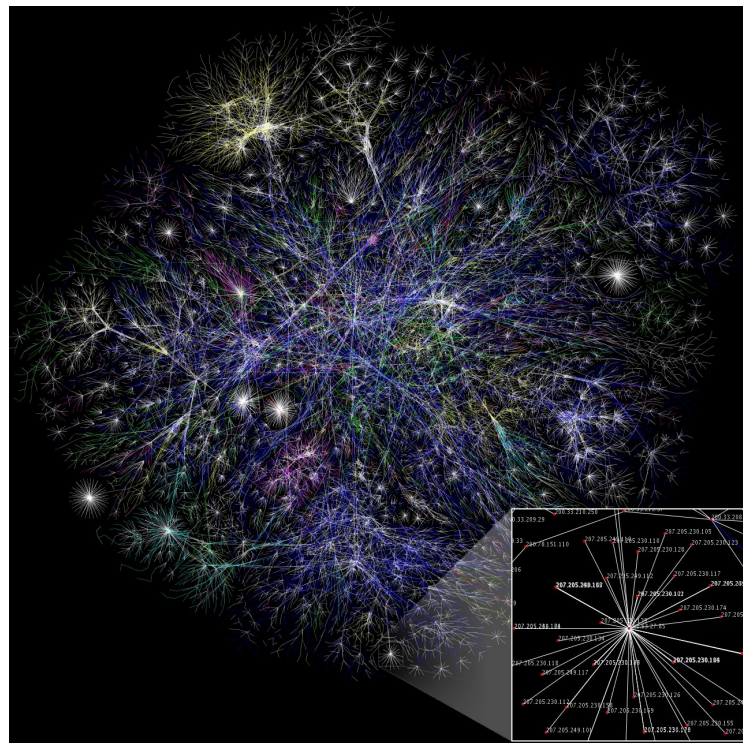
- Nodes: Events/Actions
- Edges: Transitions
- Directed
- Unweighted



Revisiting Graph Examples: The Internet

Properties

- Nodes: Devices (phones, computers, etc.)
- Edges: Connection pathways (Bluetooth, WiFi, Ethernet, cables)
- Undirected
- Can be weighted or unweighted



Graphs as Linked Data Structures

Putting it All Together

- We've seen nodes connected by edges (links) before when discussing linked lists and trees. These, along with graphs, are all **linked data structures**!

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- What differentiates each of these linked data structures?
 - **Linked lists:** Linear structure, each node connected to at most one other node.
 - **Trees:** Nodes can connect to multiple other nodes, no cycles, parent/child relationship and a single, special root node.

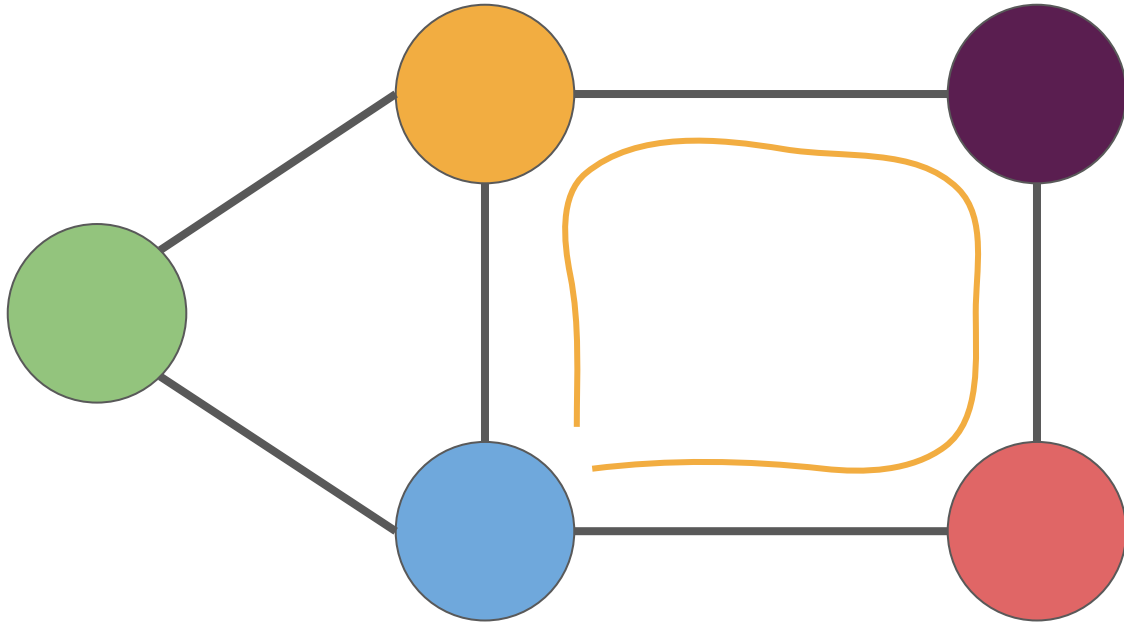


Putting it All Together

- We've seen nodes connected by edges (links) before when discussing linked lists and trees. These, along with graphs, are all **linked data structures**!
- What differentiates each of these linked data structures?
 - **Linked lists:** Linear structure, each node connected to at most one other node.
 - **Trees:** Nodes can connect to multiple other nodes, no cycles, parent/child relationship and a single, special root node.
 - **Graphs:** No restrictions. It's the wild, wild west of the node-based world!

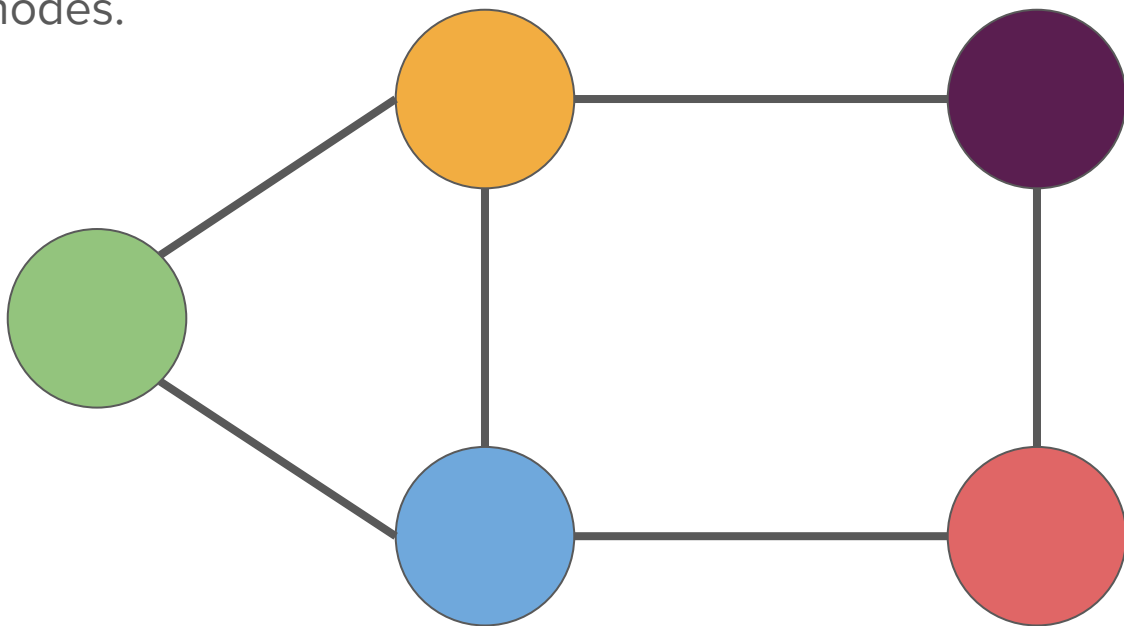
The Wild World of Graphs

- Graphs can have **cycles**...



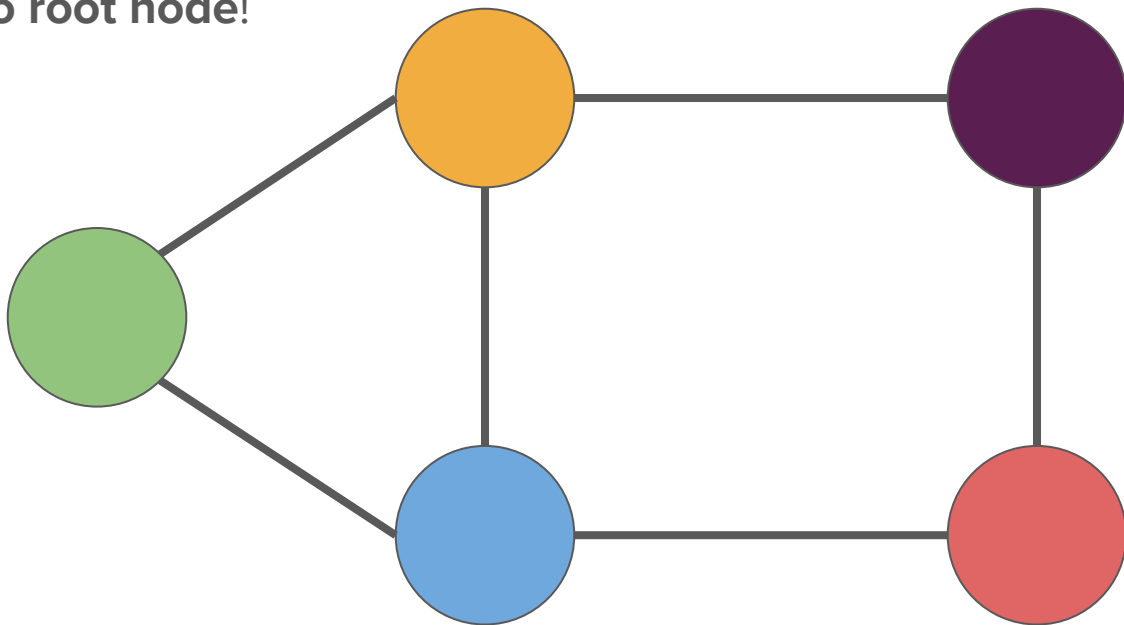
The Wild World of Graphs

- Graphs can have cycles, and there is no notion of a parent-child relationship between nodes.



The Wild World of Graphs

- Graphs have no nodes that are more important than other nodes. In particular, **there is no root node!**



Graphs are the most *powerful, flexible, and expressive* abstraction that we can use to *model relationships between different distributed entities*. You will find graphs everywhere you look!

Representing Graphs

How do we store and represent graphs in code?

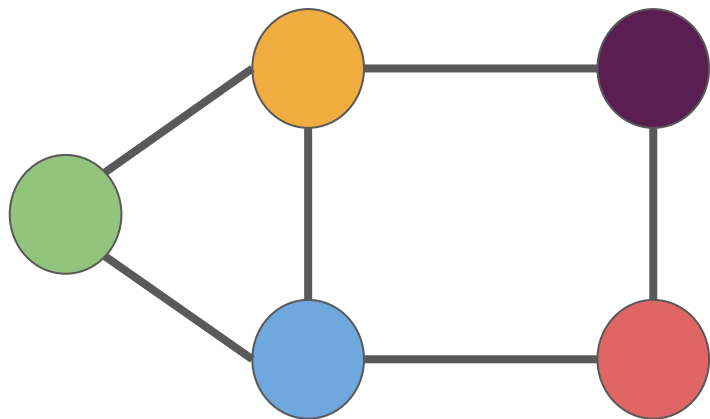
Take 1: Adjacency List

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- We can represent a graph as a map from nodes to the collection of nodes that each node is adjacent to.

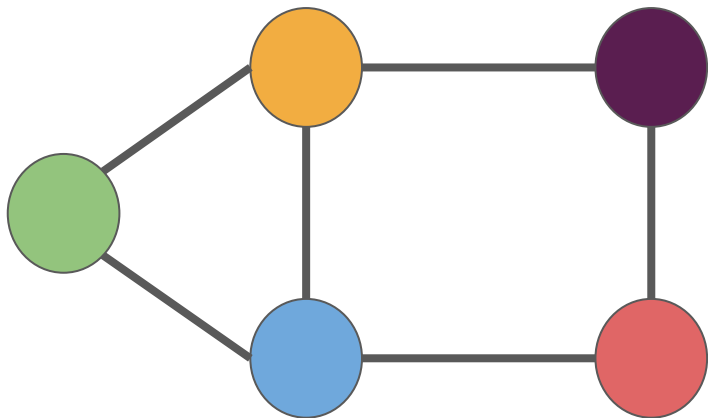
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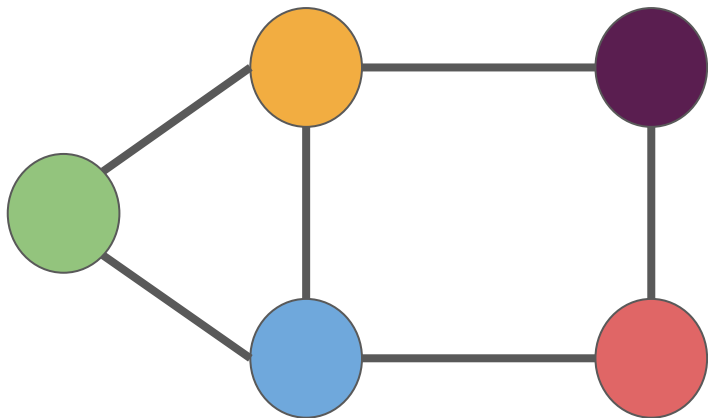
Map<Node, Set<Node>>

Node	Set<Node>>
Node	Adjacent to






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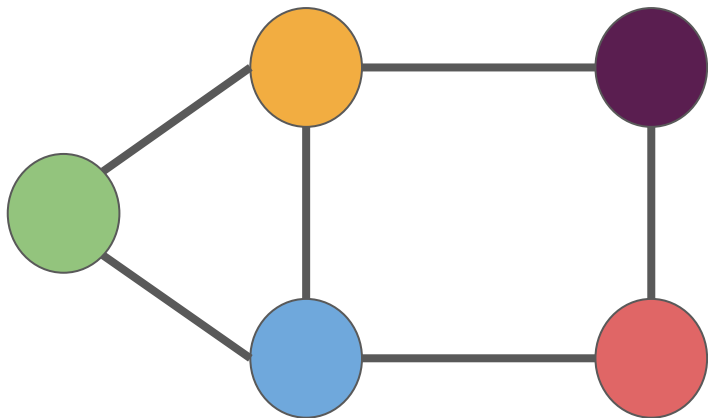


