Binary Search Trees

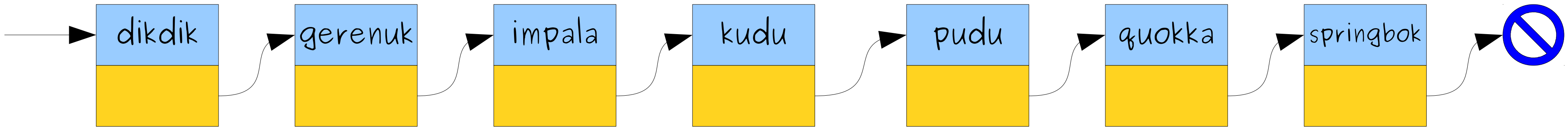
Part One

Way Back When…

Hash tables don’t store elements in sorted order. What if we want our items sorted?

Introducing `Map` and `Set`

How do `Map` and `Set` work?



What is the average cost of looking up an element in this list?

Answer: **O(*n*)**.

Intuition: Most elements are far from the front.

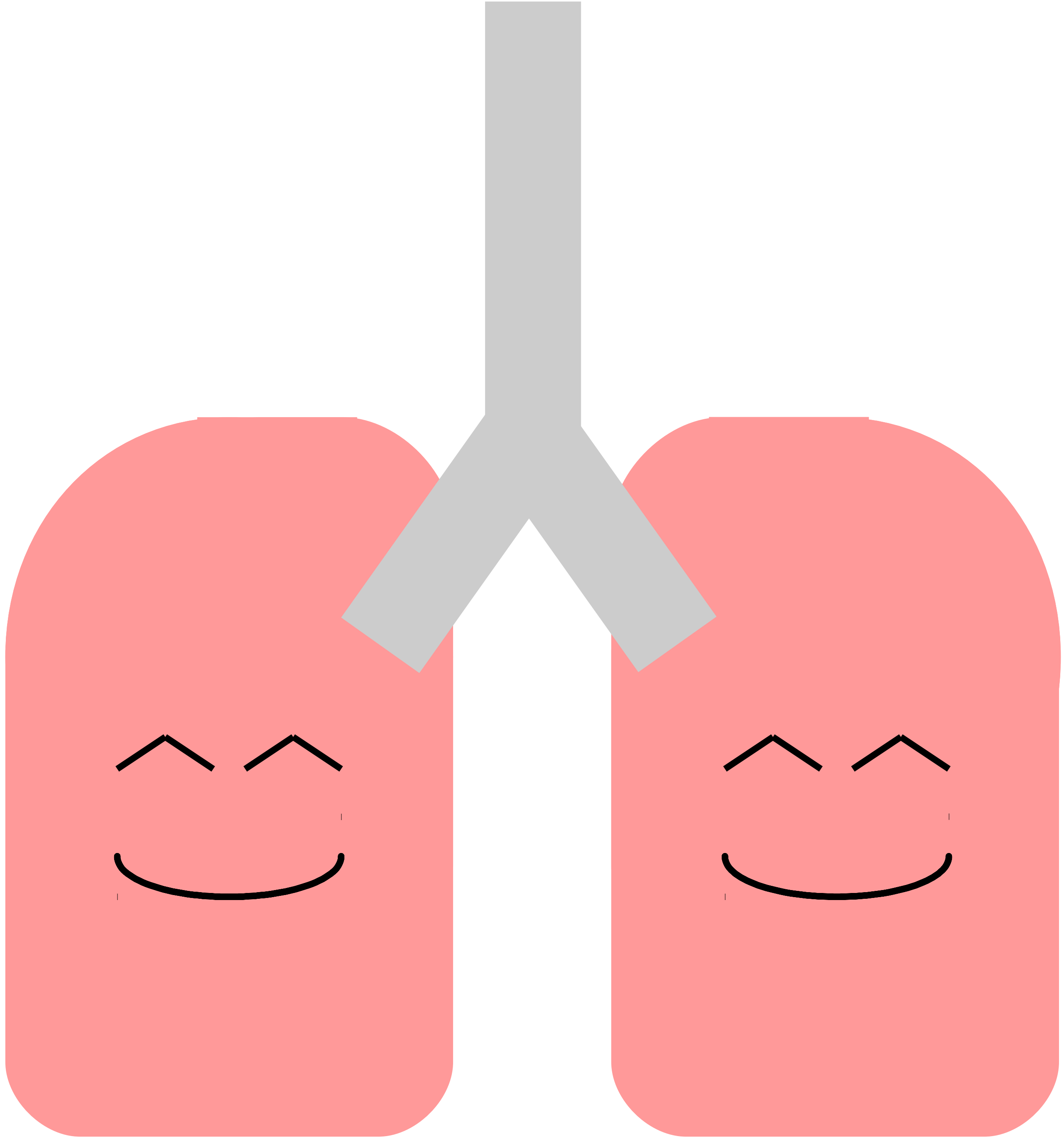
Can you chain a bunch of objects together so that most of them are near the front?

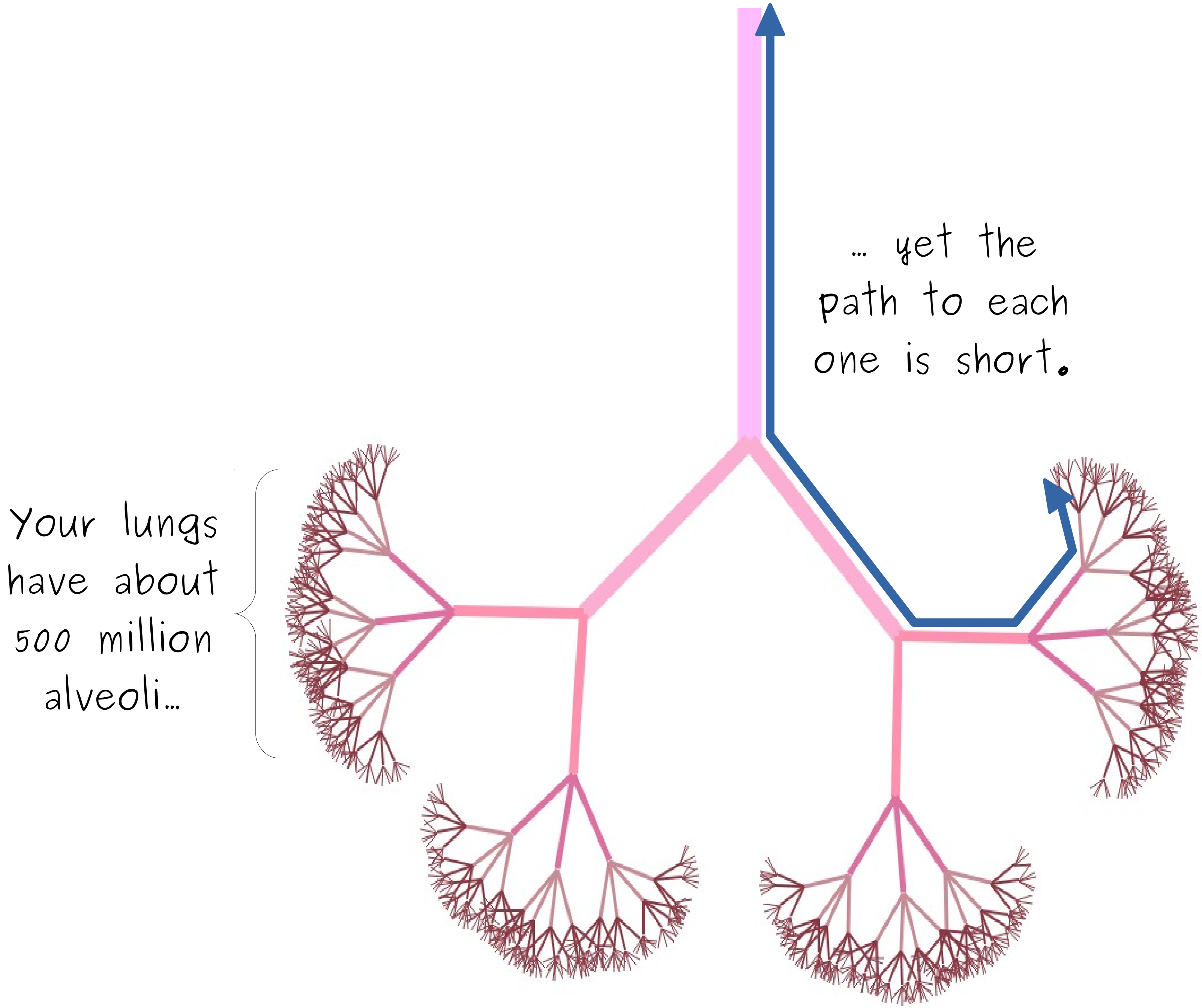
An Interactive Analogy

Take a deep breath.

And exhale.

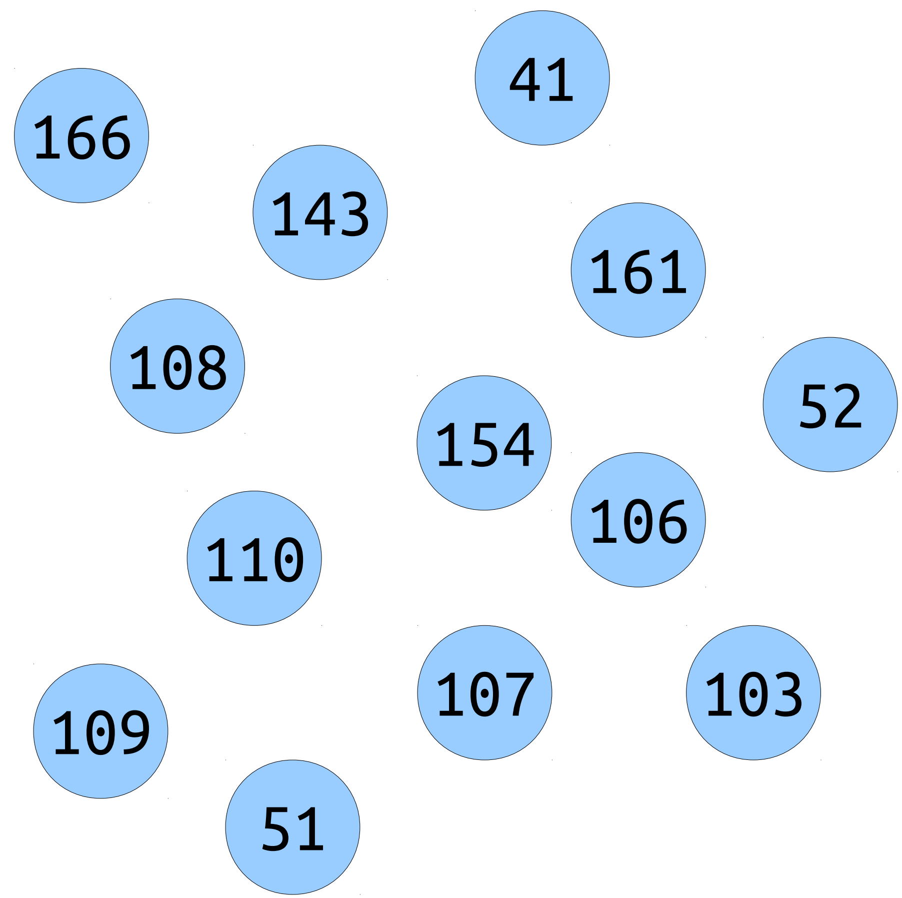
Feel nicely oxygenated?




Your lungs
have about
500 million
alveoli...
... yet the
path to each
one is short.

Key Idea: The distance from each node in a tree to the top (*root*) of the tree is small.

Harnessing this Insight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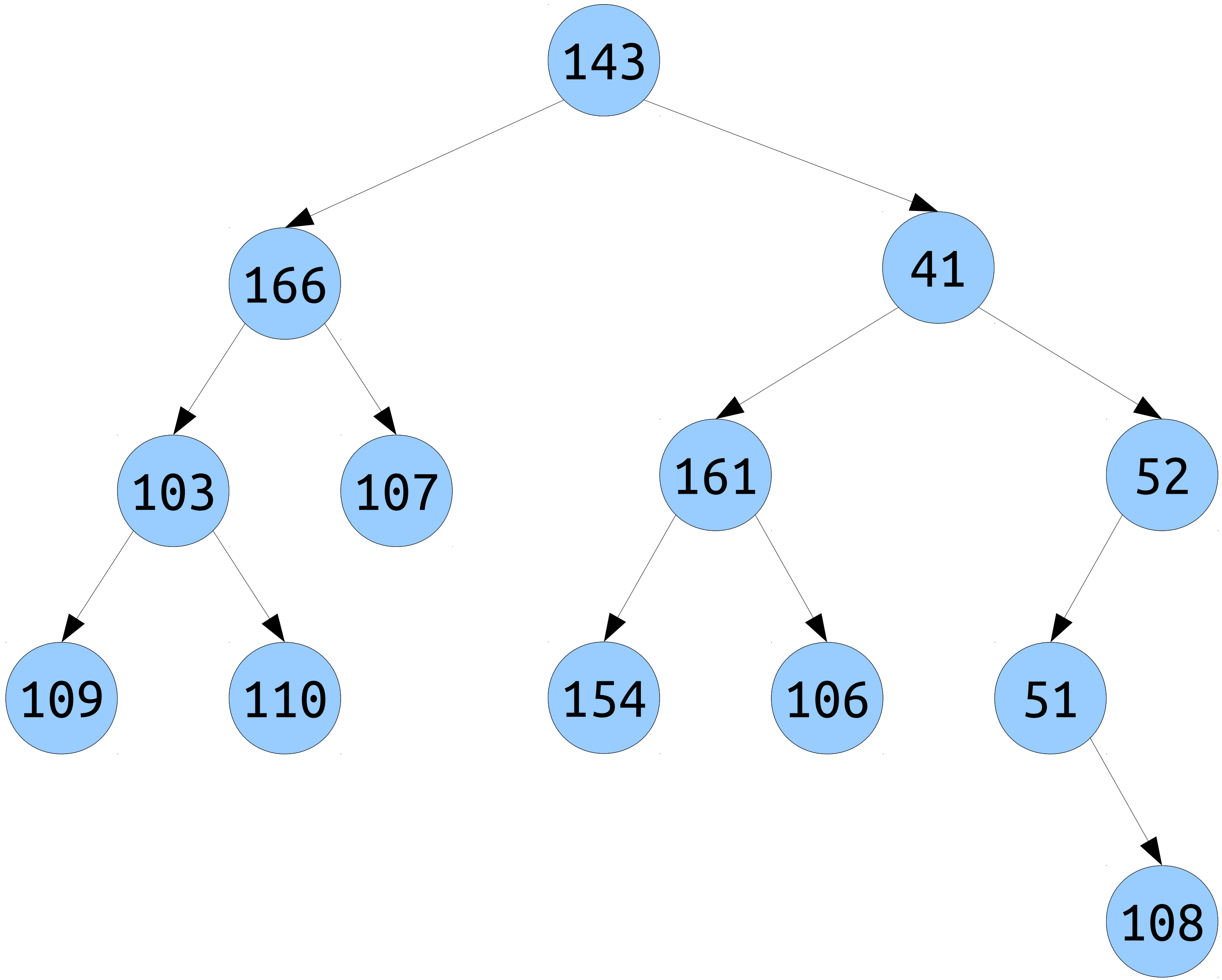
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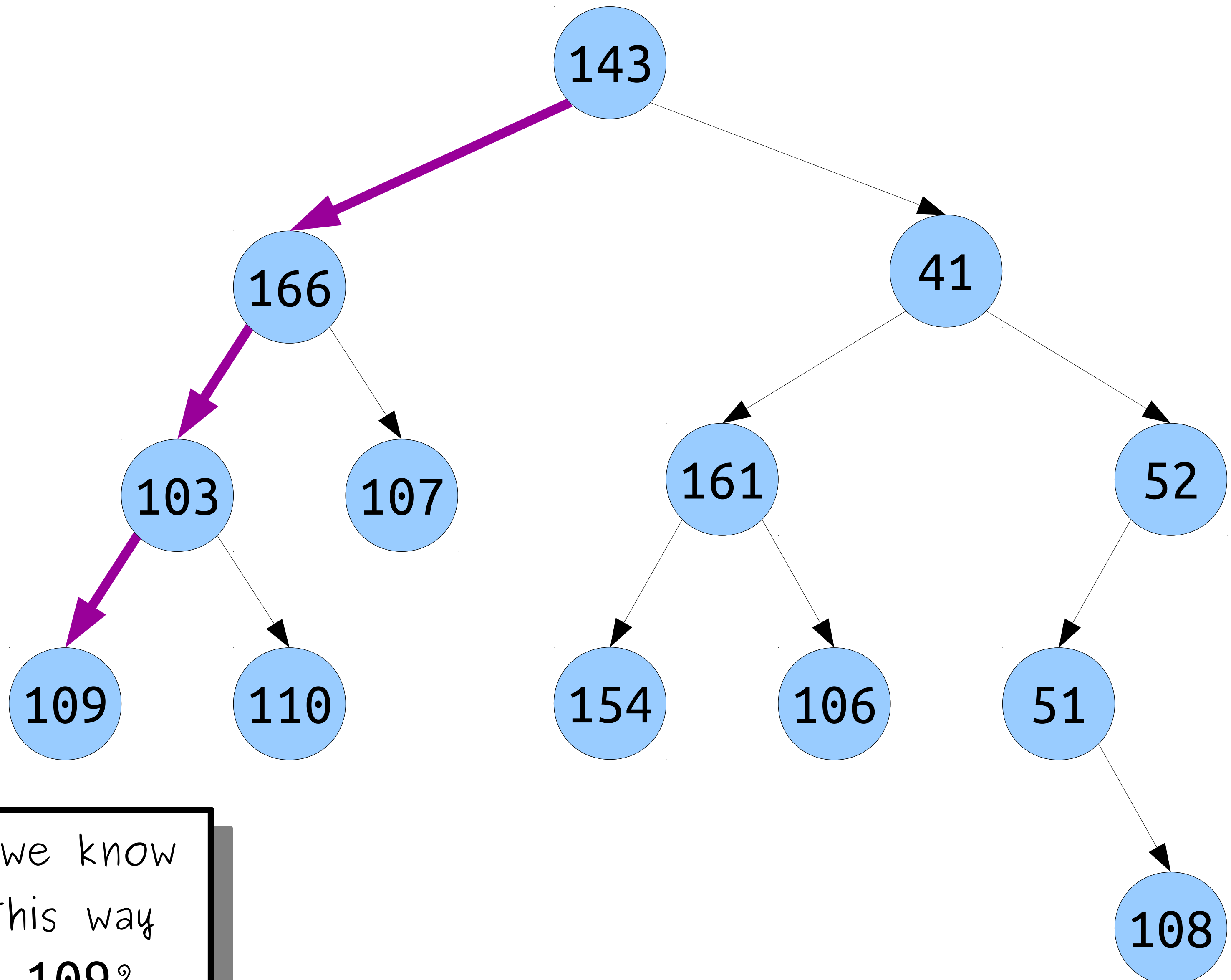
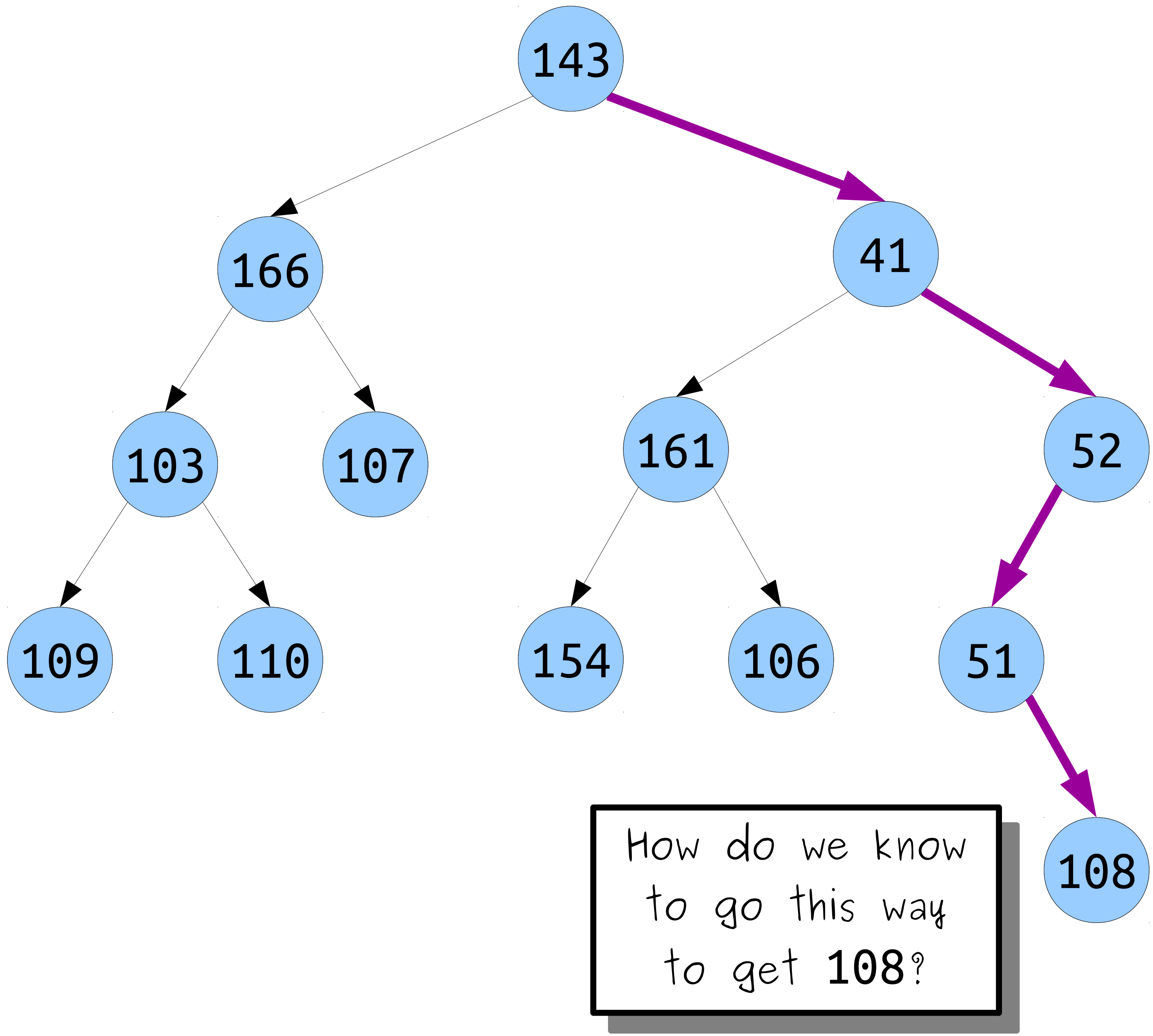


