

# Introduction to Recursion

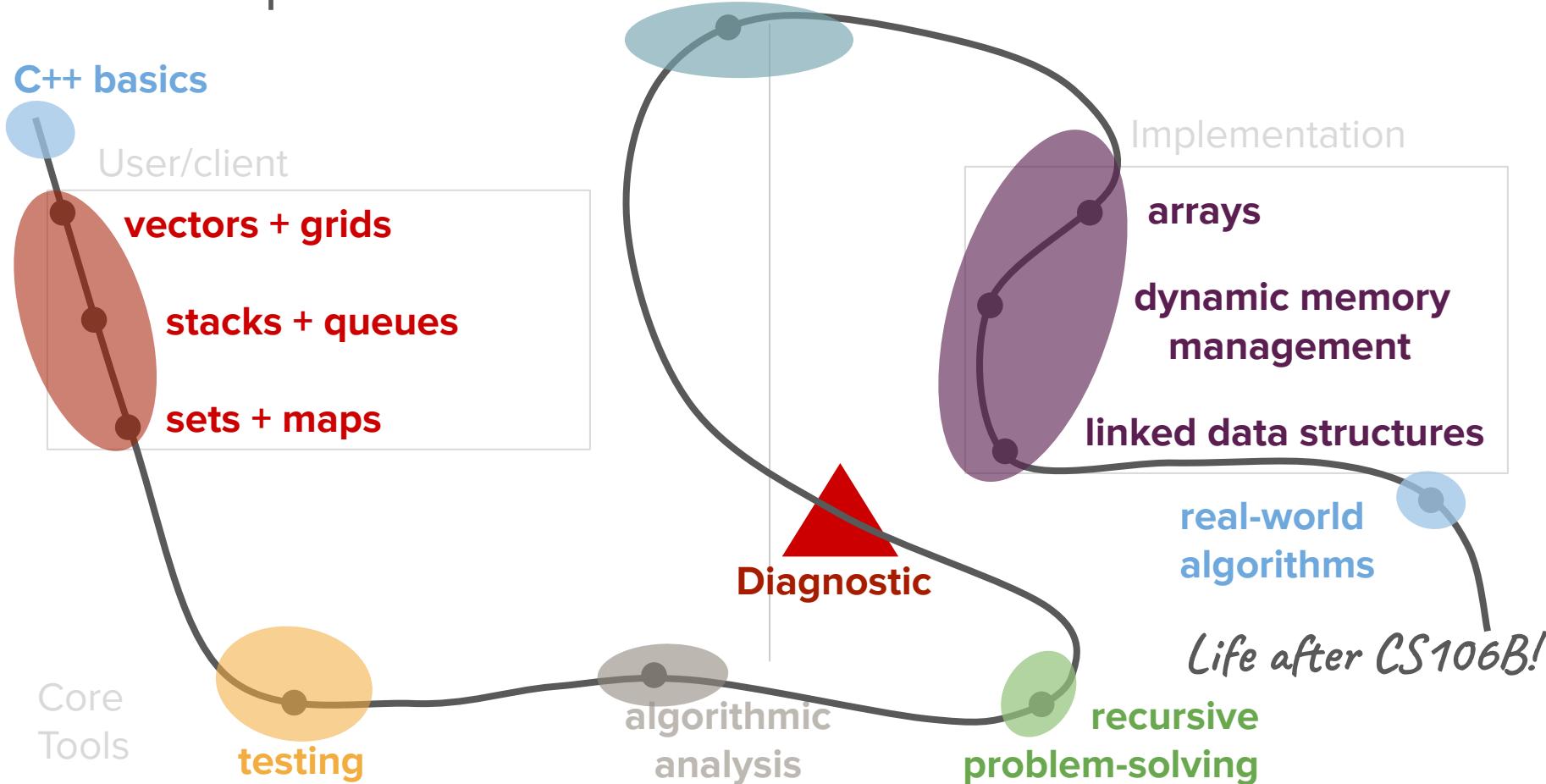
**What's been the most challenging part of  
Assignment 2 for you so far?  
(put your answers in the chat)**



# Roadmap

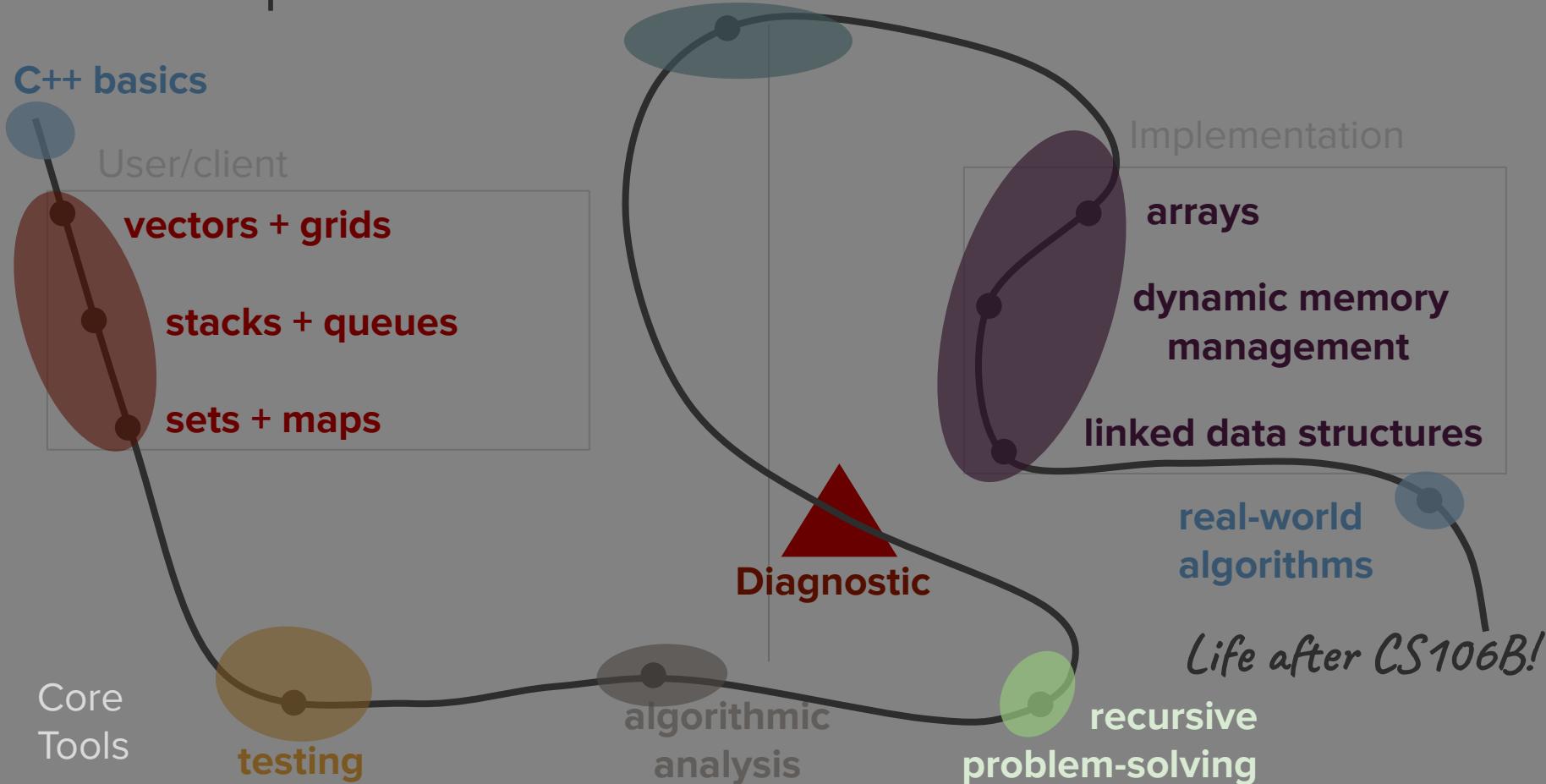
## Object-Oriented Programming

*Roadmap graphic courtesy of Nick Bowman & Kylie Jue*



# Roadmap

## Object-Oriented Programming



# Today's topics

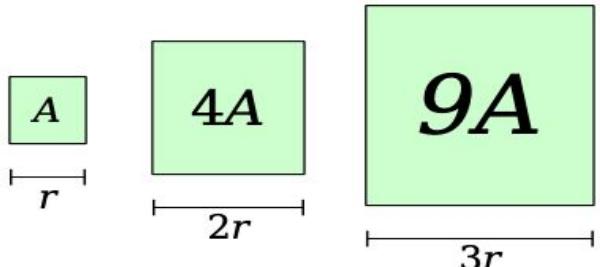
1. Review
2. Defining recursion
3. Recursion + Stack Frames  
(e.g. factorials)
4. Recursive Problem-Solving  
(e.g. string reversal)
5. Time permitting,  
introduction to Fractals

# Review

## Big O

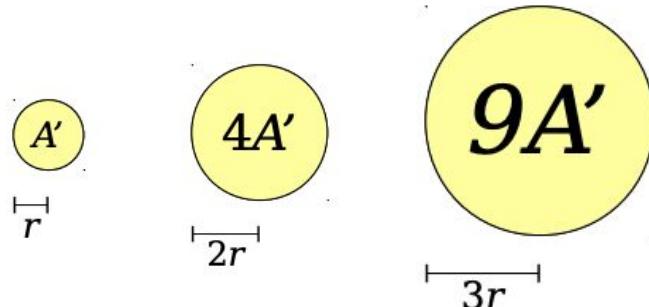
# Big-O Notation

- **Big-O notation** is a way of quantifying the **rate at which some quantity grows**.
- Example:
  - A square of side length  $r$  has area  $O(r^2)$ .
  - A circle of radius  $r$  has area  $O(r^2)$ .



*Doubling r increases area 4x*

*Tripling r increases area 9x*



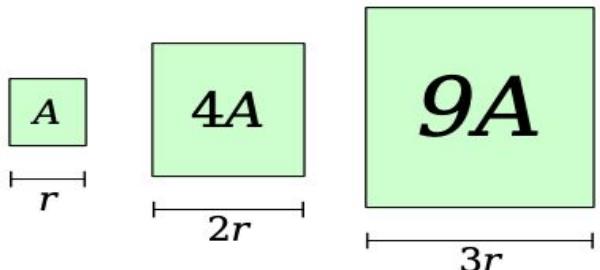
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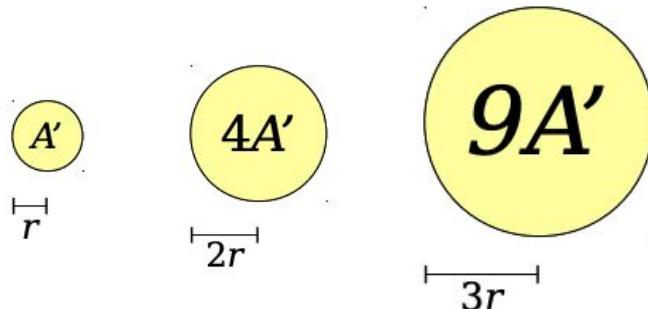
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- Example:
  - A square of side length  $r$  has area  $O(r^2)$ .
  - A circle of radius  $r$  has area  $O(r^2)$ .

*This just says that these quantities grow at the same relative rates. It does not say that they're equal!*



*Doubling  $r$  increases area 4x*

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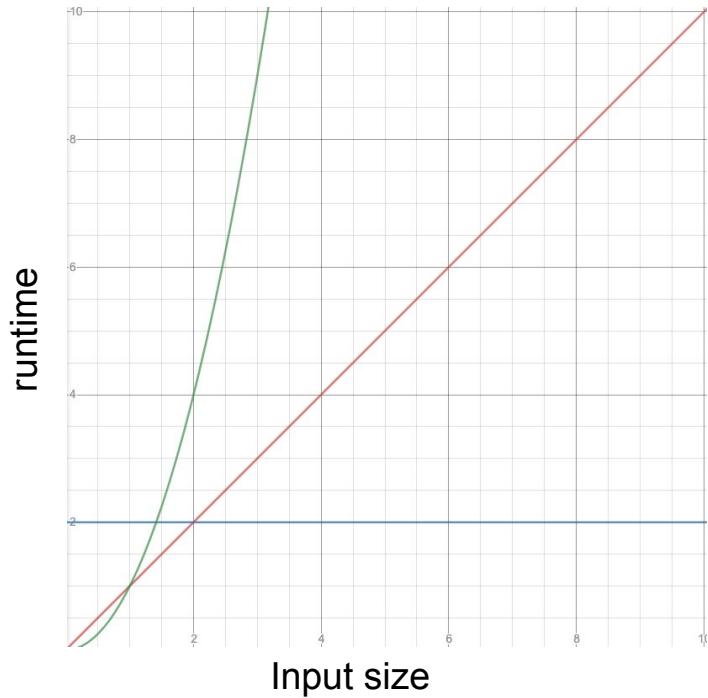


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# Efficiency Categorizations So Far

- Constant Time –  $O(1)$ 
  - Super fast, this is the best we can hope for!
  - example: Euclid's Algorithm for Perfect Numbers
- Linear Time –  $O(n)$ 
  - This is okay; we can live with this
- Quadratic Time –  $O(n^2)$ 
  - This can start to slow down really quickly
  - example: Exhaustive Search for Perfect Numbers



# ADT Big-O Matrix

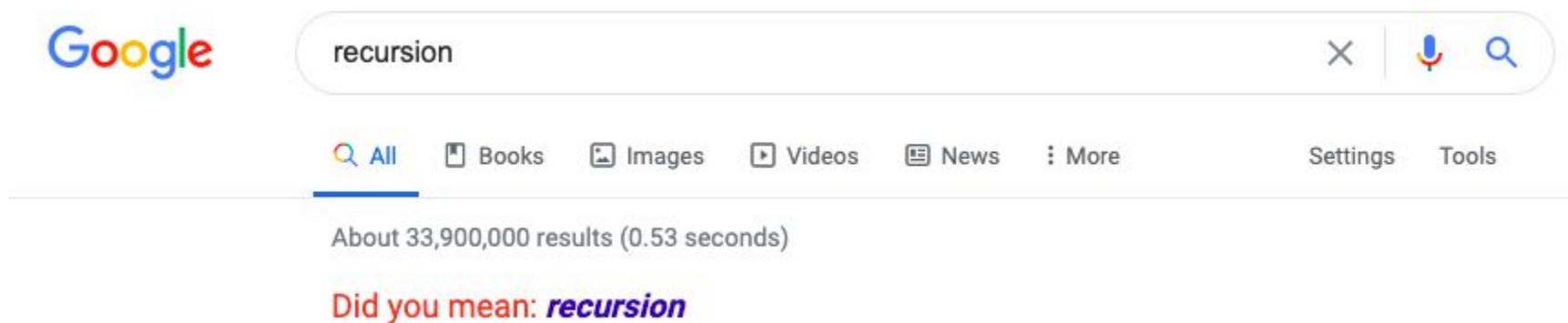
- Vectors
  - `.size()` -  $O(1)$
  - `.add()` -  $O(1)$
  - `v[i]` -  $O(1)$
  - `.insert()` -  $O(n)$
  - `.remove()` -  $O(n)$
  - `.clear()` -  $O(n)$
  - `traversal` -  $O(n)$  }
- Grids
  - `. numRows() / . numCols()` -  $O(1)$
  - `g[i][j]` -  $O(1)$
  - `.inBounds()` -  $O(1)$
  - `traversal` -  $O(n^2)$
- Queues
  - `.size()` -  $O(1)$
  - `.peek()` -  $O(1)$
  - `.enqueue()` -  $O(1)$
  - `.dequeue()` -  $O(1)$
  - `.isEmpty()` -  $O(1)$
  - `traversal` -  $O(n)$
- Stacks
  - `.size()` -  $O(1)$
  - `.peek()` -  $O(1)$
  - `.push()` -  $O(1)$
  - `.pop()` -  $O(1)$
  - `.isEmpty()` -  $O(1)$
  - `traversal` -  $O(n)$
- Sets
  - `.size()` -  $O(1)$
  - `.isEmpty()` -  $O(1)$
  - `.add()` - ???
  - `.remove()` - ???
  - `.contains()` - ???
  - `traversal` -  $O(n)$
- Maps
  - `.size()` -  $O(1)$
  - `.isEmpty()` -  $O(1)$
  - `m[key]` - ???
  - `.contains()` - ???
  - `traversal` -  $O(n)$

# What is recursion?



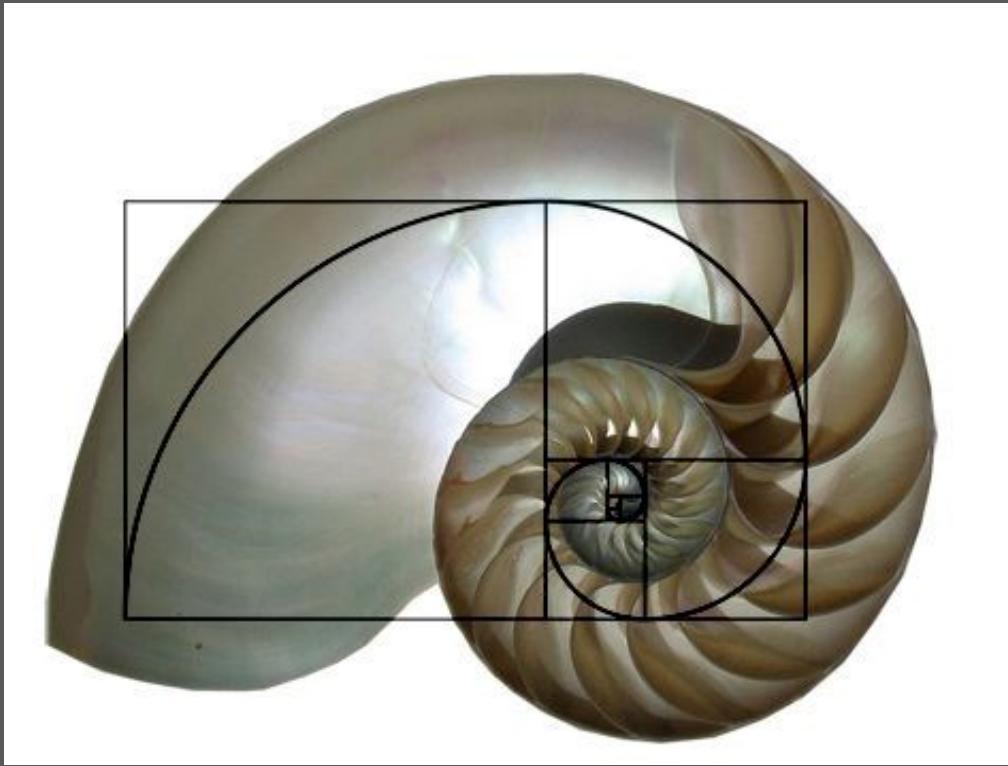
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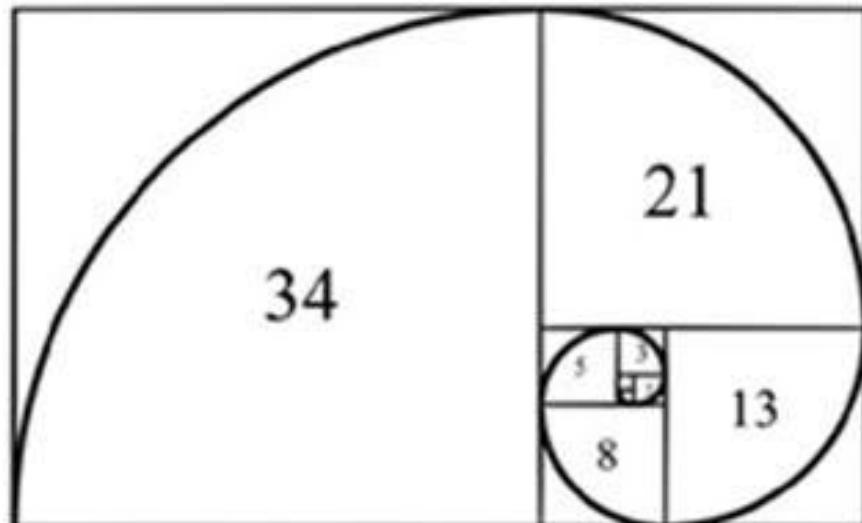
Wikipedia: “Recursion occurs when a thing is defined in terms of itself.”