Map<Node, Set<Node>>







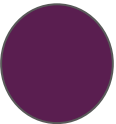
Node	Set<Node>>
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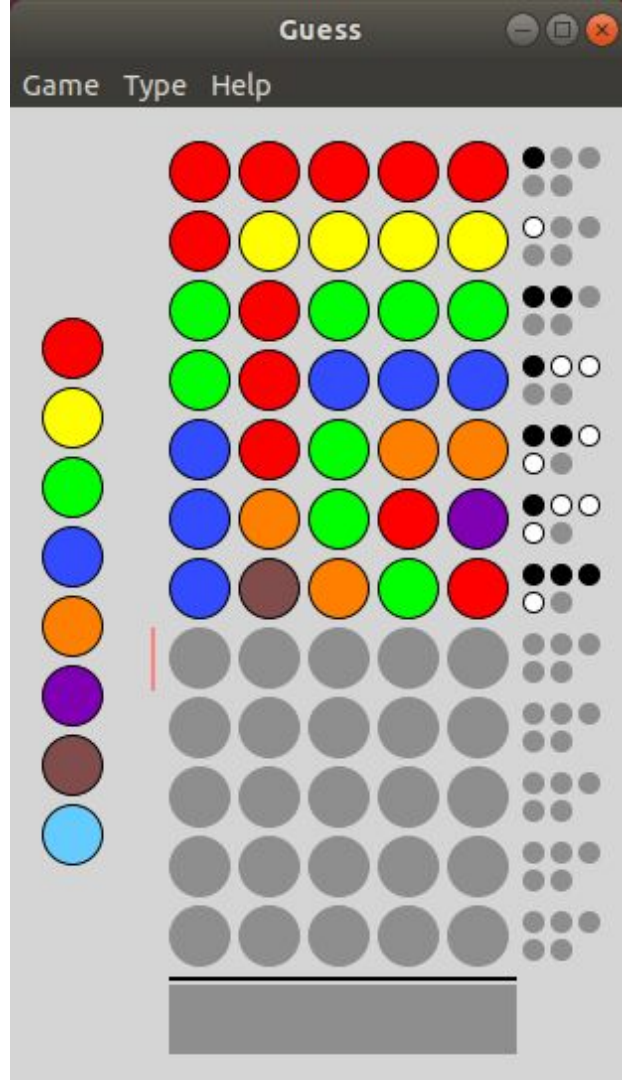
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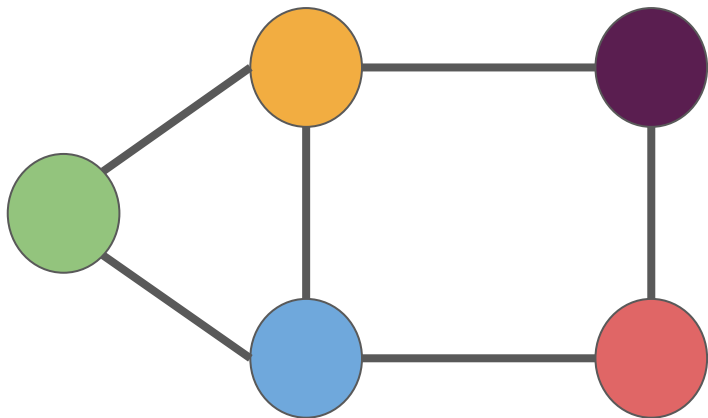
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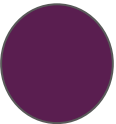


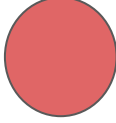

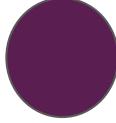


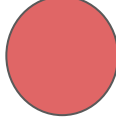
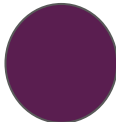



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Map<Node, Set<Node>>

Node	Set<Node>>		
Node	Adjacent to		
			
			
			
			
			

Take 1: Adjacency List

- The approach we just saw is called an adjacency list and comes in a number of different forms:
 - `Map<Node, Vector<Node>>`
 - `HashMap<Node, HashSet<Node>>`
 - `Map<Node, Set<Node>>`
 - `Vector<Node>` <- in this case, the Node struct holds collection of its adjacent neighbors

Take 1: Adjacency List

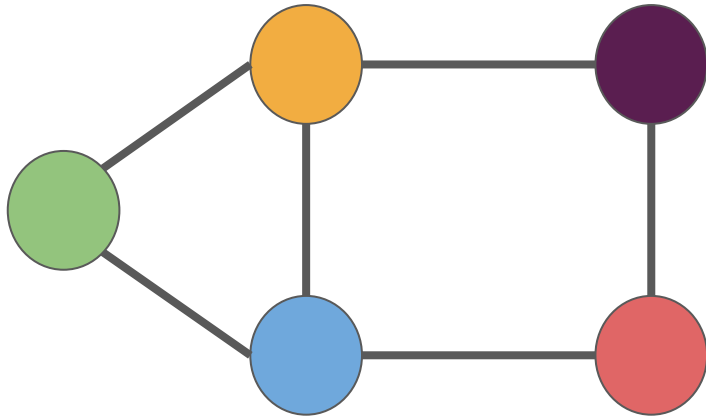
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 - `Map<Node, Set<Node>>`
 - `Vector<Node>` <- in this case, the Node struct holds collection of its adjacent neighbors
- The core idea is that we have some kind of mapping associating each node with its outgoing edges (or neighboring nodes).

Take 2: Adjacency Matrix

- We can also use a two-dimensional matrix to represent the relationships in a graph.

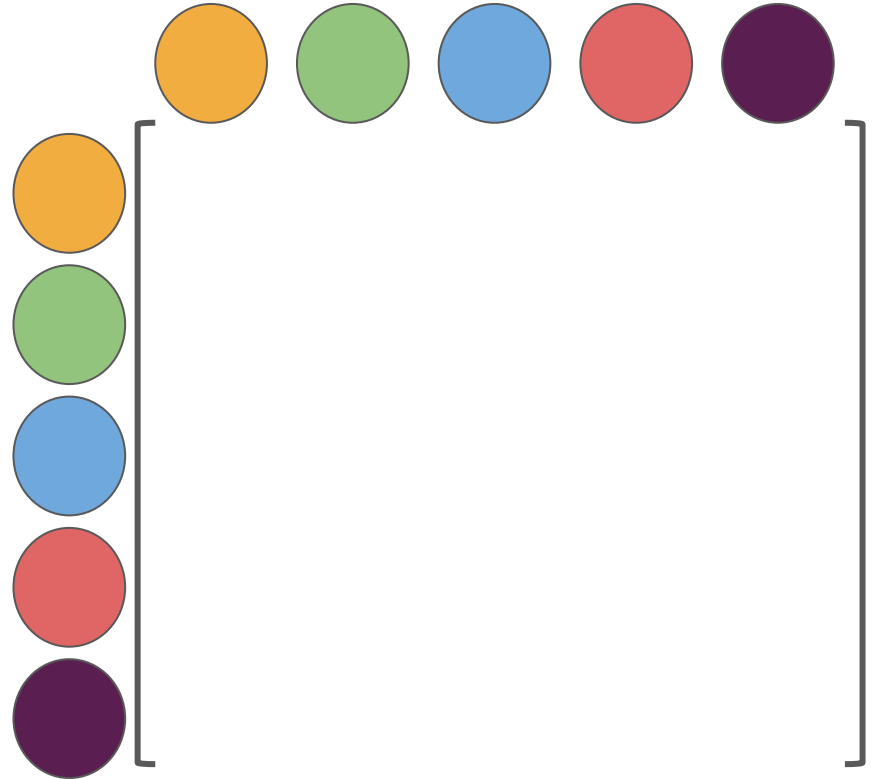
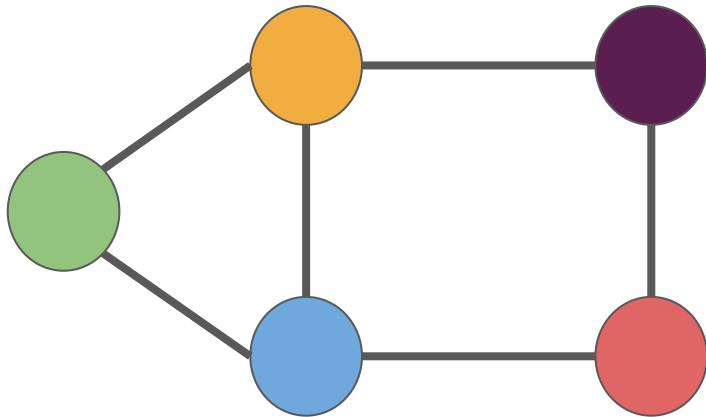
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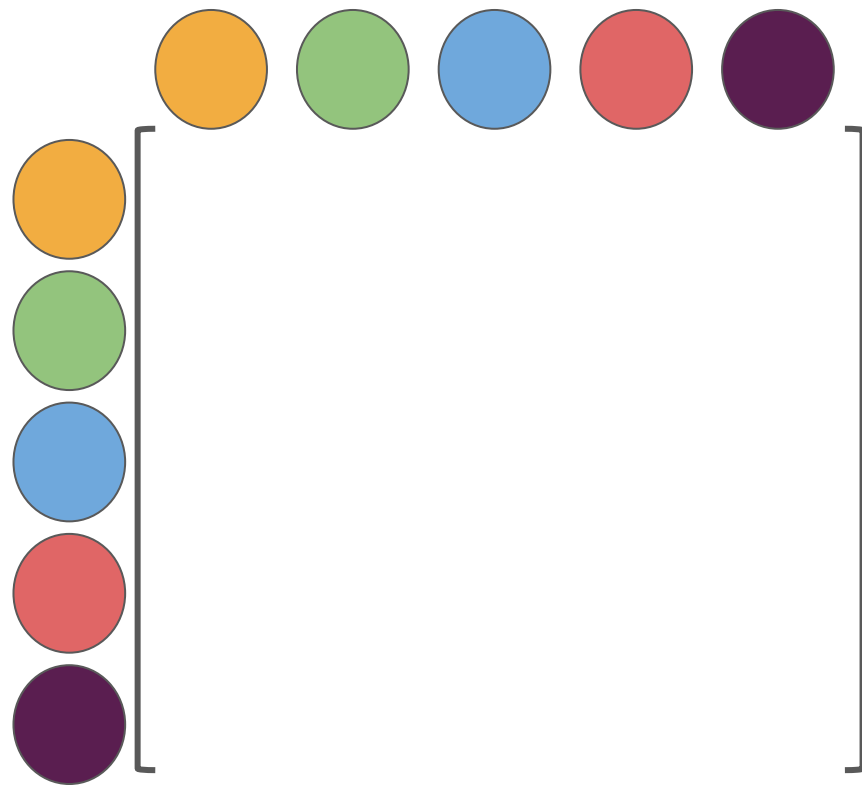
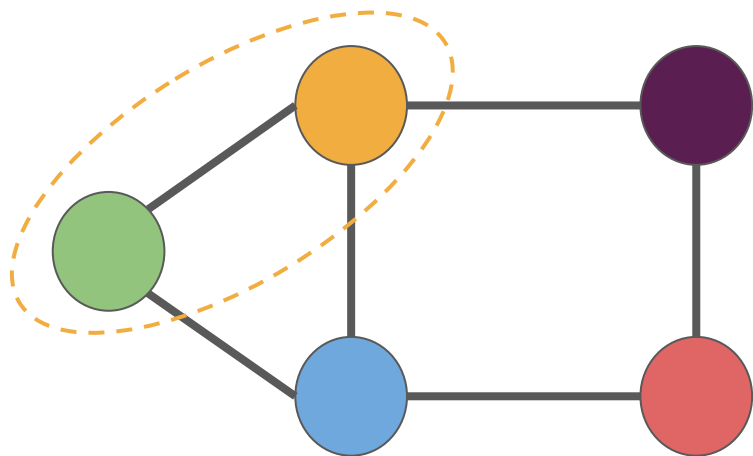
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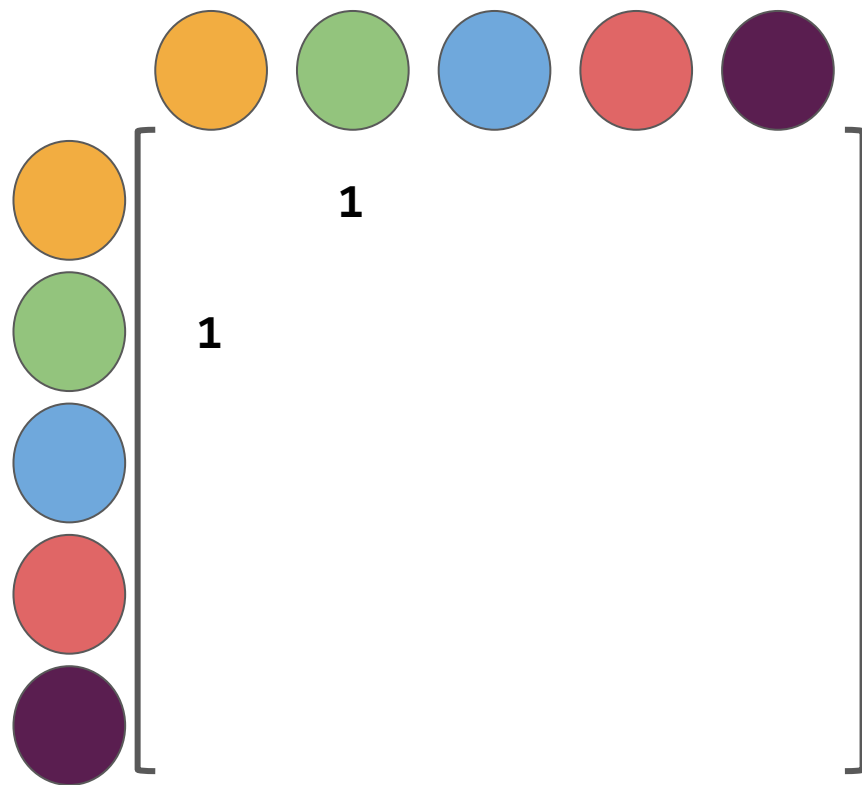
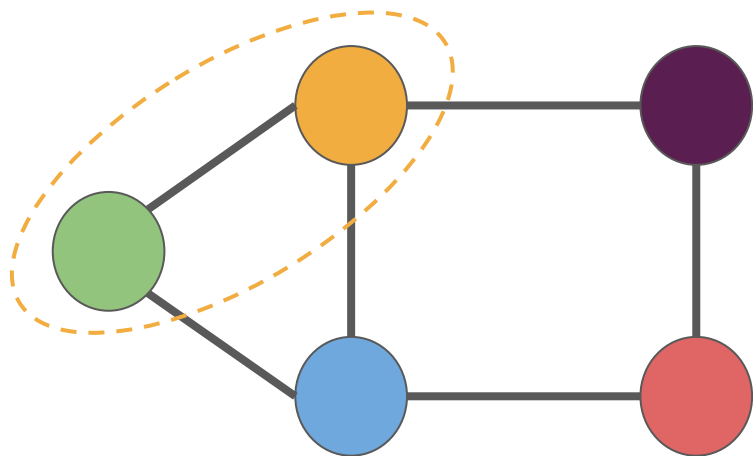
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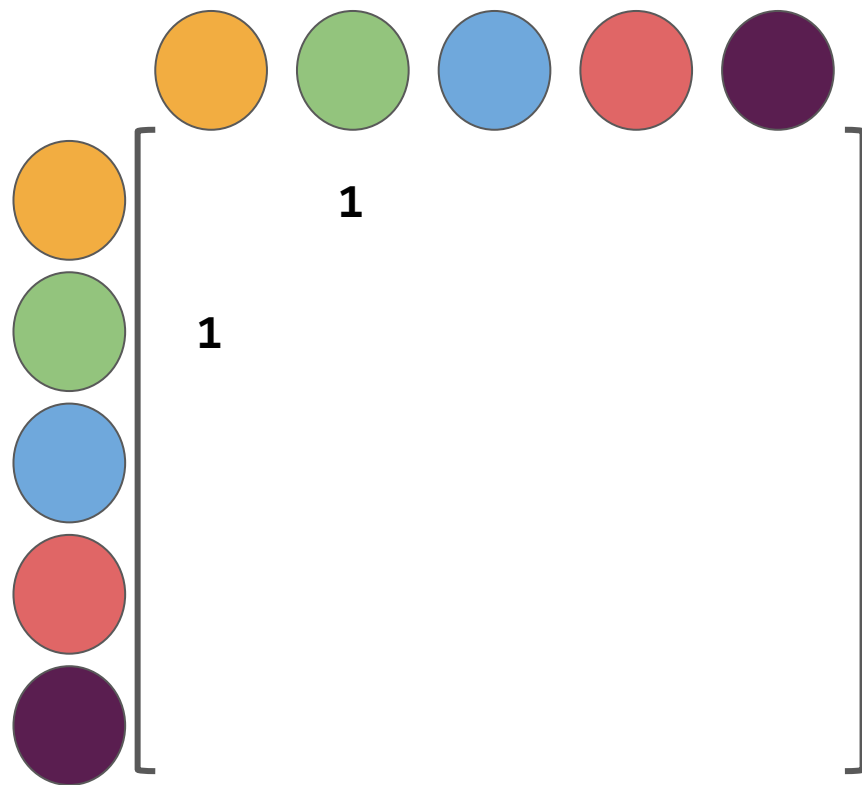
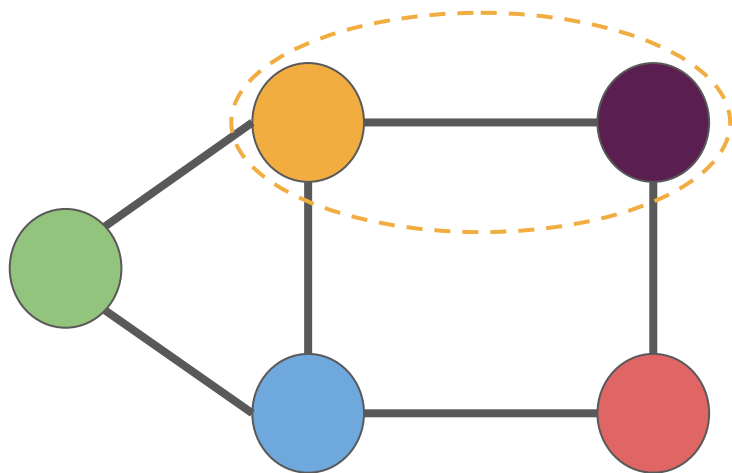
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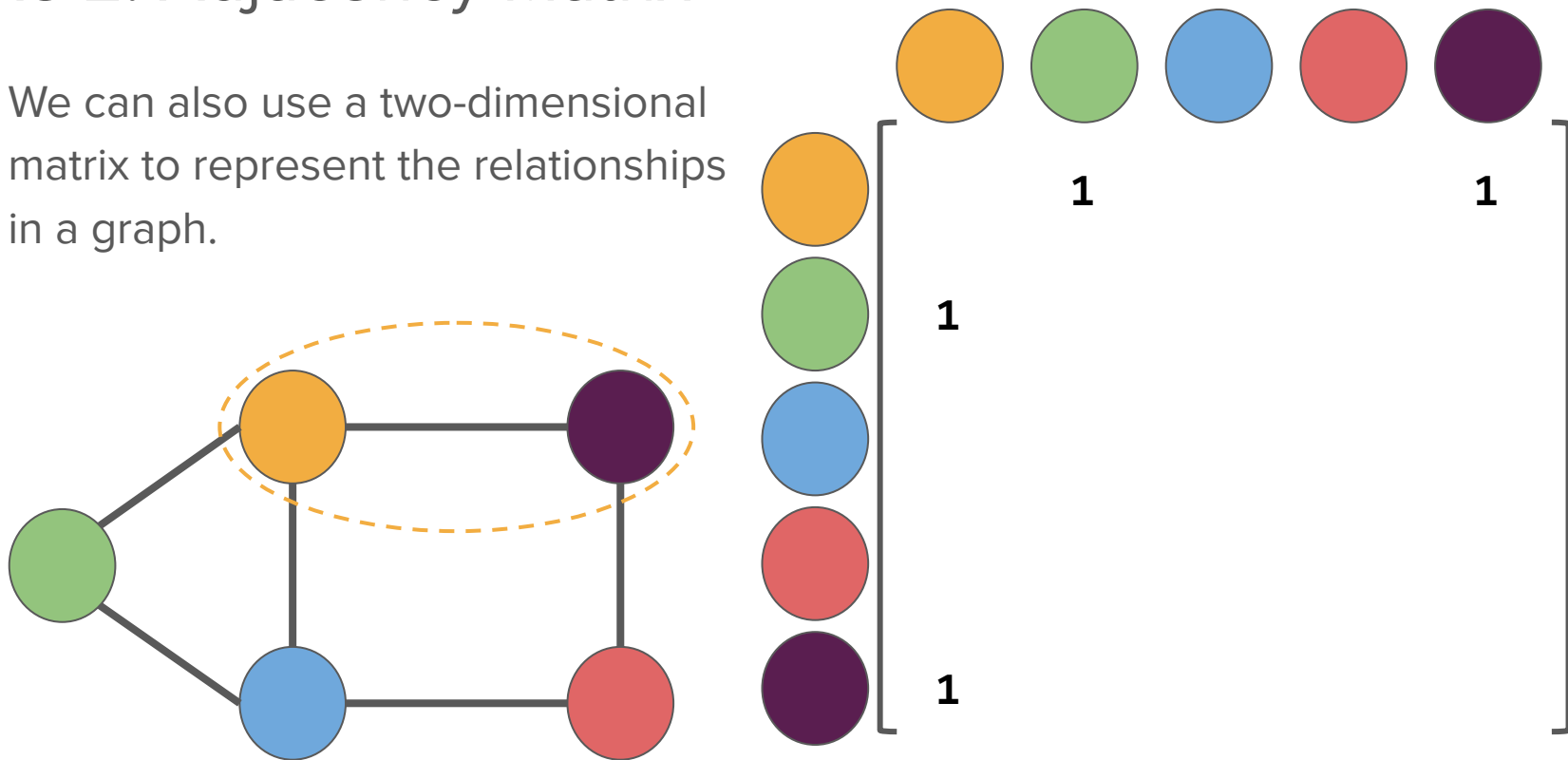
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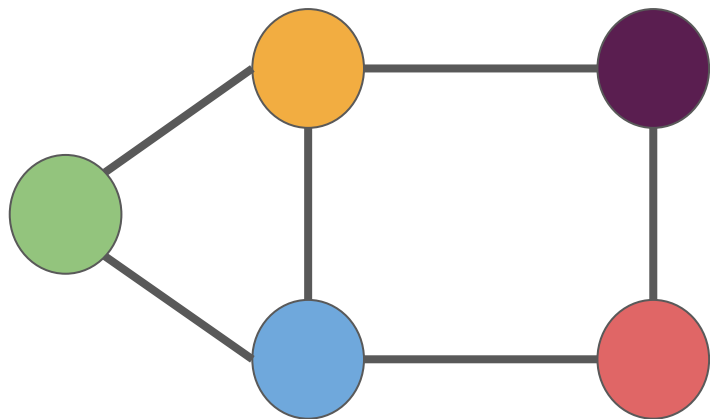
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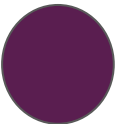


