Is there any component of "Life after CS106B" that you would like us to focus on in the final lecture next week?

(put your answers in the chat)
Roadmap

C++ basics
- vectors + grids
- stacks + queues
- sets + maps

User/client

Object-Oriented Programming
- arrays
- dynamic memory management
- linked data structures

Implementation
- real-world algorithms
- recursive problem-solving

Diagnostic

Core Tools
- testing
- algorithmic analysis

Life after CS106B!

Roadmap graphic courtesy of Nick Bowman & Kylie Jue
Roadmap

C++ basics

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arrays
dynamic memory management

linked data structures

real-world algorithms

Life after CS106B!

Core Tools

testing

algorithmic analysis

recursive problem-solving
Today’s questions

How can we better organize data stored in a linked data structure?
Today’s topics

1. Linked Data Structure Overview
2. Introduction to Trees
3. Trees in C++
Review
[linked data structures]
Linked Data Structures

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- In order to organize this data, we had to **bundle data alongside pointers** in the concept of a "node."
Linked Data Structures

- Last week, we explored linked lists, our first example of a linked data structure.
- Linked data structures are distinguished by the fact that they stored data in a distributed manner. This means that the data is stored across many different locations in computer memory.
- In order to organize this data, we had to bundle data alongside pointers in the concept of a "node."
- Using pointers lets us create links to other nodes to impose structure (why?)
Linked List Tradeoffs

- Storing data in a distributed (non-contiguous) manner had some distinct advantages over working with arrays.
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Linked List Tradeoffs

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  - Insertion/removal of elements of a linked list was very quick because it only involved fast pointer rewiring operations. We never had to "shift" elements over to make room.
  - Because all the data was stored in dynamic memory, expanding the size of the linked list was very easy and never required an expensive "re-sizing" operation that had to copy all the data.
Linked List Tradeoffs

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- However, we also ran into some limitations when it came to working with lists:
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  - Data was organized in a linear structure, which meant the path to traverse between any two nodes (specifically between the front and a node later on in the list) could get very long.
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  - Finding elements in a linked list is an $O(n)$ operation, which can get slow when we want to store many elements.
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- However, we also ran into some limitations when it came to working with lists:
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  - Finding elements in a linked list is an $O(n)$ operation, which can get slow when we want to store many elements.
  - We couldn't feasibly write recursive algorithms that traversed linked lists, due to stack frame limits that came into play since traversal algorithms required one stack frame per node.
Linked List Tradeoffs

- Storing data in a distributed (non-contiguous) manner had some distinct advantages over working with arrays.

- However, we also ran into some limitations when it came to working with lists.

- **Question:** Can we organize data in a linked data structure in such a way that the path between the "front" and any element in the structure is short (better than $O(n)$) even if there are many elements?
How can we better organize data stored in a linked data structure?
Interactive Exercise
[borrowed from Keith Schwarz]
Take a deep breath.
And exhale...
Feel nicely oxygenated?
Beautiful art by Keith
Your lungs have about 500 million alveoli...
Your lungs have about 500 million alveoli...

... yet the path to each one is short.
Key Idea: The distance from each element in this structure to the top of the structure is small, even if there are many elements.
Trees
Throwback Thursday (on Monday)

- We've already seen trees before in this class... decision trees!
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Trees in the Wild

- Trees are useful in other ways besides just visualizing recursive backtracking.
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Trees can be used to describe hierarchies.
Trees in the Wild

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Trees are used to model the structure of websites.
Trees in the Wild

- Trees are useful in other ways besides just visualizing recursive backtracking.

```python
def run() {
    move();
    while (notFinished()) {
        if (isPathClear()) {
            move();
        } else {
            turnLeft();
        }
    }
    move();
}
```

Trees describe the syntax structure of programs.
Trees in the Wild

- Trees are useful in other ways besides just visualizing recursive backtracking.

- But, it is not a coincidence that we first saw them appear in conjunction with recursion.
Trees in the Wild

- Trees are useful in other ways besides just visualizing recursive backtracking.

- But, it is not a coincidence that we first saw them appear in conjunction with recursion.

- Trees are inherently defined recursively!
What is a tree?

A tree is either...
What is a tree?

A tree is either...

An empty data structure, or...
What is a tree?

A tree is either...