How do we know to go this way to get **109**?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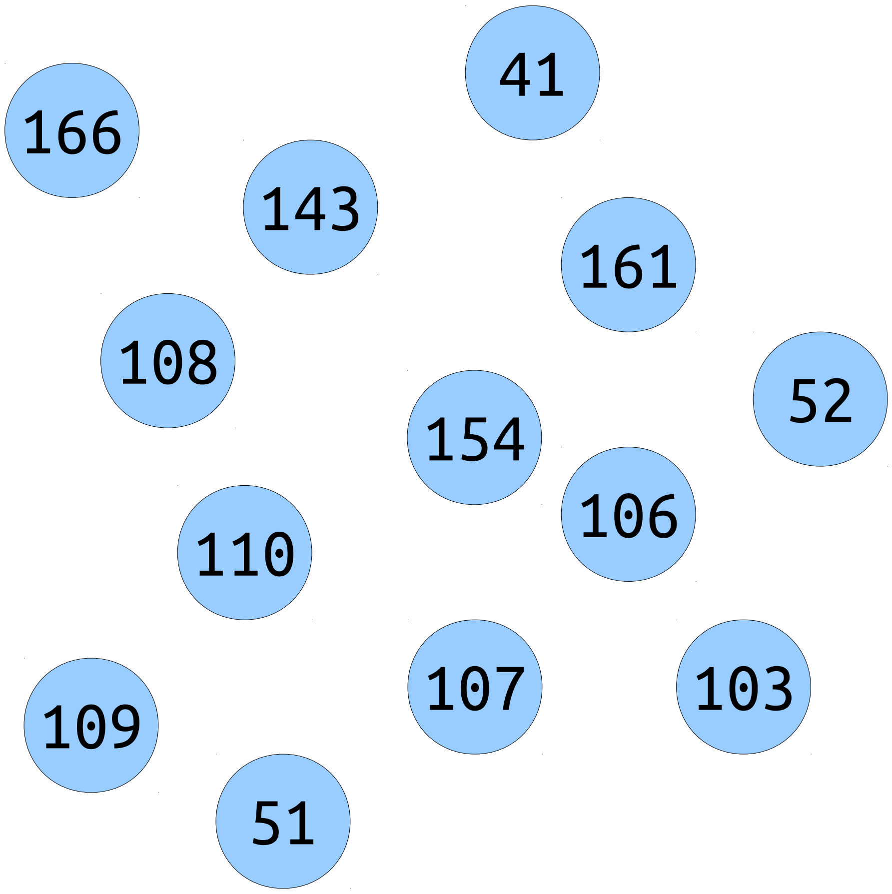
How do we know to go this way to get **108**?