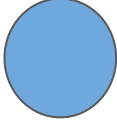
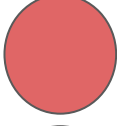
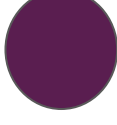
- We can also use a two-dimensional matrix to represent the relationships in a graph.



Take 2: Adjacency Matrix

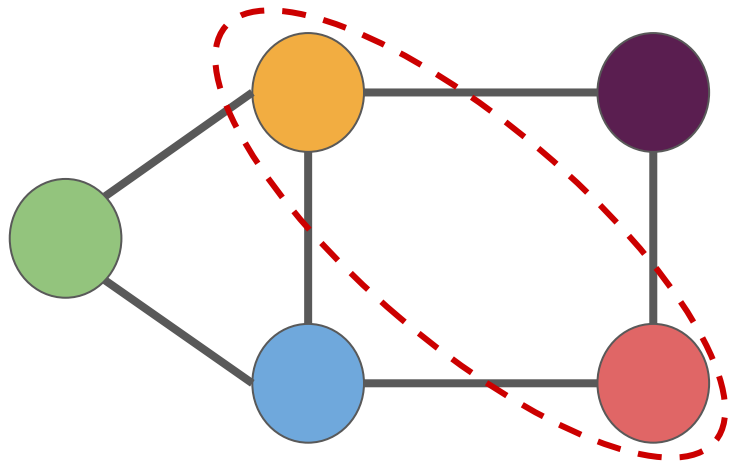
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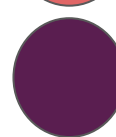


					
		1	1		1
	1		1		
	1	1		1	
			1		1
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Take 2: Adjacency Matrix

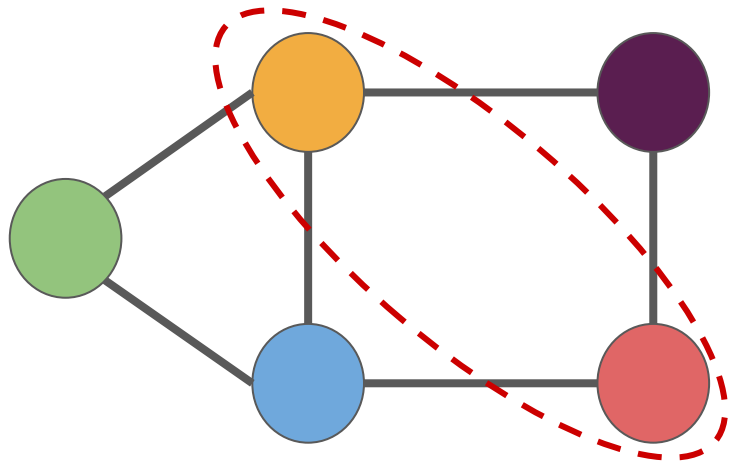
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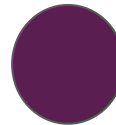


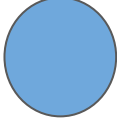
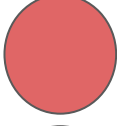
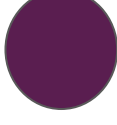


					
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	1		1		
	1	1		1	
			1		1
	1			1	

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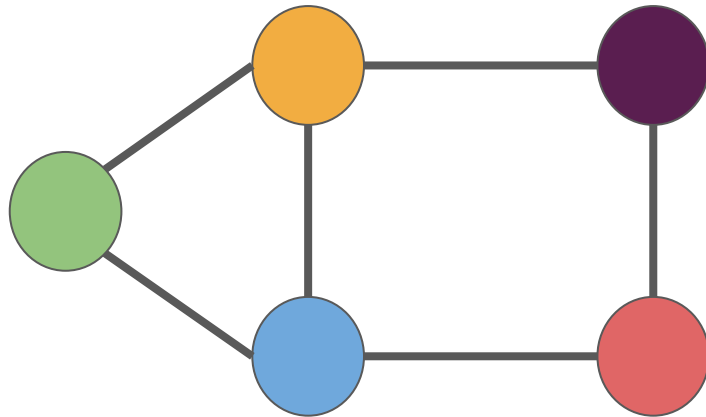
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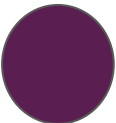


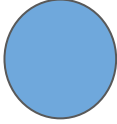
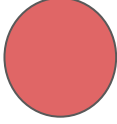
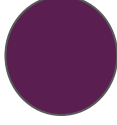


					
		1	1	0	1
	1		1		
	1	1		1	
	0		1		1
	1			1	

Take 2: Adjacency Matrix

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	0	1	1	0	1
	1	0	1	0	0
	1	1	0	1	0
	0	0	1	0	1
	1	0	0	1	0

Going forward, unless stated otherwise, assume we're using an **adjacency list** representation.

Announcements

Announcements

- Assignment 6 is due on **Wednesday, August 11 at 11:59pm PDT**. This is a hard deadline – there is **no grace period, and no submissions will be accepted after this time**.
- Your last section is this week!
- Thursday: **No class!** Use the time to review for the End of Quarter Assessment.
- End of Quarter Assessment will **be released on Friday, August 13** and is **due on Sunday, August 15 at 11:59PM**.

Graph Algorithms

Graph Traversal

Iterating over a Graph

- In a singly-linked list, there's pretty much one way to iterate over the list: start at the front and go forward!

Iterating over a Graph

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 - Post-order traversal

Iterating over a Graph

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- In a tree, there are many traversal strategies:
 - Pre-order traversal
 - In-order traversal
 - Post-order traversal
- There are many ways to iterate over a graph, each of which have different properties.
 - First idea: Let's revisit breadth-first search!