A screenshot of a Google search results page. The search bar at the top contains the query "recursion". Below the search bar, the "All" tab is selected, followed by "Books", "Images", "Videos", "News", and "More". To the right of these tabs are "Settings" and "Tools" buttons. A message "About 33,900,000 results (0.53 seconds)" is displayed. Below this, a red "Did you mean" message suggests the misspelled term "recursion".







0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144...

$$0 + 1 = 1$$

$$1 + 1 = 2$$

$$2 + 1 = 3$$

$$3 + 2 = 5$$

$$5 + 3 = 8$$

$$8 + 5 = 13$$

$$13 + 8 = 21$$

$$21 + 13 = 34$$

$$34 + 21 = 55$$

$$55 + 34 = 89$$

$$89 + 55 = 144$$



# Today's question

How can we take  
advantage of self-similarity  
within a problem to solve it  
more elegantly?

## *Definition*

### **recursion**

A problem-solving technique in which tasks are completed by reducing them into repeated, smaller tasks of the same form.

# What is recursion?

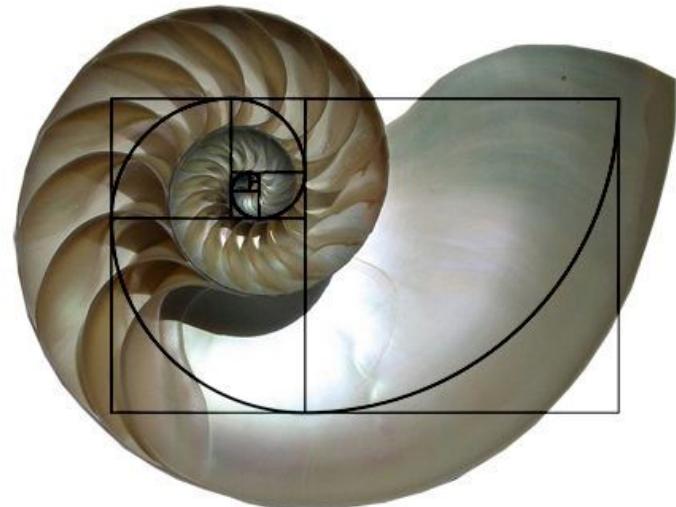
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- Results in elegant, often shorter code when
- Often applied to sorting and searching problems and can be used to express patterns seen in nature



# What is recursion?

- A powerful substitute for iteration (loops)
  - We'll start off with seeing the difference between iterative vs. recursive solutions
  - Later in the week we'll see problems/tasks that can only be solved using recursion
- Results in elegant, often shorter code when used well
- Often applied to sorting and searching problems and can be used to express patterns seen in nature
- Will be part of many of our future assignments!

# How many students are in a lecture hall?

a [non-Zoom] analogy

# How many students are in the lecture hall?

- Let's suppose I want to find out how many people are at lecture today, but I don't want to walk around and count each person.
- I want to recruit your help, but I also want to minimize each individual's amount of work.

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*We can solve this problem recursively!*

# How many students are in the lecture hall?

- We'll focus on solving the problem for single “column” of students.



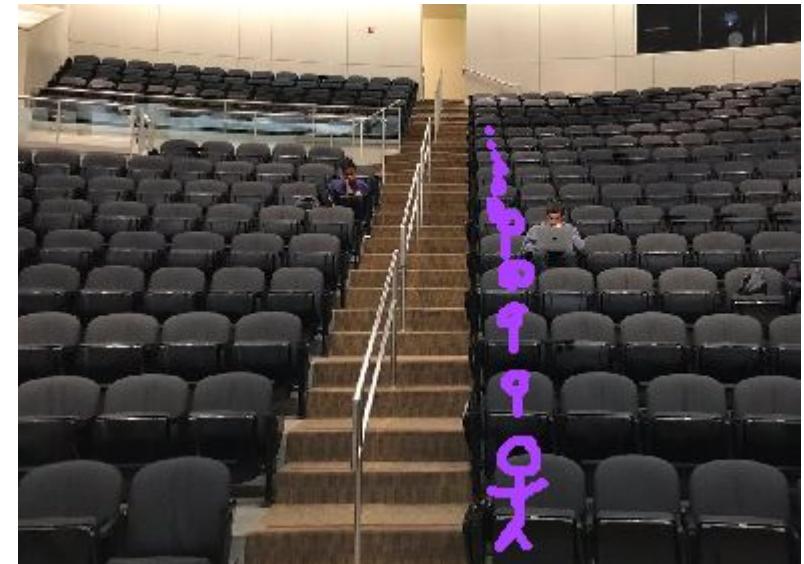
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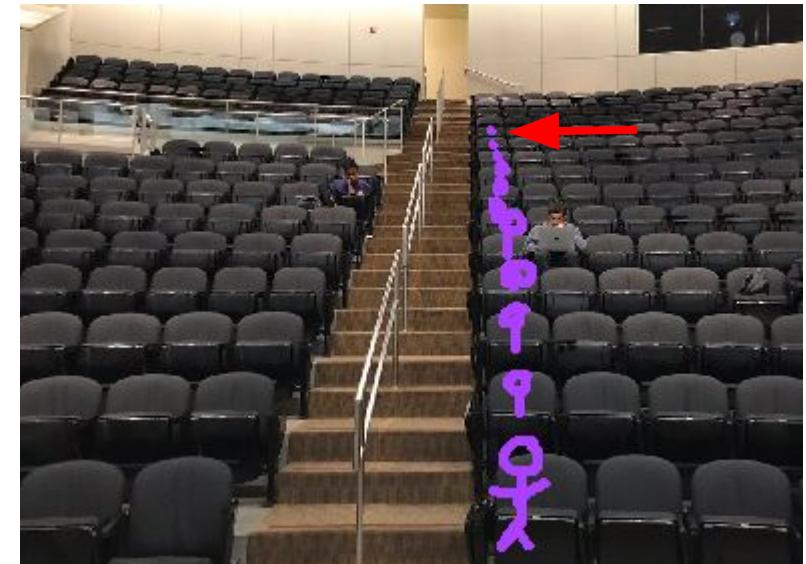
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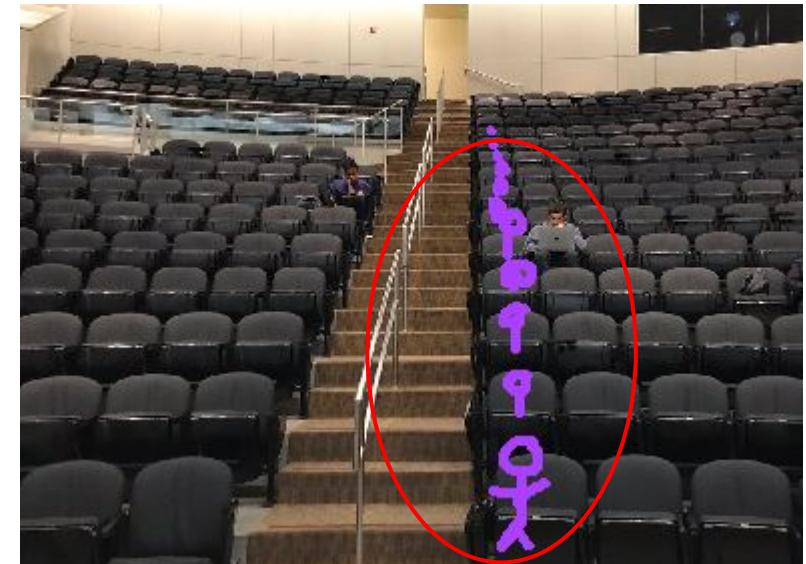
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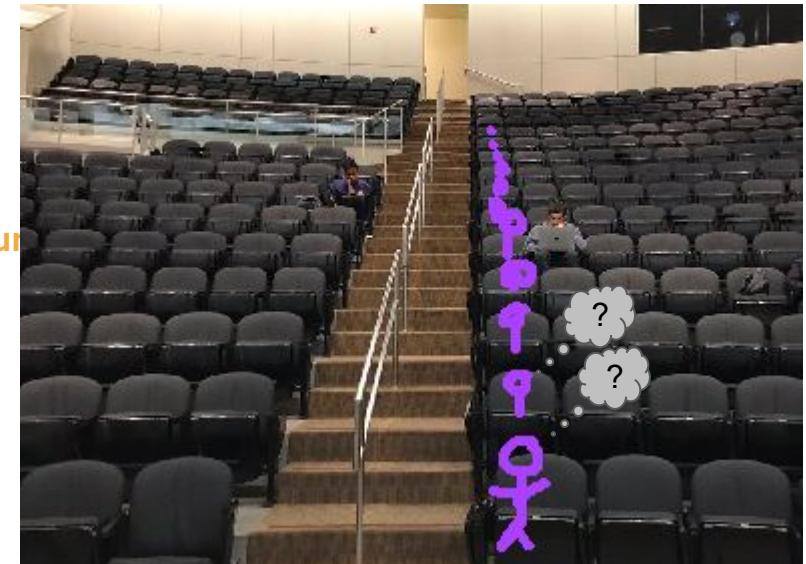
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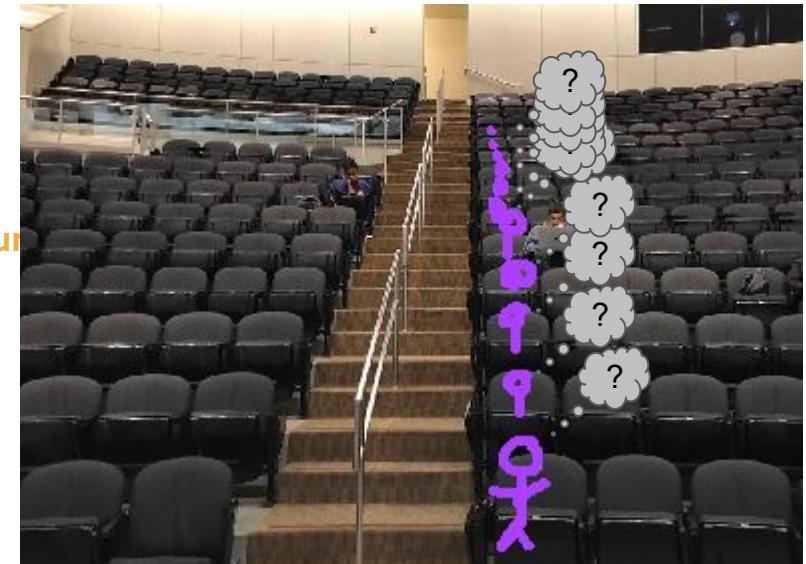
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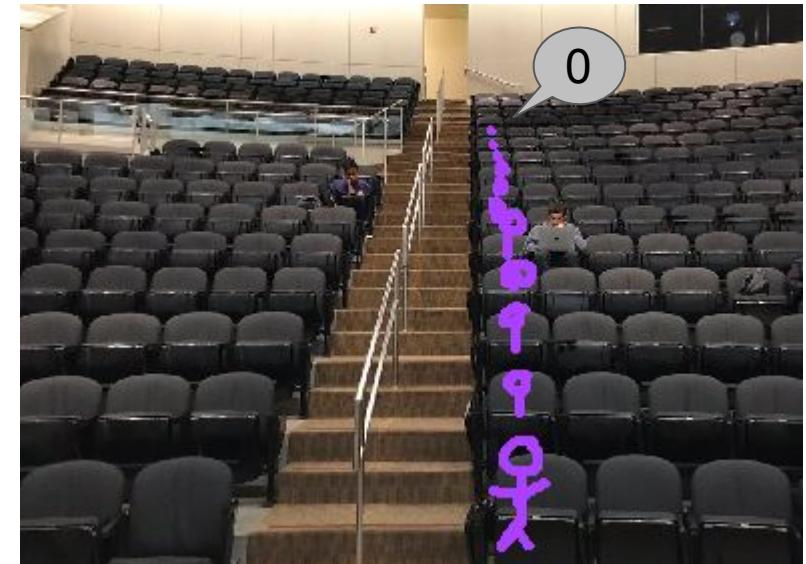
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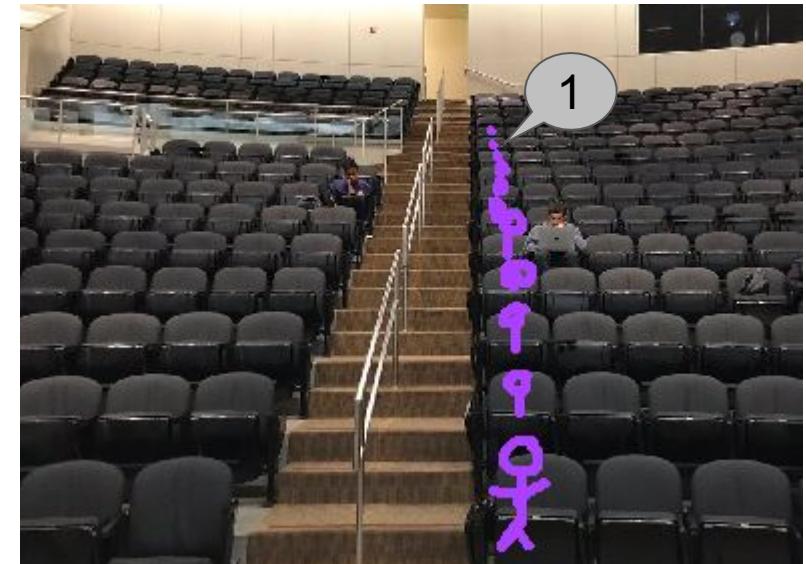
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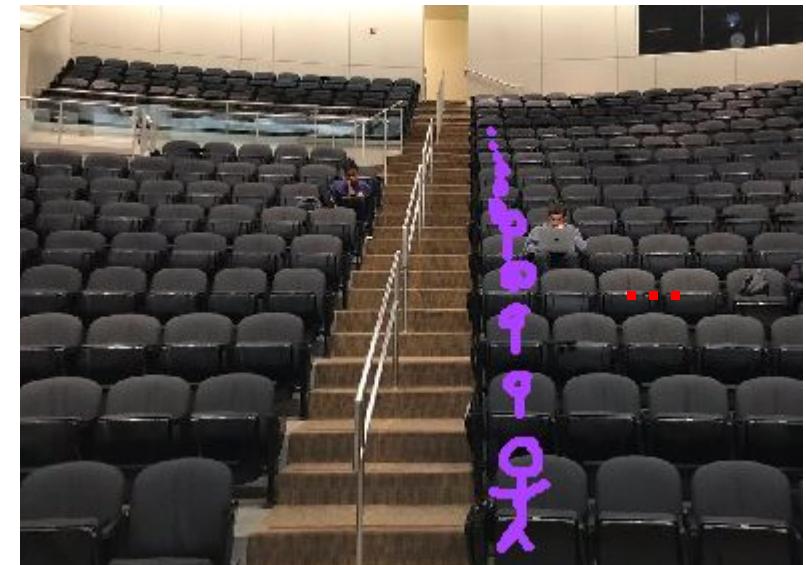
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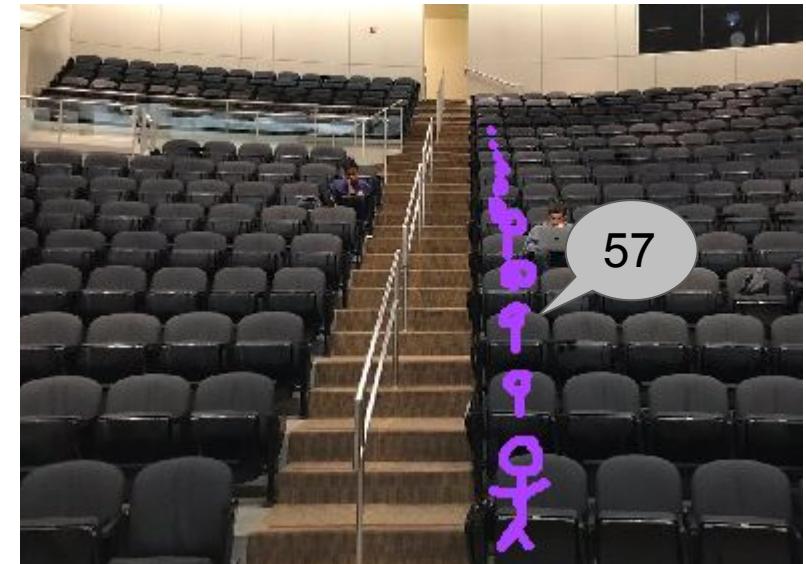
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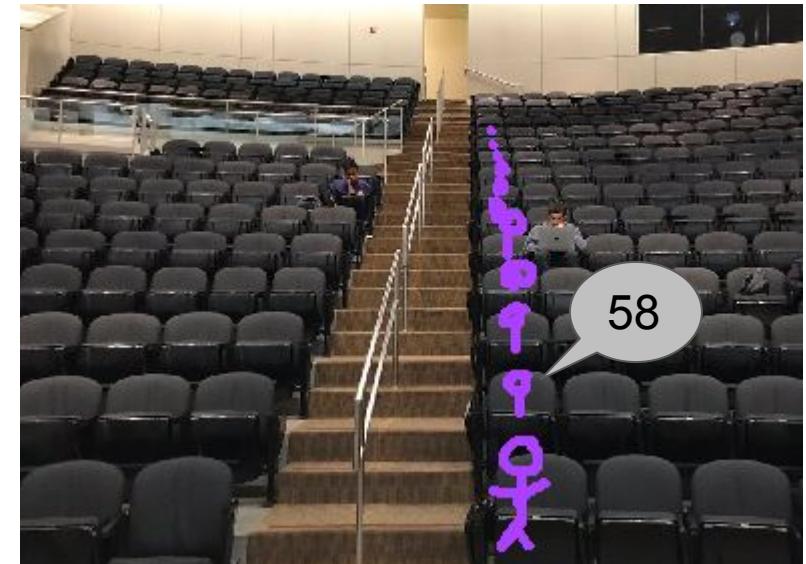
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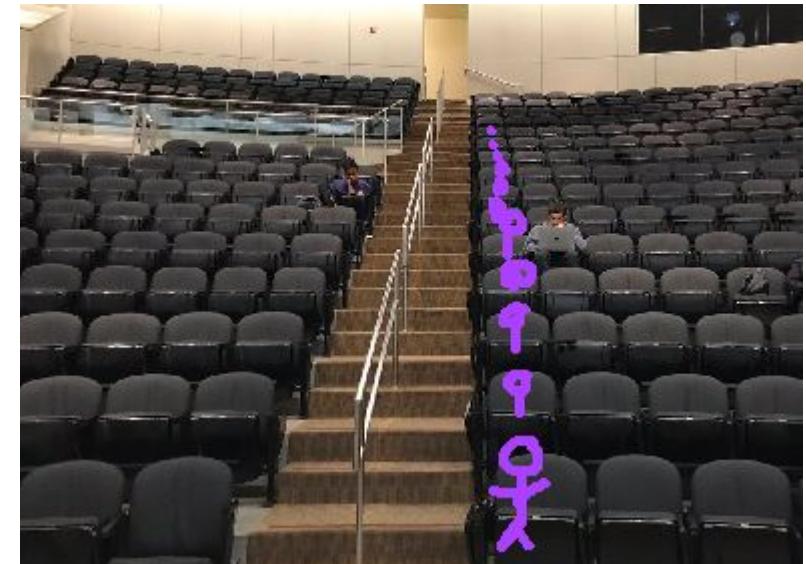
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- Can generalize to the entire lecture hall!