Goal: Store elements in a tree structure where there's an easy way to find them.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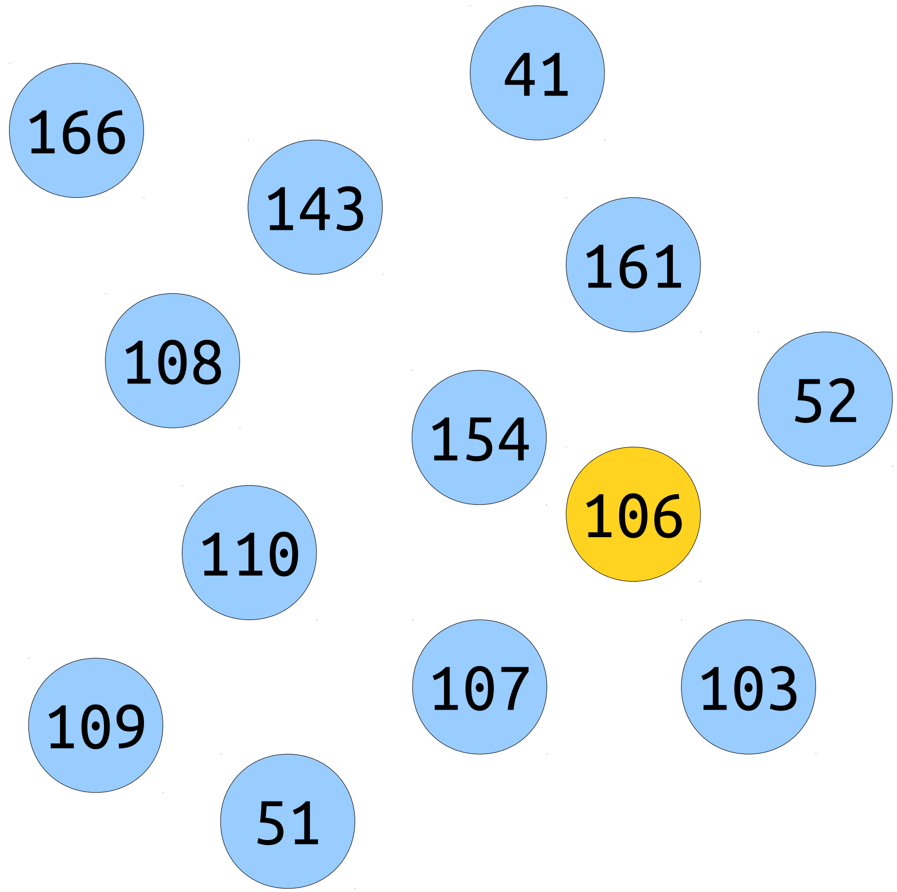
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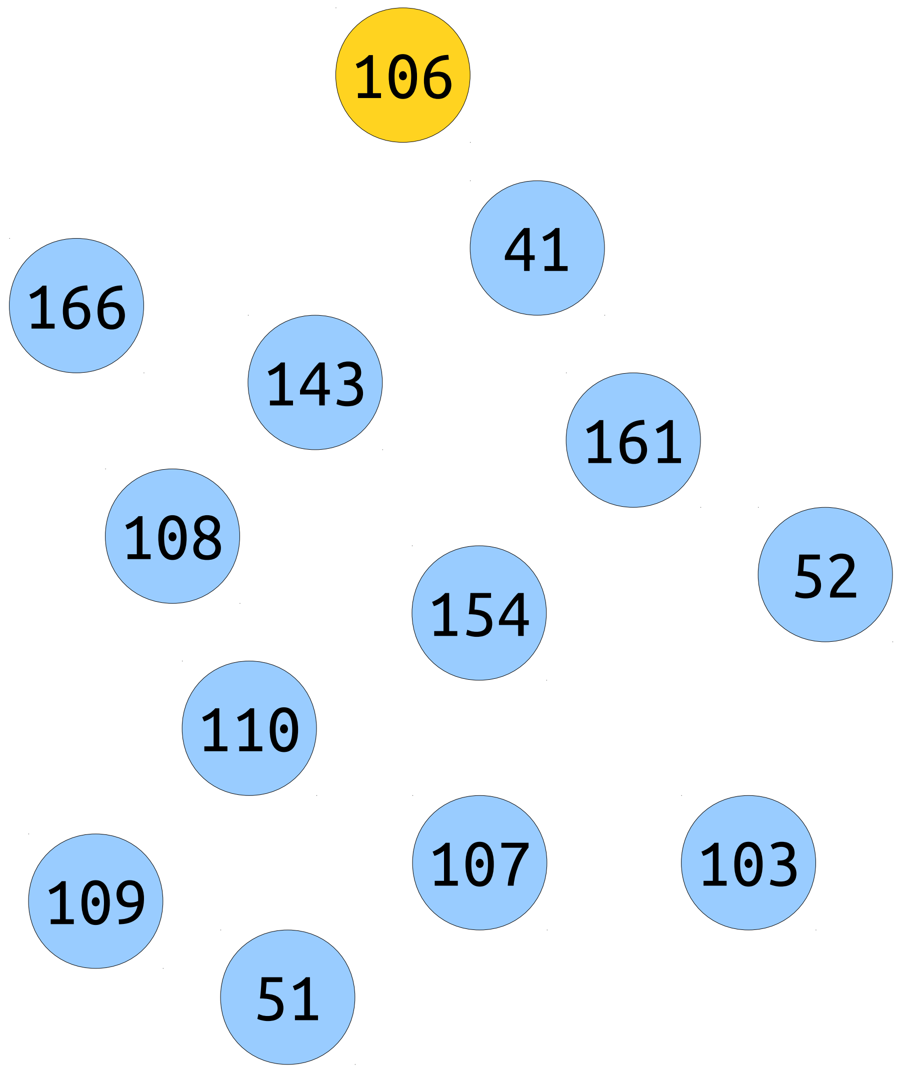
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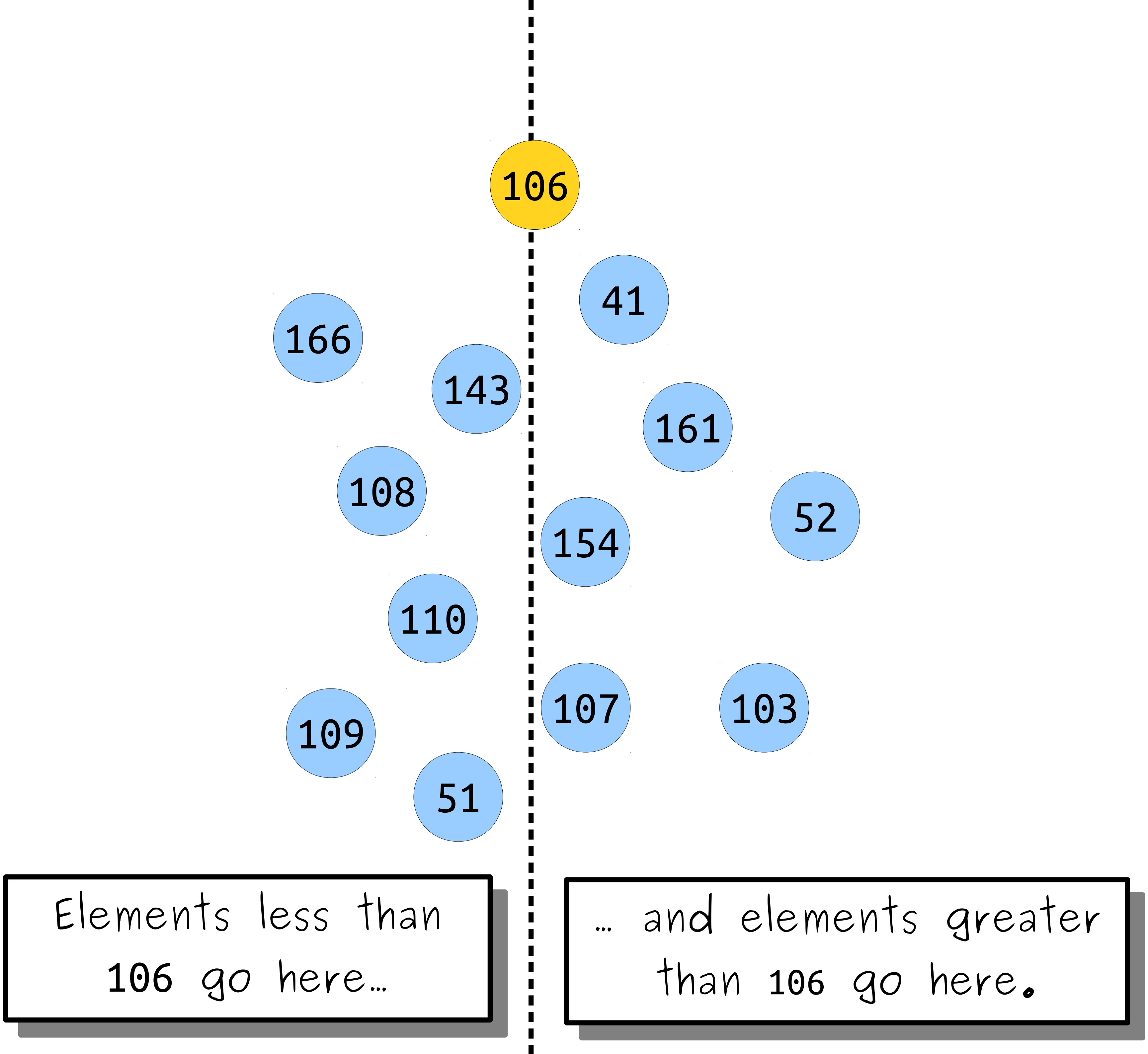
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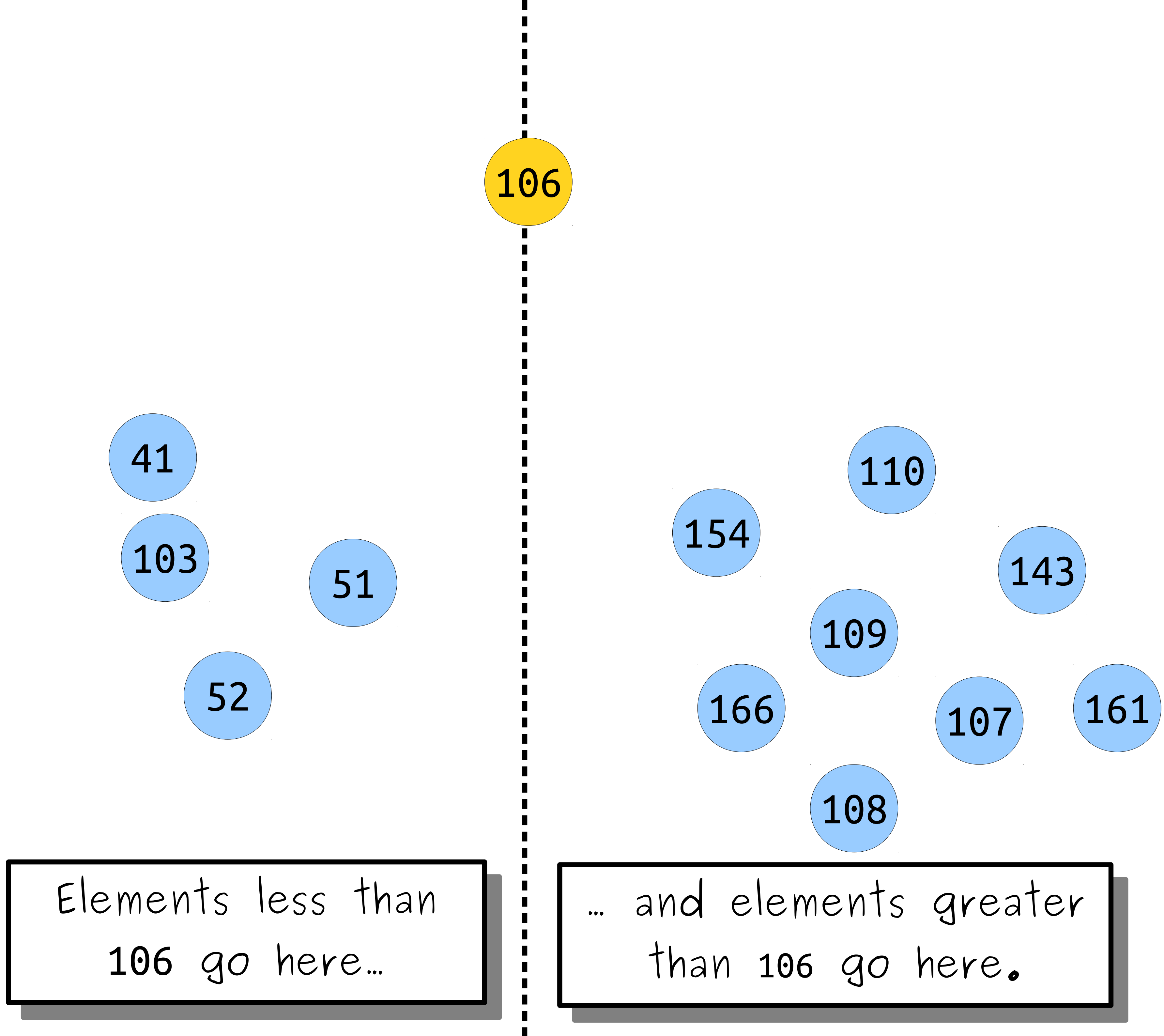
Elements less than 106 go here...
... and elements greater than 106 go here.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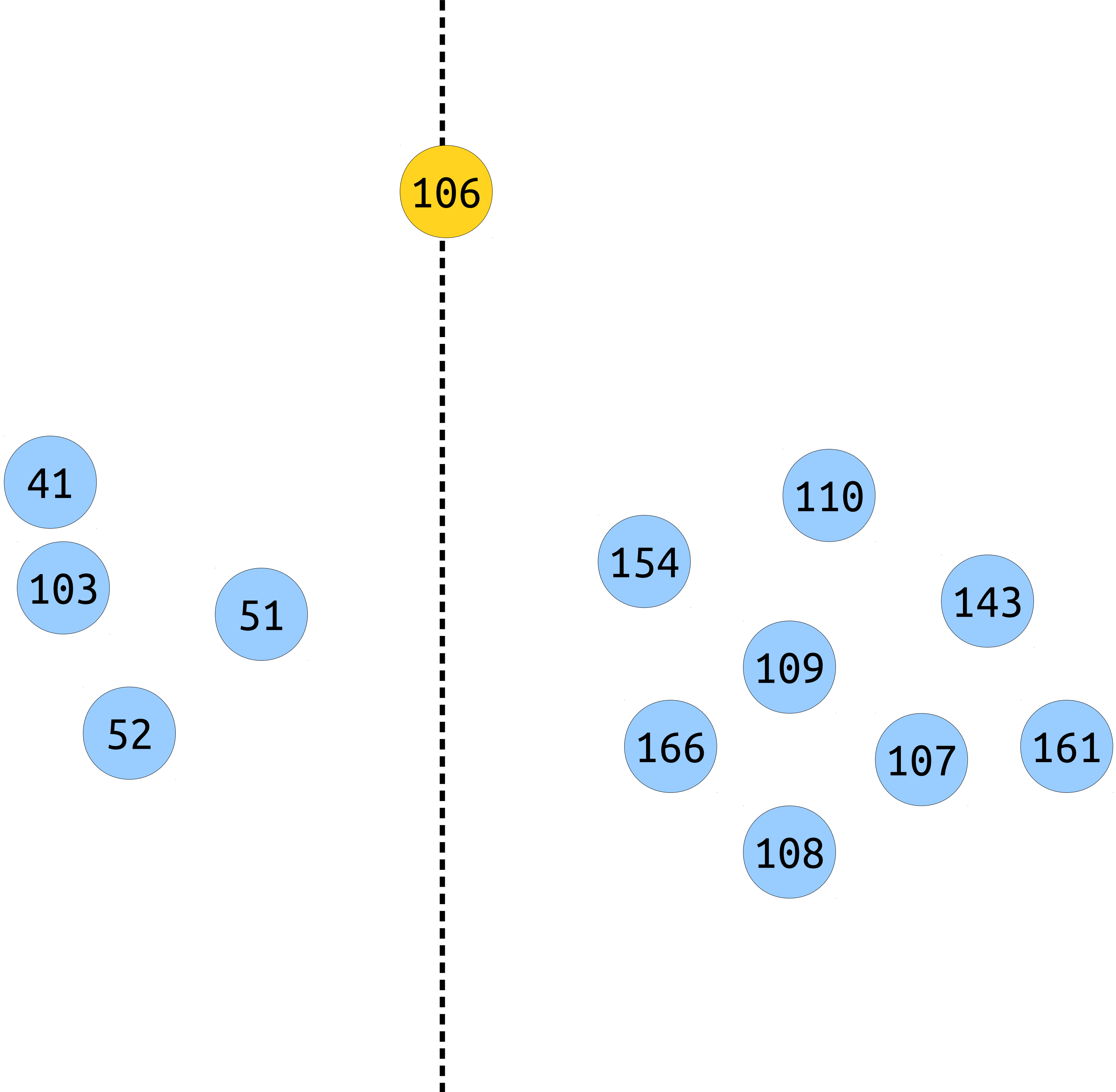
Elements less than 106 go here…
… and elements greater than 106 go here.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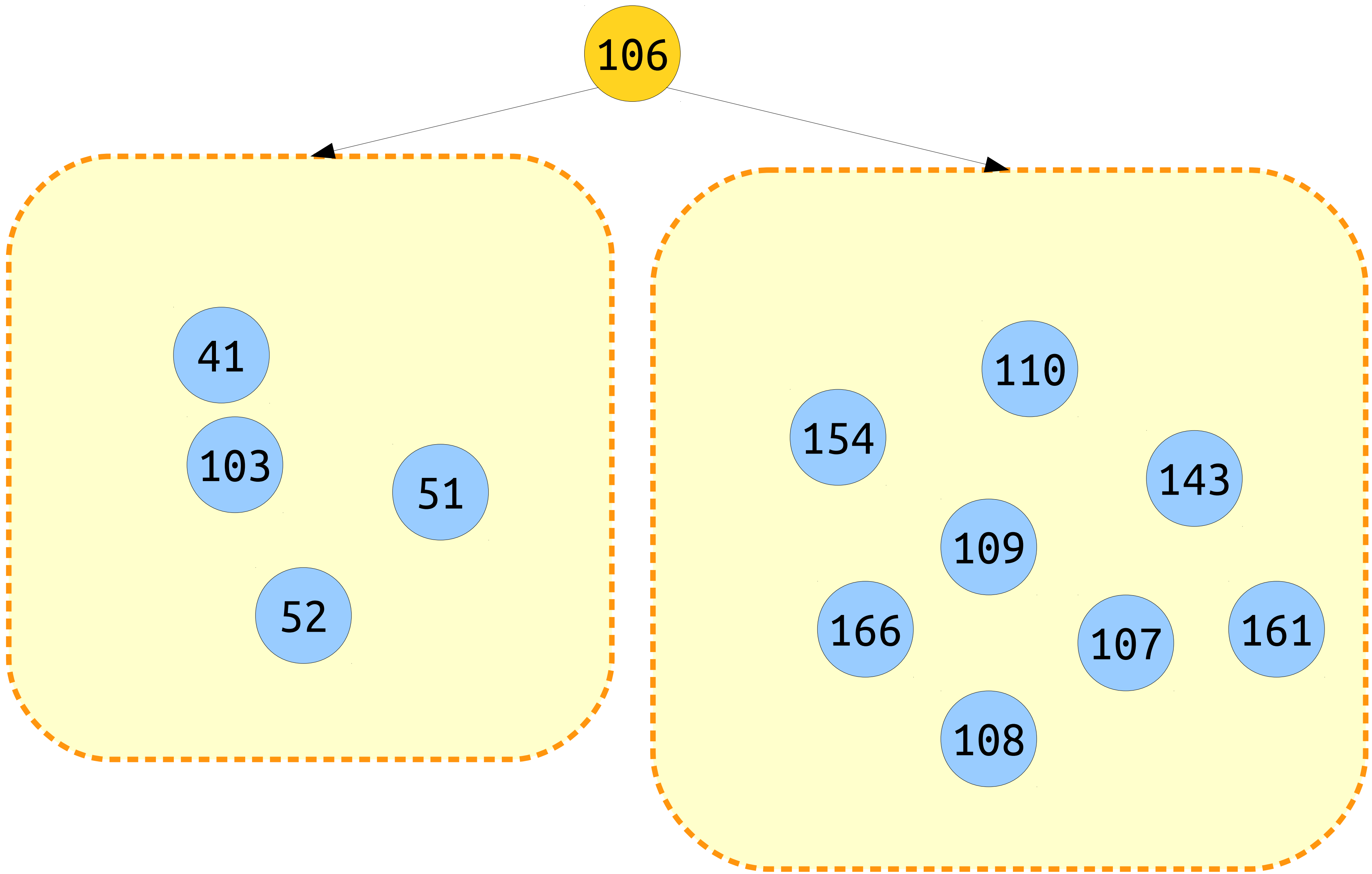
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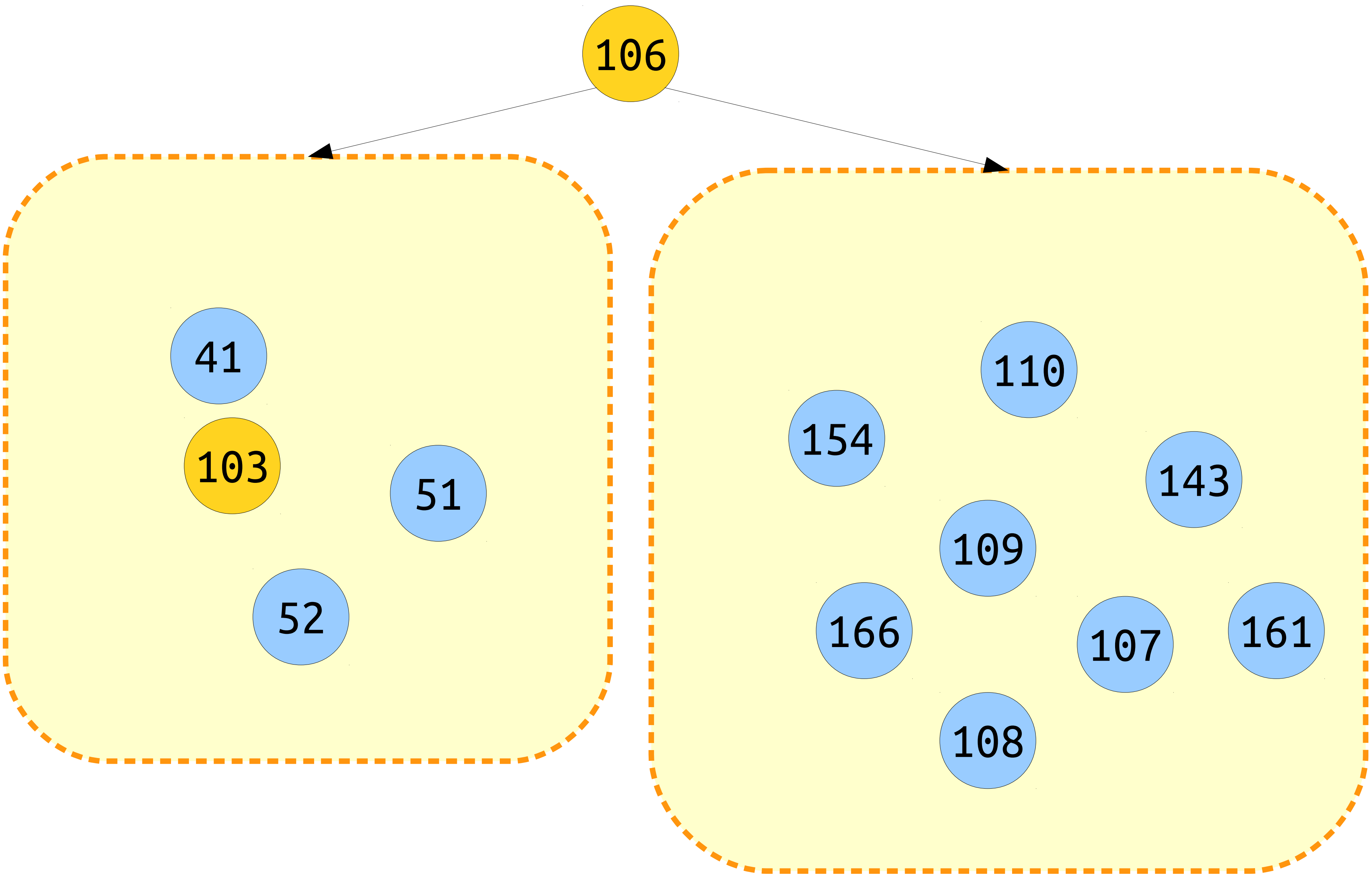
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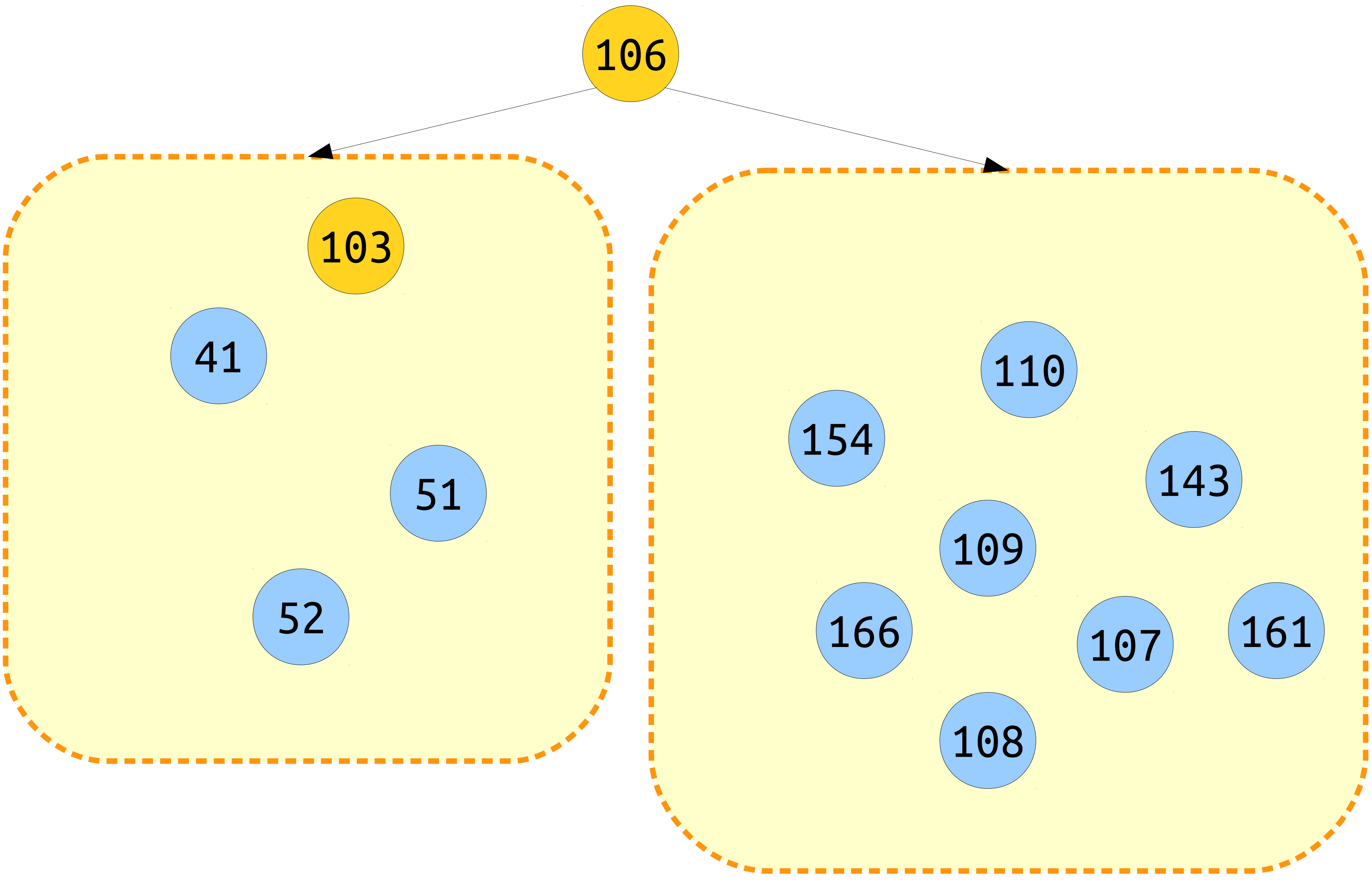
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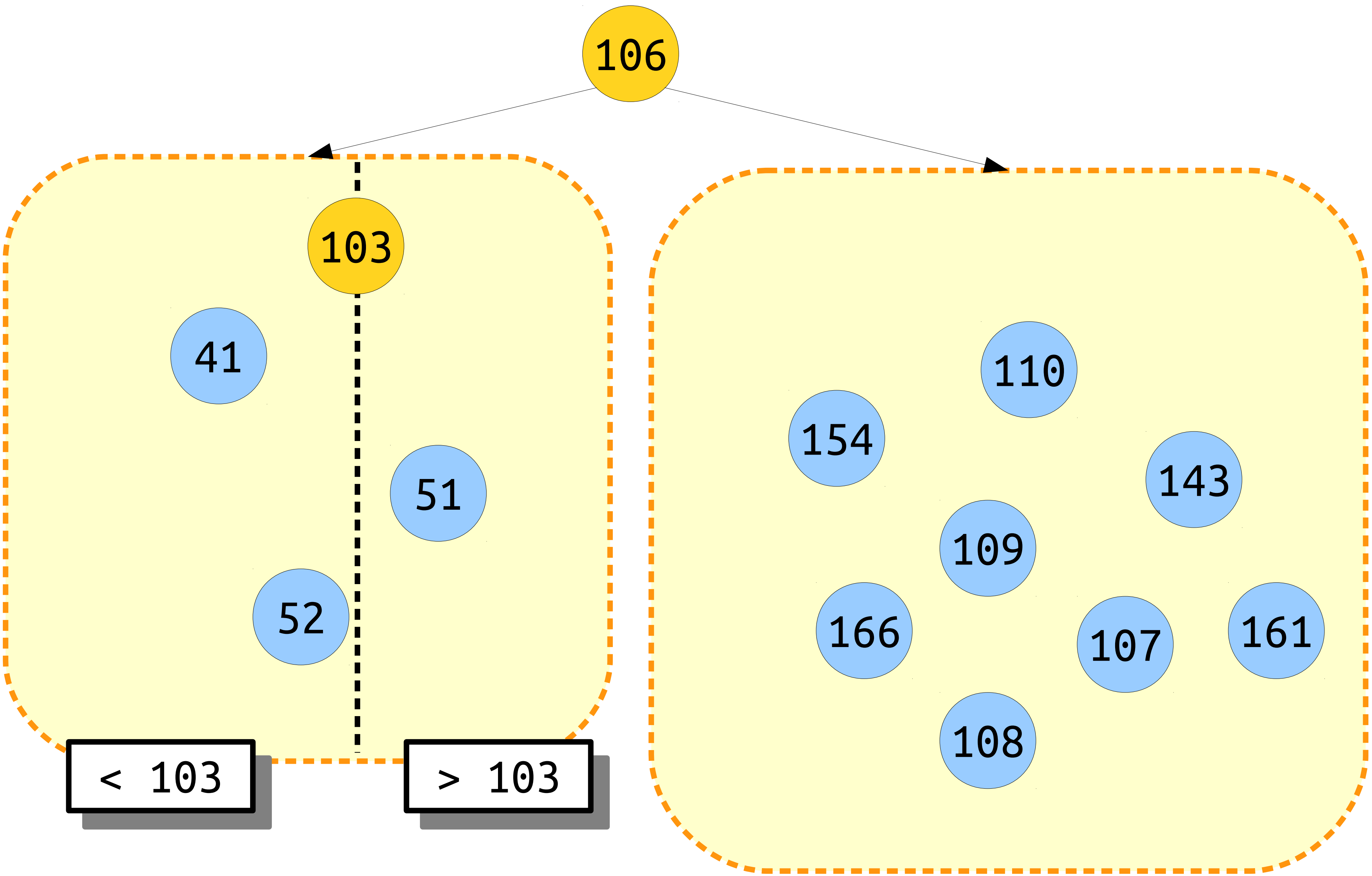
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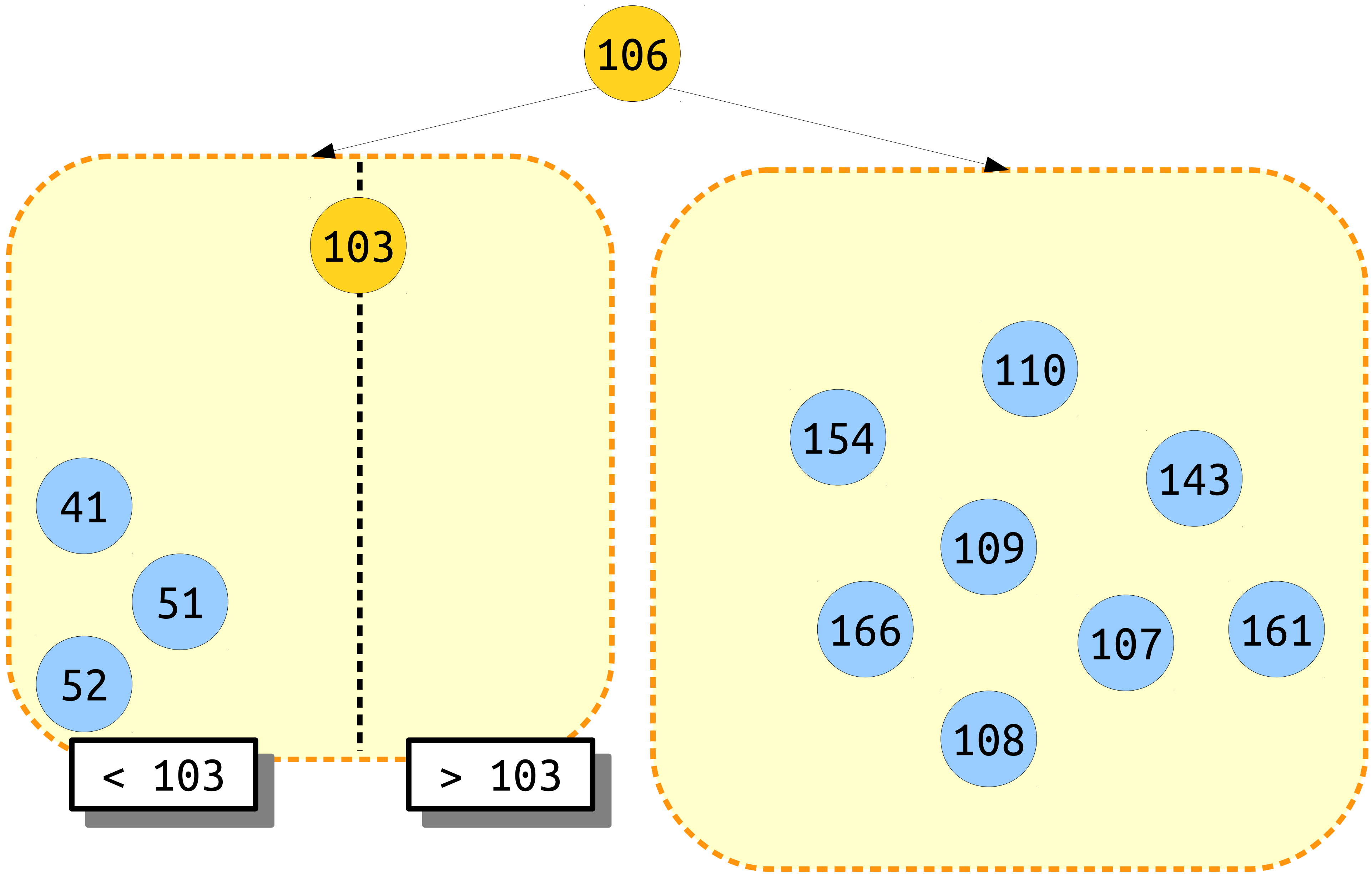
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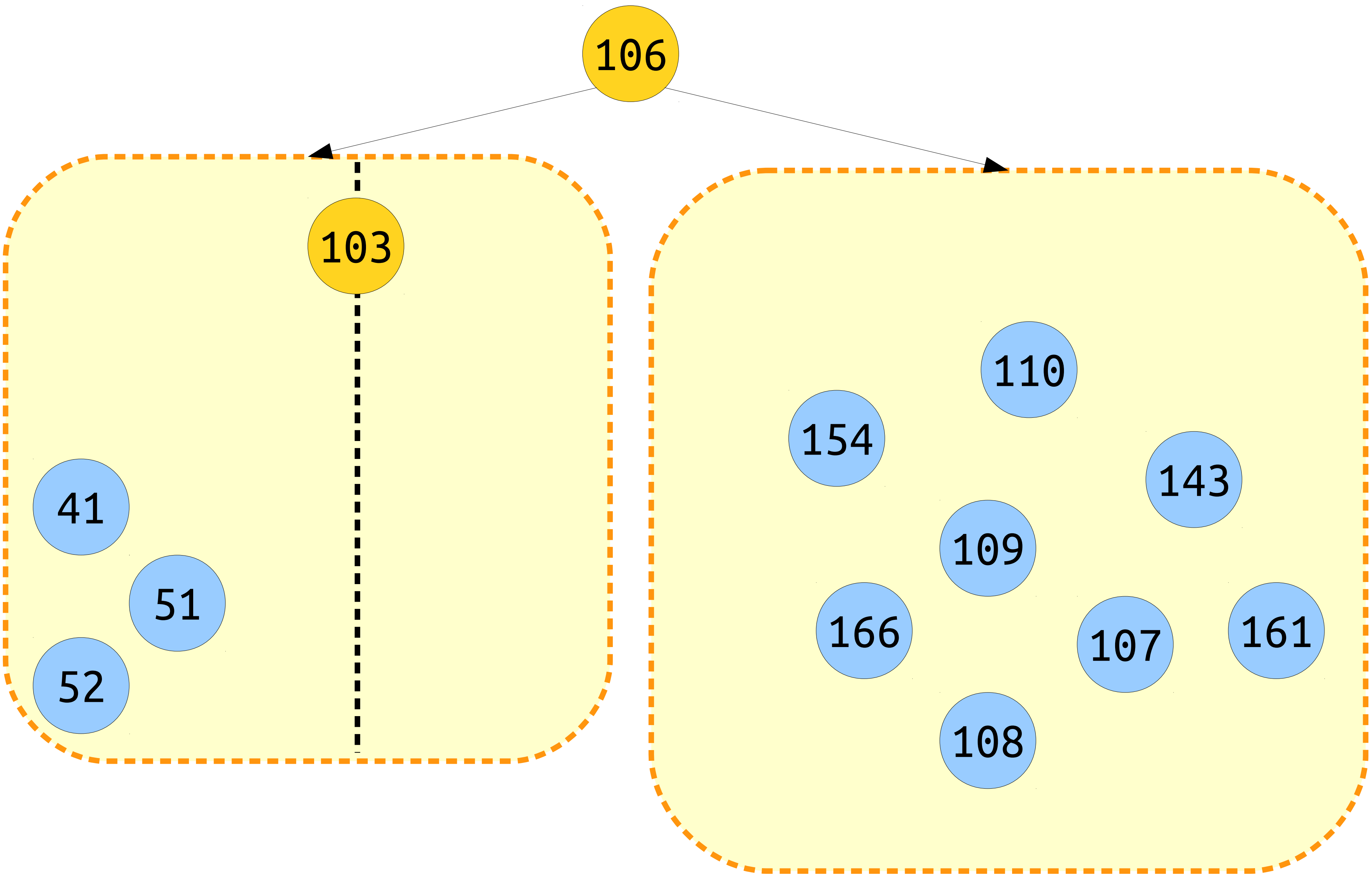
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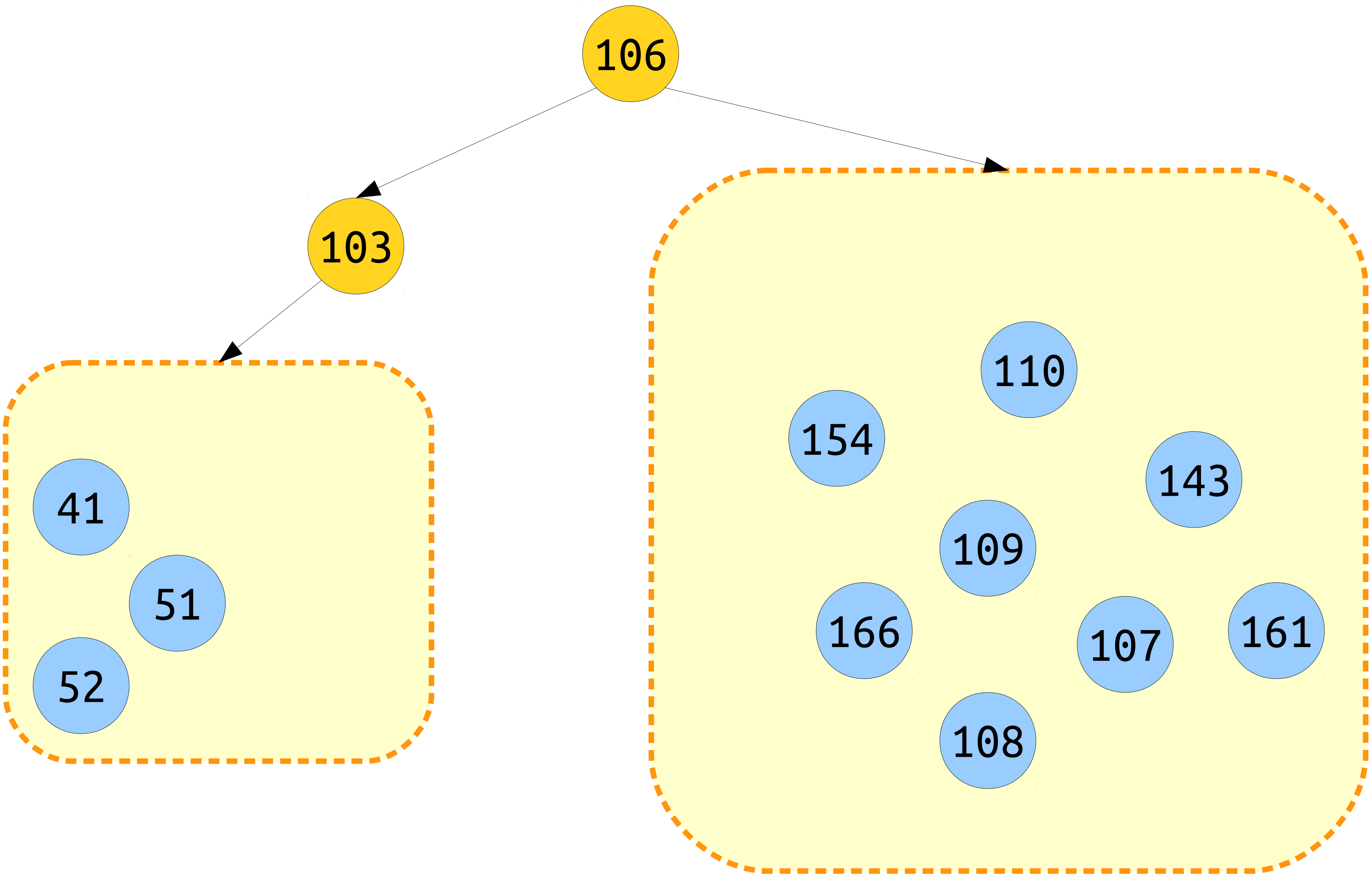
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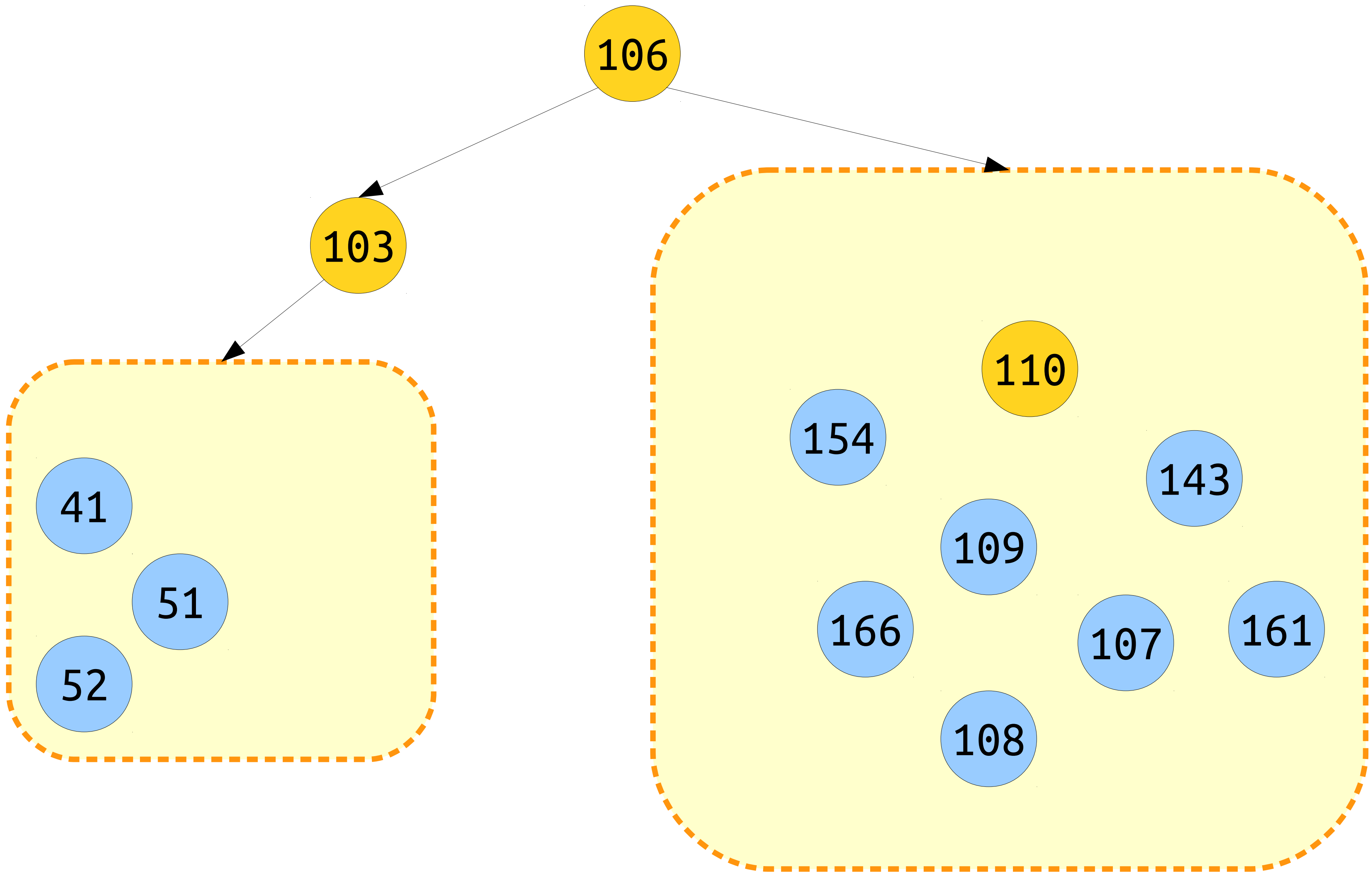
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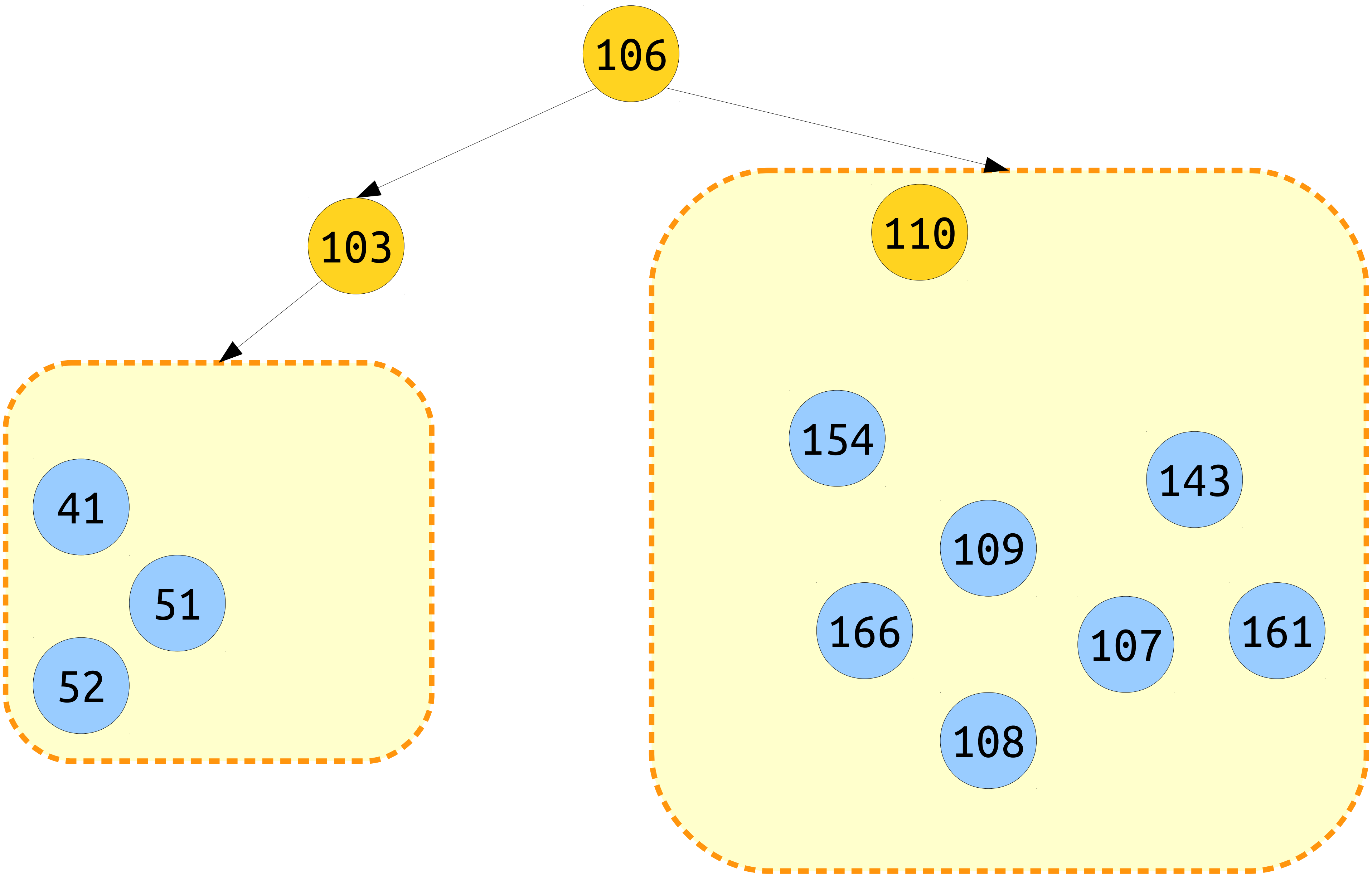