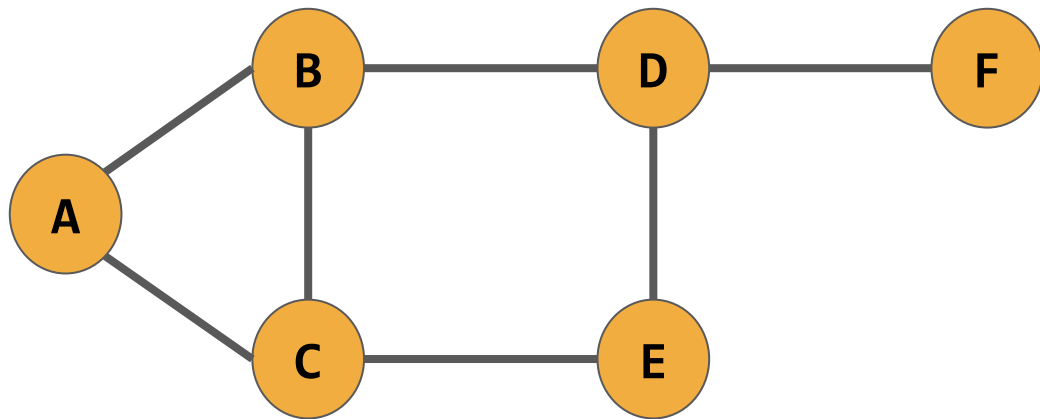
Breadth-First Search

Revisiting Breadth-First Search

- Core Idea: Find everything one hop away from the start, then two hops away, then three hops away, etc.

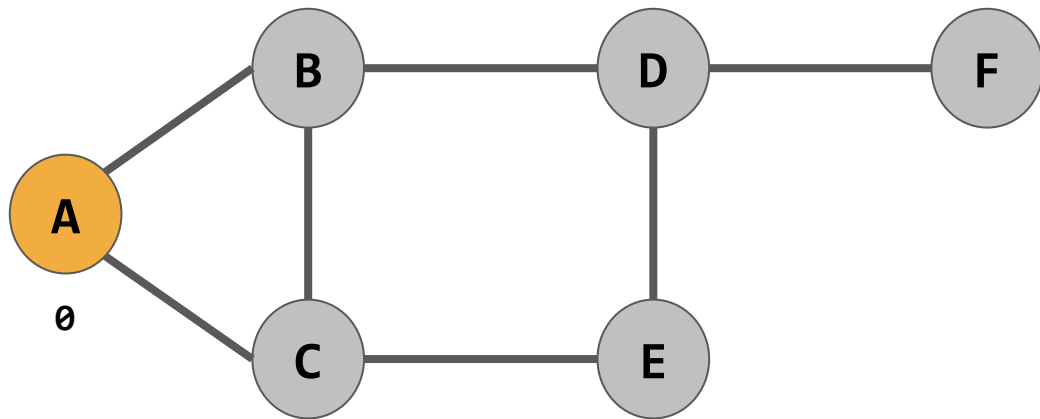
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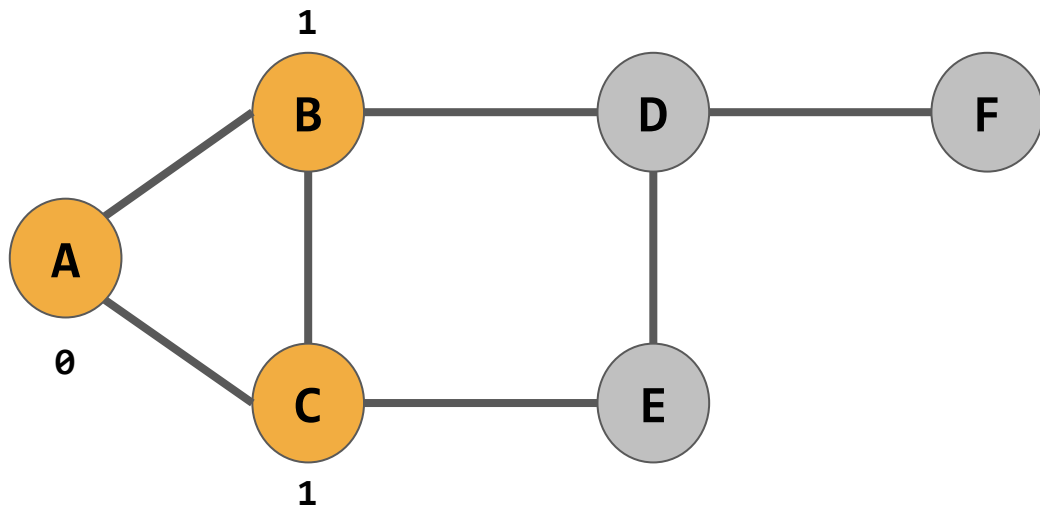
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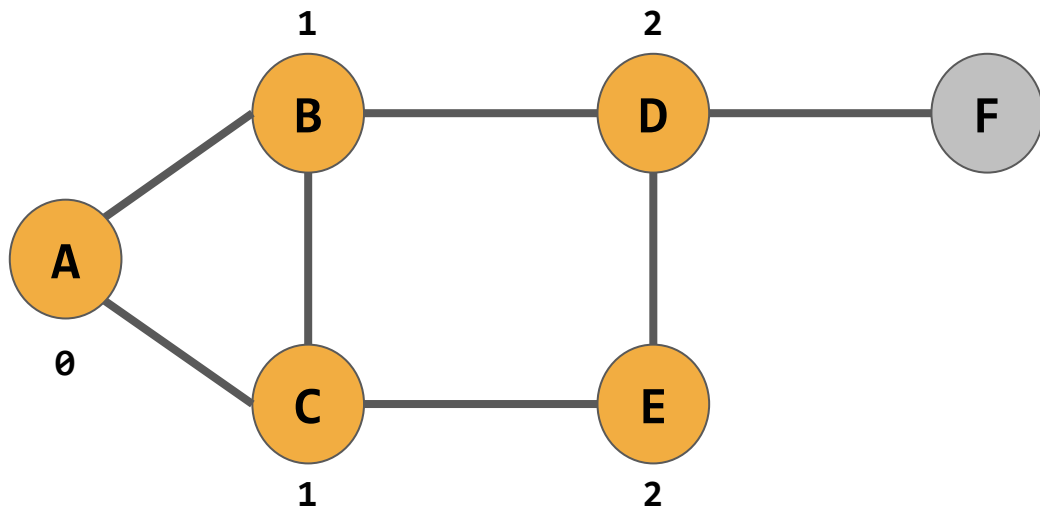
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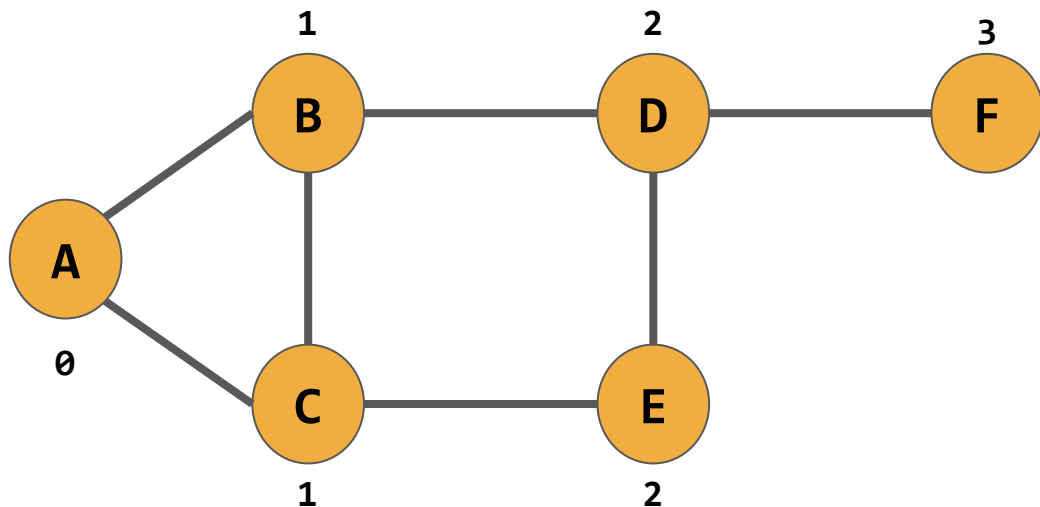
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Graph Breadth-First Search

- The BFS algorithm on graphs looks very similar to what we saw way back in Week 2. The main difference is we just keep track of nodes rather than partial paths.

- BFS Pseudocode

```
bfs-from(node v) {  
    make a queue of nodes, initially seeded with v  
  
    while (queue not empty) {  
        curr = dequeue from queue  
        "process" curr  
        for each node adjacent to curr {  
            if that node hasn't yet been visited, enqueue it  
        }  
    }  
}
```
- [Visualization 1](#)
- [Visualization 2](#)

Breadth-First Search Properties

- Breadth-First Search allows us to find the shortest path/distance between any two nodes in an **unweighted graph**.
- However, BFS doesn't do anything to incorporate edge weights when applied to a weighted graph.
- Most real-world applications of finding the shortest path between two nodes in a graph occur on weighted graphs.
- **How can we improve BFS to take into account edge weights?**

Dijkstra's Algorithm

The Problem

- Let's implement Google Maps!

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- As we've previously discussed, a road network can be thought of as a weighted graph between many different destination points.
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- We want to prioritize finding the quickest route between our starting point and our destination point, on this weighted graph.
- How can we do it?

The Idea

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- The seed node (starting point) is enqueued with priority 0. Every subsequent node is enqueued with priority equal to the current node's priority + the weight of the edge being traversed.

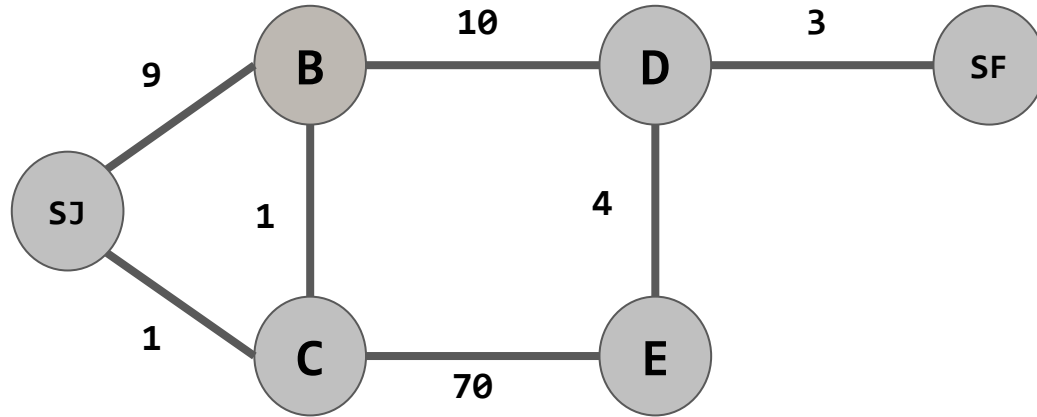
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- What data structure will be useful for this? A priority queue!
- The seed node (starting point) is enqueued with priority 0. Every subsequent node is enqueued with priority equal to the current node's priority + the weight of the edge being traversed.
- The priority queue guarantees we will visit nodes in order of increasing distance from the seed node.

Dijkstra's Algorithm Pseudocode

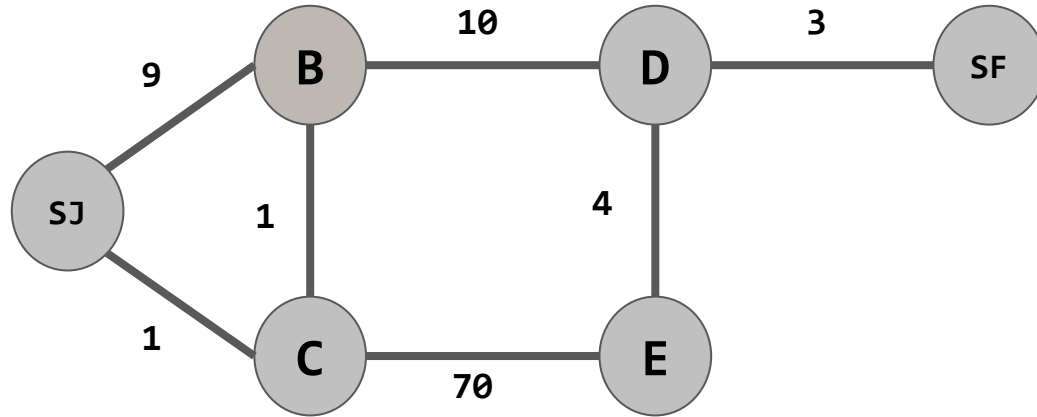
```
dijkstras-from(node v) {  
    Initialize an empty priority queue of nodes  
    Add v to the priority queue with priority 0  
  
    while (queue not empty) {  
        currPriority = peek priority of first element in queue  
        curr = dequeue from queue  
        "process" curr  
        for each node adjacent to curr {  
            if that node hasn't yet been visited, enqueue it with priority  
            equal to currPriority + edge weight between curr and node  
  
            if that node has been visited and is still in the priority queue,  
            update its priority to be currPriority + edge weight  
        }  
    }  
}
```