## *Definition*

### **recursion**

A problem-solving technique in which tasks are completed by reducing them into repeated, smaller tasks of the same form.

# Two main cases (components) of recursion

- Base case
  - The simplest version(s) of your problem that all other cases reduce to
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*“If there is no one behind me, answer 0.”*

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*“If someone is sitting behind me...”*

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# Factorial example

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- The number **n factorial**, denoted **n!**, is

$$n \times (n - 1) \times \dots \times 3 \times 2 \times 1$$

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- For example,
  - $3! = 3 \times 2 \times 1 = 6.$
  - $4! = 4 \times 3 \times 2 \times 1 = 24.$
  - $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120.$
  - $0! = 1.$  (by definition)

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- Factorials show up in unexpected places. We'll see one later this quarter when we talk about sorting algorithms.
- Let's implement a function to compute factorials!

# Computing factorials

$$5! = 5 \times 4 \times 3 \times 2 \times 1$$

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## Computing factorials

$$5! = 5 \times 4 \times 3 \times 2 \times 1$$

$$4!$$

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$$1! = 1 \times 0!$$

## Computing factorials

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*By definition!*



## Another view of factorials

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ n \times (n - 1)! & \text{otherwise} \end{cases}$$

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```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```

# Recursion in action

```
int main() {  
    int n = factorial(5);  
    cout << "5! = " << n << endl;  
    return 0;  
}
```

# Recursion in action

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int main() {  
    int n = factorial(5);  
    cout << "5! = " << n << endl;  
    return 0;  
}
```



This is a “**stack frame**.” One gets created each time a function is called.

- The “stack” is where in your computer’s memory the information is stored.
- A “frame” stores all of the data (variables) for that particular function call.

# Recursion in action

```
int main() {  
    int n = factorial(5);  
    cout << "5! = " << n << endl;  
    return 0;  
}
```

# Recursion in action

```
int main() {  
  
    int factorial (int n) {  
        if (n == 0) {  
            return 1;  
        } else {  
            return n * factorial(n-1);  
        }  
    }  
}
```



When a function gets called, a new stack frame gets created.

# Recursion in action

```
int main() {  
  
    int factorial (int n) {  
        if (n == 0) {  
            return 1;  
        } else {  
            return n * factorial(n-1);  
        }  
    }  
}
```



n

# Recursion in action

```
int main() {  
  
    int factorial (int n) {  
        if (n == 0) {  
            return 1;  
        } else {  
            return n * factorial(n-1);  
        }  
    }  
}
```



# Recursion in action

```
int main() {  
  
    int factorial (int n) {  
        if (n == 0) {  
            return 1;  
        } else {  
            return n * factorial(n-1);  
        }  
    }  
}
```



n

# Recursion in action

```
int main() {  
  
    int factorial (int n) {  
        if (n == 0) {  
            return 1;  
        } else {  
            return n * factorial(n-1);  
        }  
    }  
}
```



n

5

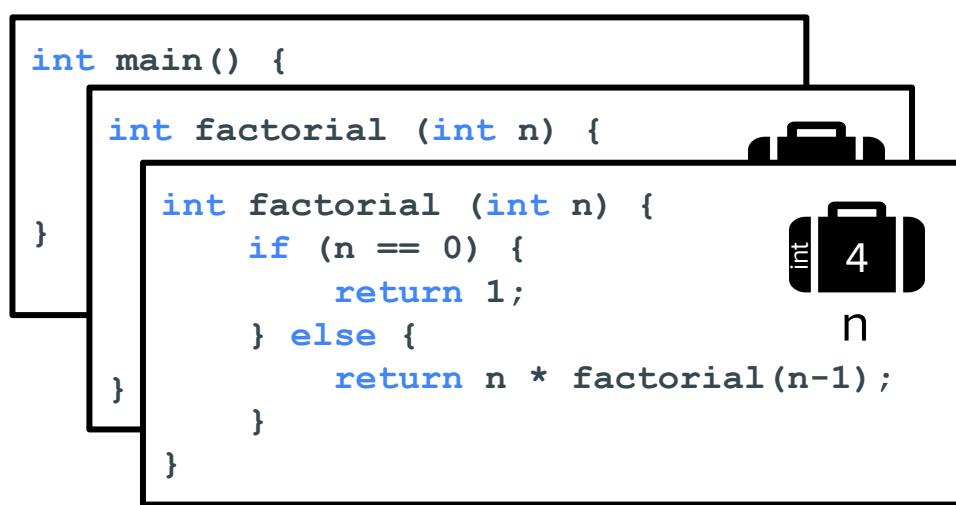
# Recursion in action

```
int main() {  
  
    int factorial (int n) {  
        if (n == 0) {  
            return 1;  
        } else {  
            return n * factorial(n-1);  
        }  
    }  
}
```



The diagram illustrates a recursive call to the factorial function. A black suitcase labeled "int 5" is shown above the variable "n" in the code, indicating the current argument value. The recursive call "factorial(n-1)" is highlighted with an orange box.

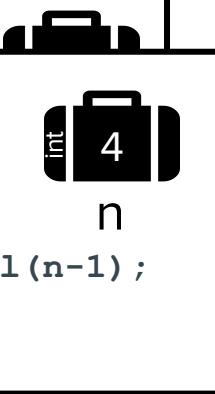
# Recursion in action



Every time we call **factorial()**, we get a new copy of the local variable **n** that's independent of all the previous copies because it exists inside the new frame.

# Recursion in action

```
int main() {  
    int factorial (int n) {  
        int factorial (int n) {  
            if (n == 0) {  
                return 1;  
            } else {  
                return n * factorial(n-1);  
            }  
        }  
    }  
}
```



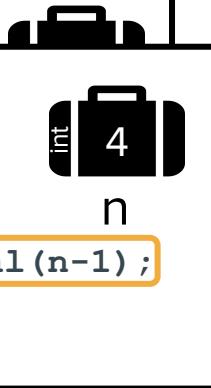
# Recursion in action

```
int main() {  
    int factorial (int n) {  
        int factorial (int n) {  
            if (n == 0) {  
                return 1;  
            } else {  
                return n * factorial(n-1);  
            }  
        }  
    }  
}
```



# Recursion in action

```
int main() {  
    int factorial (int n) {  
        int factorial (int n) {  
            if (n == 0) {  
                return 1;  
            } else {  
                return n * factorial(n-1);  
            }  
        }  
    }  
}
```



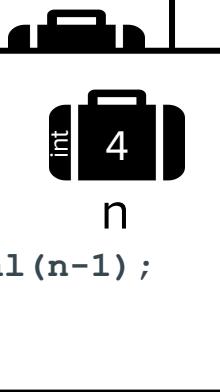
# Recursion in action

```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}
```



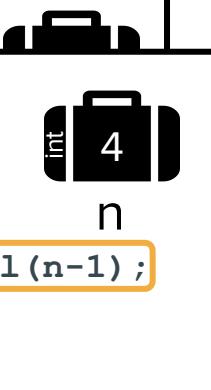
# Recursion in action

```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}
```



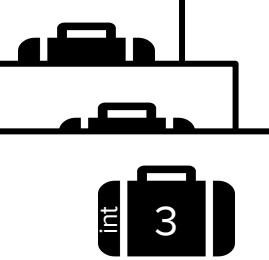
# Recursion in action

```
int main() {  
    int factorial (int n) {  
        int factorial (int n) {  
            if (n == 0) {  
                return 1;  
            } else {  
                return n * factorial(n-1);  
            }  
        }  
    }  
}
```



# Recursion in action

```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}
```



# Recursion in action

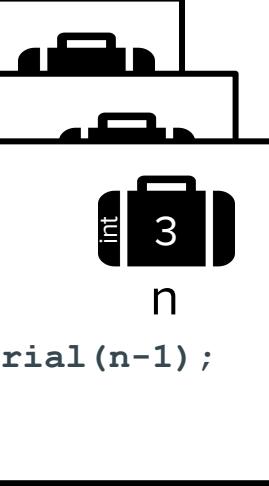
```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}
```



n

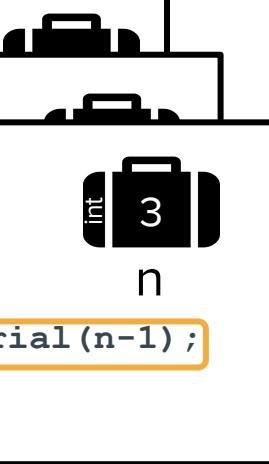
# Recursion in action

```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}
```



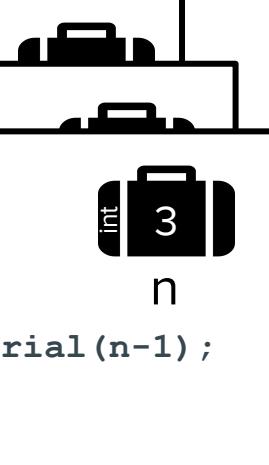
# Recursion in action

```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}
```



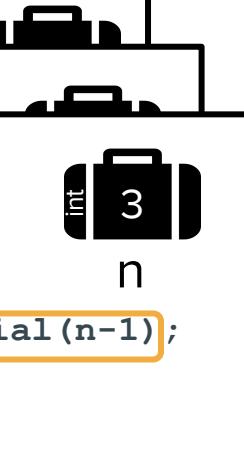
# Recursion in action

```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}
```



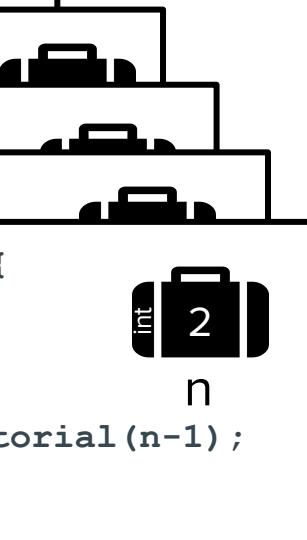
# Recursion in action

```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}
```



# Recursion in action

```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}
```



# Recursion in action

```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}
```

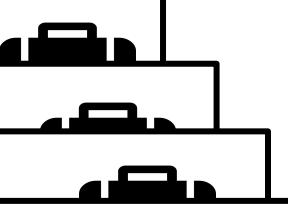
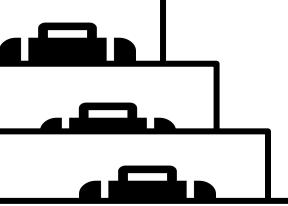


Diagram illustrating the execution stack for a recursive factorial function. The stack grows from bottom to top, with each frame containing the current value of  $n$ . The bottom frame ( $n=2$ ) has a yellow box around the `if (n == 0)` condition, indicating the current execution point.

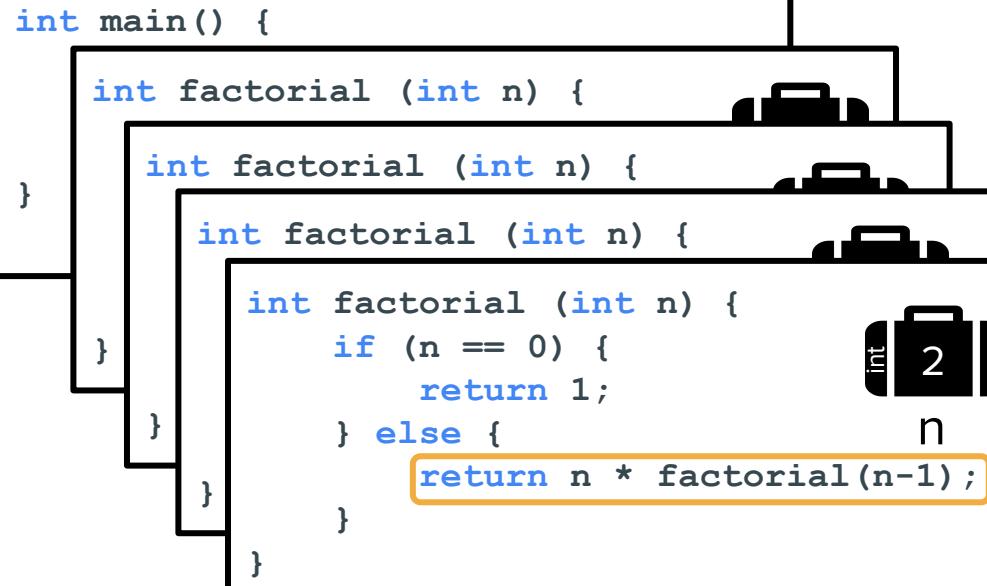
# Recursion in action

```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}
```



# Recursion in action

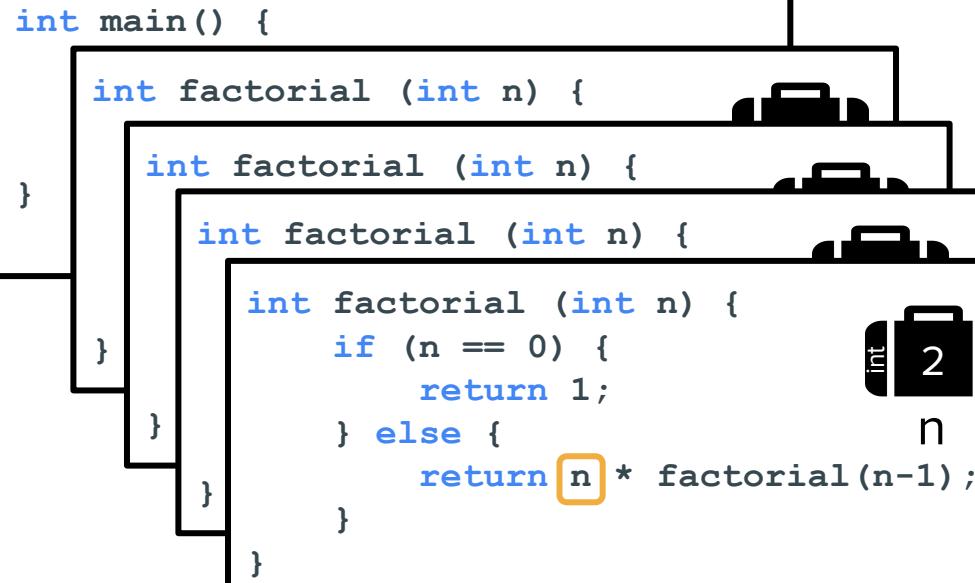
```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}
```



The diagram illustrates the execution stack for a recursive factorial function. The stack grows from bottom to top, with each frame containing the current value of  $n$ . The base case  $n=0$  is reached in the fourth frame, where the return value is 1. The recursive call  $n * \text{factorial}(n-1)$  is highlighted in orange in the fifth frame, with the arguments  $n=2$  and the return value 1 shown to its right.

# Recursion in action

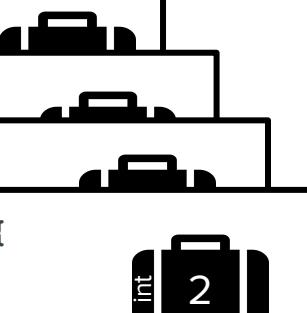
```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}
```



The diagram illustrates the recursive call `n * factorial(n-1)` in the factorial function. The variable `n` is represented by a black suitcase labeled `n`, and the value `2` is represented by a black suitcase labeled `int 2`. The recursive call is highlighted with a yellow box around the `n` in `n * factorial(n-1)`.

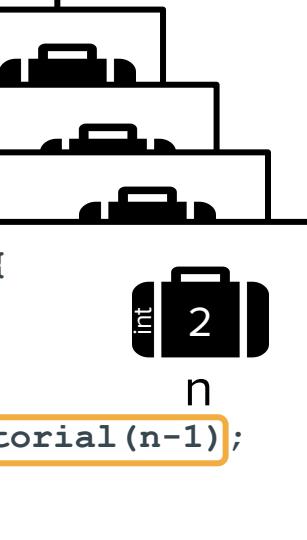
# Recursion in action

```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}
```



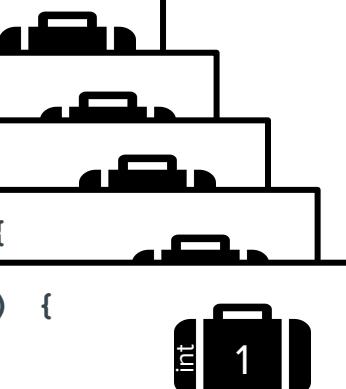
# Recursion in action

```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}
```



# Recursion in action

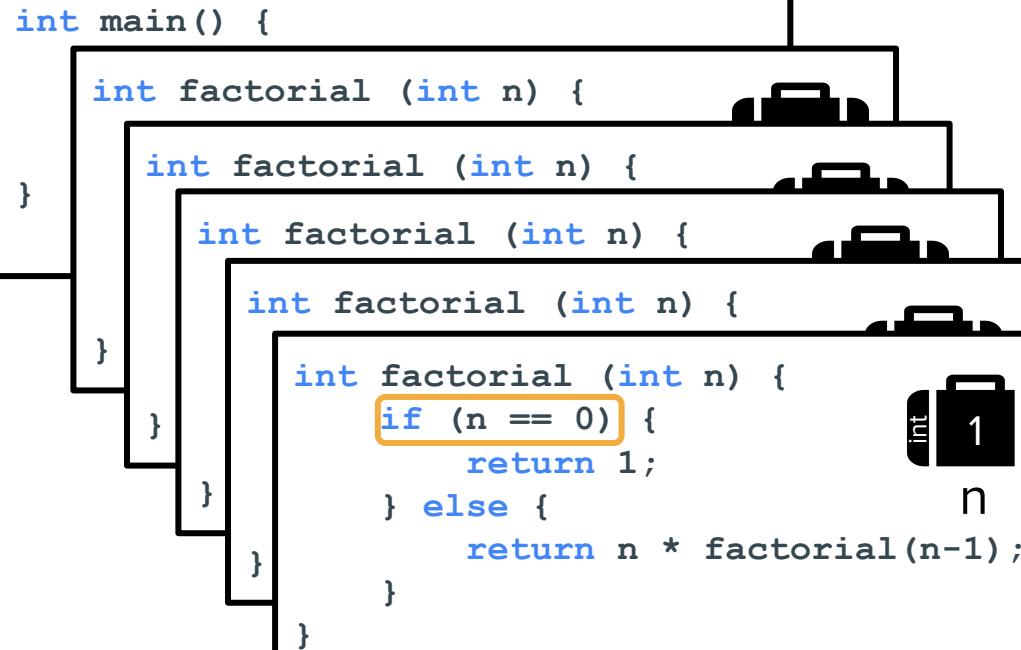
```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}
```



The diagram shows a stack of five nested function frames for the factorial function. The bottom frame (n=5) is the active call, with a return value of 120. The stack frames are represented as suitcases.

# Recursion in action

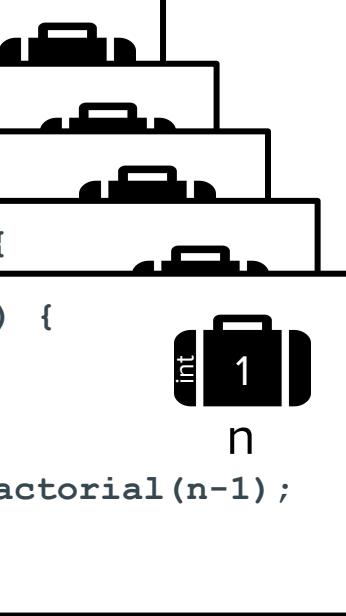
```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}
```



The diagram shows a vertical stack of rectangular frames, each representing a recursive call to the `factorial` function. The frames are arranged from bottom to top, corresponding to increasing values of `n`. The bottom-most frame is the initial call from `main`. Each subsequent frame is nested within the previous one. The `n` parameter is shown in the bottom right corner of each frame. The base case, where `n` is 0, is highlighted with an orange box around the `if (n == 0)` condition. The `return 1;` statement is also highlighted with an orange box. The recursive call `return n * factorial(n-1);` is shown in the frame below, indicating the function calls itself with a smaller value of `n`.

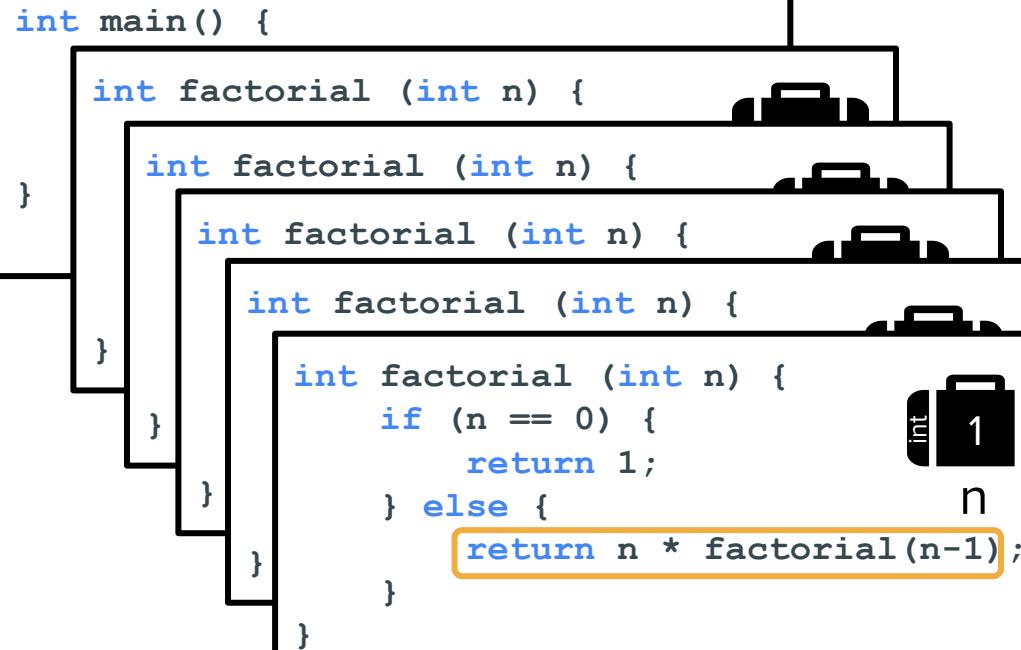
# Recursion in action