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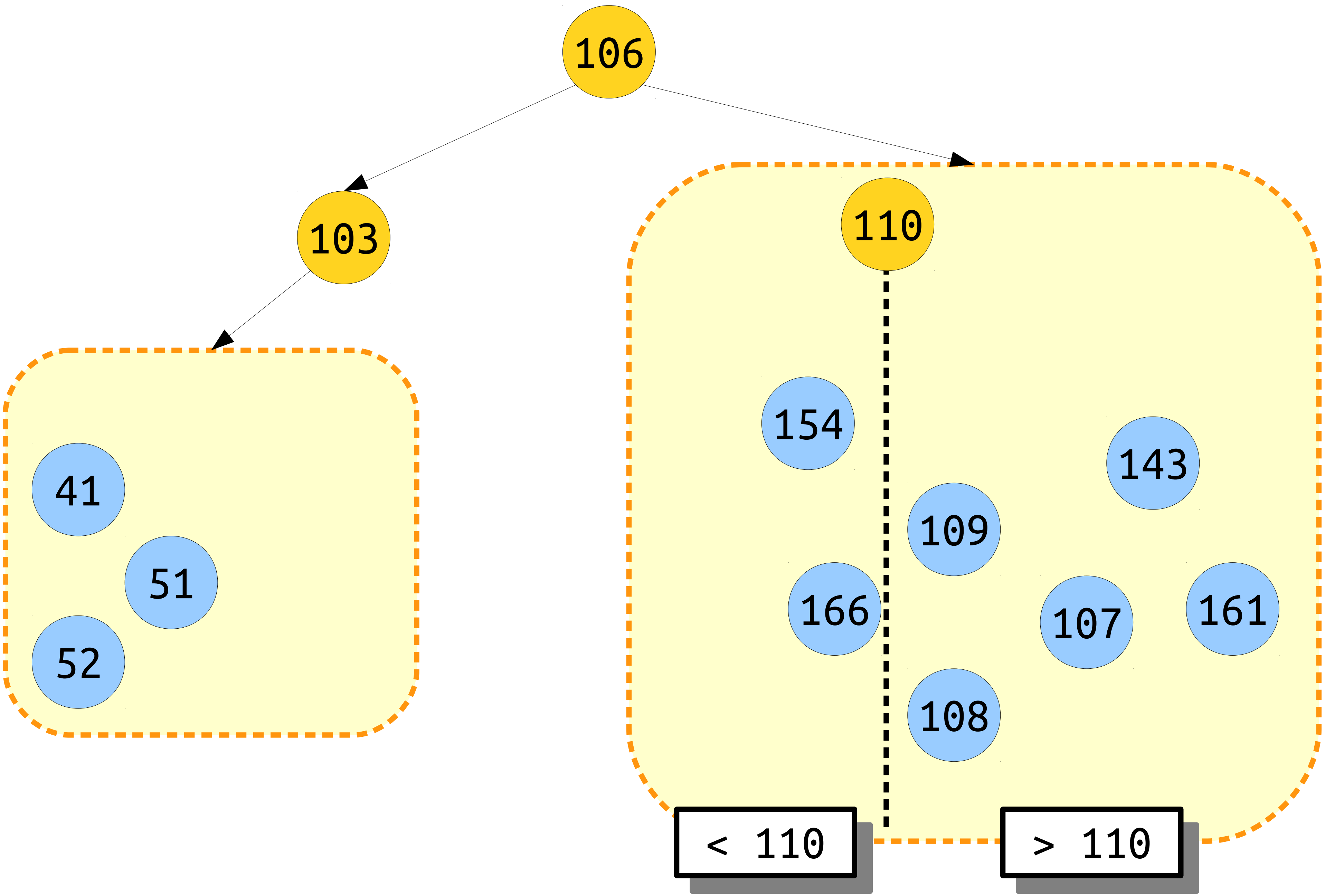
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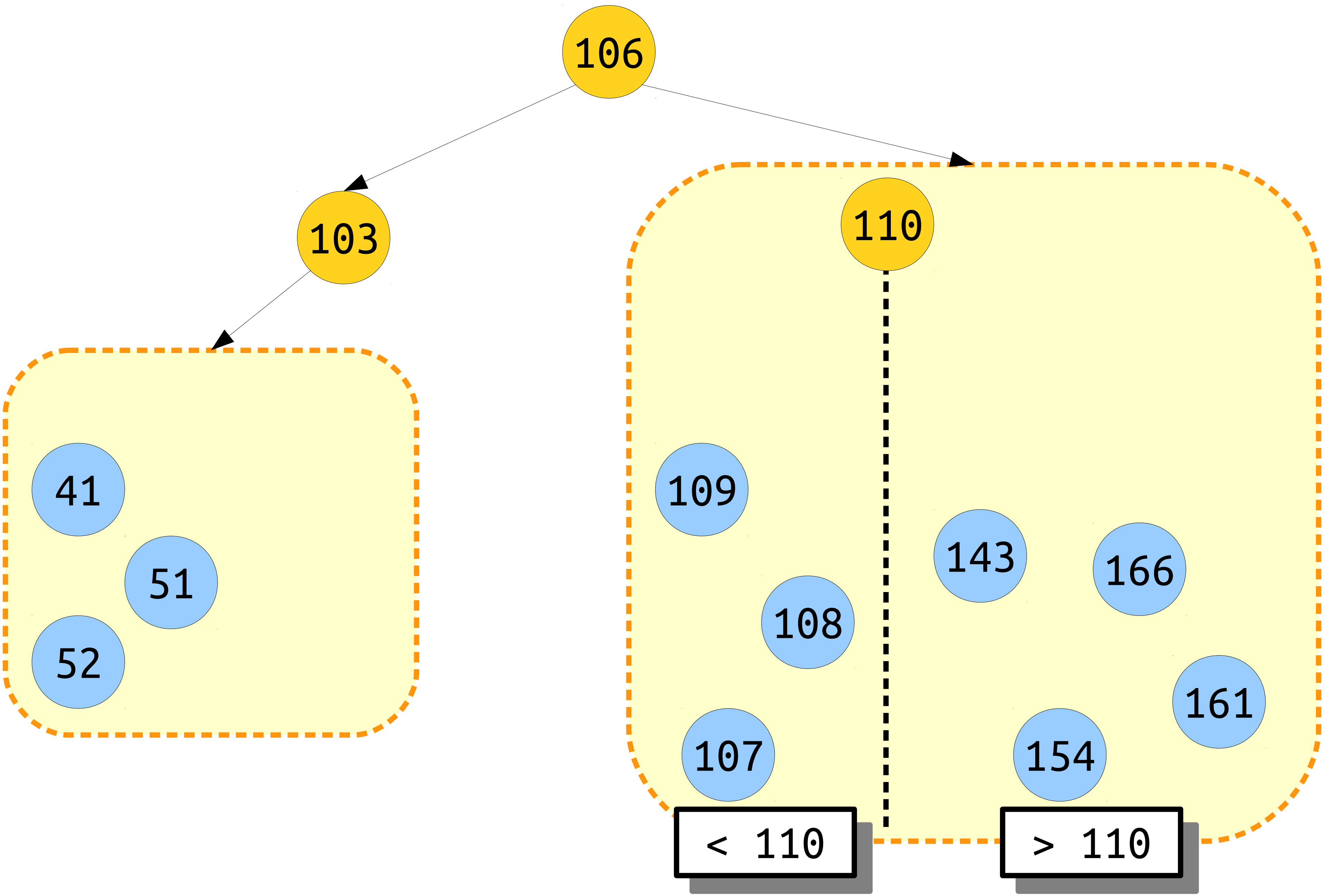