Dijkstra's In Practice



Goal: Find the shortest path/distance from SJ to SF

Dijkstra's In Practice



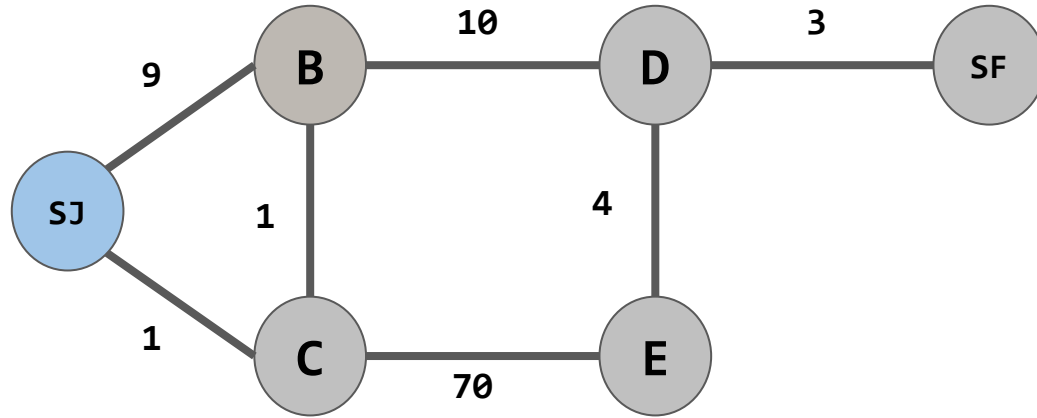
higher priority

lower priority

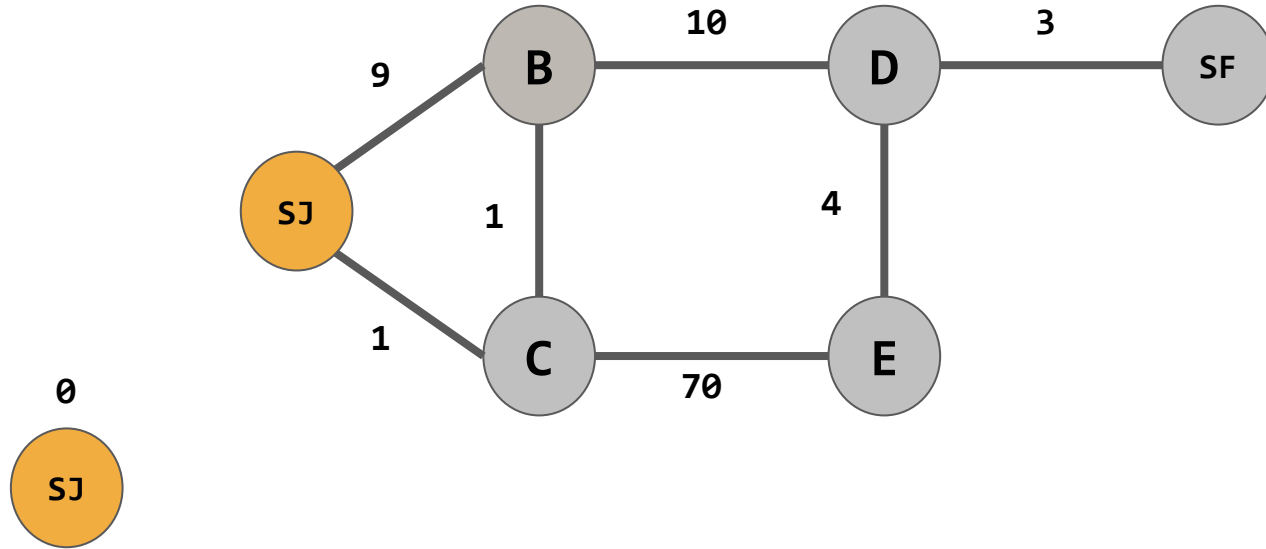
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Dijkstra's In Practice



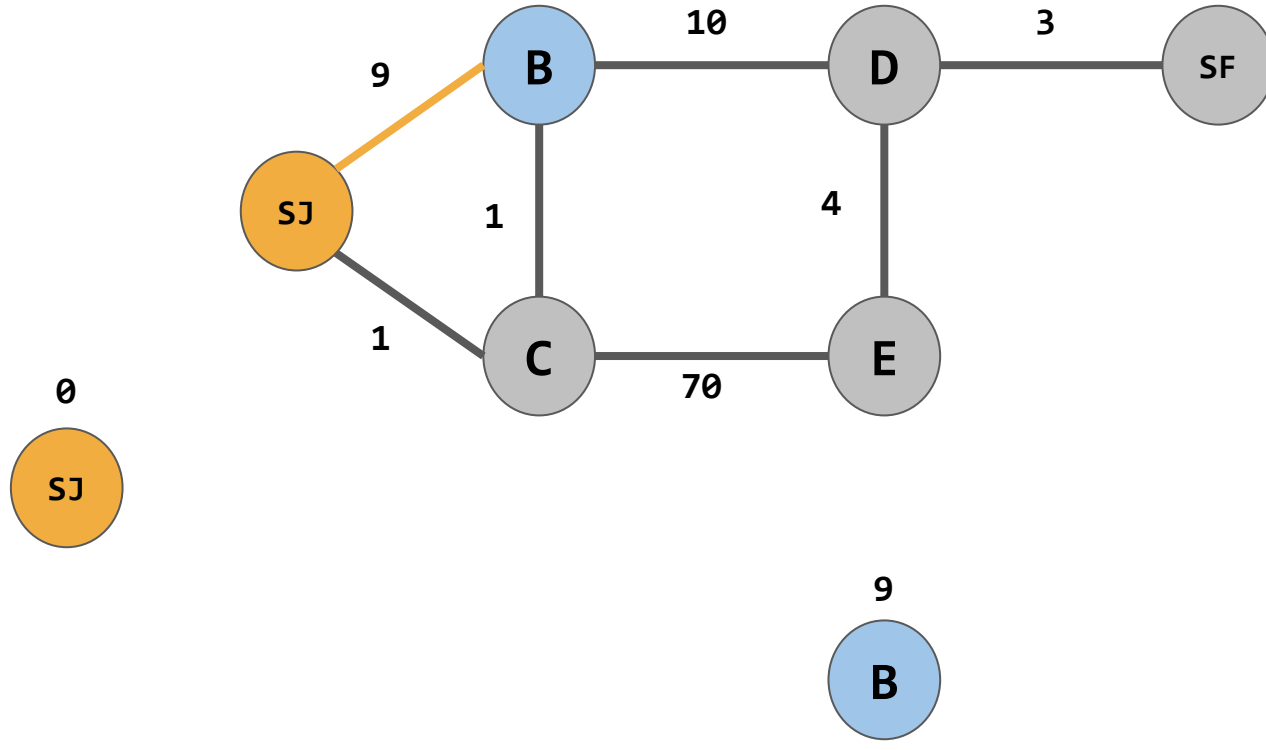
Dijkstra's In Practice



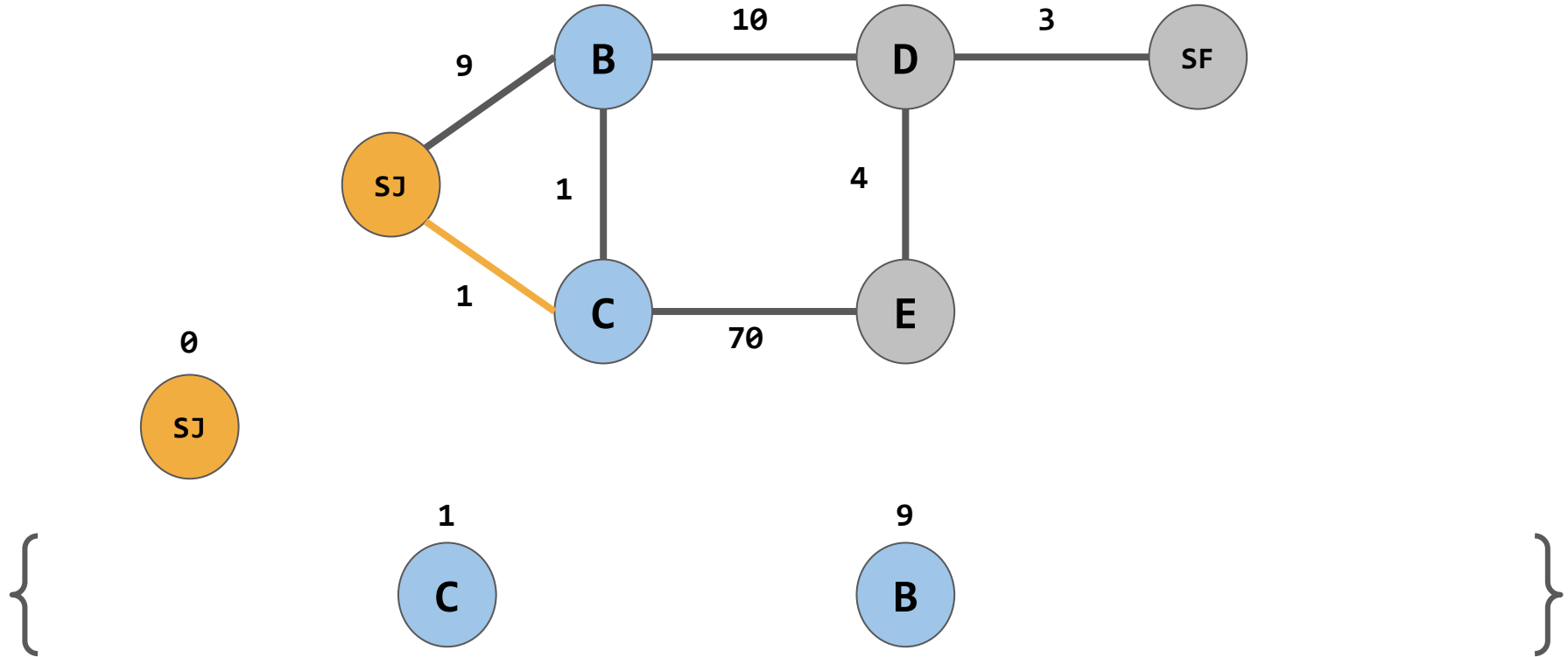
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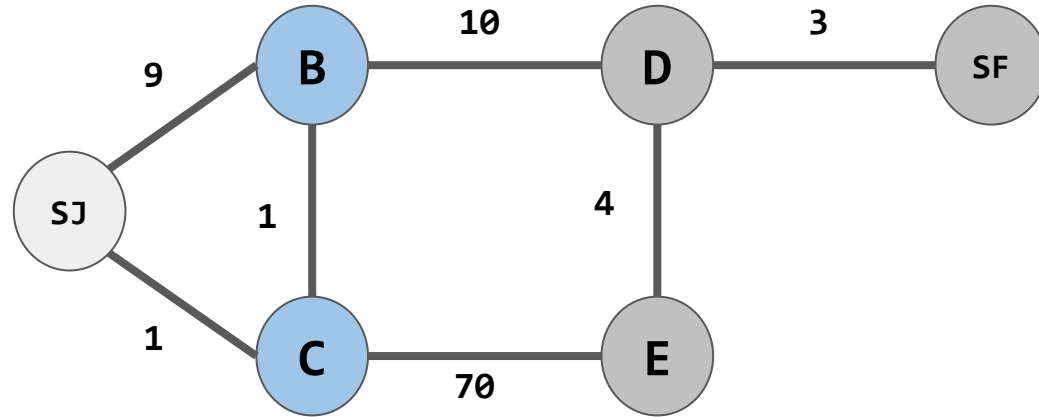
Dijkstra's In Practice