```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}
```



# Recursion in action

```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}
```



The diagram shows a stack of five frames, each representing a recursive call to the factorial function. The frames are arranged vertically, with the base case at the bottom and subsequent recursive calls above it. Each frame contains the current value of the parameter `n`. The base case `n == 0` is at the bottom, and the recursive call `n * factorial(n-1)` is highlighted in orange. The frames are represented by rectangular boxes with a black border, and the value of `n` is shown in a small black box to the right of each frame.

# Recursion in action

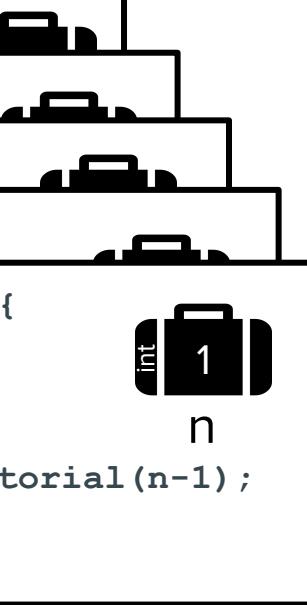
```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}
```



The diagram illustrates the recursive call `n * factorial(n-1)` in the factorial function. The variable `n` is highlighted with an orange box, indicating it is the current argument being passed to the recursive call. The recursive call is shown as a black suitcase icon, representing the function call stack.

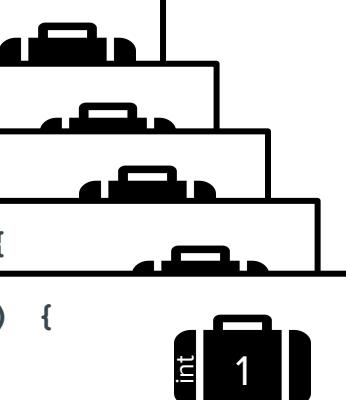
# Recursion in action

```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}
```



# Recursion in action

```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}
```

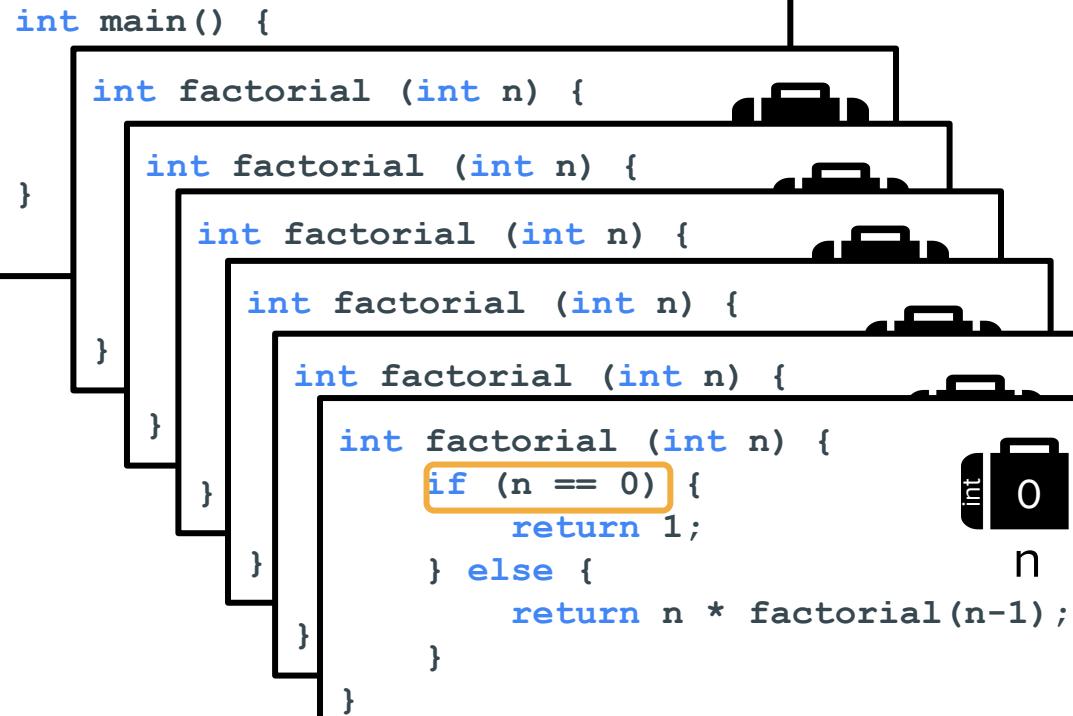


# Recursion in action

```
int main() {  
    int factorial (int n) {  
        int factorial (int n) {  
            int factorial (int n) {  
                int factorial (int n) {  
                    int factorial (int n) {  
                        if (n == 0) {  
                            return 1;  
                        } else {  
                            return n * factorial(n-1);  
                        }  
                    }  
                }  
            }  
        }  
    }  
}
```

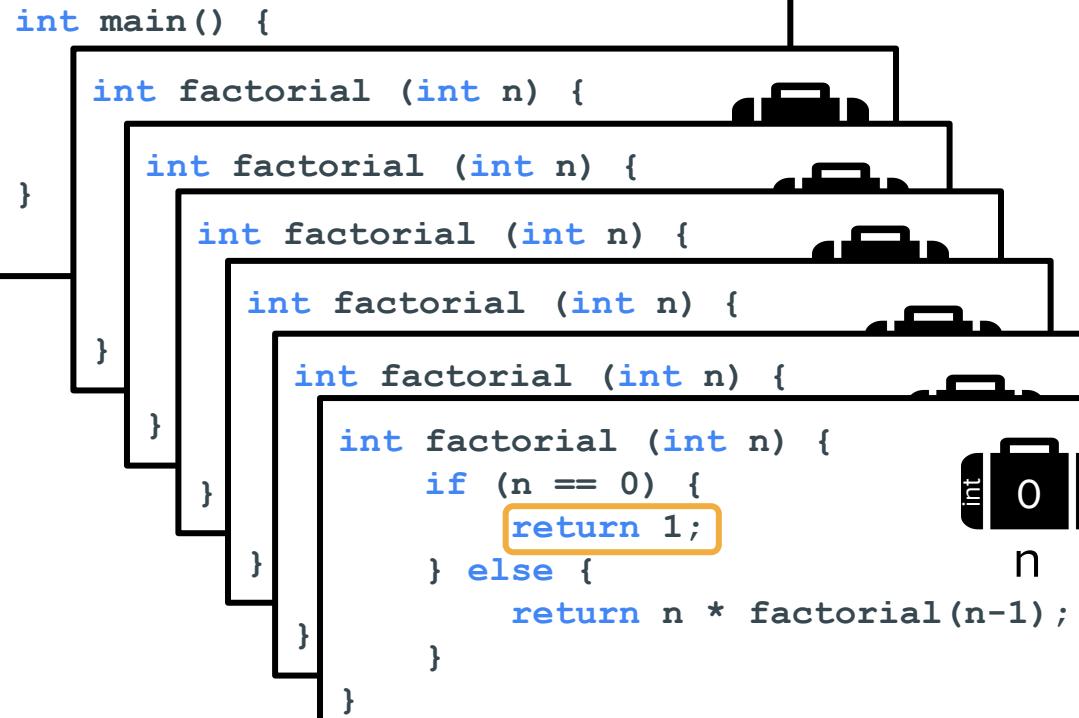
# Recursion in action

```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}
```



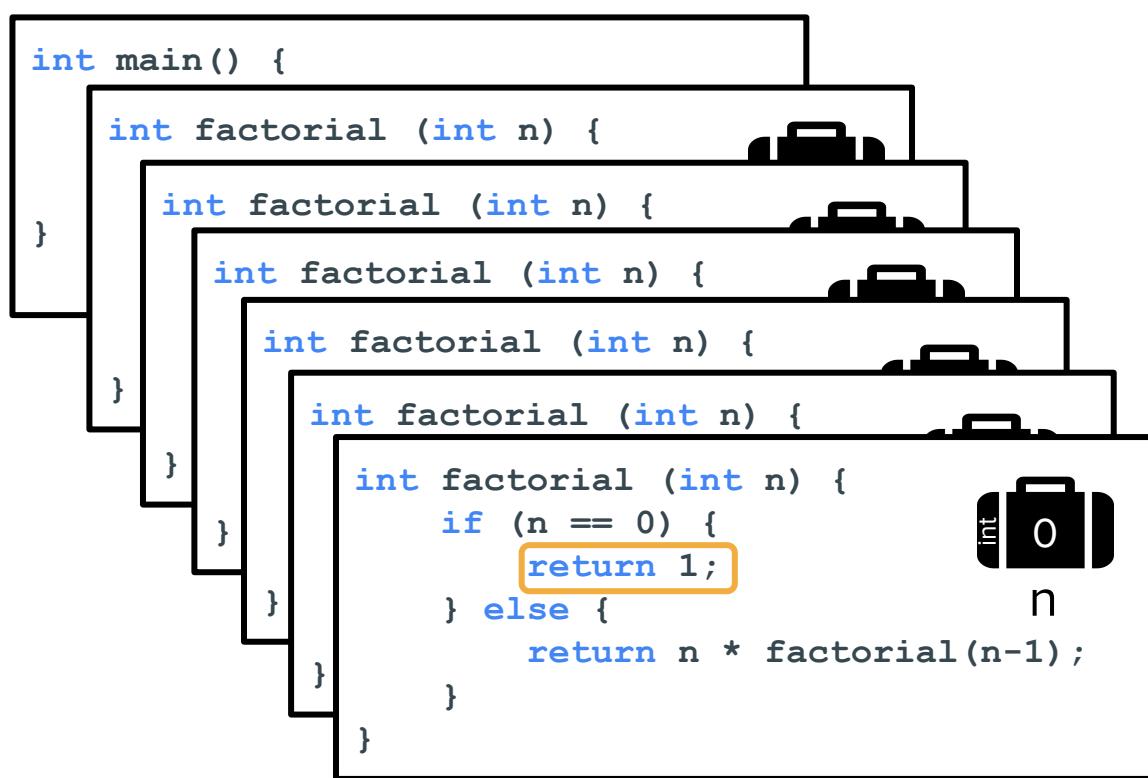
# Recursion in action

```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}
```



The diagram shows a stack of recursive function frames for a factorial function. The stack grows from bottom to top, with each frame containing the current value of  $n$ . The base case is highlighted with a yellow box around the `return 1;` statement.

# Recursion in action

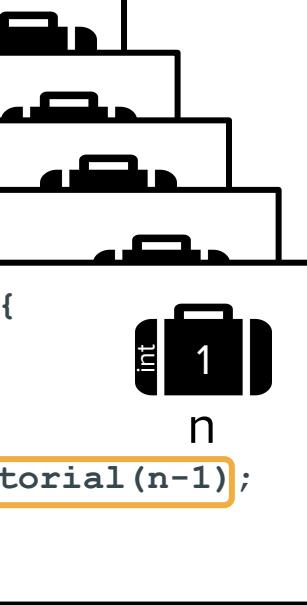


Stack frames go away (get cleared from memory) once they return.



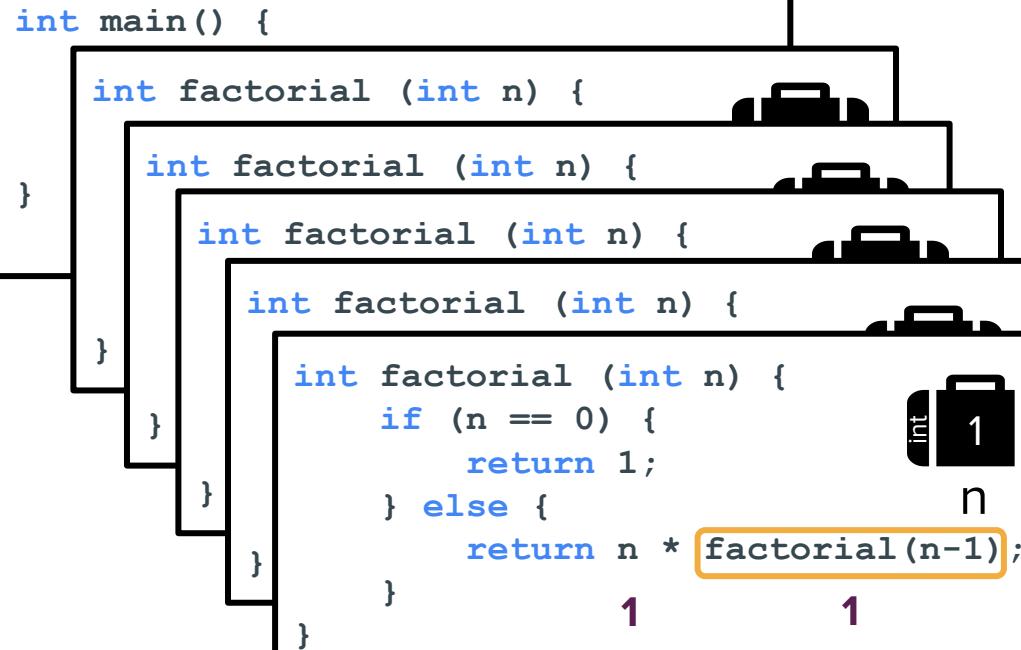
# Recursion in action

```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}
```



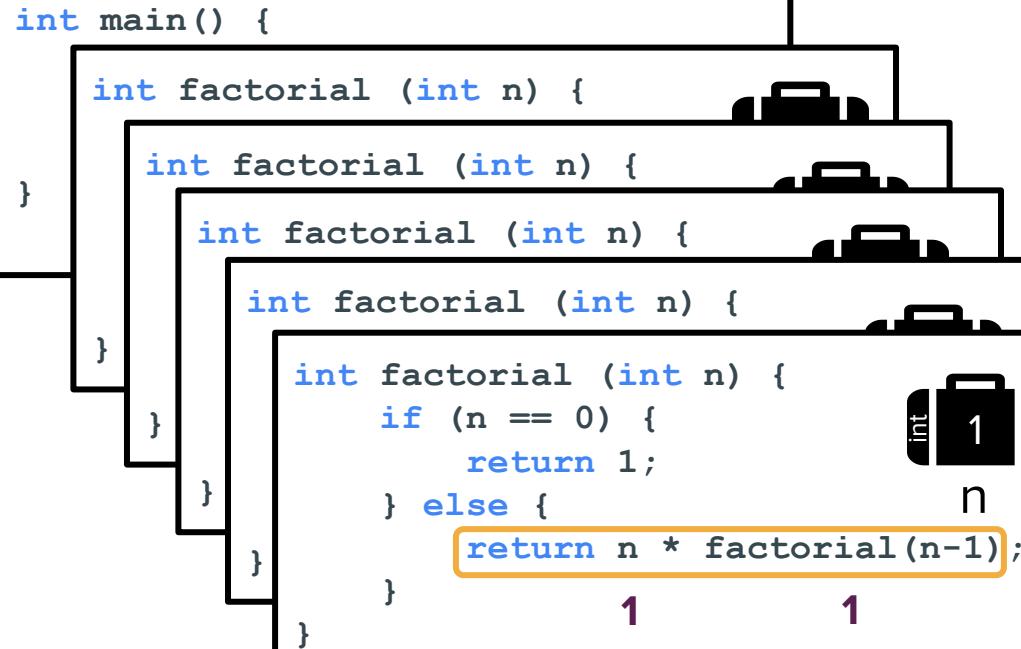
# Recursion in action

```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}
```



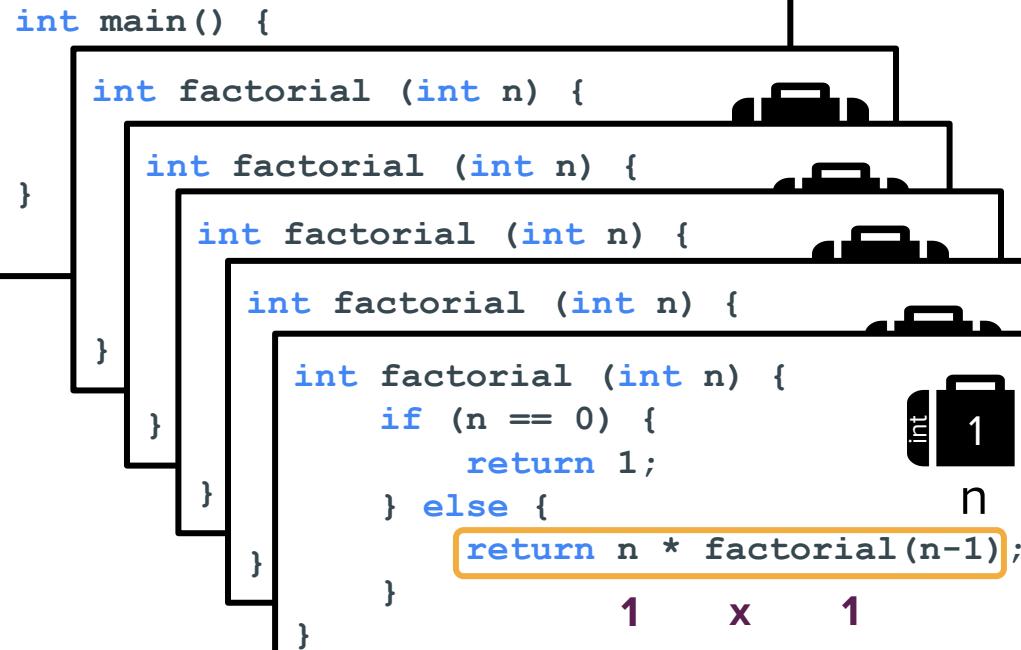
# Recursion in action