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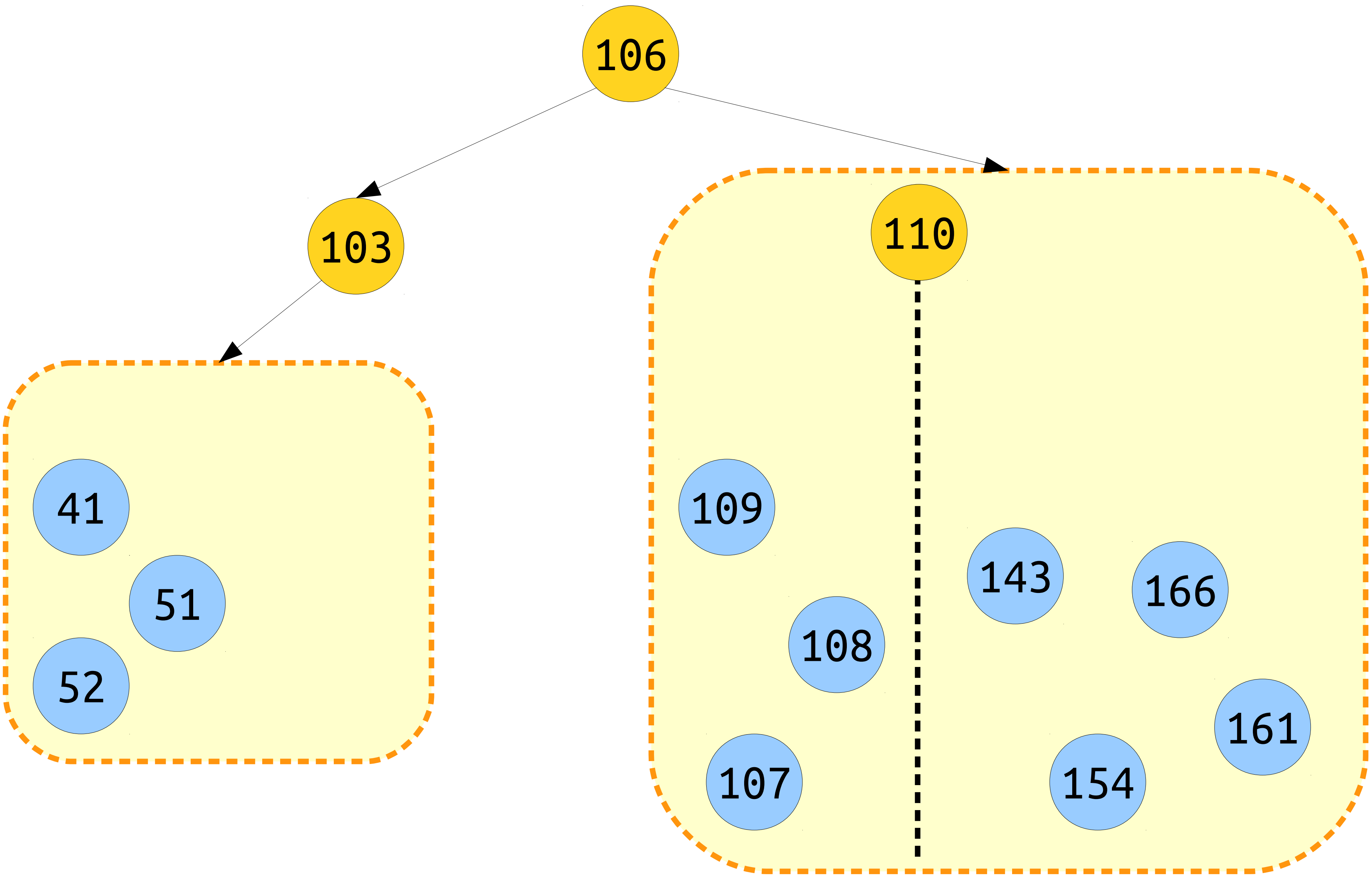
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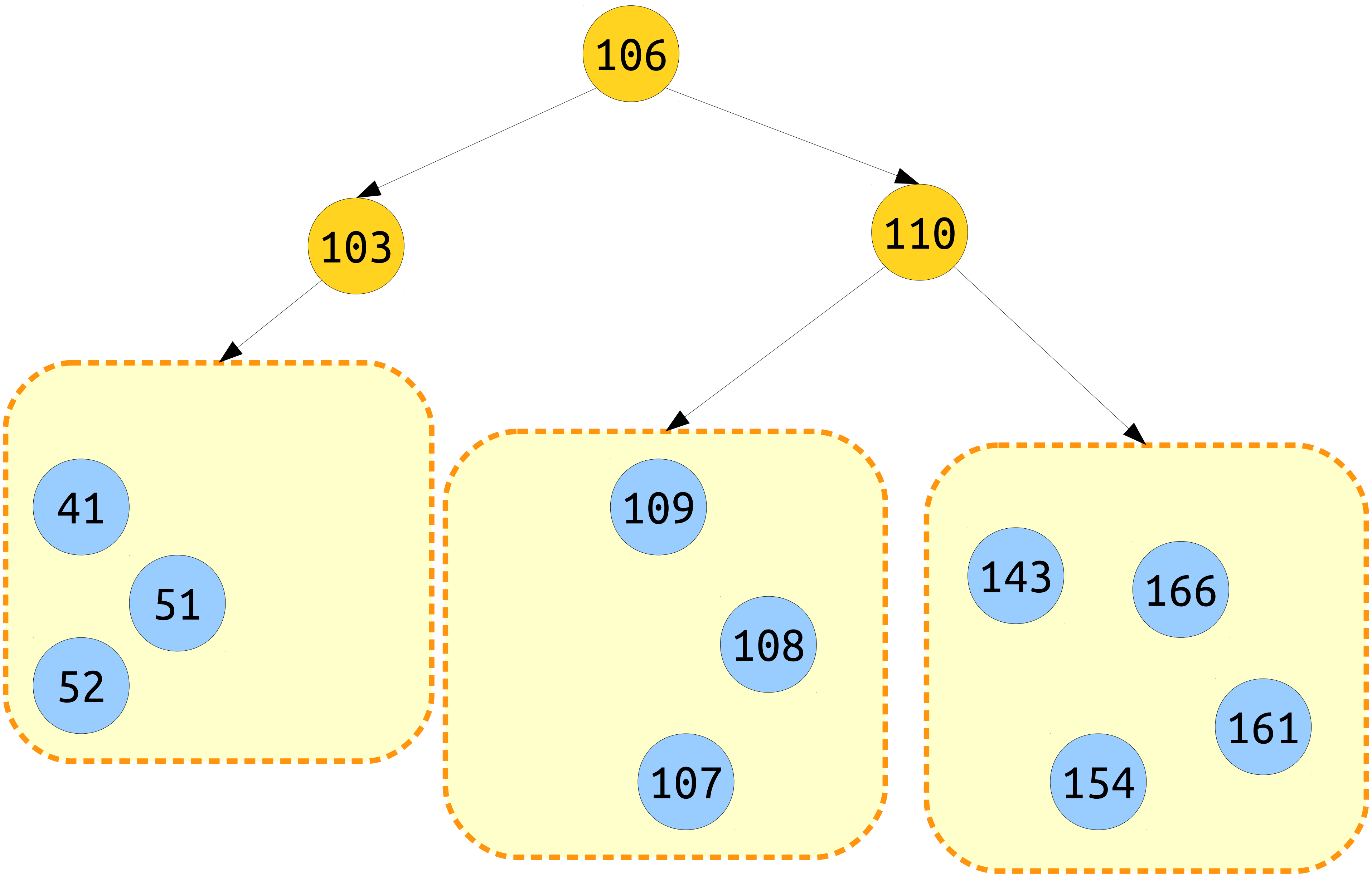
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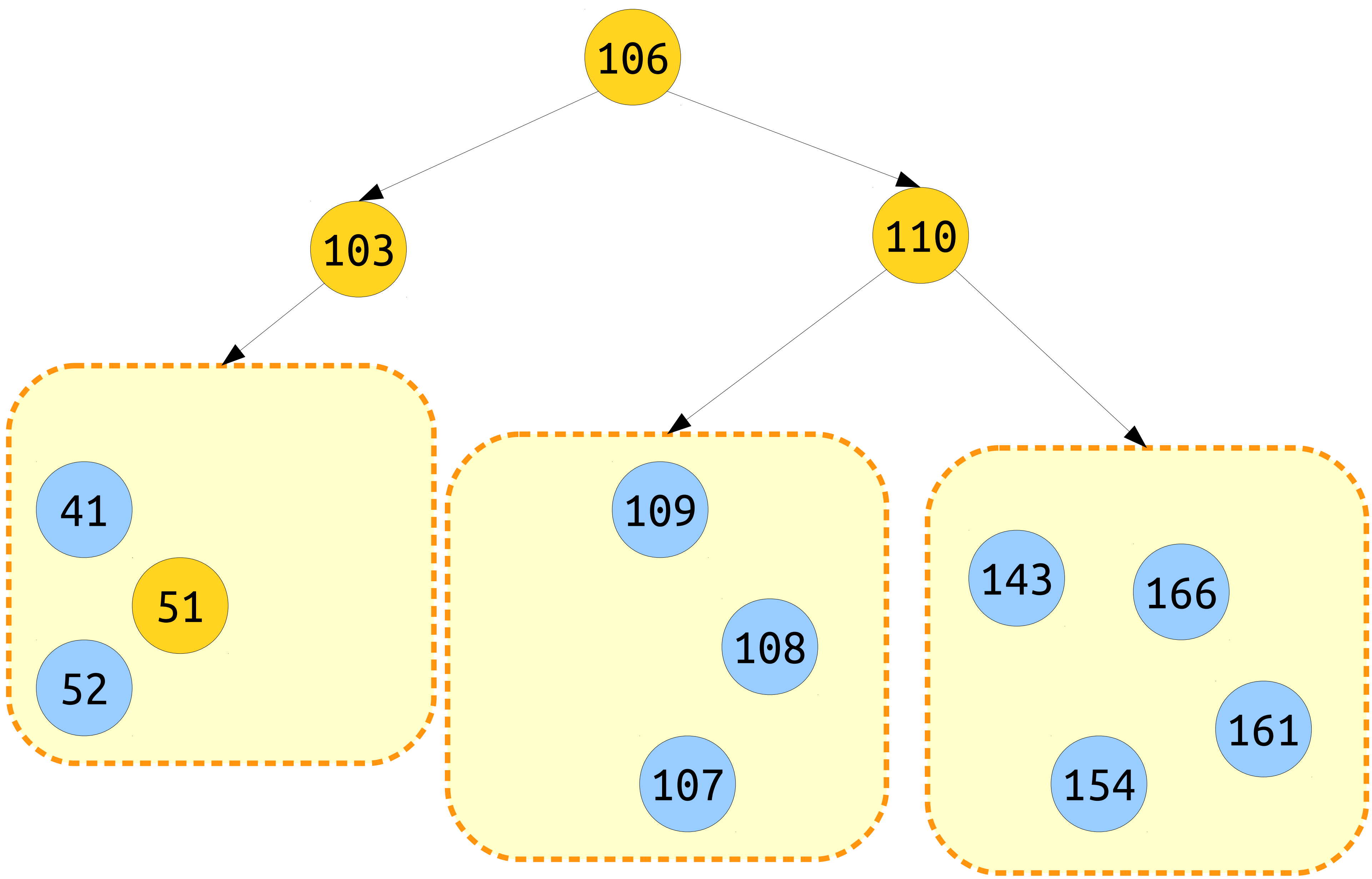
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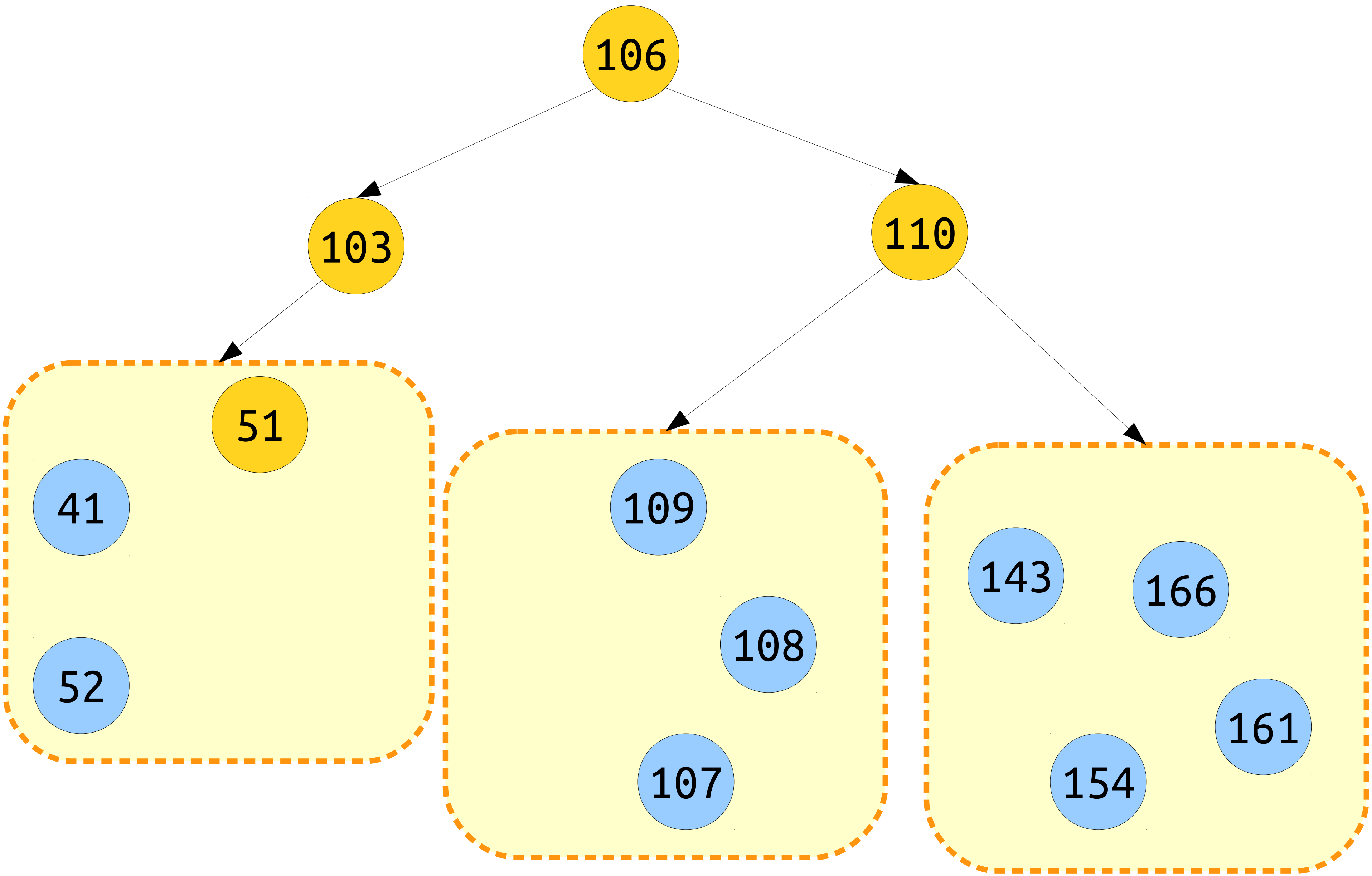
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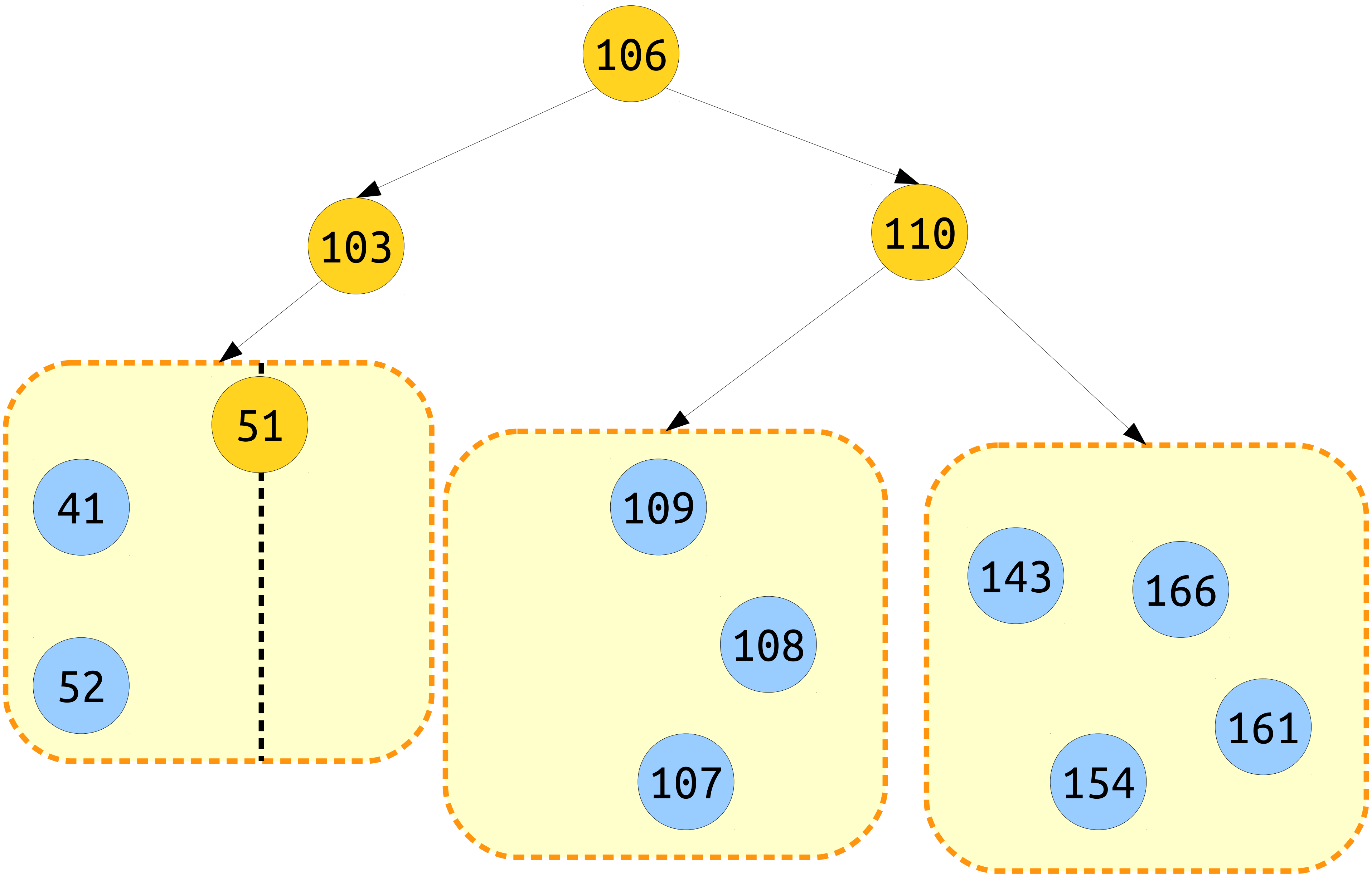
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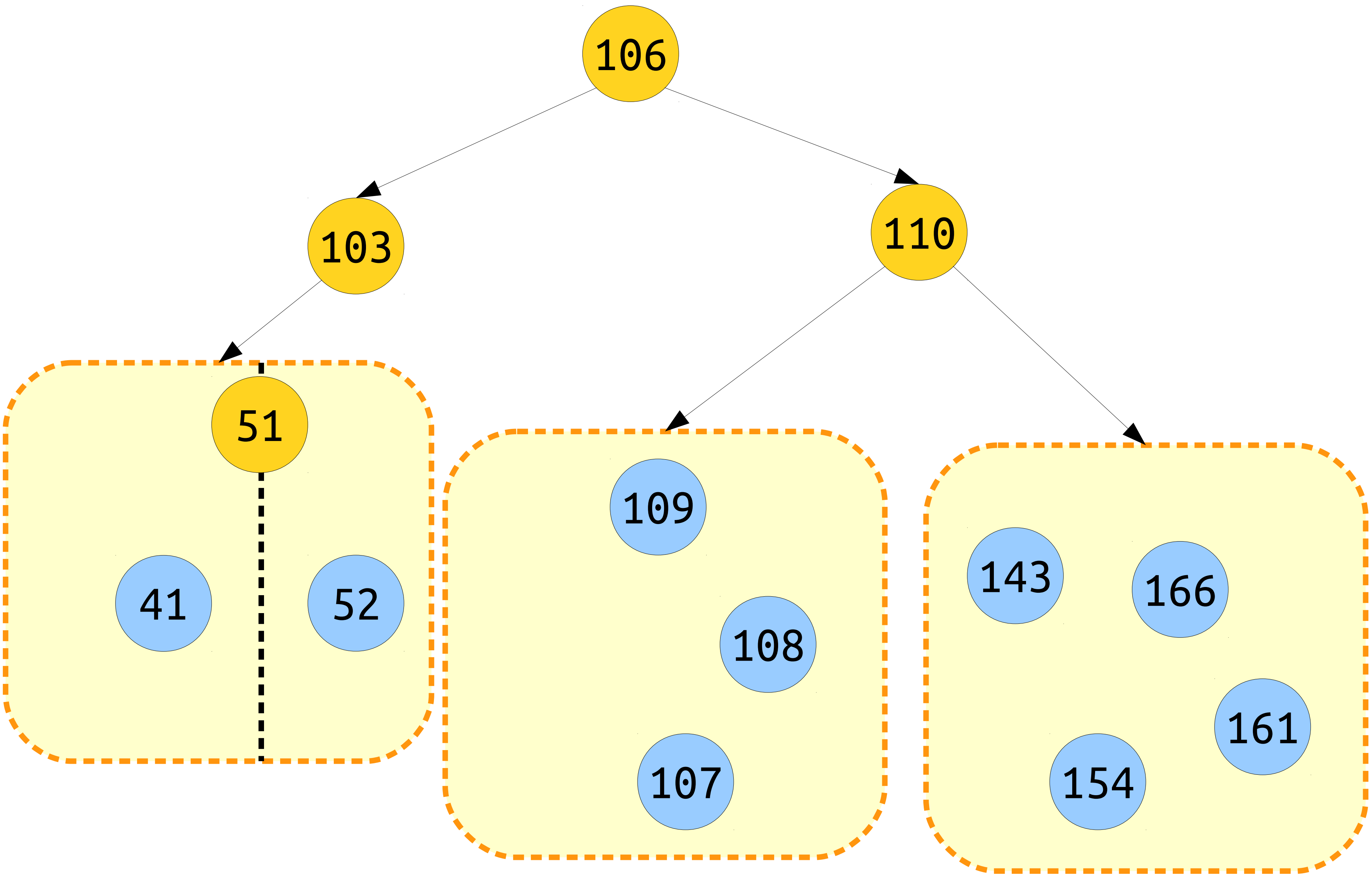
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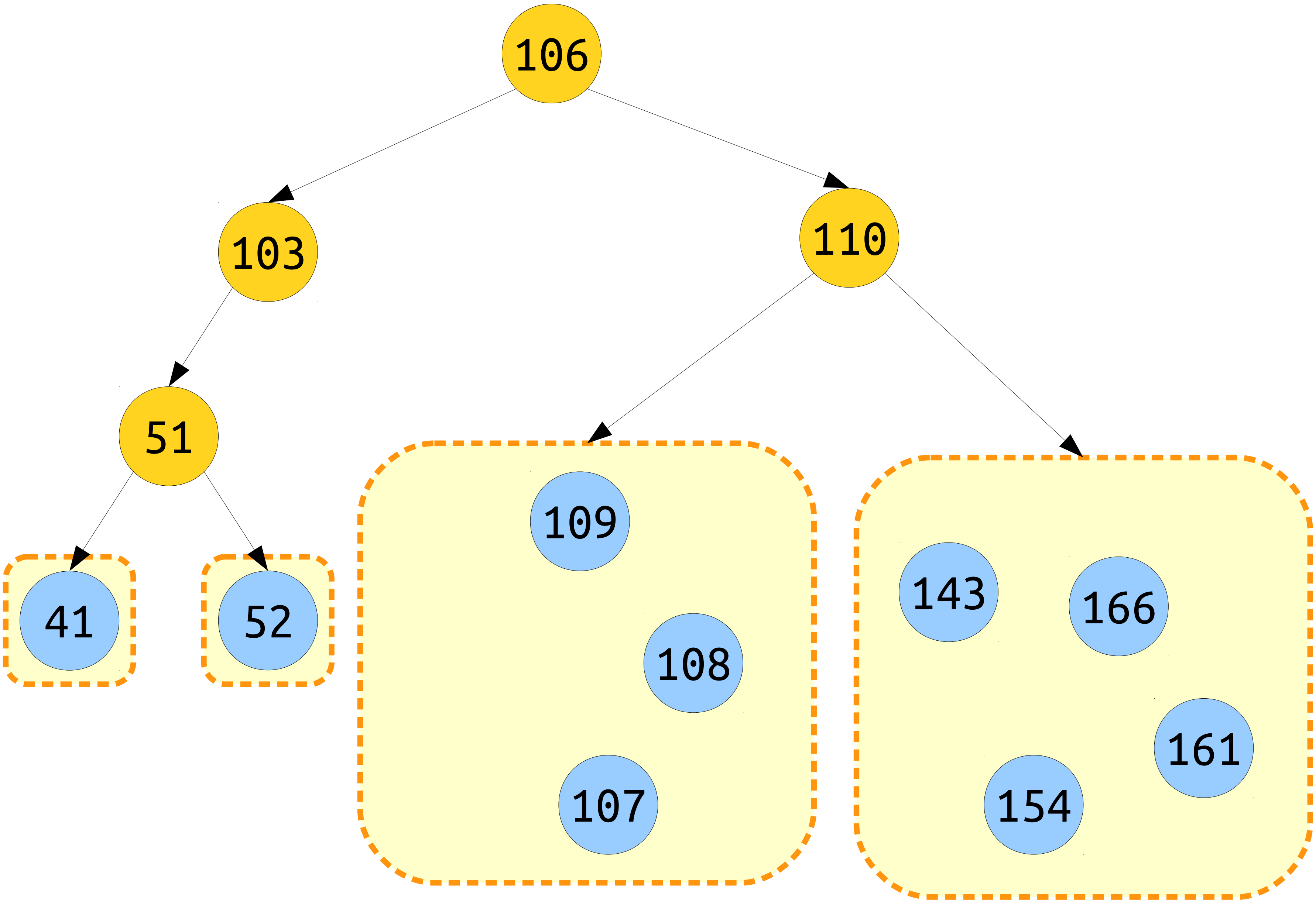