- An empty data structure, or...
- A single node (parent), with zero or more non-empty subtrees (children)
Definition

tree
A tree is hierarchical data organization structure composed of a root value linked to zero or more non-empty subtrees.
Tree Terminology
Tree Terminology

A

B
C
D
E
F

G
H
I

J
K
L
Tree Terminology

A node...
Tree Terminology

A node with 0 or more non-empty subtrees
Tree Terminology

A node with 0 or more non-empty subtrees
Tree Terminology

A node with 0 or more non-empty subtrees
Tree Terminology

A **node** with 0 or more non-empty subtrees
Tree Terminology

A node with 0 or more non-empty subtrees
Tree Terminology
A is the root node of the tree
Tree Terminology

B, C, D, E, and F are children of A
Tree Terminology

A is the parent of B, C, D, E, and F
Tree Terminology

Node B has no children. A node with no children is called a **leaf node**.
B, G, H, I, D, E, J, and L are all leaf nodes.
**Tree Terminology**

G, H and I all have the same parent. Nodes with the same parent are **siblings**.
We can define a path through the tree between two nodes.
Tree Terminology

We can define a path through the tree between two nodes.

Note: We can only follow the links in the direction the arrow points!
Tree Terminology

The path from A to L is A -> F -> K -> L
Tree Terminology

The **length** of the path is **number of edges it contains**. The path from **A** to **L** has length 3.
Tree Terminology

The depth of a node is the length of its path to the root.
Tree Terminology

The depth of a node is the length of its path to the root.

depth: 0
The depth of a node is the length of its path to the root.
Tree Terminology

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Tree Terminology

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The height of a tree is defined to be the maximal depth that a tree has.
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Tree Terminology Summary

- Every non-empty tree has a root node that defines the "top" of the tree.

- Every node has 0 or more children nodes descended from it. Nodes with no children are called leaf nodes.

- Every node in a tree has exactly one parent node (except for the root node).

- A path through the tree traverses edges between parents and their children.

- The depth of a node is the number of edges between the root and that node. A tree's height is the number of edges in the longest path through the tree.
Tree Properties
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Not a tree!
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Not a tree!
Announcements
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● Assignment 5 is due on **Tuesday, August 3 at 11:59pm PDT**.
  ○ On the short answer, problem 7 asks about 2 sort prototypes. You should pretend that question only says one!

● Trip’s group OH will be moved From Wednesday, 8/4 to Thursday 8/5, still from 10am-12pm PT
  ○ Trip still has office hours on 8/3 from 9-11am PT. Come thru!

● Assignment 6 will be released on Wednesday and will be due on **Wednesday, August 11 at 11:59pm PDT**. This is a hard deadline – there is **no grace period and no submissions will be accepted after this time**.

● The End-quarter Assessment will take place over 3 days from **Friday, August 13 to Sunday, August 15**. More information will be released soon.
Trees in C++
Binary Trees

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- A binary tree is a tree where every node has either 0, 1, or 2 children. No node in a binary tree can have more than 2 children.
Binary Trees

- In general, we've seen that nodes in a tree can have variable numbers of children (subtrees) and sometimes very, very many.

- However, when working with trees in computer programs, it is common to work mostly with binary trees.

- A binary tree is a tree where every node has either 0, 1, or 2 children. No node in a binary tree can have more than 2 children.

- Typically, the two children of a node in a binary tree are referred to as the left child and the right child.
Binary Trees

A

B
C
D
Binary Trees

Binary Tree!
Binary Trees

Binary Tree!
Binary Trees

Binary Tree!

Not a binary tree!
Building Trees Programmatically

● To build a tree in C++, we need a new version of the Node struct we've seen before.
Building Trees Programmatically

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- In this case, we want each Node to have a data value (like a linked list), but now we want two pointers, one to the left child, and one to the right child.
Building Trees Programmatically

- To build a tree in C++, we need a new version of the Node struct we've seen before.

- In this case, we want each Node to have a data value (like a linked list), but now we want two pointers, one to the left child, and one to the right child.

```cpp
struct TreeNode {
    string data;
    TreeNode* left;
    TreeNode* right;
};
```
What is a tree in C++?

A tree is either...

An empty data structure, or...

A single node (parent), with zero or more non-empty subtrees (children)
What is a tree in C++?

A tree is either...

An empty tree represented by `nullptr`, or...

A single node (parent), with zero or more non-empty subtrees (children)
What is a tree in C++?

A tree is either...

An empty tree represented by `nullptr`, or...