```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}
```



# Recursion in action

```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}  
}
```



# Recursion in action

```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}
```

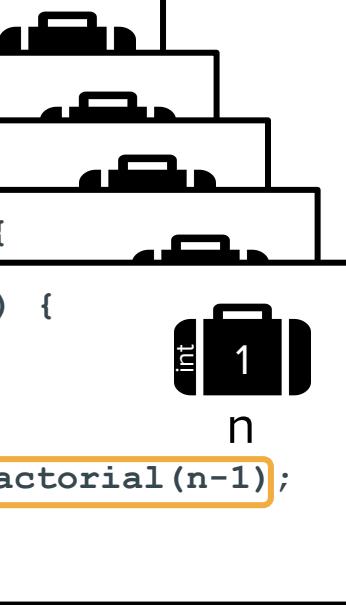
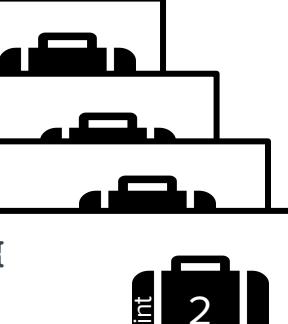


Diagram illustrating the recursive call stack for the factorial function. The stack has five levels, each representing a recursive call. The bottom level (n=5) is highlighted with a yellow box around the recursive call line. The stack grows from the bottom to the top, with each level having a suitcase icon above it. The variable 'n' is labeled with the value 5 at the bottom level and 1 at the top level. The variable '1' is labeled with the value 1 at both the bottom and top levels of the stack.

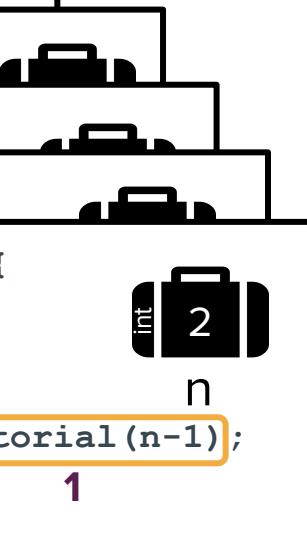
# Recursion in action

```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}
```



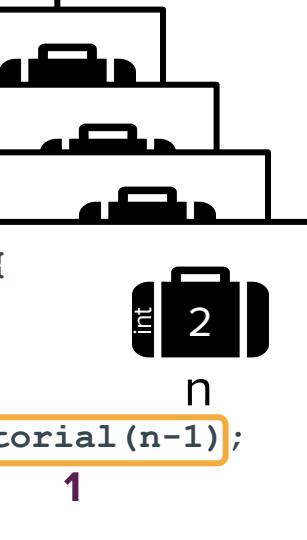
# Recursion in action

```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}
```



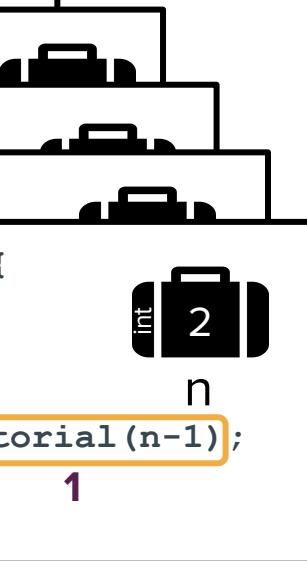
# Recursion in action

```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}
```



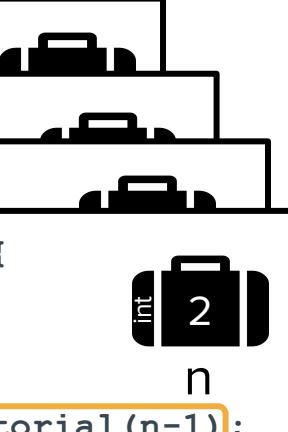
# Recursion in action

```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}
```



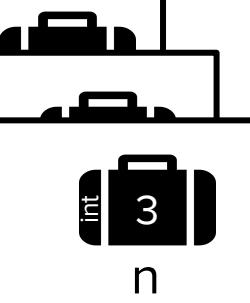
# Recursion in action

```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}
```



# Recursion in action

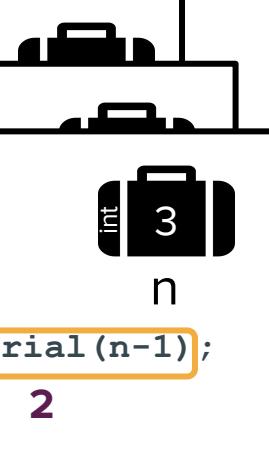
```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}
```



The diagram illustrates the execution of a recursive factorial function. It shows three nested function calls. The innermost call is for n=3. The recursive call `factorial(n-1)` is highlighted with a yellow box. To the right, three black suitcases are arranged vertically, with the top one labeled "int", the middle one labeled "3", and the bottom one labeled "n".

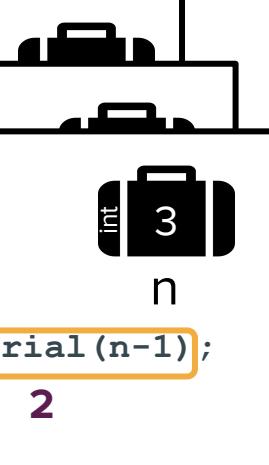
# Recursion in action

```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}  
3 2
```



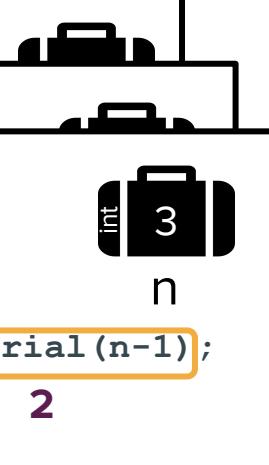
# Recursion in action

```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}  
3 2
```



# Recursion in action

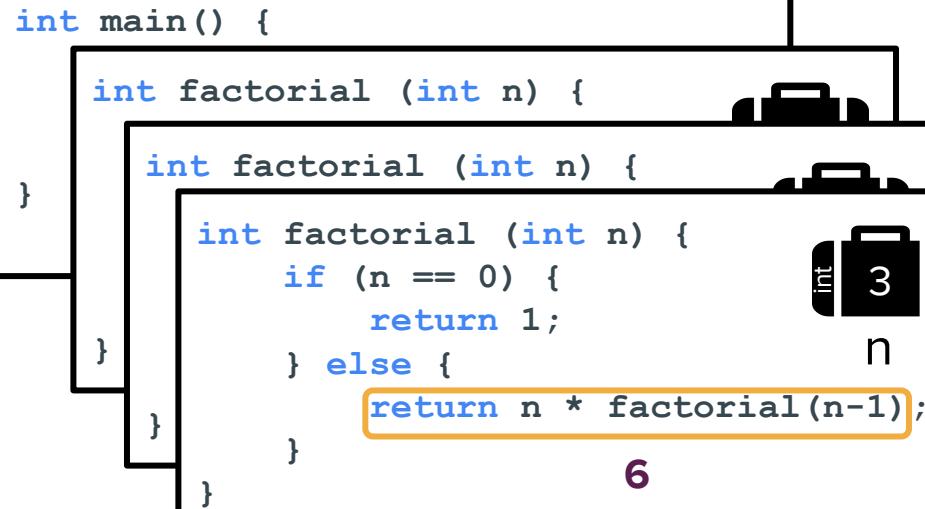
```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}  
3 x 2
```



# Recursion in action

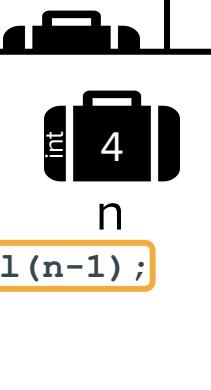
```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}
```

3  
n  
6



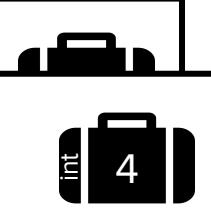
# Recursion in action

```
int main() {  
    int factorial (int n) {  
        int factorial (int n) {  
            if (n == 0) {  
                return 1;  
            } else {  
                return n * factorial(n-1);  
            }  
        }  
    }  
}
```



# Recursion in action

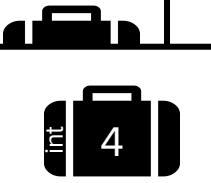
```
int main() {  
    int factorial (int n) {  
        int factorial (int n) {  
            if (n == 0) {  
                return 1;  
            } else {  
                return n * factorial(n-1);  
            }  
        }  
    }  
}
```



The diagram illustrates a recursive call in the factorial function. A call to `factorial(4)` is shown, with the argument `4` in a box and the variable `n` above it. The recursive call `factorial(n-1)` is highlighted with an orange box.

# Recursion in action

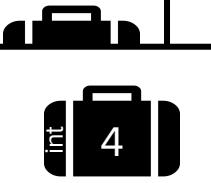
```
int main() {  
    int factorial (int n) {  
        int factorial (int n) {  
            if (n == 0) {  
                return 1;  
            } else {  
                return n * factorial(n-1);  
            }  
        }  
    }  
}
```



The diagram illustrates a recursive call in the factorial function. A call to `factorial(4)` is shown with a suitcase labeled `n` containing the value `4`. The recursive call `factorial(n-1)` is highlighted with an orange box.

# Recursion in action

```
int main() {  
    int factorial (int n) {  
        int factorial (int n) {  
            if (n == 0) {  
                return 1;  
            } else {  
                return n * factorial(n-1);  
            }  
        }  
    }  
}
```

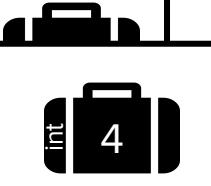


4    x    6

# Recursion in action

```
int main() {  
    int factorial (int n) {  
        int factorial (int n) {  
            if (n == 0) {  
                return 1;  
            } else {  
                return n * factorial(n-1);  
            }  
        }  
    }  
}
```

24



# Recursion in action

```
int main() {  
  
    int factorial (int n) {  
        if (n == 0) {  
            return 1;  
        } else {  
            return n * factorial(n-1);  
        }  
    }  
}
```



The diagram illustrates a recursive call to the factorial function. A black suitcase labeled "int 5" is shown above the variable "n" in the code, indicating the current argument value. The recursive call "factorial(n-1)" is highlighted with an orange box.

# Recursion in action

```
int main() {  
  
    int factorial (int n) {  
        if (n == 0) {  
            return 1;  
        } else {  
            return n * factorial(n-1);  
        }  
    }  
}
```

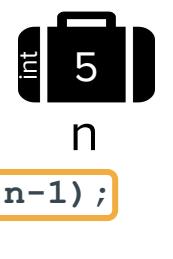
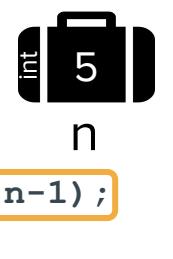


Diagram illustrating the recursive call stack for `factorial(5)`. The stack frames are represented as suitcases. The top suitcase contains the value `int 5`. Below it, the next suitcase contains the value `24`.

# Recursion in action

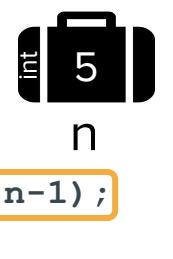
```
int main() {  
  
    int factorial (int n) {  
        if (n == 0) {  
            return 1;  
        } else {  
            return n * factorial(n-1);  
        }  
    }  
}
```



int 5  
n  
5      24

# Recursion in action

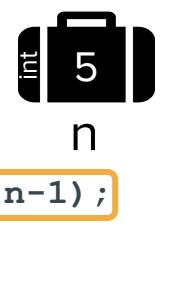
```
int main() {  
  
    int factorial (int n) {  
        if (n == 0) {  
            return 1;  
        } else {  
            return n * factorial(n-1);  
        }  
    }  
}
```



int 5  
n  
5 x 24

# Recursion in action

```
int main() {  
  
    int factorial (int n) {  
        if (n == 0) {  
            return 1;  
        } else {  
            return n * factorial(n-1);  
        }  
    }  
}
```



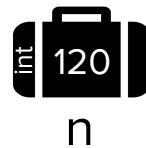
int 5  
n  
120

# Recursion in action

```
int main() {  
    int n = factorial(5);  
    cout << "5! = " << n << endl;  
    return 0;  
}
```

# Recursion in action

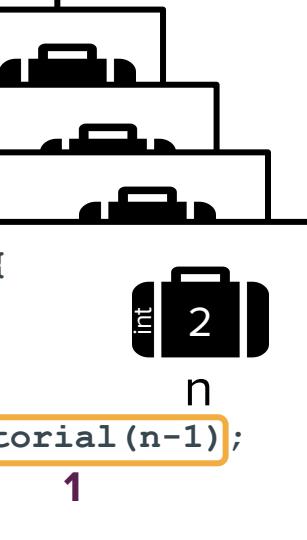
```
int main() {  
    int n = factorial(5);  
    cout << "5! = " << n << endl;  
    return 0;  
}
```





# Summary of Recursion in action

```
int main() {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    int factorial (int n) {  
        }  
    }  
}  
  
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}
```



```
int factorial(int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}
```

# Recursion in action

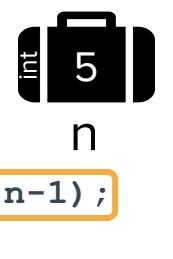
```
int main() {  
    int factorial (int n) {  
        int factorial (int n) {  
            if (n == 0) {  
                return 1;  
            } else {  
                return n * factorial(n-1);  
            }  
        }  
    }  
}
```



4    x    6

# Recursion in action

```
int main() {  
  
    int factorial (int n) {  
        if (n == 0) {  
            return 1;  
        } else {  
            return n * factorial(n-1);  
        }  
    }  
}
```



int 5  
n  
5 x 24

# Recursive vs. Iterative

```
d  
int factorial(int n) {  
    if (n == 0) {  
        return 1;  
    } else {  
        return n * factorial(n-1);  
    }  
}
```