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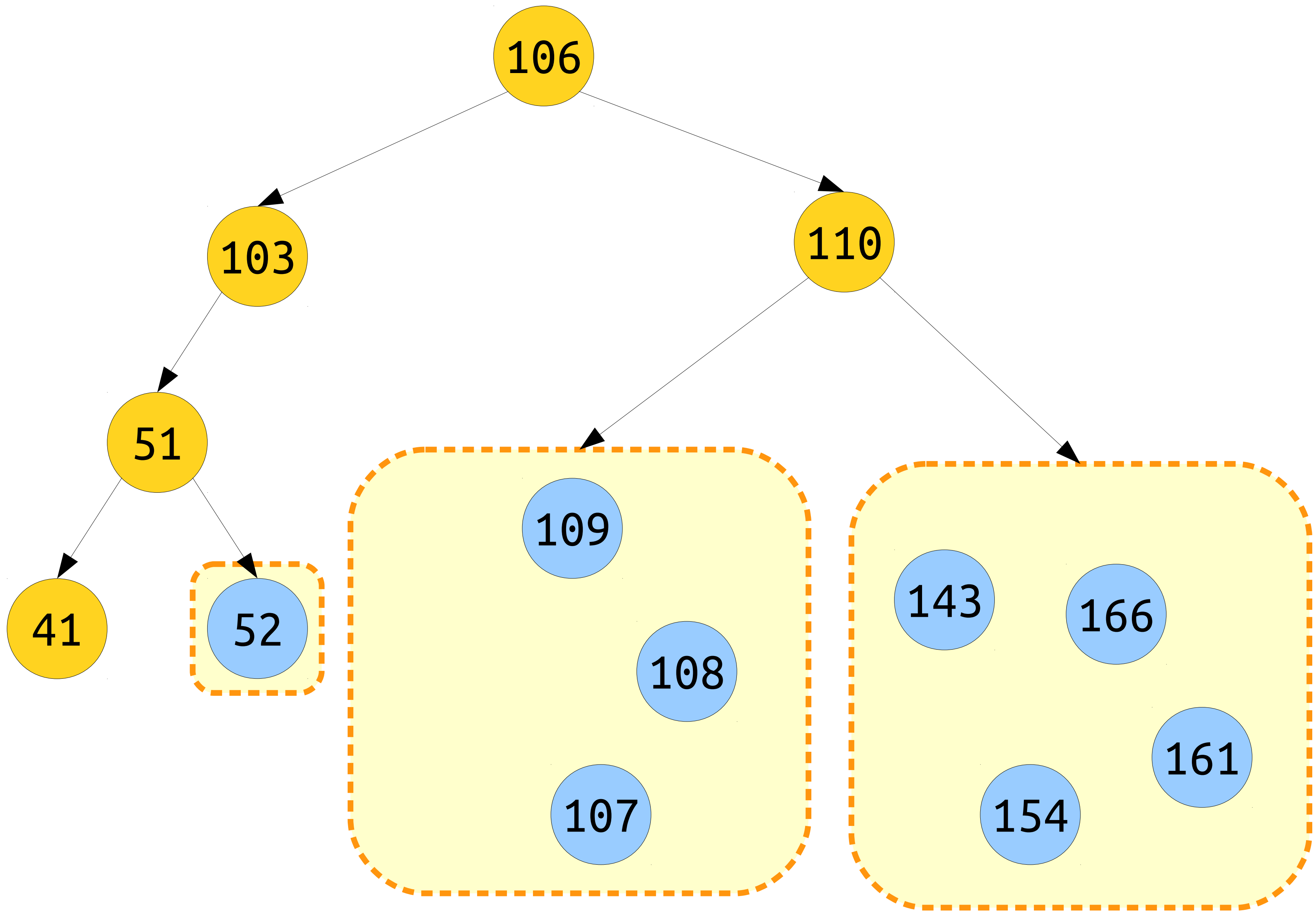
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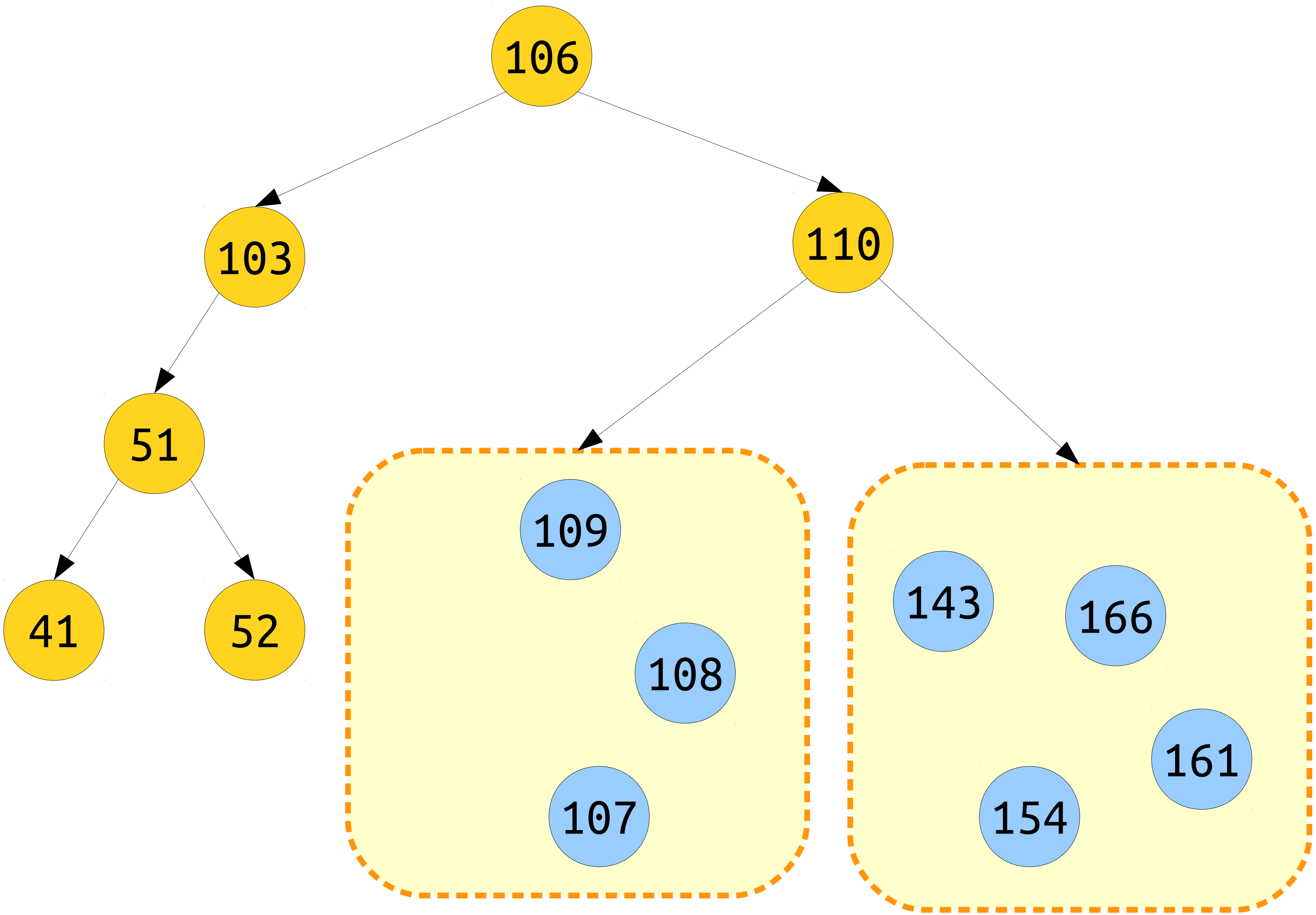
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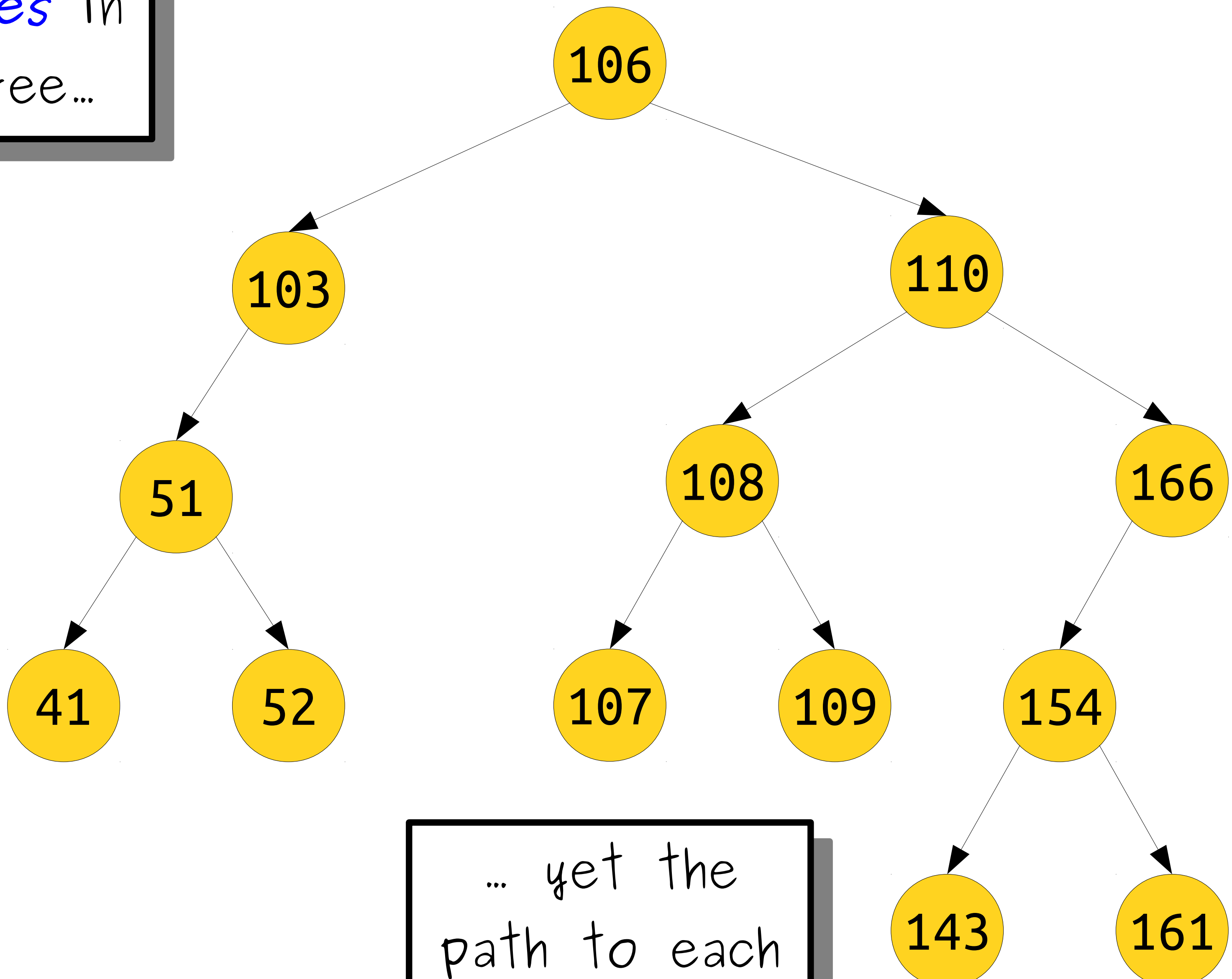
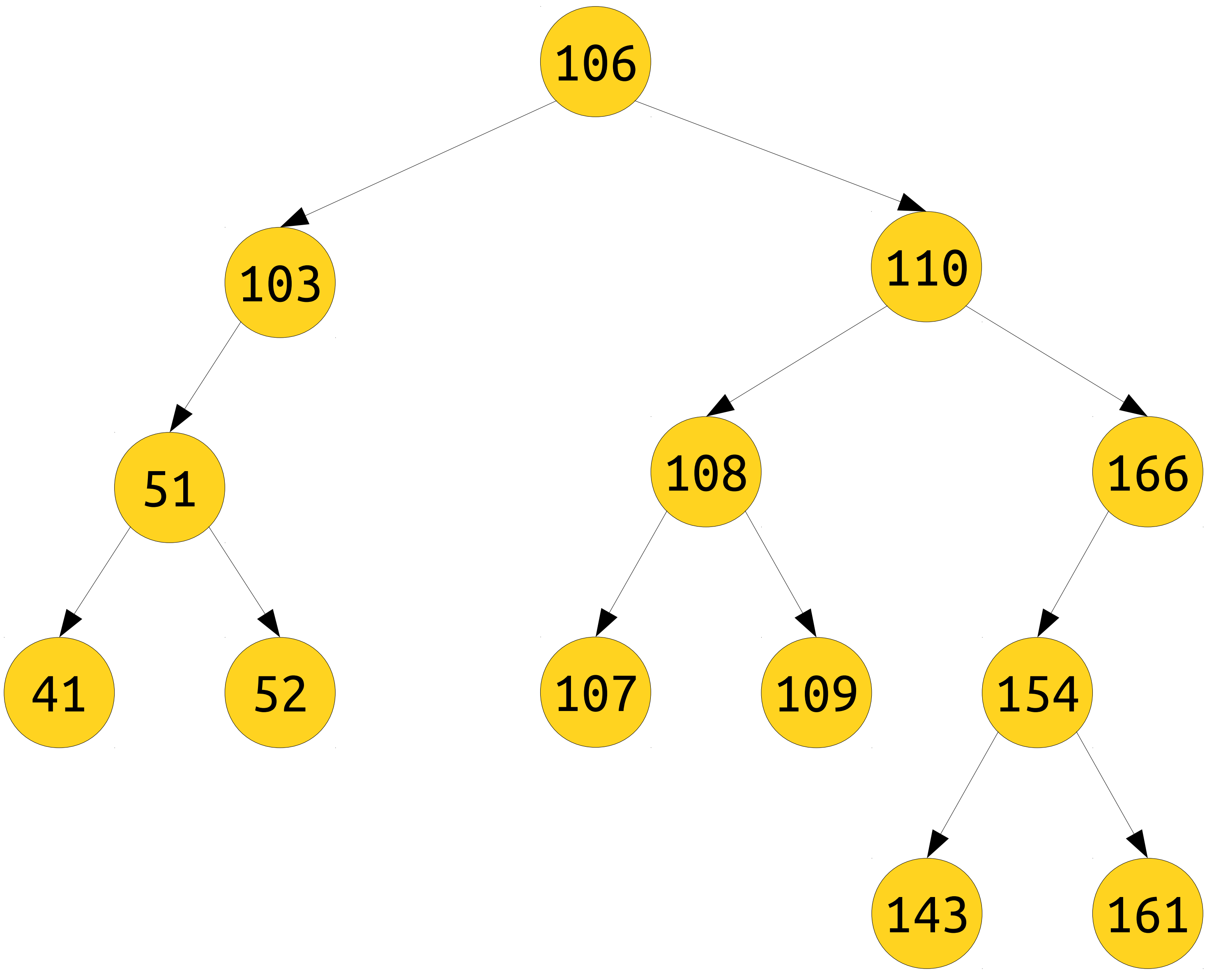
There are 13 *nodes* in this tree...