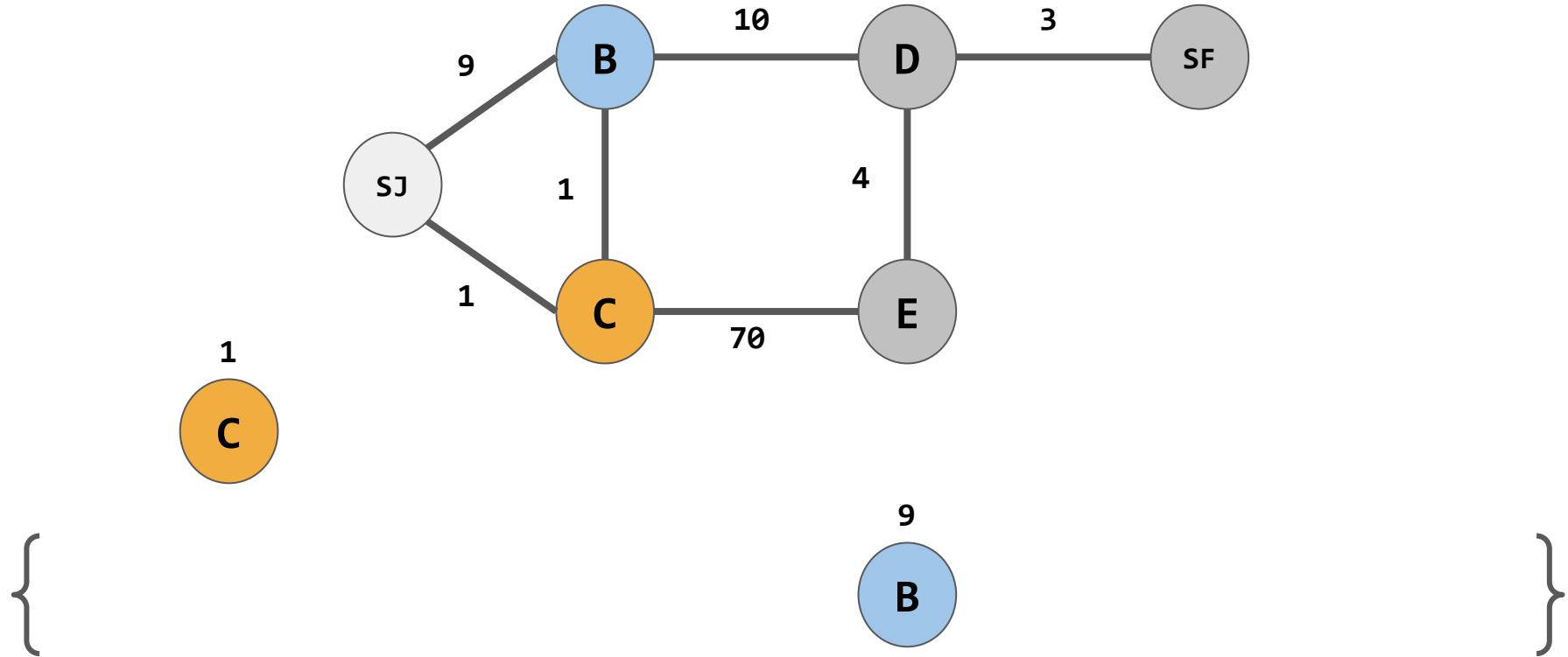
Dijkstra's In Practice



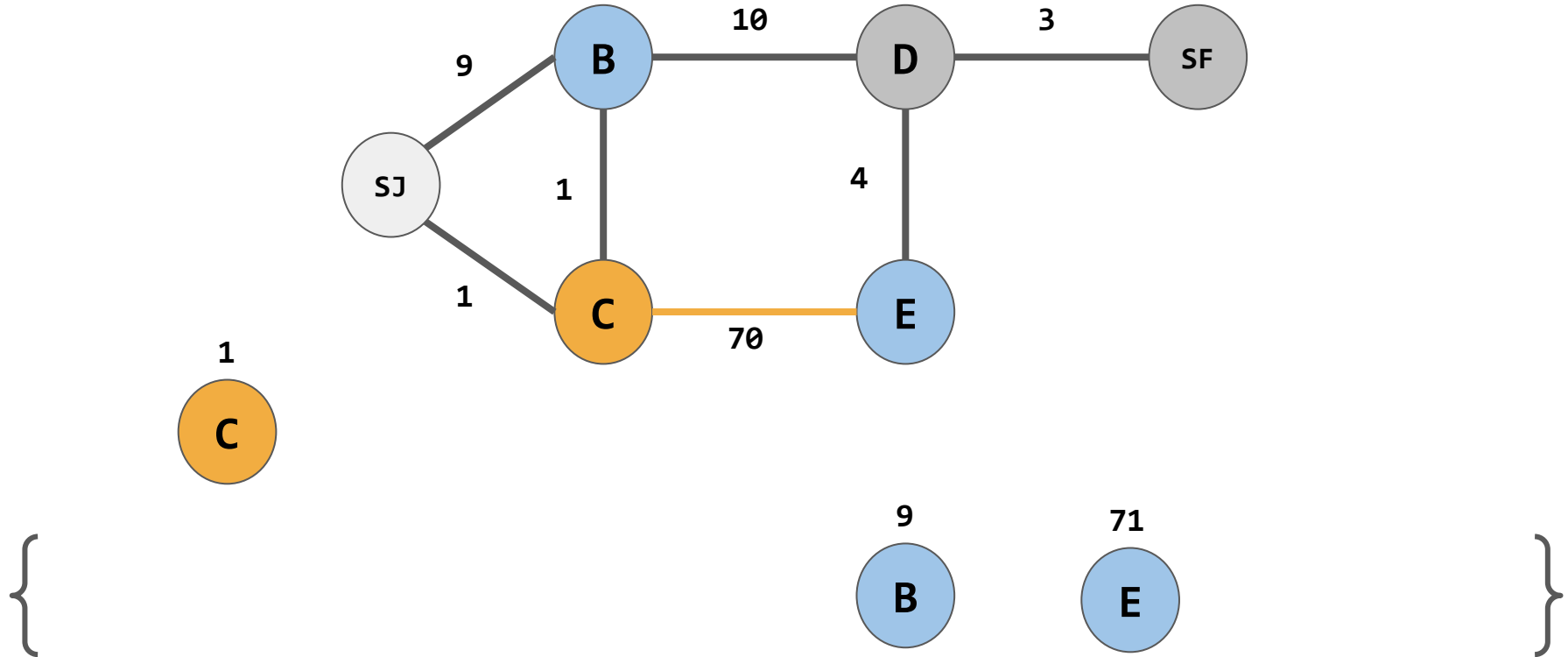
Dijkstra's In Practice



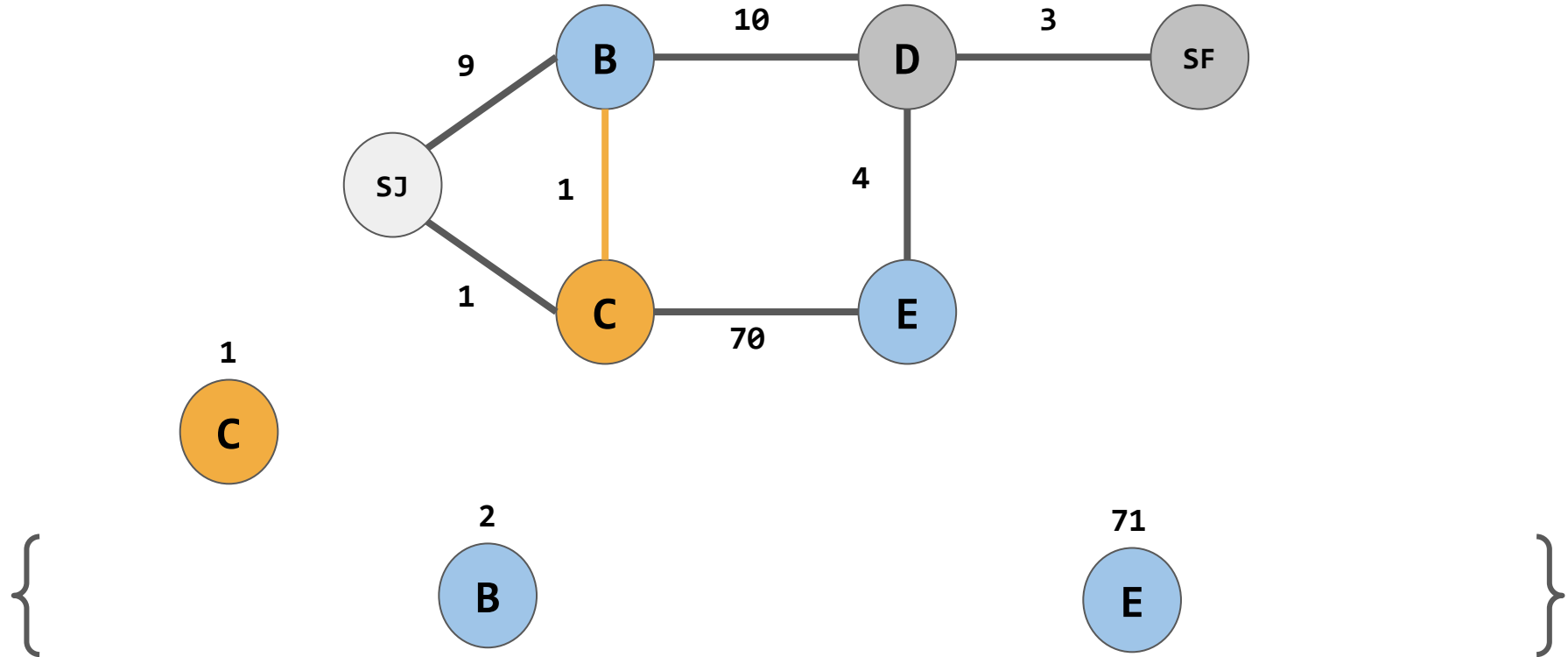
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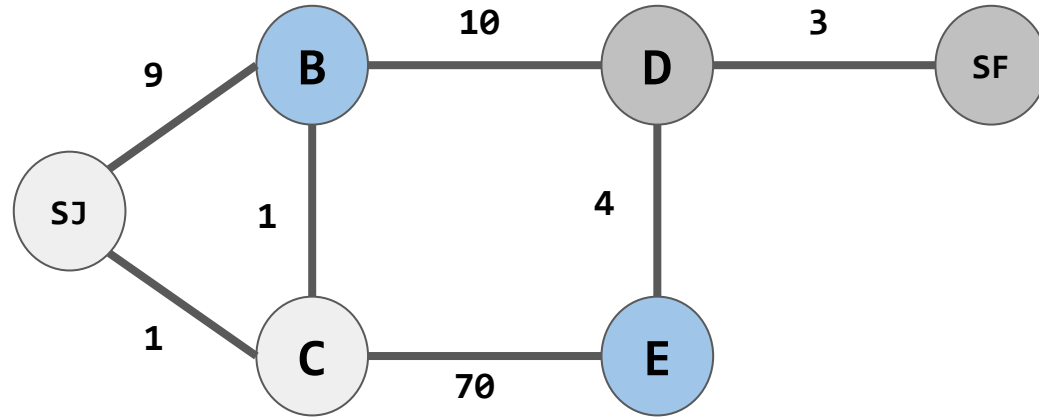
Dijkstra's In Practice



Dijkstra's In Practice



Dijkstra's In Practice



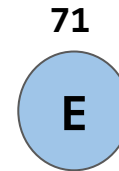
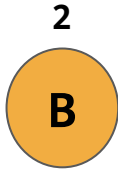
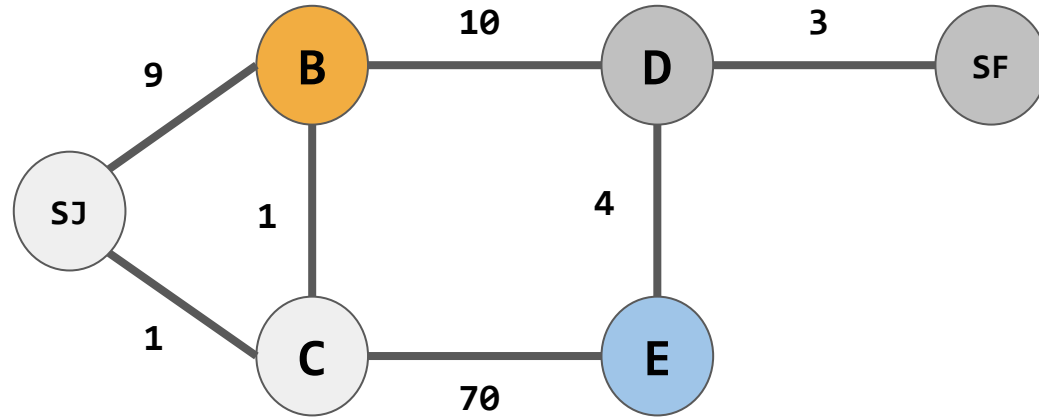
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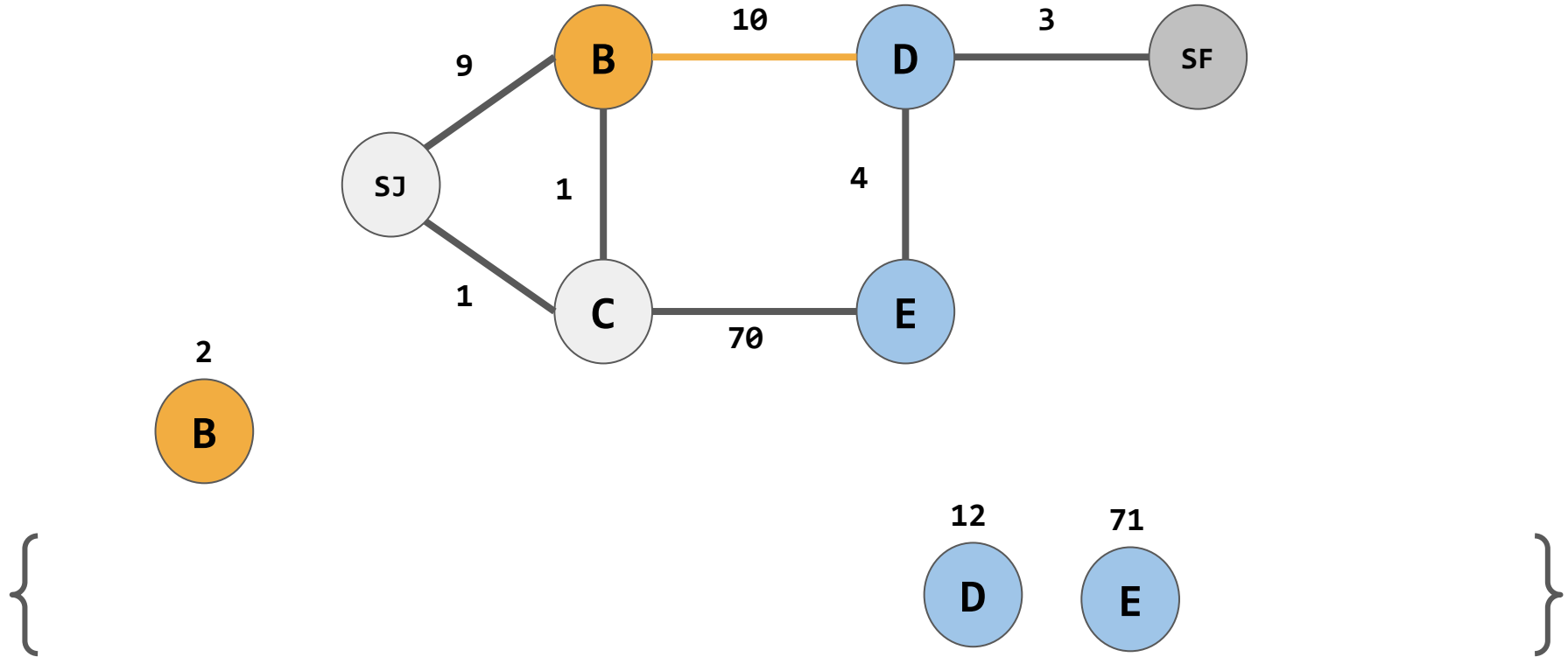
Dijkstra's In Practice



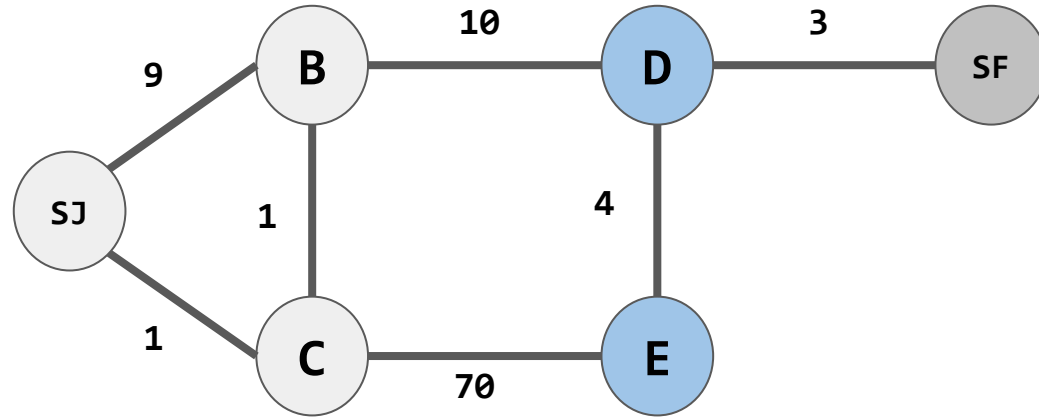
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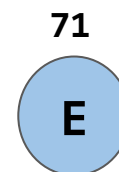
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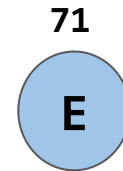
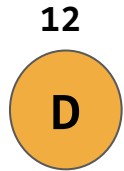
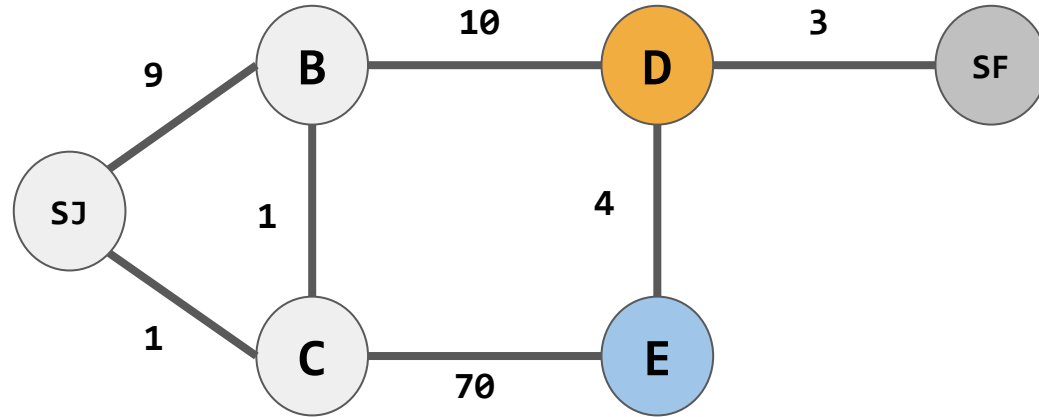