```
int factorialIterative(int n) {  
    int result = 1;  
    for (int i = 1; i <= n; i++) {  
        result = result * i;  
    }  
    return result;  
}
```

# Announcements

# Announcements

- Assignment 2 is due **Tomorrow**, 7/7 at 11:59pm PT. The grace period expires at the same time on **Friday**. After that, we will **not** be accepting submissions.
- Assignment 3 will be released by the end of the day on Thursday and will be due 8 days later on a Friday.
- The mid-quarter diagnostic will cover through the middle of next week (7/14 will be the last day of content covered).
  - We'll have practice problems ready by this weekend, with more next week.

# Announcements II

- Here's the LaIR schedule for the quarter:

<b>Day</b>	<b>Time</b>
Monday	5-7pm Pacific
Tuesday	7-9pm Pacific
Wednesday	5-7pm Pacific
Thursday	7-9pm Pacific

- Recall that we have a special queue in the LaIR for conceptual questions. If you want to review lecture material, LaIR is a great place to get extra practice with concepts.

# Reverse string example



# How can we reverse a string?

Suppose we want to reverse strings like in the following examples:

“dog” → “god”

“stressed” → “desserts”

“recursion” → “noisrucer”

“level” → “level”

“a” → “a”

# Approaching recursive problems

- Look for self-similarity.
- Try out an example.
  - Work through a simple example and then increase the complexity.
  - Think about what information needs to be “stored” at each step in the recursive case (like the current value of **n** in each **factorial** stack frame).
- Ask yourself:
  - What is the base case? (What is the simplest case?)
  - What is the recursive case? (What pattern of self-similarity do you see?)

# Discuss:

What are the base and  
recursive cases?

(breakout rooms)

# How can we reverse a string?

- Look for self-similarity: **stressed** → **desserts**

# How can we reverse a string?

- Look for self-similarity: **stressed** → **desserts**
  - What's the first step you would take to reverse “stressed”?

# How can we reverse a string?

- Look for self-similarity: **stressed** → **desserts**
  - Take the s and put it at the end of the string.

# How can we reverse a string?

- Look for self-similarity: **stressed** → **desserts**
  - Take the s and put it at the end of the string.
  - Then reverse “tressed”

# How can we reverse a string?

- Look for self-similarity: **stressed** → **desserts**
  - Take the s and put it at the end of the string.
  - Then reverse “tressed”:
    - Take the t and put it at the end of the string.
    - Then reverse “ressed”

# How can we reverse a string?

- Look for self-similarity: **stressed** → **desserts**
  - Take the s and put it at the end of the string.
  - Then reverse “tressed”:
    - Take the t and put it at the end of the string.
    - Then reverse “ressed”:
      - Take the r and put it at the end of the string.
      - Then reverse “essed”

# How can we reverse a string?

- Look for self-similarity: **stressed** → **desserts**
  - Take the s and put it at the end of the string.
  - Then reverse “tressed”:
    - Take the t and put it at the end of the string.
    - Then reverse “ressed”:
      - Take the r and put it at the end of the string.
      - Then reverse “essed”:
        - ...
        - Take the d and put it at the end of the string.
        - **Base case**: reverse “” → get “”