... yet the path to each one is short.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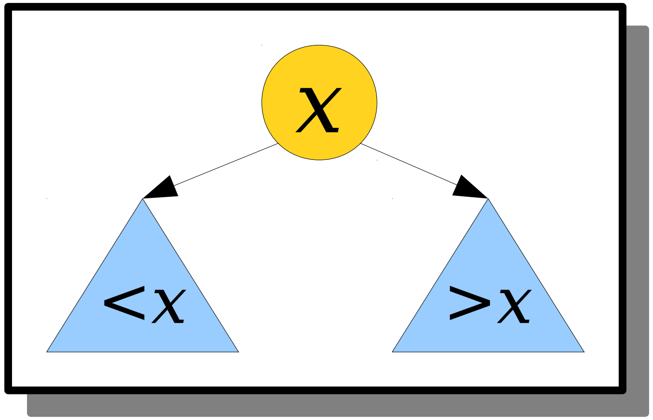
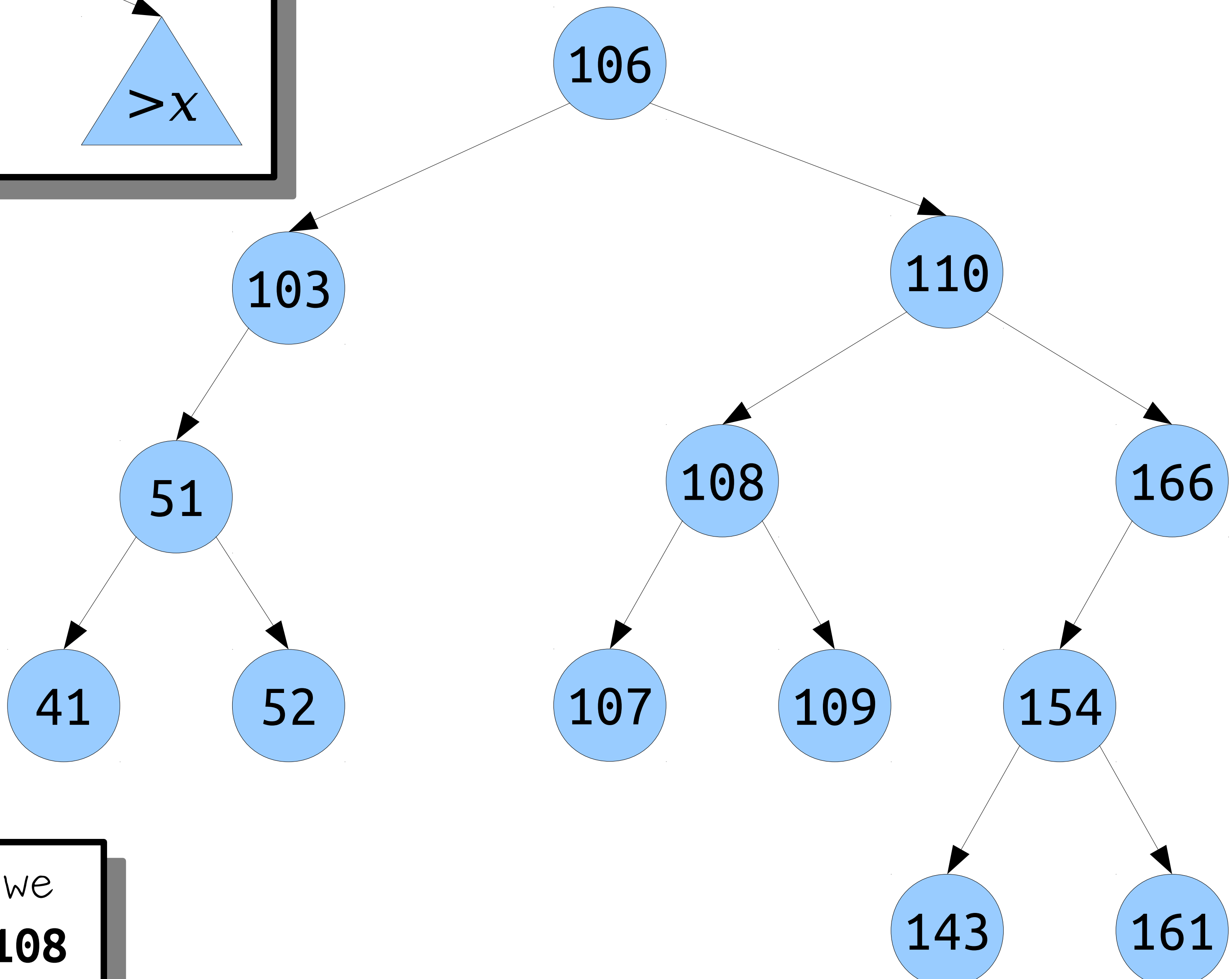
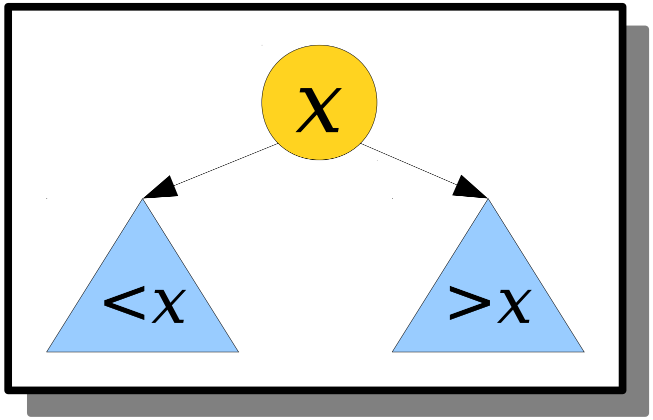
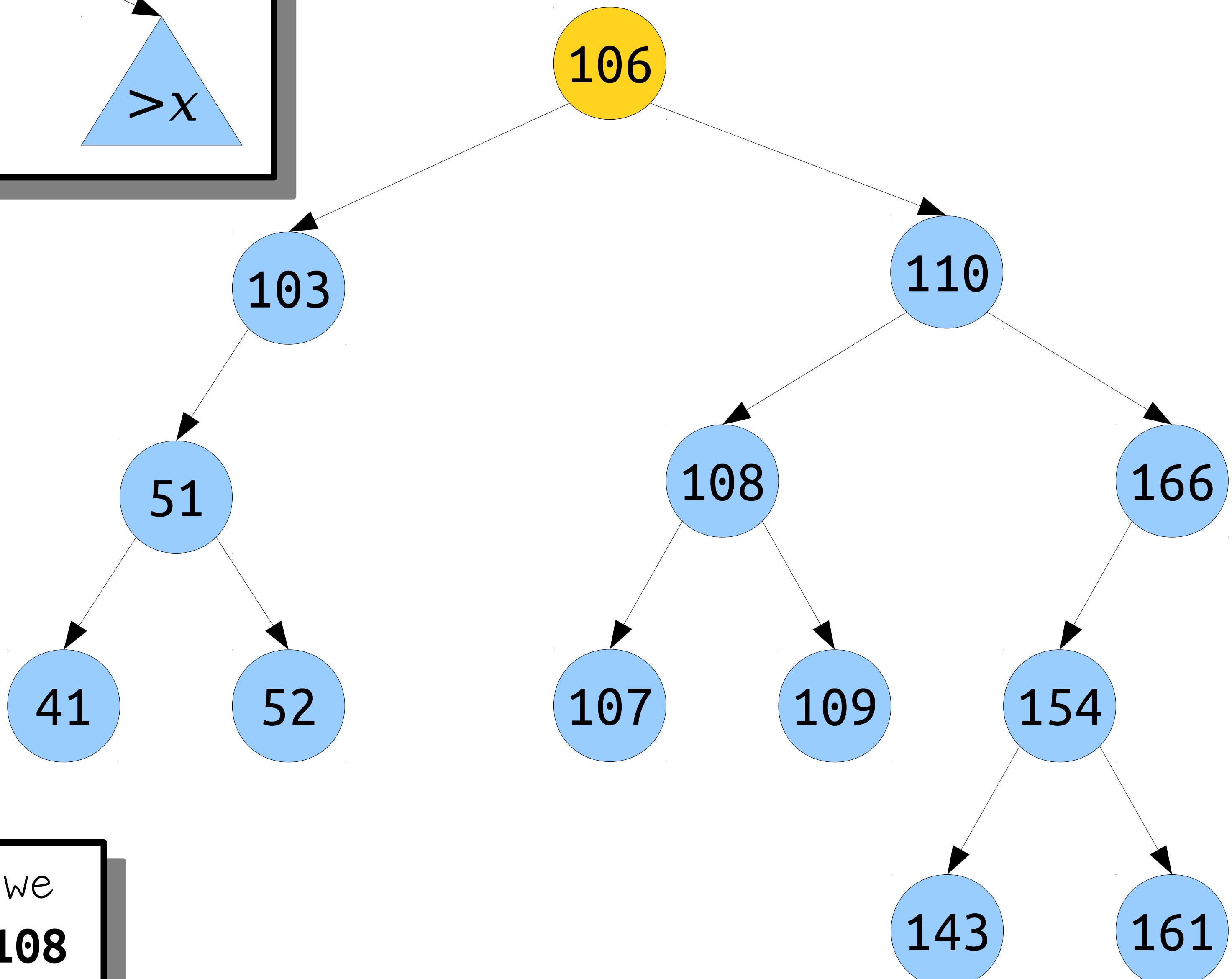