A single `TreeNode`, with 0, 1, or 2 non-null pointers to other `TreeNode`s.
Building Trees Programmatically

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struct TreeNode {
    string data;
    TreeNode* left;
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}
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Building Trees Programmatically

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struct TreeNode {
    string data;
    TreeNode* left;
    TreeNode* right;
}
```
Building Trees Programmatically

```
struct TreeNode {
    string data;
    TreeNode* left;
    TreeNode* right;
}
```

"pineapple"
Building Trees Programatically

```c
struct TreeNode {
    string data;
    TreeNode* left;
    TreeNode* right;
}
```

```
"pineapple"
```

```
"coconut"
```
Building Trees Programmatically

```
struct TreeNode {
    string data;
    TreeNode* left;
    TreeNode* right;
}
```
Building Trees Programmatically

```c
struct TreeNode {
    string data;
    TreeNode* left;
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    TreeNode* right;
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```

Note: Trees do not have to be complete, like heaps. Any node can have 0, 1, or 2 children.
Let's code it!

buildExampleTree()
Building a Tree Takeaways

- Building a tree is very similar to the process of building a linked list.

- We create new nodes of the tree by dynamically allocating memory.

- We integrate these new nodes into the tree by rewiring the left and right pointers of existing nodes in the tree.
Tree Traversals
Tree Traversals

- Often, we will want to "do something" with each node in a tree. Like linked lists, we can do so by **traversing the tree**. With the branching involved, this is a slightly more involved process than traversing a linked list!
Tree Traversals

● Often, we will want to "do something" with each node in a tree. Like linked lists, we can do so by traversing the tree. With the branching involved, this is a slightly more involved process than traversing a linked list!

● There are three main ways to traverse a binary tree:
  ○ Pre-order traversal
  ○ In-order traversal
  ○ Post-order traversal
Tree Traversals

- Often, we will want to "do something" with each node in a tree. Like linked lists, we can do so by traversing the tree. With the branching involved, this is a slightly more involved process than traversing a linked list!

- There are three main ways to traverse a binary tree:
  - Pre-order traversal
  - In-order traversal
  - Post-order traversal

- Due to the recursive nature of trees, all of these algorithms are most easily defined recursively.
Pre-order Traversal

- The algorithm for a pre-order traversal is defined as follows:
  - "Do something" with the current node
  - Traverse the left subtree
  - Traverse the right subtree

- For example purposes, let's have our "do something" to be printing the contents of the current node, which will allow us to print the overall tree.
Let's code it!
preorderPrintTree()
Pre-order Traversal

- The algorithm for a pre-order traversal is defined as follows:
  - "Do something" with the current node
  - Traverse the left subtree
  - Traverse the right subtree

- For example purposes, let's have our "do something" be printing the contents of the current node, which will allow us to print the overall tree.

- Output: pineapple coconut banana durian strawberry taro
In-order Traversal

- The algorithm for an in-order traversal is defined as follows:
  - Traverse the left subtree
  - "Do something" with the current node
  - Traverse the right subtree
Let's code it!
inorderPrintTree()
In-order Traversal

- The algorithm for an in-order traversal is defined as follows:
  - Traverse the left subtree
  - "Do something" with the current node
  - Traverse the right subtree

- Output: **banana coconut durian pineapple strawberry taro**

- Observation: The output of this traversal gives as all the values in alphabetical order. Is this a coincidence?
  - No! We'll see why tomorrow! (for now, just note that this phenomena is not guaranteed for all binary trees.)
Post-order Traversal

- The algorithm for a post-order traversal is defined as follows:
  - Traverse the left subtree
  - Traverse the right subtree
  - "Do something" with the current node
Try it yourself!
postorderPrintTree()
Post-order Traversal

- The algorithm for a post-order traversal is defined as follows:
  - Traverse the left subtree
  - Traverse the right subtree
  - "Do something" with the current node

- Output: banana durian coconut taro strawberry pineapple

- Application: Freeing trees! (we'll see this in lecture tomorrow)
Summary
Trees Summary

- Trees allow us to organize information in a linked data structure such that the distance to any element is short, even if there are many elements.
- Trees organize nodes in a hierarchical manner, where each element contains connections to children nodes that exist "lower" in the tree.
- There are three main ways to traverse the nodes in a tree, and each type of traversal visits the nodes of the tree in a distinctly different order.
What’s next?
Roadmap

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C++ basics
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sets + maps

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testing

Algorithmic analysis

Implementation
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Recursive problem-solving

Life after CS106B!
Binary Search Trees