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- Look for self-similarity: **stressed** → **desserts**
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# How can we reverse a string?

- **Recursive case:**  $\text{reverse}(\text{str}) = \text{reverse}(\text{str without first letter}) + \text{first letter of str}$
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- **Recursive case:**  $\text{reverse}(\text{str}) = \text{reverse}(\text{str without first letter}) + \text{first letter of str}$
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Depending on how you thought of the problem, you may have also come up with:

- **Recursive case:**  $\text{reverse}(\text{str}) = \text{last letter of str} + \text{reverse}(\text{str without last letter})$
- **Base case:**  $\text{reverse}("") = ""$

# Let's code it!

(live coding)

# Summary

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- Recursion is a problem-solving technique in which tasks are completed by reducing them into **repeated, smaller tasks of the same form**.
  - A recursive operation (function) is defined in terms of itself (i.e. it calls itself).

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- Recursion is a problem-solving technique in which tasks are completed by reducing them into **repeated, smaller tasks of the same form**.
- Recursion has two main parts: the **base case** and the **recursive case**.
  - Base case: Simplest form of the problem that has a direct answer.
  - Recursive case: The step where you break the problem into a smaller, self-similar task.

# Summary

- Recursion is a problem-solving technique in which tasks are completed by reducing them into **repeated, smaller tasks of the same form**.
- Recursion has two main parts: the **base case** and the **recursive case**.
- The solution will get built up **as you come back up the call stack**.
  - The base case will define the “base” of the solution you’re building up.
  - Each previous recursive call contributes a little bit to the final solution.
  - The initial call to your recursive function is what will return the completely constructed answer.

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- The solution will get built up **as you come back up the call stack**.
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How can we use visual  
representations to understand  
recursion?

# Self-Similarity

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- An object is **self-similar** if it contains a smaller copy of itself.

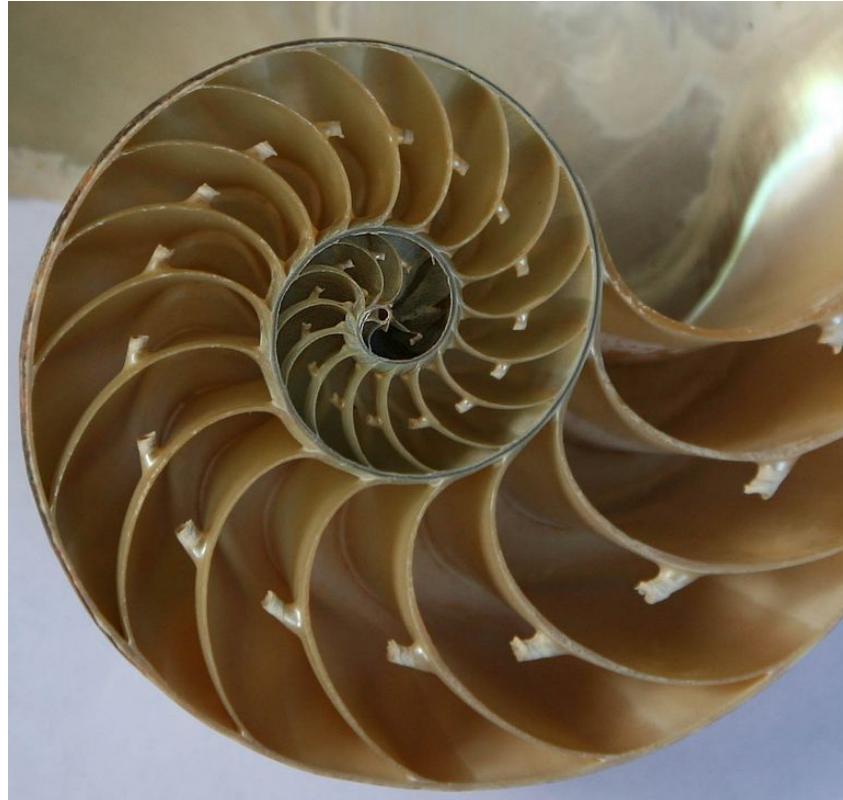
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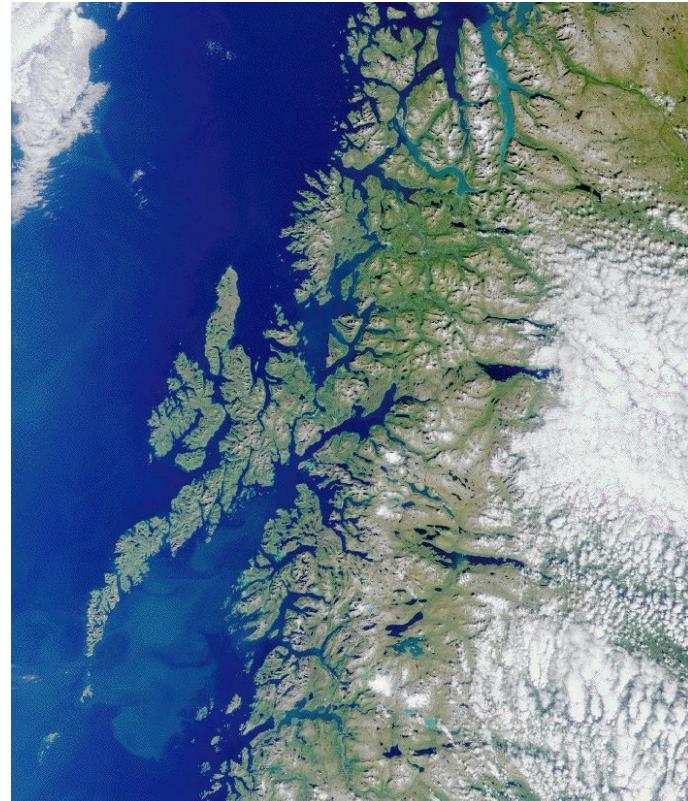
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*Self-similarity shows up in many real-world objects and phenomena, and is the key to truly understanding their formation and existence.*

# Fractals

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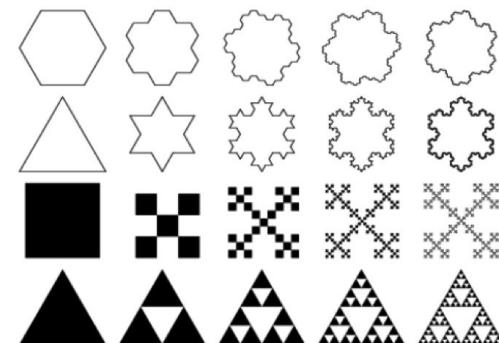
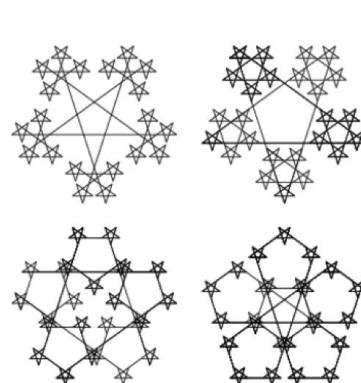
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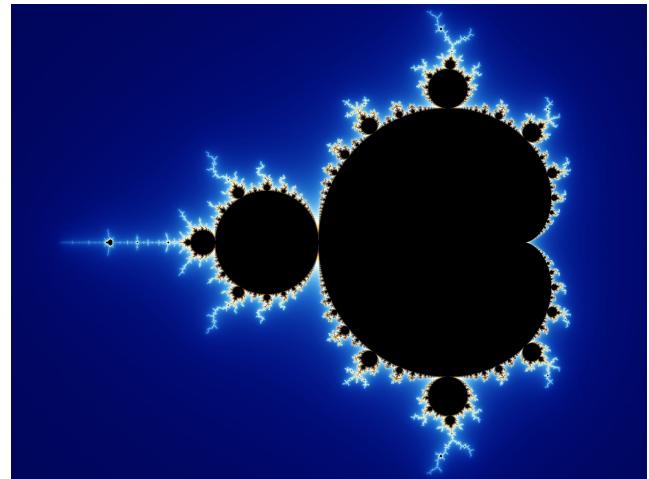
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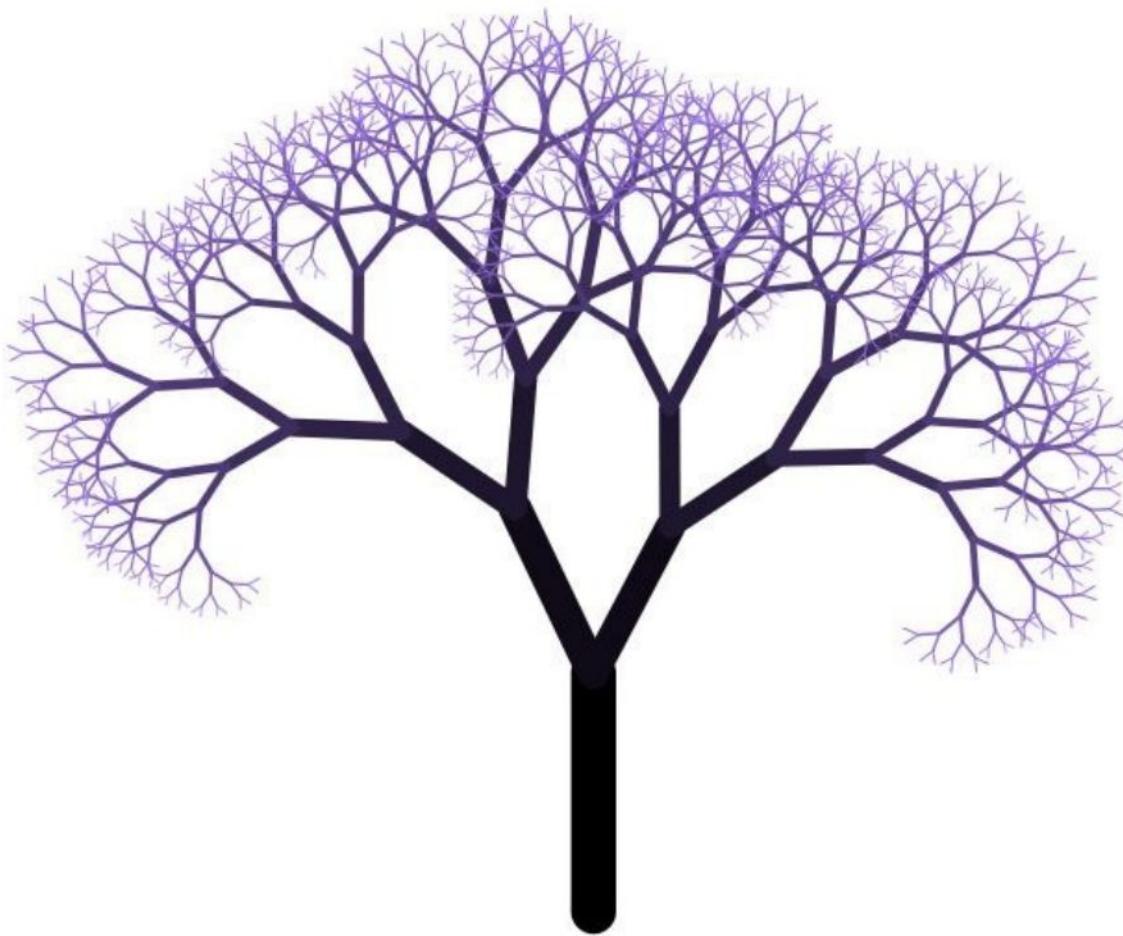


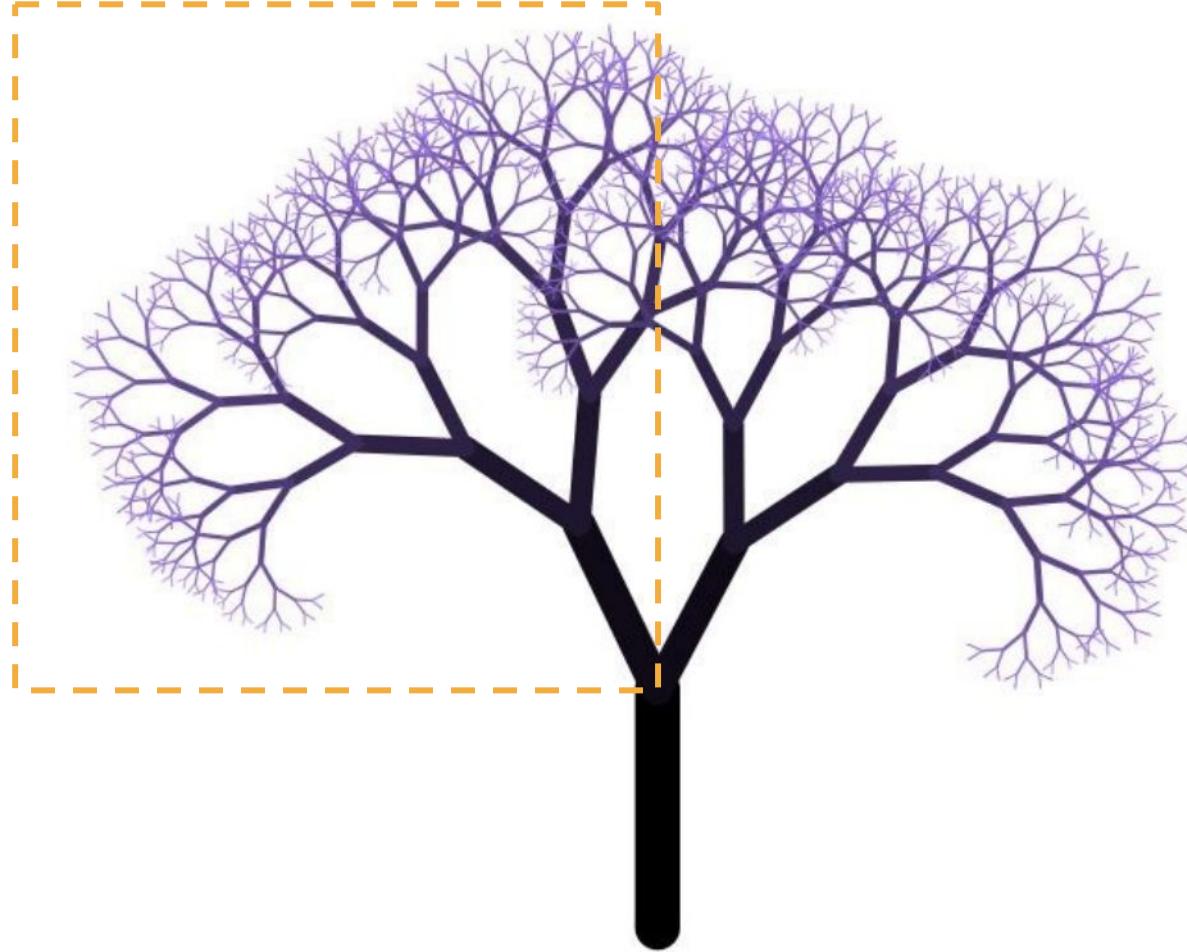
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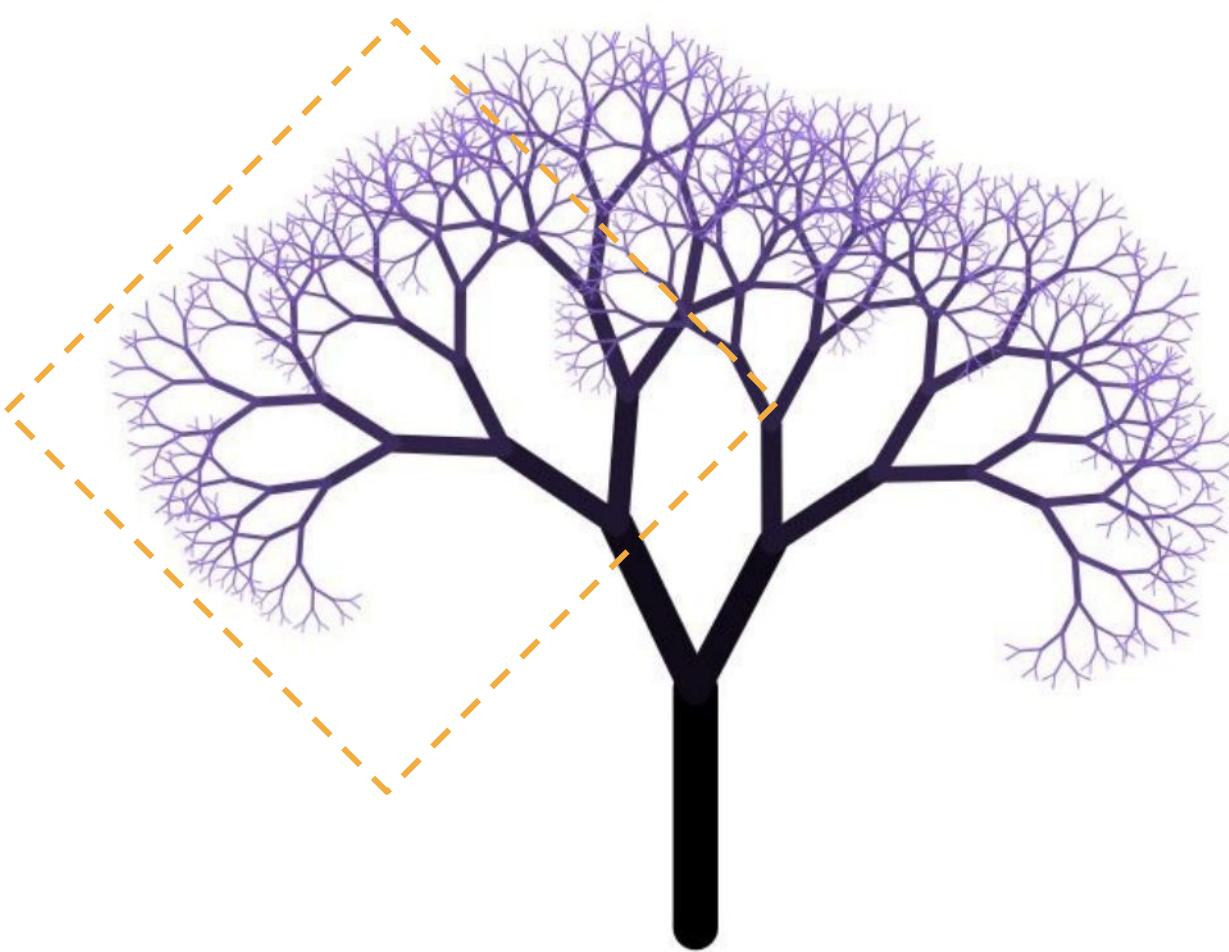
