Problem One: Recursion Tracing

Our initial call to `reverseOf("stop")` looks like this:

```java
string reverseOf(string s) {
    if (s == "") {
        return "";
    } else {
        return reverseOf(s.substr(1)) + s[0];
    }
}
```

string s = "stop"

This call then fires off a call to `reverseOf("top")`, which looks like this:

```java
string reverseOf(string s) {
    if (s == "") {
        return "";
    } else {
        return reverseOf(s.substr(1)) + s[0];
    }
}
```

string s = "top"

This call fires off a call to `reverseOf("op")`:

```java
string reverseOf(string s) {
    if (s == "") {
        return "";
    } else {
        return reverseOf(s.substr(1)) + s[0];
    }
}
```

string s = "op"

This in turn calls `reverseOf("p")`:
We now append p to return "p":

This in turn calls reverseOf(""):

This triggers the base case and returns the empty string. (Notice that the reverse of the empty string "" is indeed the empty string ")":

We now append p to return "p":
We now append o to return "po":

We append t to return "pot":

And finally we append s to return "pots" back to whoever called us. Yay!

**Problem Two: Human Pyramids**

The key recursive insight here is that a human pyramid of height 0 is a pyramid of no people, and that a human pyramid of height \( n \) is a group of \( n \) people supporting a human pyramid of \( n-1 \) people. Using that idea, we can write this function:

```cpp
int peopleInPyramidOfHeight(int n) {
    if (n == 0) {
        return 0;
    } else {
        return n + peopleInPyramidOfHeight(n - 1);
    }
}
```

As a note, you can directly evaluate `peopleInPyramidOfHeight(n)` by computing \( n(n + 1) / 2 \). We'll see a really cool intuition for this later in the quarter!
Problem Three: Random Shuffling

Here is one possible solution:

```cpp
string randomShuffle(string input) {
    /* Base case: There is only one possible permutation of a string
       * with no characters in it.
    */
    if (input == "") {
        return input;
    } else {
        /* Choose a random index in the string. */
        int i = randomInteger(0, input.length() - 1);
        /* Pull that character to the front, then permute the rest of
           * the string.
        */
        return input[i] + randomShuffle(input.substr(0, i) + input.substr(i + 1));
    }
}
```

This function is based on the recursive observation that there is only one possible random shuffle of the empty string (namely, itself), and then using the algorithm specified in the handout for the recursive step.
Problem Four: Computing Averages

There are many ways to write this function. Here's one.

```c++
Statistics documentStatisticsFor(istream& input) {
    /* Read an initial value out of the file. We're going to use this to seed
    * the minimum and maximum values.
    */
    double val;
    input >> val; // Should do error-checking, but we'll skip that here.

    /* Create a Statistics object and initialize its min and max values. */
    Statistics info;
    info.min = val;
    info.max = val;

    /* Keep track of the sum of the values and the number of values so that we
    * can report back the average value. Initially, the sum is whatever value
    * we just read, and the number of values read is 1.
    */
    double total = val;
    int numValues = 1;

    /* Read the rest of the file, updating the min/max as we go along with the
    * sum and number of values.
    */
    while (input >> val) {
        numValues++;
        total += val;

        if (val > info.max) info.max = val;
        if (val < info.min) info.min = val;
    }

    /* Compute the average. */
    info.average = total / numValues;
    return info;
}
```
Problem Five: Haiku Detection

/* We will assume that
 * this function works. You'll build it
 * in assignment 1.
 */
int syllablesIn(string word);

/* These are prototypes.
 * They let us call these functions
 * Before they're defined.
 */
bool isHaiku(string line1, string line2, string line3);
int syllablesInLine(string line);

int main() {
    string line1 = getLine("Enter the first line: ");
    string line2 = getLine("Now, enter the second line: ");
    string line3 = getLine("Enter the third line: ");

    /* Given these three lines,
     * check whether they're a haiku,
     * then show the result.
     */
    if (isHaiku(line1, line2, line3)) {
        cout << "The text you entered" << endl;
        cout << "Goes 5 - 7 - 5, so it" << endl;
        cout << "is a haiku. Yay!" << endl;
    } else {
        cout << "Though you have tried hard," << endl;
        cout << "The three lines you entered are" << endl;
        cout << "Not a haiku. Awww." << endl;
    }

    return 0;
}

/* Given a poem
 * of three lines, returns whether
 * it is a haiku.
 */
bool isHaiku(string line1, string line2, string line3) {
    return syllablesInLine(line1) == 5 &&
           syllablesInLine(line2) == 7 &&
           syllablesInLine(line3) == 5;
}
/ Counts the number of
* syllables in a line of
* text, then returns it.
*/
int syllablesInLine(string text) {
    /* To split apart the
     * text, make a TokenScanner
     * and configure it.
     */
    TokenScanner scanner(text);
    scanner.ignoreWhitespace();

    int numSyllables = 0;
    while (scanner.hasMoreTokens()) {
        /* If this token is
         * a word, count its syllables
         * and update total.
         */
        string token = scanner.nextToken();
        if (scanner.getTokenType(token) == WORD) {
            numSyllables += syllablesIn(token);
        }
    }

    return numSyllables;
}

/* Did you notice that
 * all the comments are haikus?
 * Same with the output!
 */