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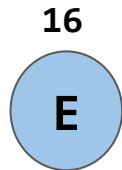
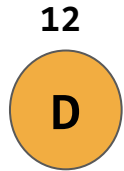
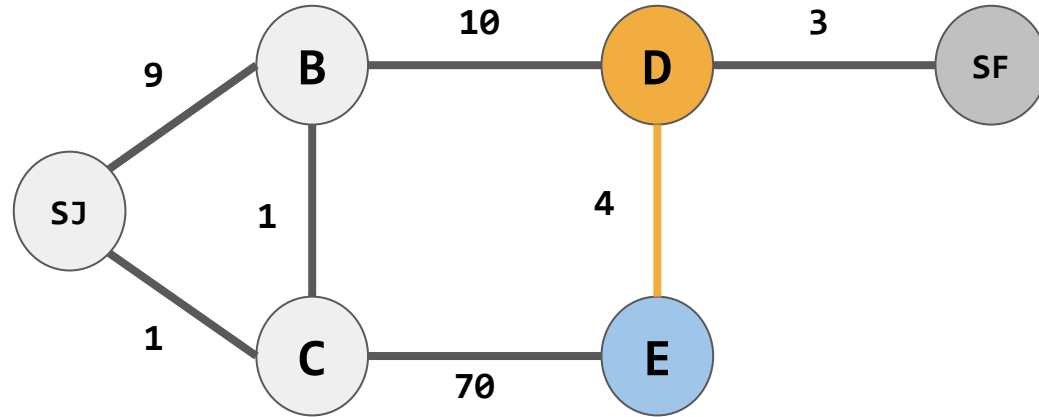
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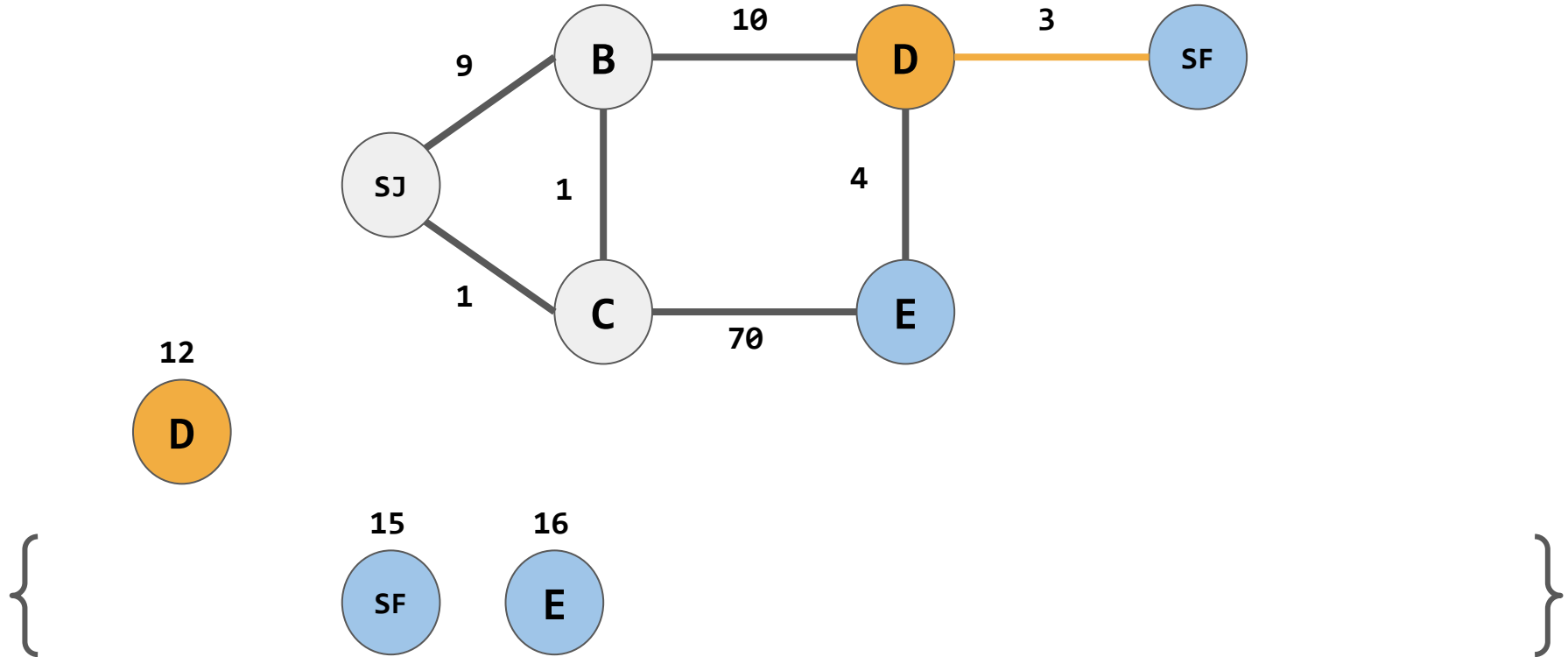
Dijkstra's In Practice



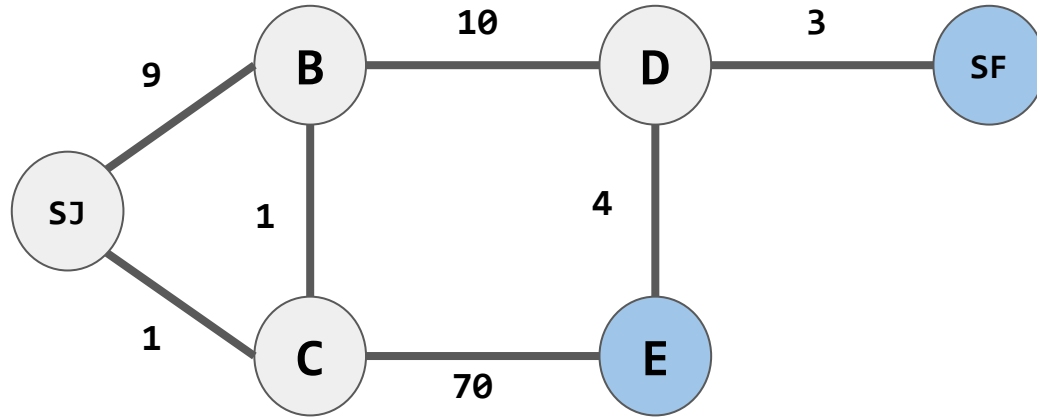
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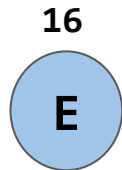
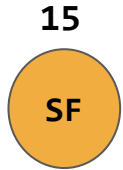
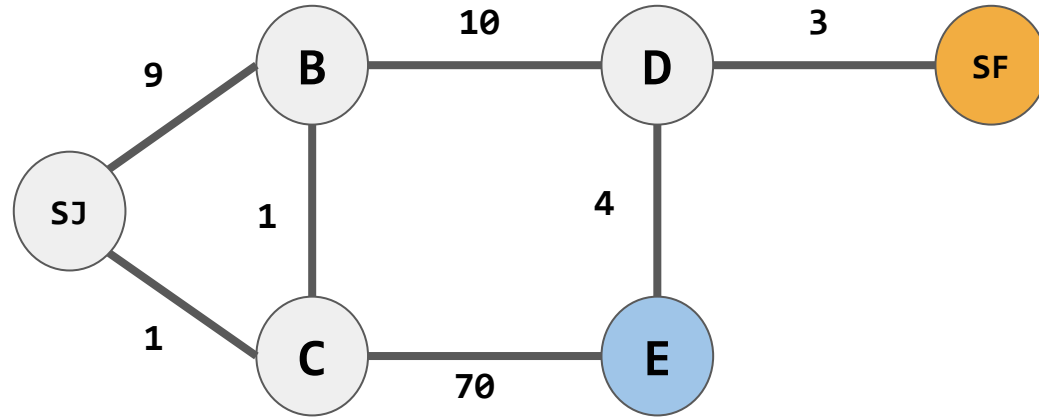
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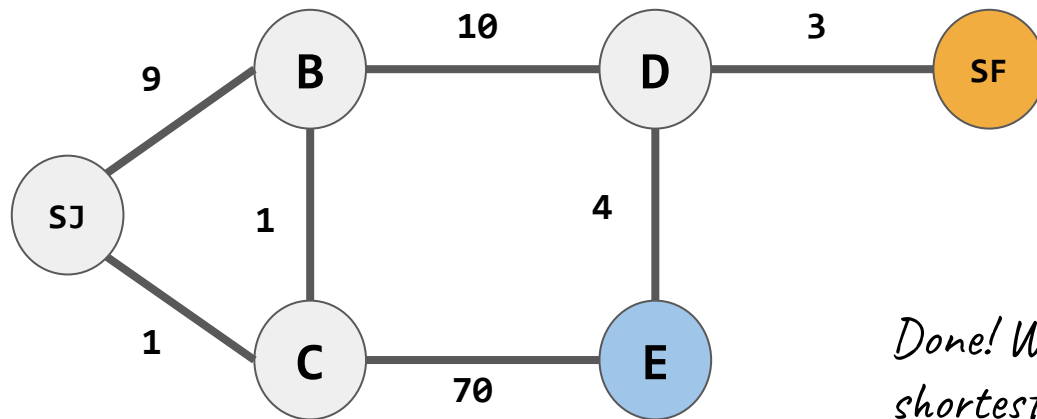
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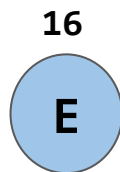
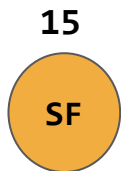
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Dijkstra's In Practice



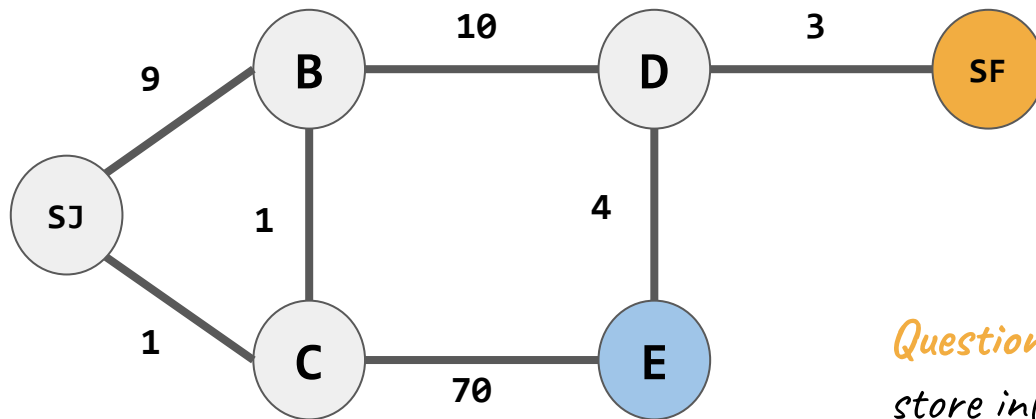
Done! We know the shortest path from SJ to SF has a total path weight of 15.



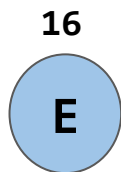
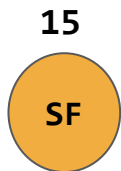
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Dijkstra's In Practice



Question: How would you store information along the way to be able to reconstruct the path?



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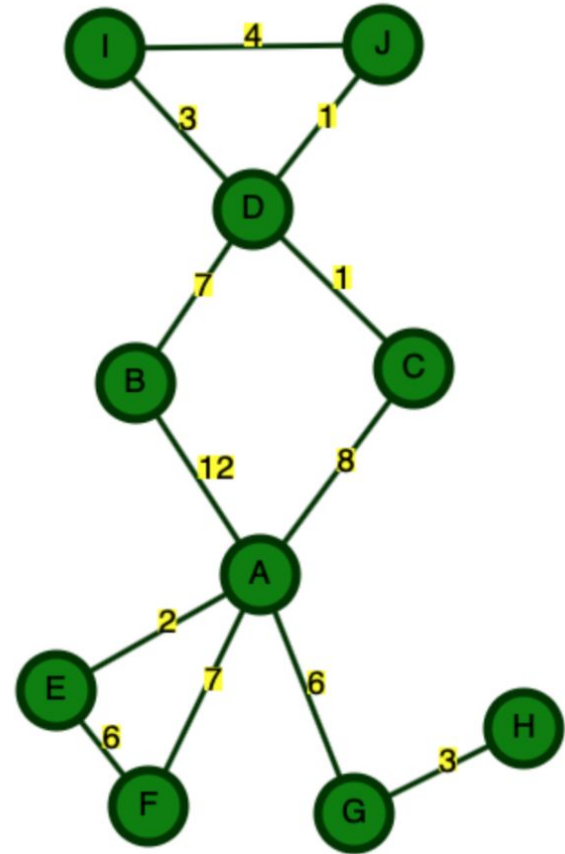
Dijkstra's Algorithm Properties

- Dijkstra's Algorithm allows us to find the shortest path/distance between any two nodes in a **weighted graph**.
- Dijkstra's Algorithm forms the basis of many powerful real-world systems that are built on top of graphs!
- However, one of the downsides to Dijkstra's algorithm is that it can, in many circumstances, ignore **other sources of information** that might prove useful to finding the shortest path in the fewest number of steps.
- Can we find the solution while using less steps than with Dijkstra's Algorithm?