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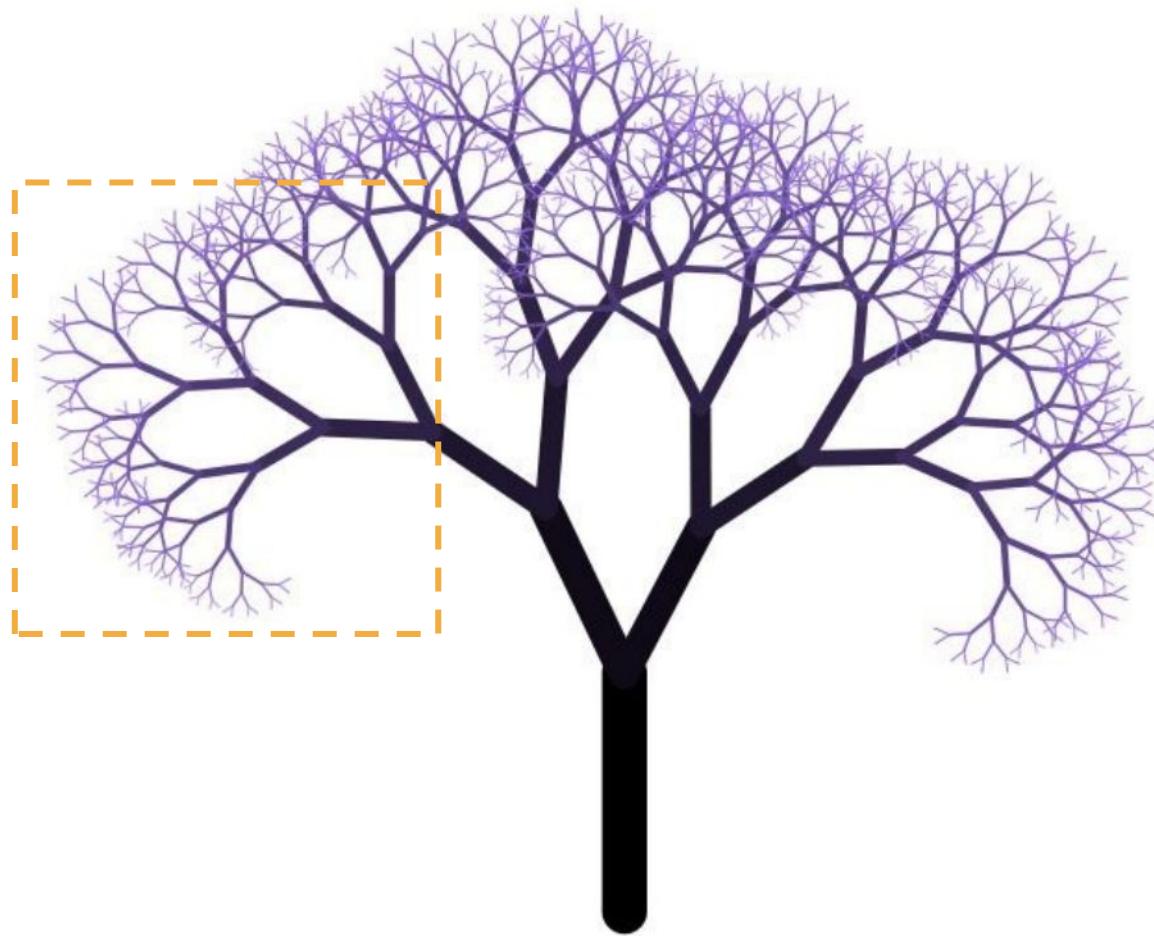


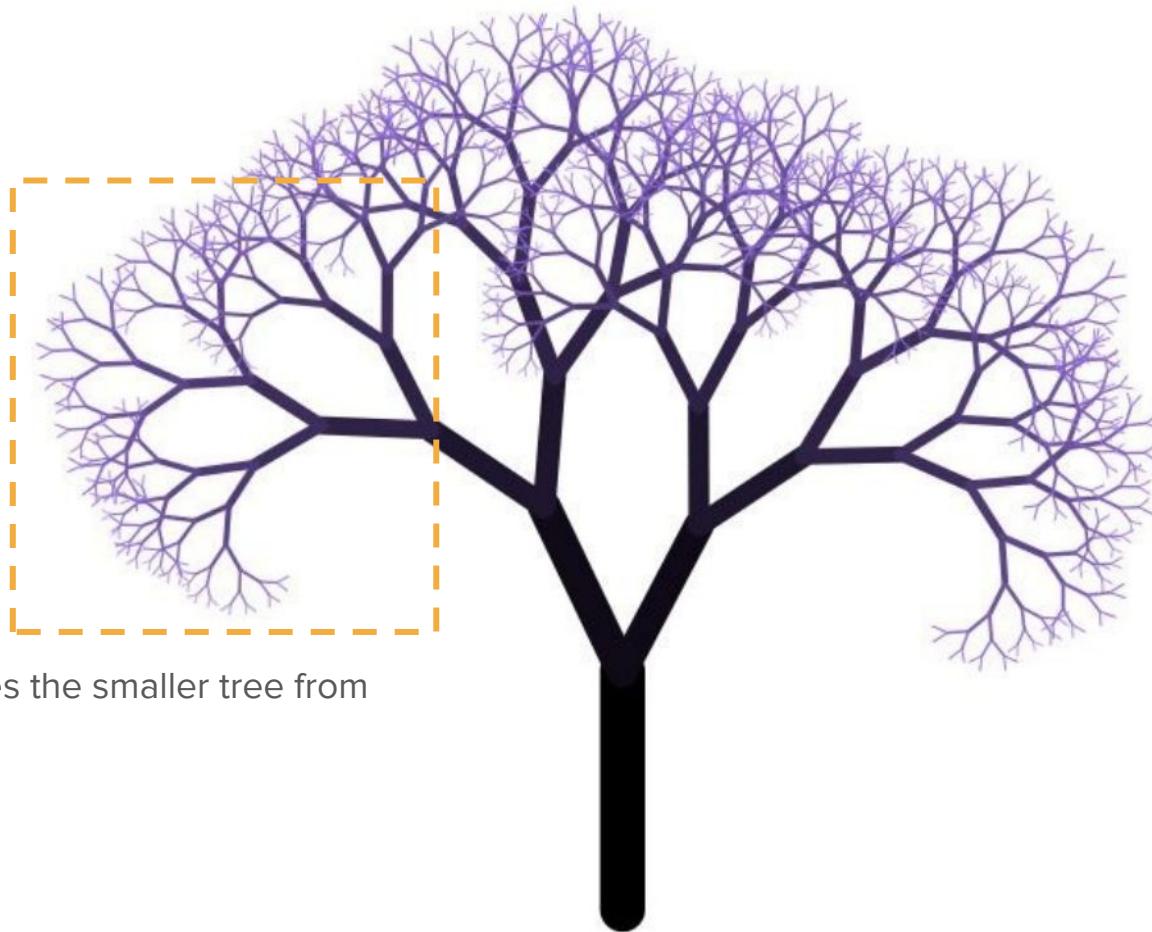
# Understanding Fractal Structure



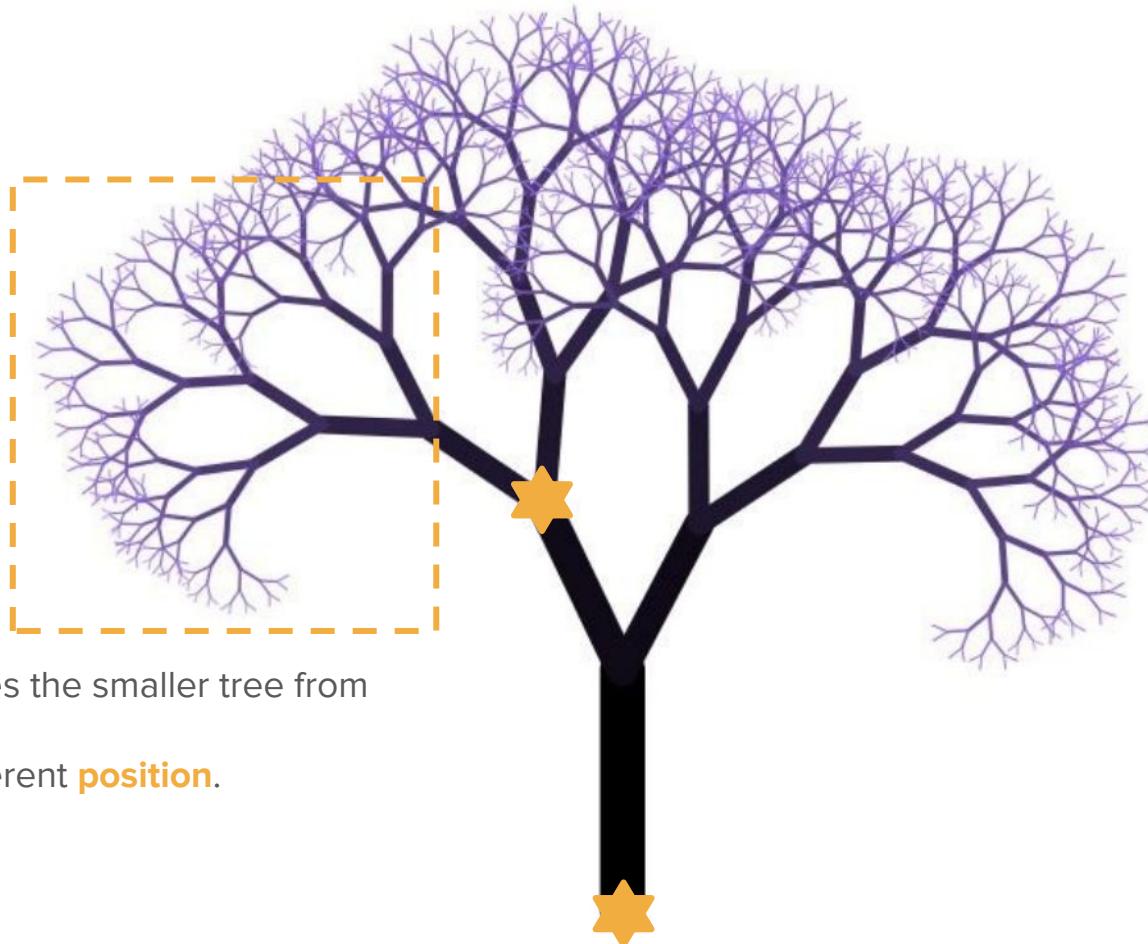






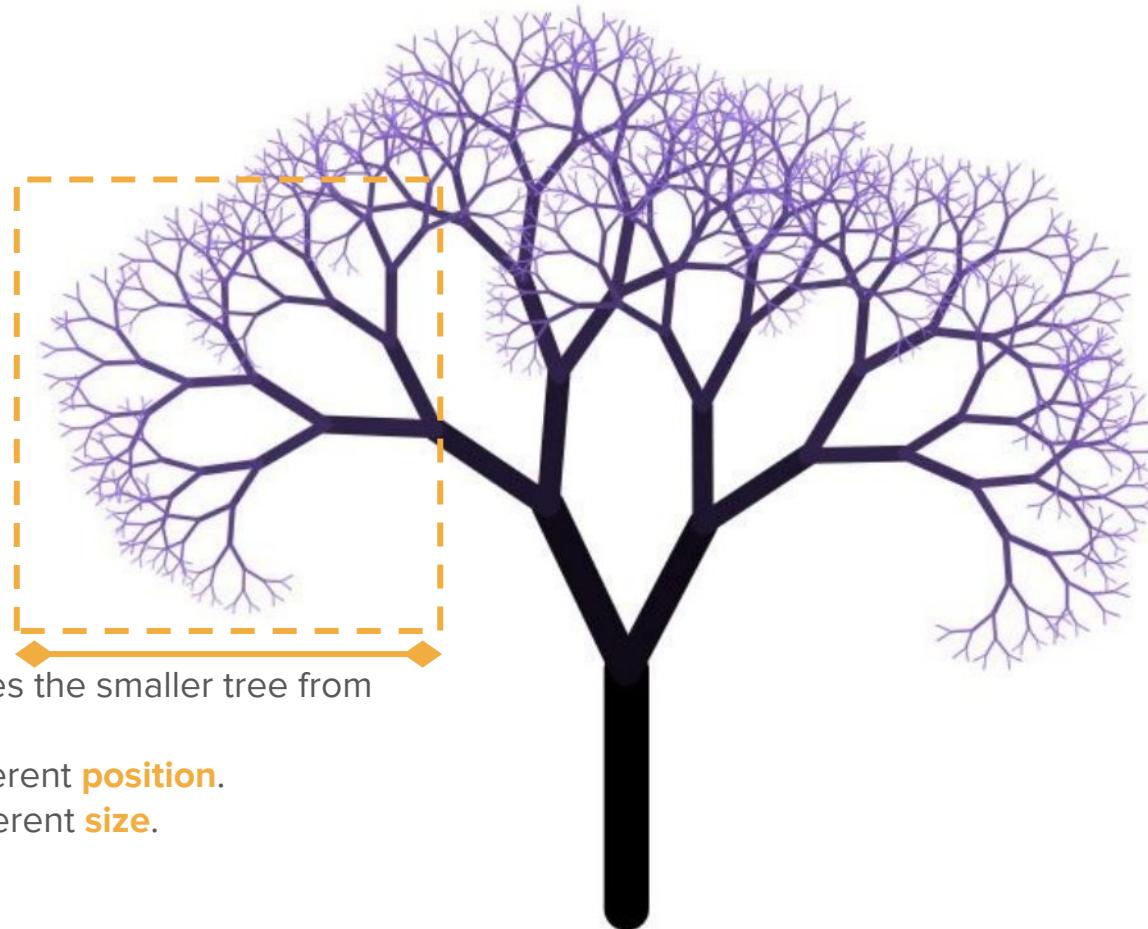


What differentiates the smaller tree from  
the bigger one?



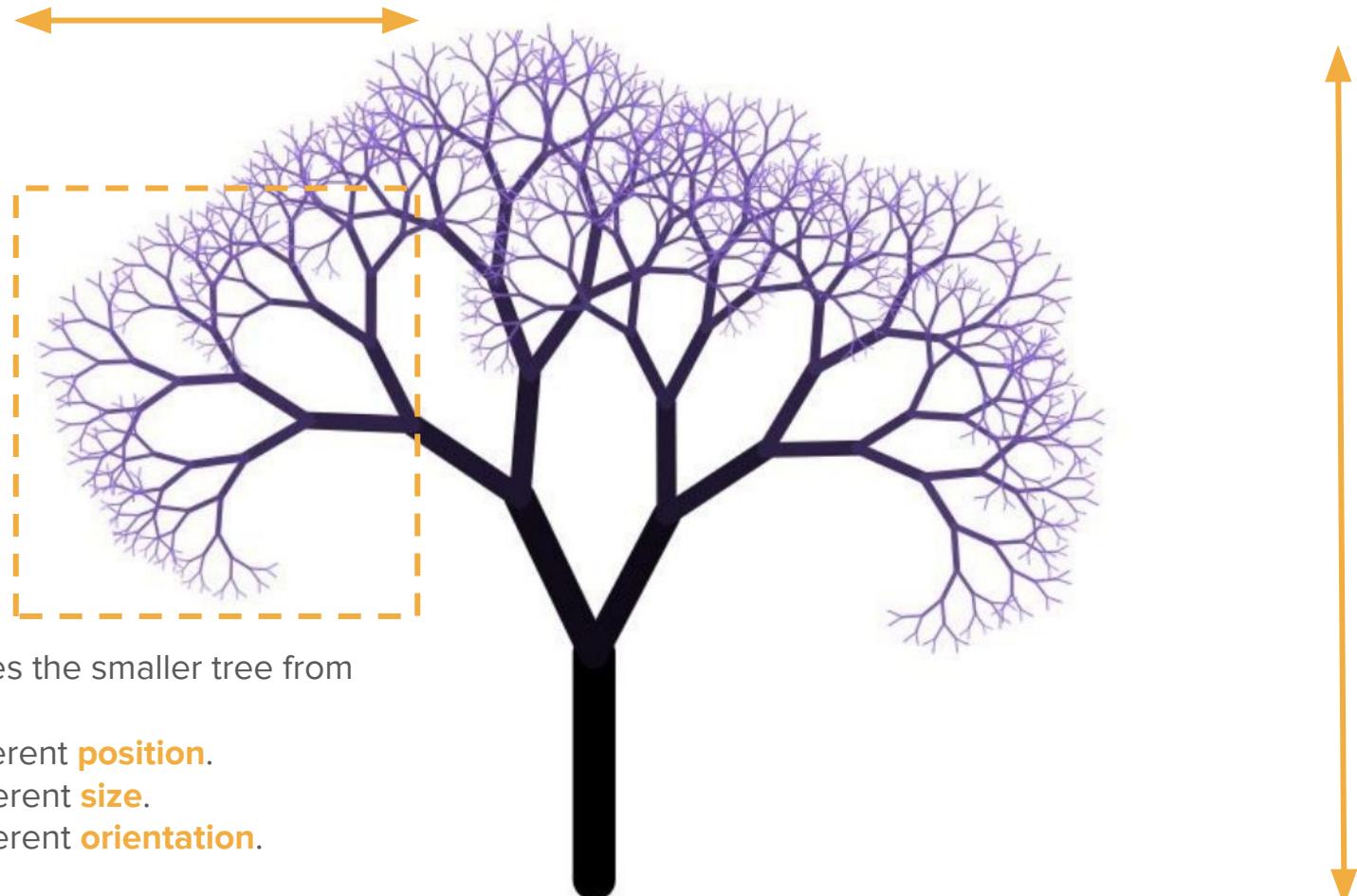
What differentiates the smaller tree from the bigger one?

1. It's at a different **position**.



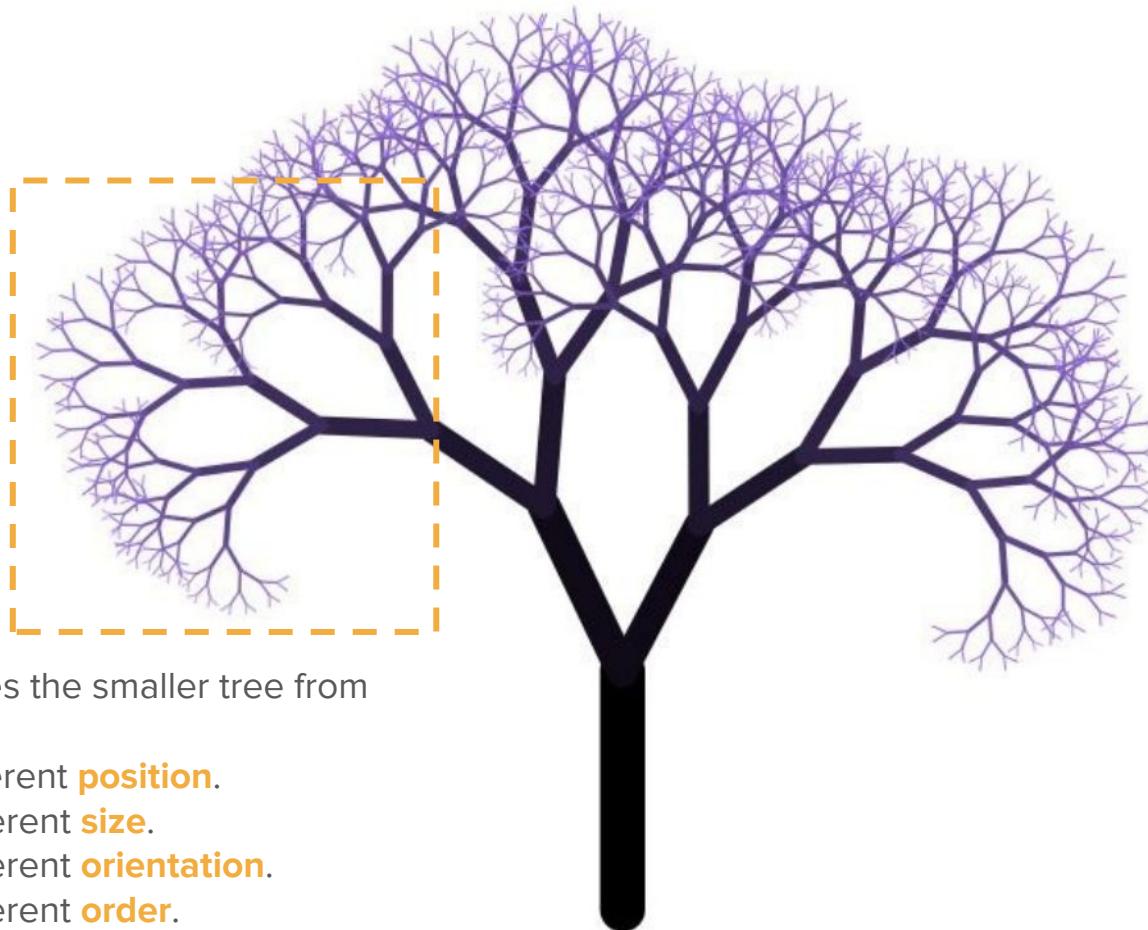
What differentiates the smaller tree from the bigger one?

1. It's at a different **position**.
2. It has a different **size**.



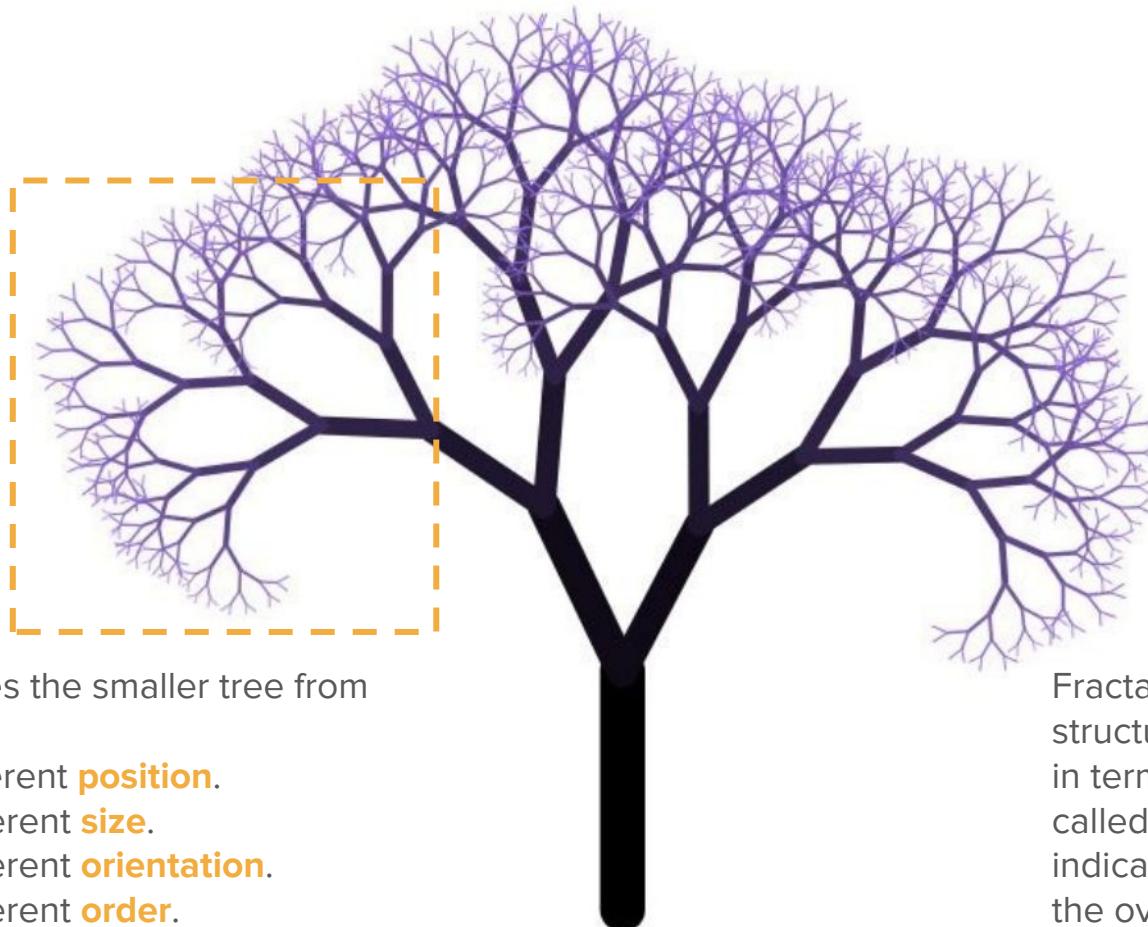
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Fractals and self-similar structures are often defined in terms of some parameter called the **order**, which indicates the complexity of the overall structure.

# An order-0 tree

What differentiates the smaller tree from the bigger one?

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# An order-1 tree

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# An order-2 tree

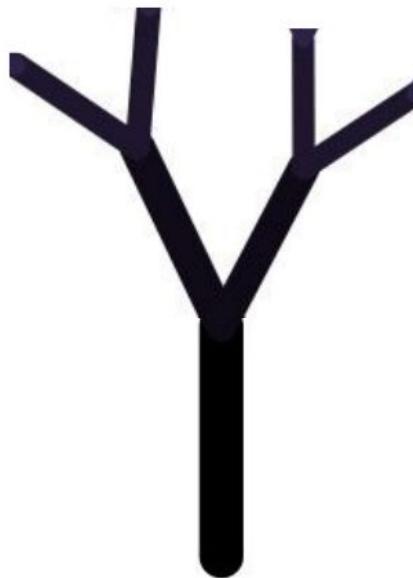
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# An order-3 tree

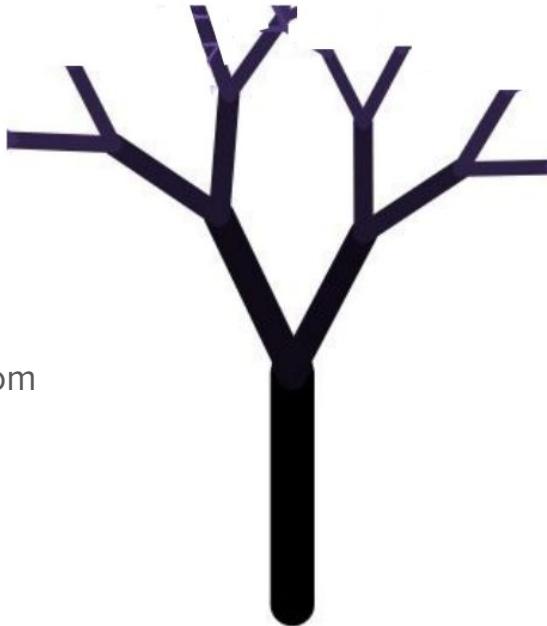


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# An order-4 tree



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# An order-11 tree

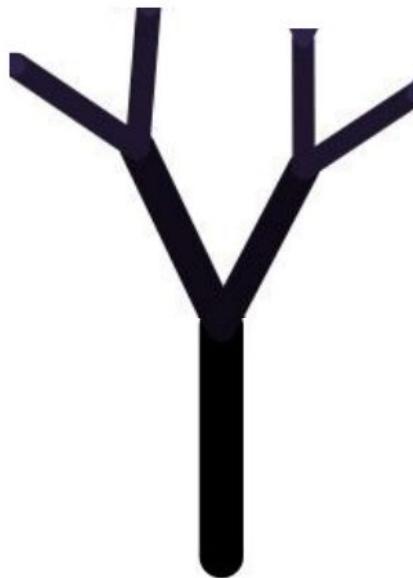


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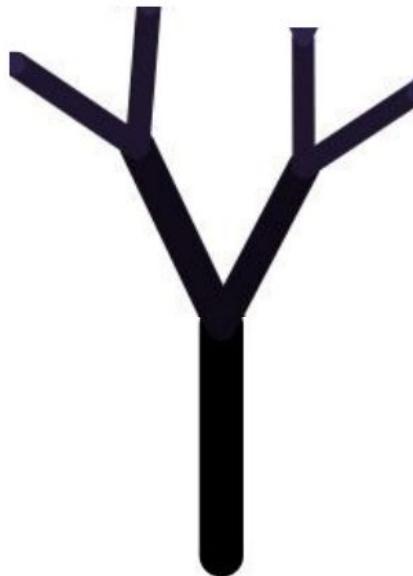
# An order-3 tree

An order-0 tree is nothing at all.

An order-**n** tree is a line with two smaller order- (**n-1**) trees starting at the end of that line.

What differentiates the smaller tree from the bigger one?

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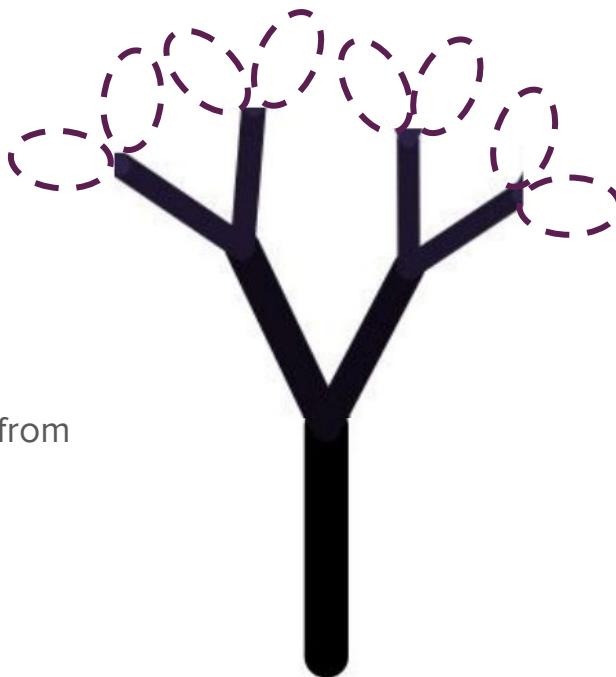
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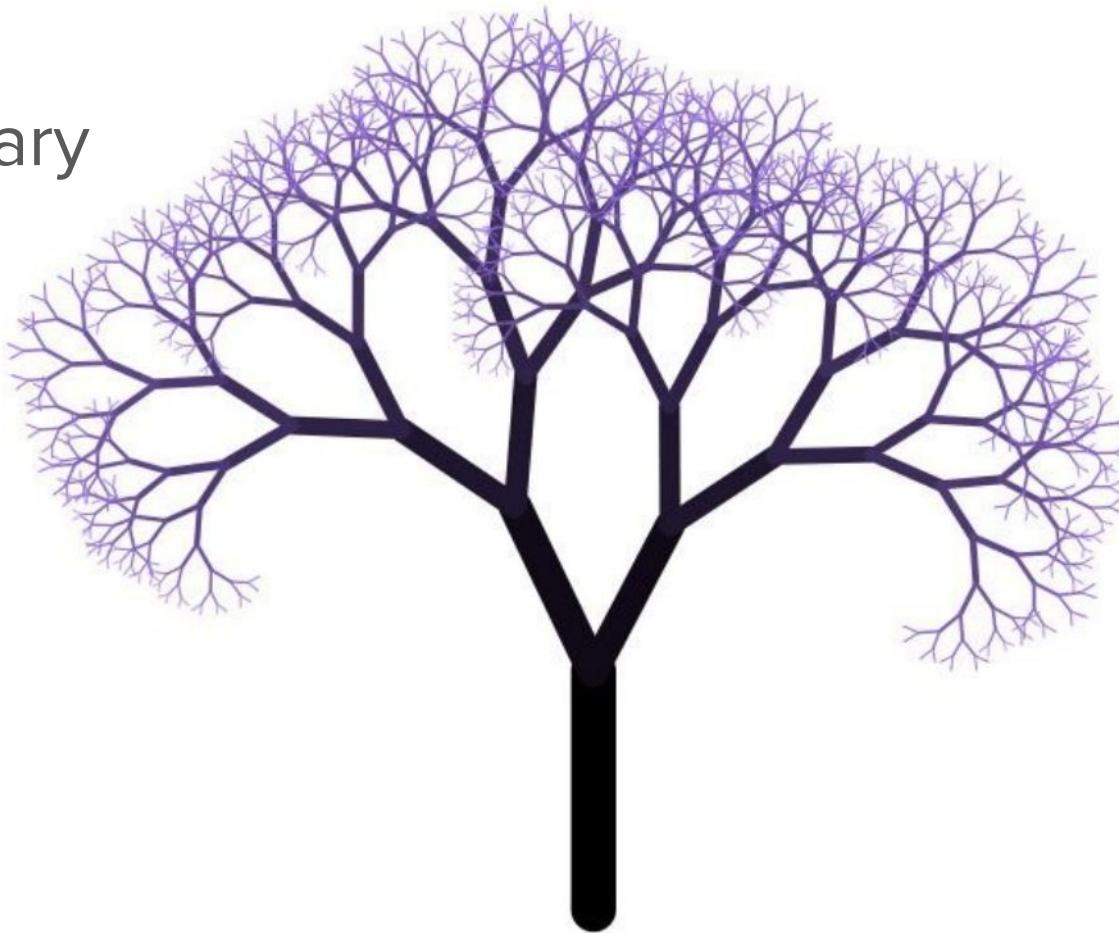
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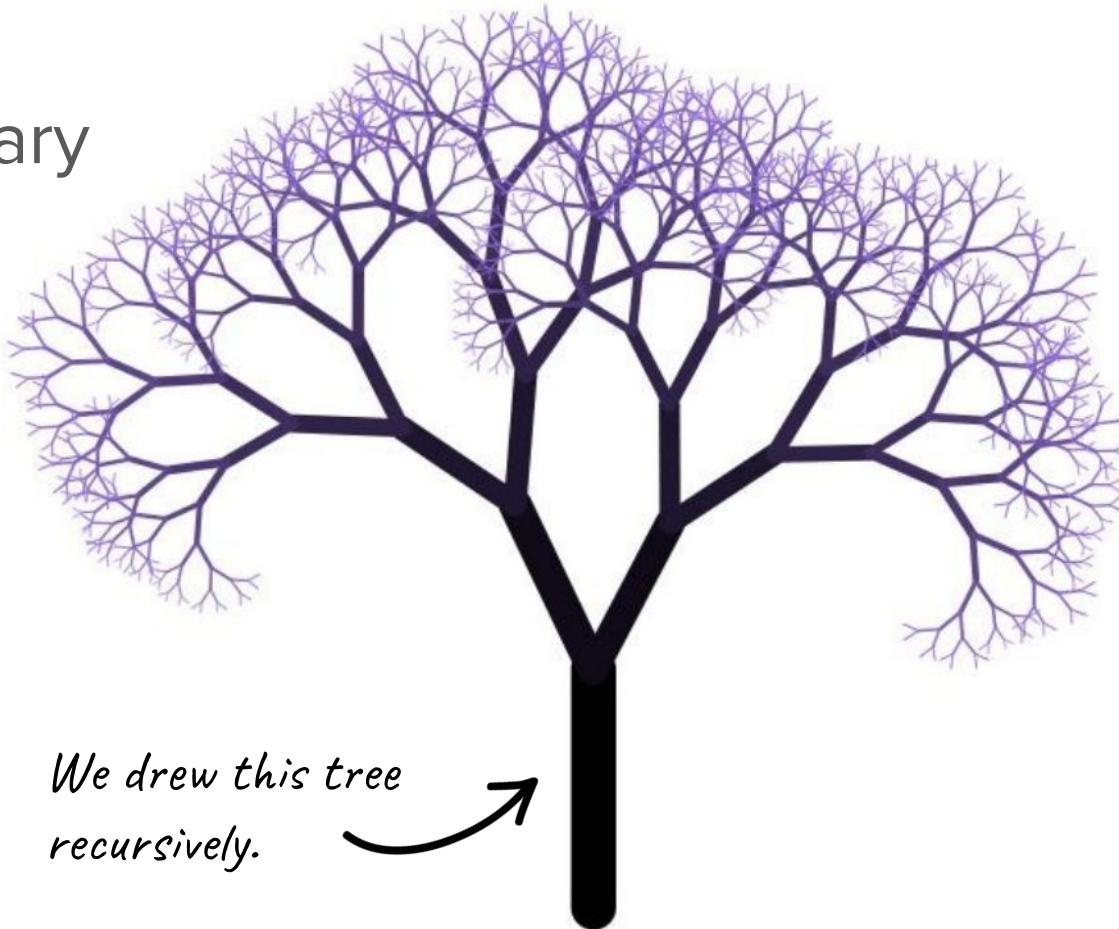


Fractals and self-similar structures are often defined in terms of some parameter called the **order**, which indicates the complexity of the overall structure.

# In Summary

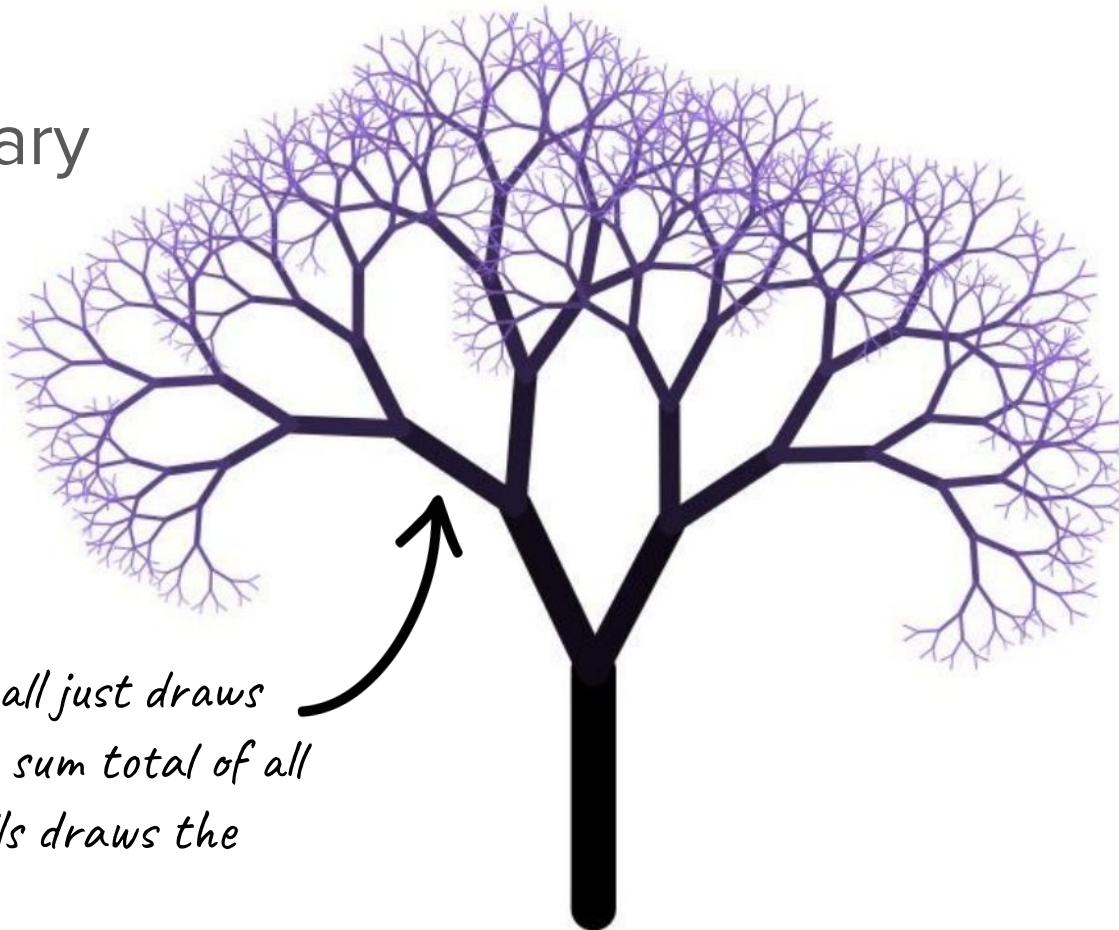


# In Summary



*We drew this tree  
recursively.*

# In Summary

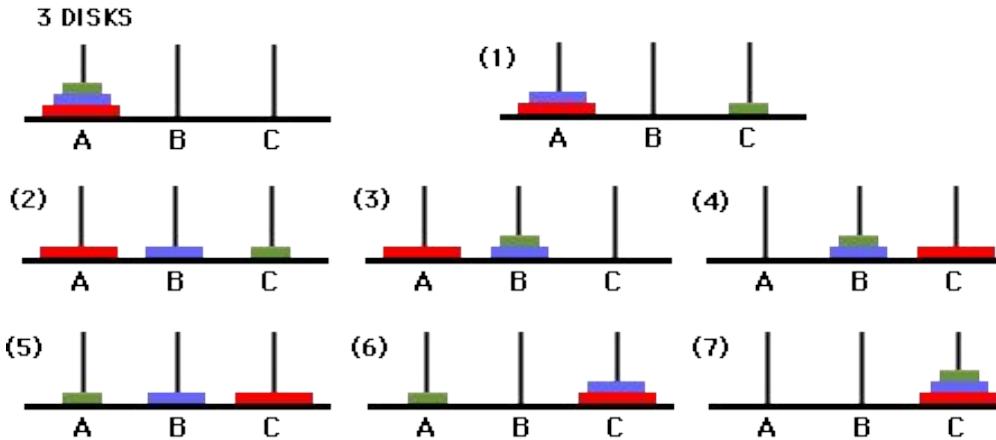


*Each recursive call just draws one branch. The sum total of all the recursive calls draws the whole tree.*

# Revisiting the Towers of Hanoi

[Recursive Part 2: Electric Boogaloo]

# Pseudocode for 3 disks



- (1) Move disk 1 to destination
- (2) Move disk 2 to auxiliary
- (3) Move disk 1 to auxiliary
- (4) Move disk 3 to destination
- (5) Move disk 1 to source
- (6) Move disk 2 to destination
- (7) Move disk 1 to destination

# To Do before tomorrow's lecture

- Play Towers of Hanoi:  
<https://www.mathsisfun.com/games/towerofhanoi.html>
- Look for and write down patterns in how to solve the problem as you increase the number of disks. Try to get to at least 5 disks!
- **Extra challenge** (optional): How would you define this problem recursively?
  - Don't worry about data structures here. Assume we have a function `moveDisk(X, Y)` that will handle moving a disk from the top of post X to the top of post Y.

# An Awesome Website!

<http://recursivedrawing.com/>

# What's next?

# Roadmap

## Object-Oriented Programming