A* Search

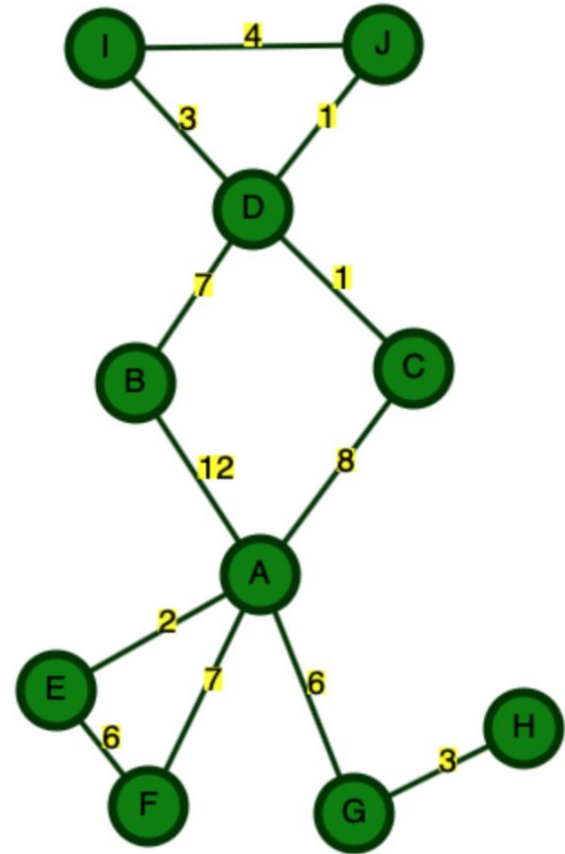
A* Search

- Suppose we wanted to find the shortest path from **A** to **J** in the graph to the right.
 - Given no other information, we can do no better than using Dijkstra's.



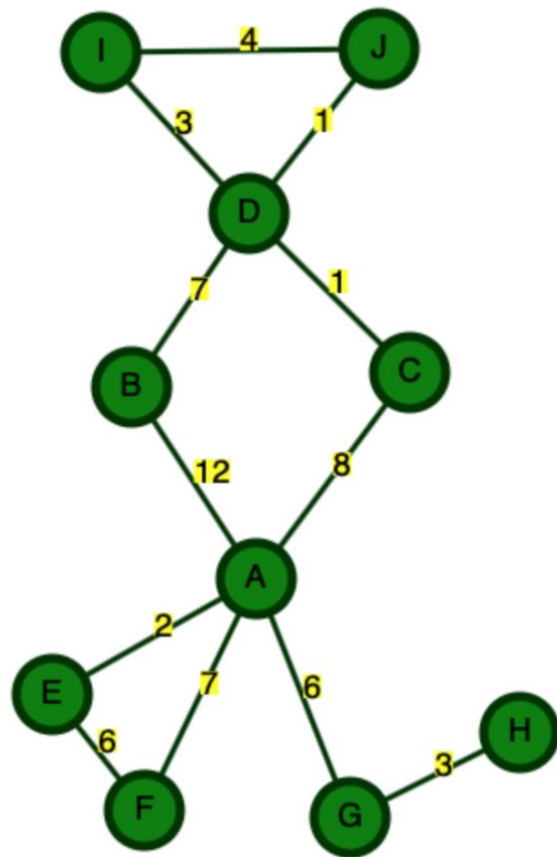
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A* Search

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 - Given no other information, we can do no better than using Dijkstra's.
- But, if we know that this graph represents a map, we can start **reasoning about cardinal directions**.
- **Idea:** If we our goal is to go north from A to J, exploring paths to the south probably doesn't make sense!



Heuristics

- We call the idea of using external information about a graph a **heuristic**.
 - The heuristic estimates the cost of the cheapest path to the goal.
 - It is different for every problem and corresponds to some real-world information.

Heuristics

- We call the idea of using external information about a graph a **heuristic**.
- A heuristic should always **underestimate the distance to the goal**.
 - If it overestimates the distance, it could end up finding a solution that is not actually optimal (though it will do so relatively fast).

Heuristics

- We call the idea of using external information about a graph a **heuristic**.
- A heuristic should always **underestimate the distance to the goal**.
- We use the heuristic as an **addition to the value for the priority**.
 - For the case of maps, if the distance to the destination is closer, this will weight the nodes in that direction to be preferable (i.e., they will actually have a smaller numerical priority value).
 - In other words, **$\text{priority}(\mathbf{u}) = \text{weight}(\mathbf{s}, \mathbf{u}) + \text{heuristic}(\mathbf{u}, \mathbf{d})$** , where **s** is the start, **u** is the node we are considering, and **d** is the destination.

Heuristics

- We call the idea of using external information about a graph a **heuristic**.
- A heuristic should always **underestimate the distance to the goal**.
- We use the heuristic as an **addition to the value for the priority**.
- Common heuristics for distance-based graphs include Manhattan distance, as-the-crow-flies distance, and Chebyshev distance.

Graph Search Demo

<https://qiao.github.io/PathFinding.js/visual/>

Beyond Traversal

More Graph Algorithms

- There are many, many different graph algorithms out there.

More Graph Algorithms

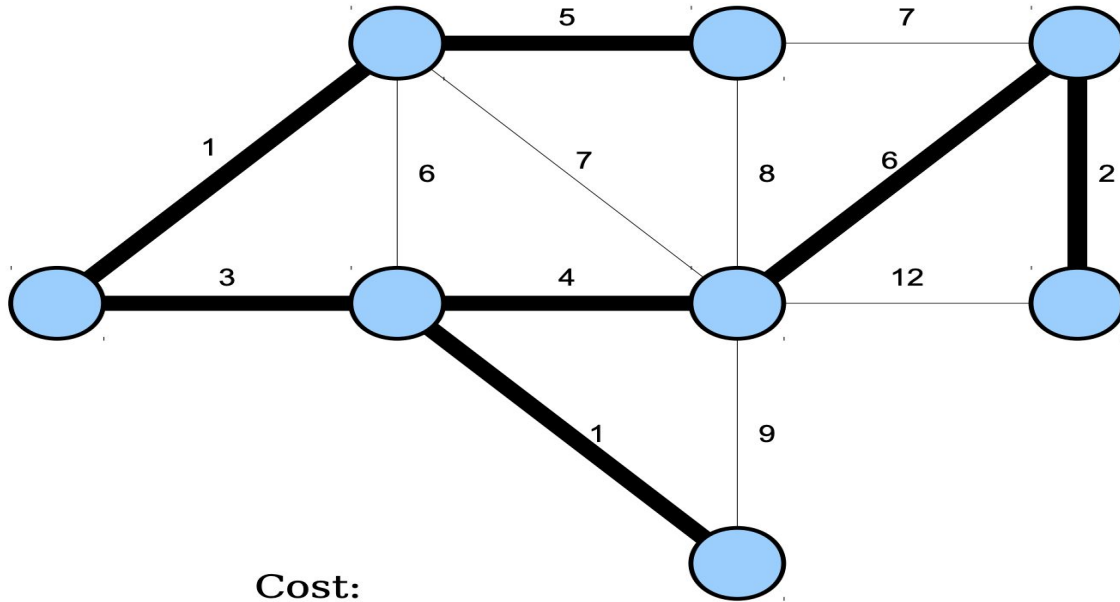
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- Some famous examples include:

More Graph Algorithms

- There are many, many different graph algorithms out there.
- Some famous examples include:
 - **BFS, Dijkstra's algorithm, and A* Search:** Find the shortest path between two nodes in a graph.

Kruskal's Algorithm

Find a minimum spanning tree from a given graph.

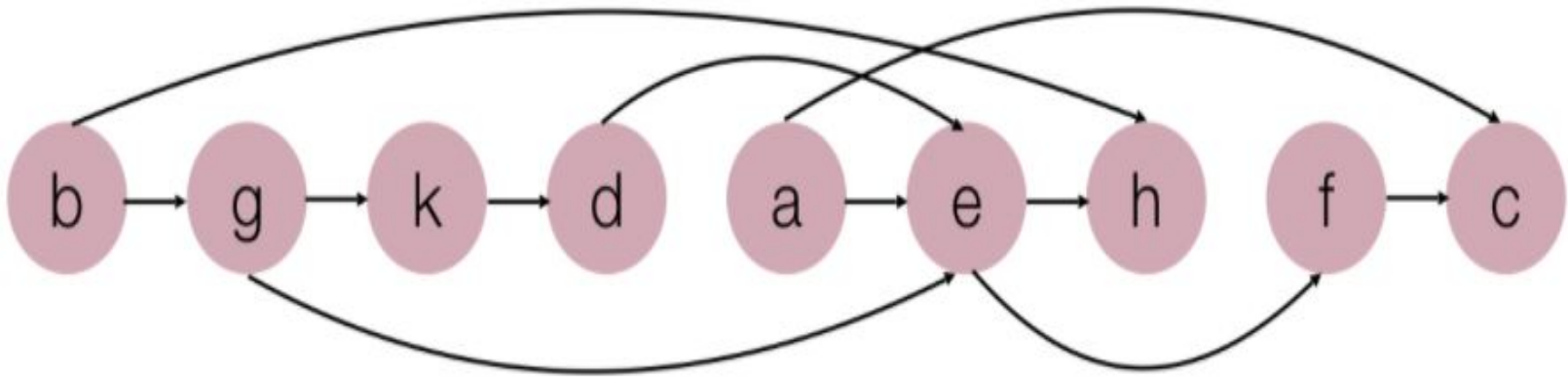


Cost:

$$1 + 3 + 5 + 4 + 1 + 6 + 2 = \mathbf{22}$$

Topological Sort

"Sort" the nodes in a dependency graph in such a way that traversing the nodes in order results in all dependencies being fulfilled at each point in time.



Traveling Salesman Algorithm

Given a map of cities and the distances between them, find the shortest path that traverses all cities in the map.



More Graph Algorithms

- There are many, many different graph algorithms out there.
- Some famous examples include:
 - **BFS, Dijkstra's algorithm, and A* Search:** Find the shortest path between two nodes in a graph.
 - **Kruskal's Algorithm:** Find a minimum spanning tree from a given graph.
 - **Topological Sort:** "Sort" the nodes in a dependency graph in such a way that traversing the nodes in order results in all dependencies being fulfilled at each point in time.
 - **Traveling salesman:** Given a map of cities and the distances between them, find the shortest path that traverses all cities in the map.
- Graphs can also be used in conjunction with machine learning algorithms to accomplish cool things.

Summary

Graphs Summary

- Graphs are the most powerful and flexible manner for organizing data in a linked data structure, particularly when expressing complex patterns and relationships between different data entities.
- Graphs are composed of nodes connected by edges.
- Graphs can be directed, undirected, weighted, or unweighted.
- Graph algorithms can be used to find interesting properties of graphs. BFS, Dijkstra's Algorithm, and A* Search are three ways to find the shortest path between two nodes in a graph.

What's next?

Roadmap

C++ basics

User/client

vectors + grids

stacks + queues

sets + maps

Core
Tools

testing

algorithmic
analysis

**recursive
problem-solving**

Object-Oriented
Programming

Implementation

arrays

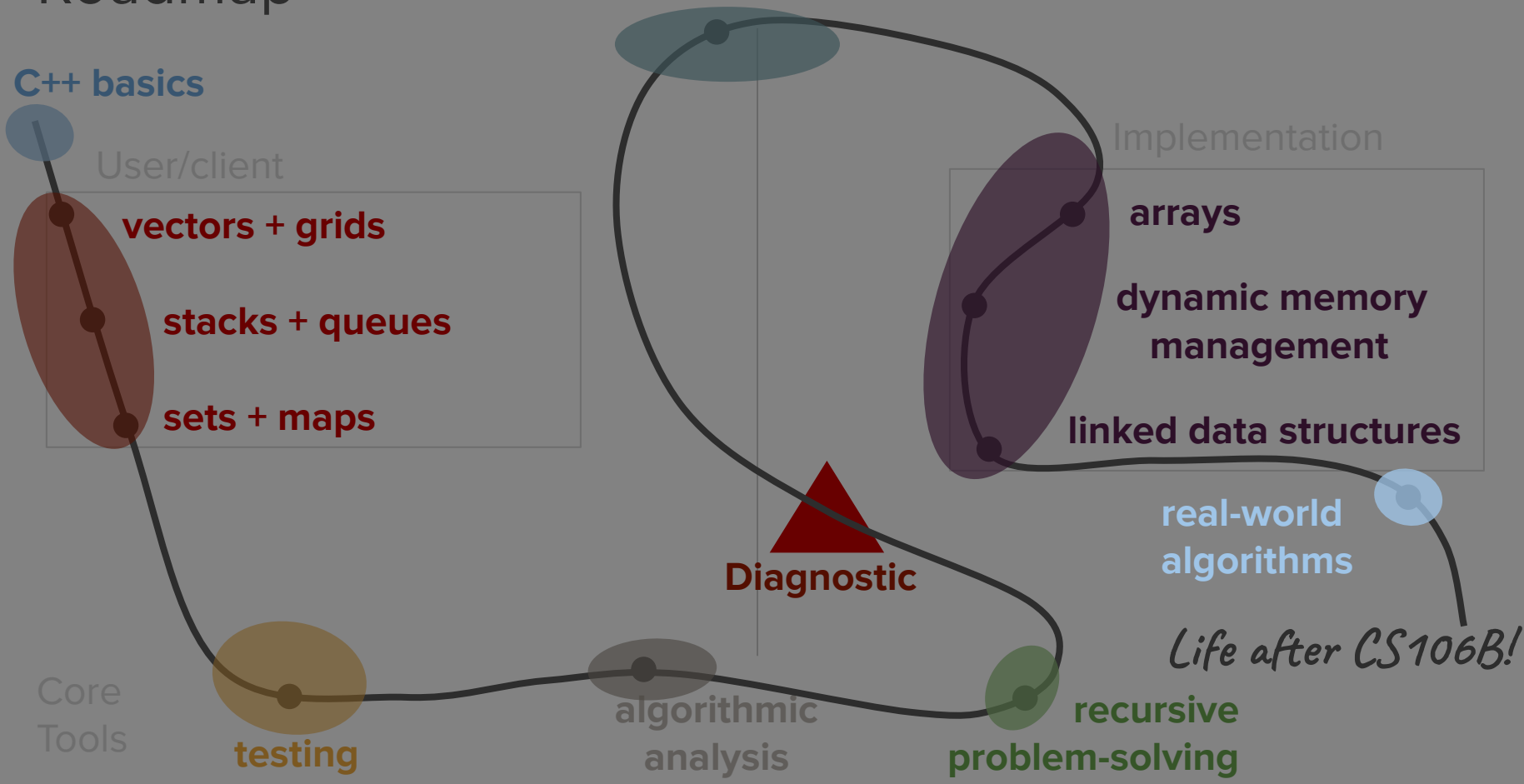
**dynamic memory
management**

linked data structures

**real-world
algorithms**

Life after CS106B!

Diagnostic



Multithreading and Parallel Computing