How can we check if **108** is in this tree?





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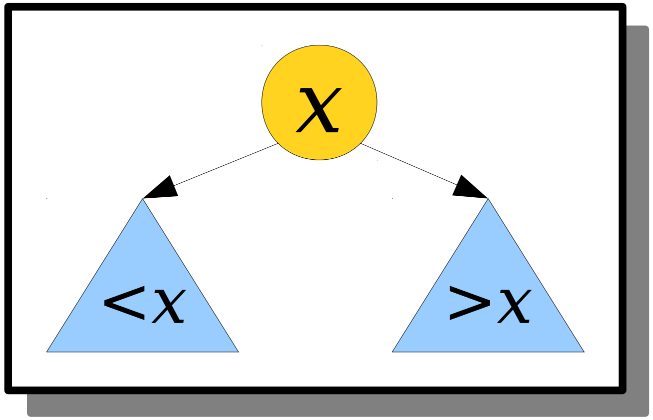

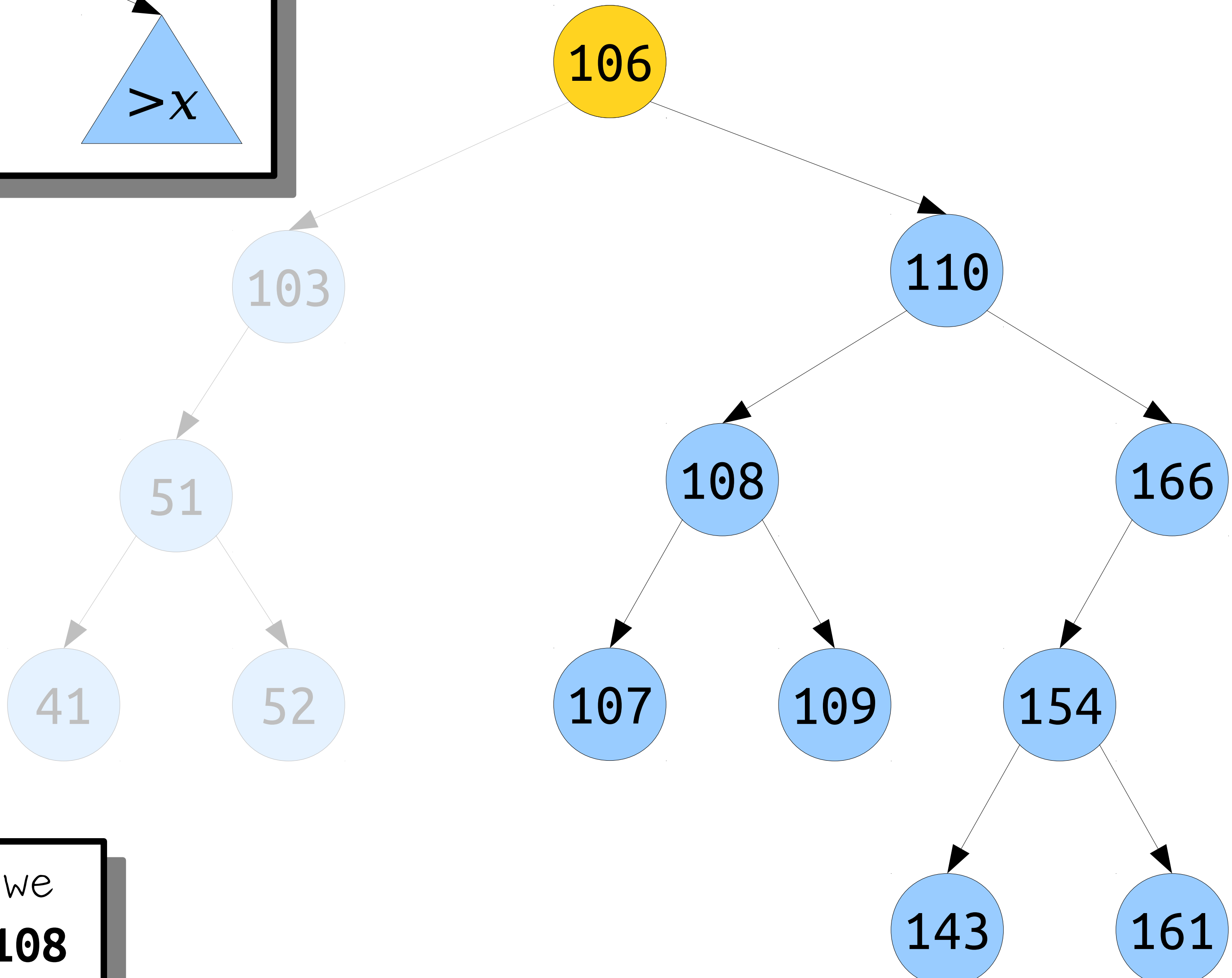

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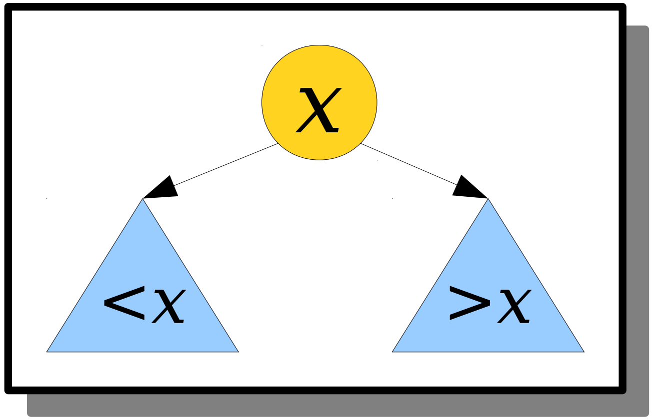
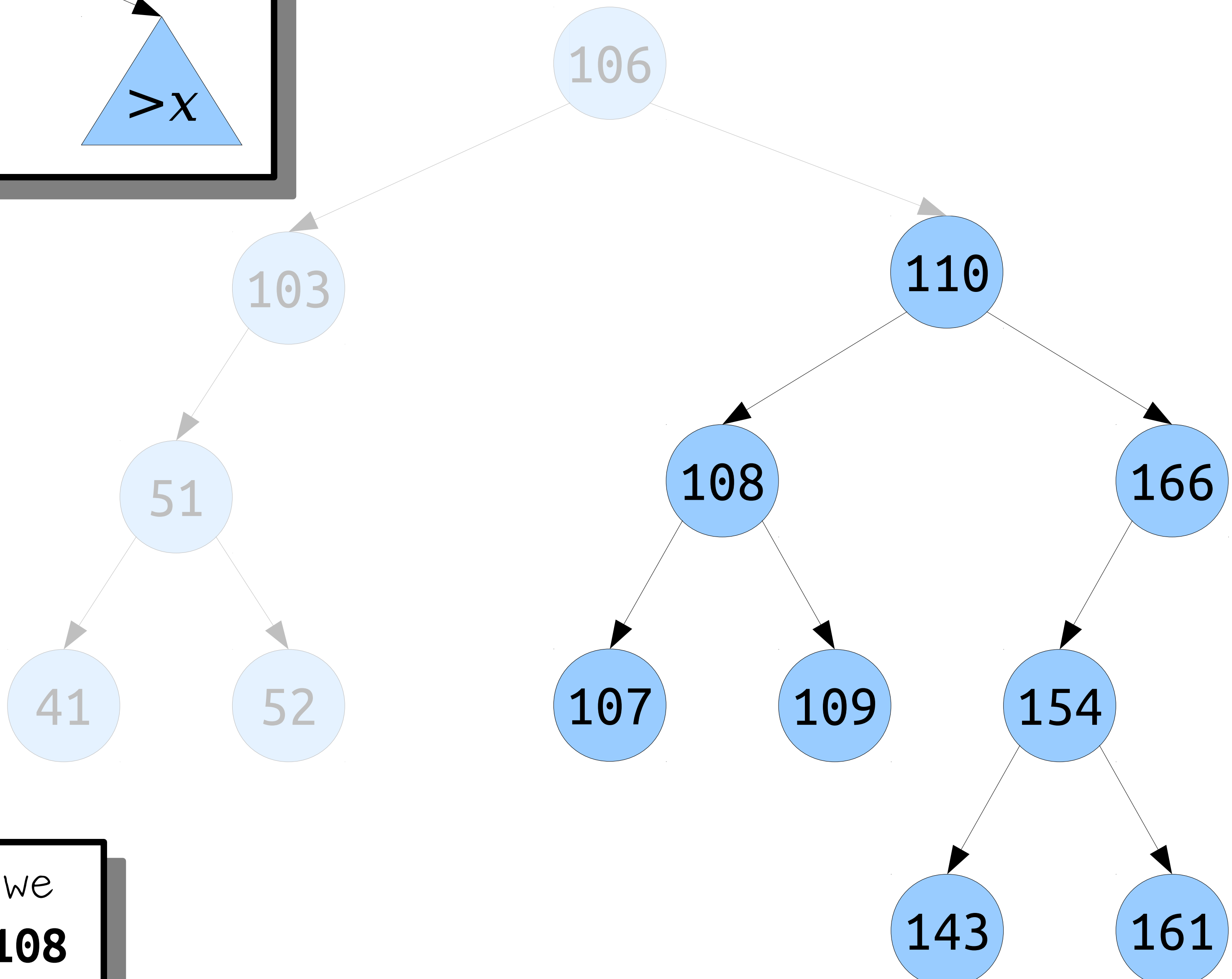
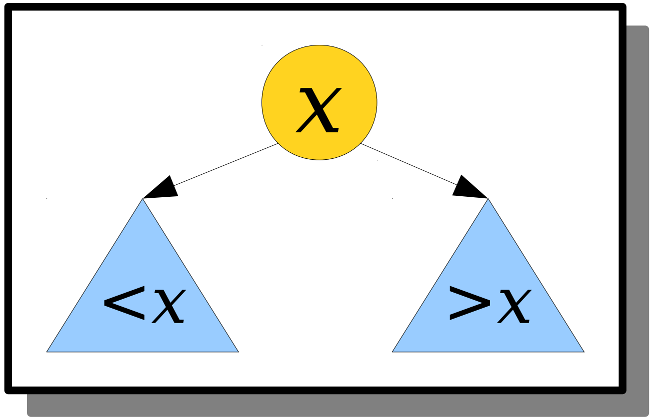
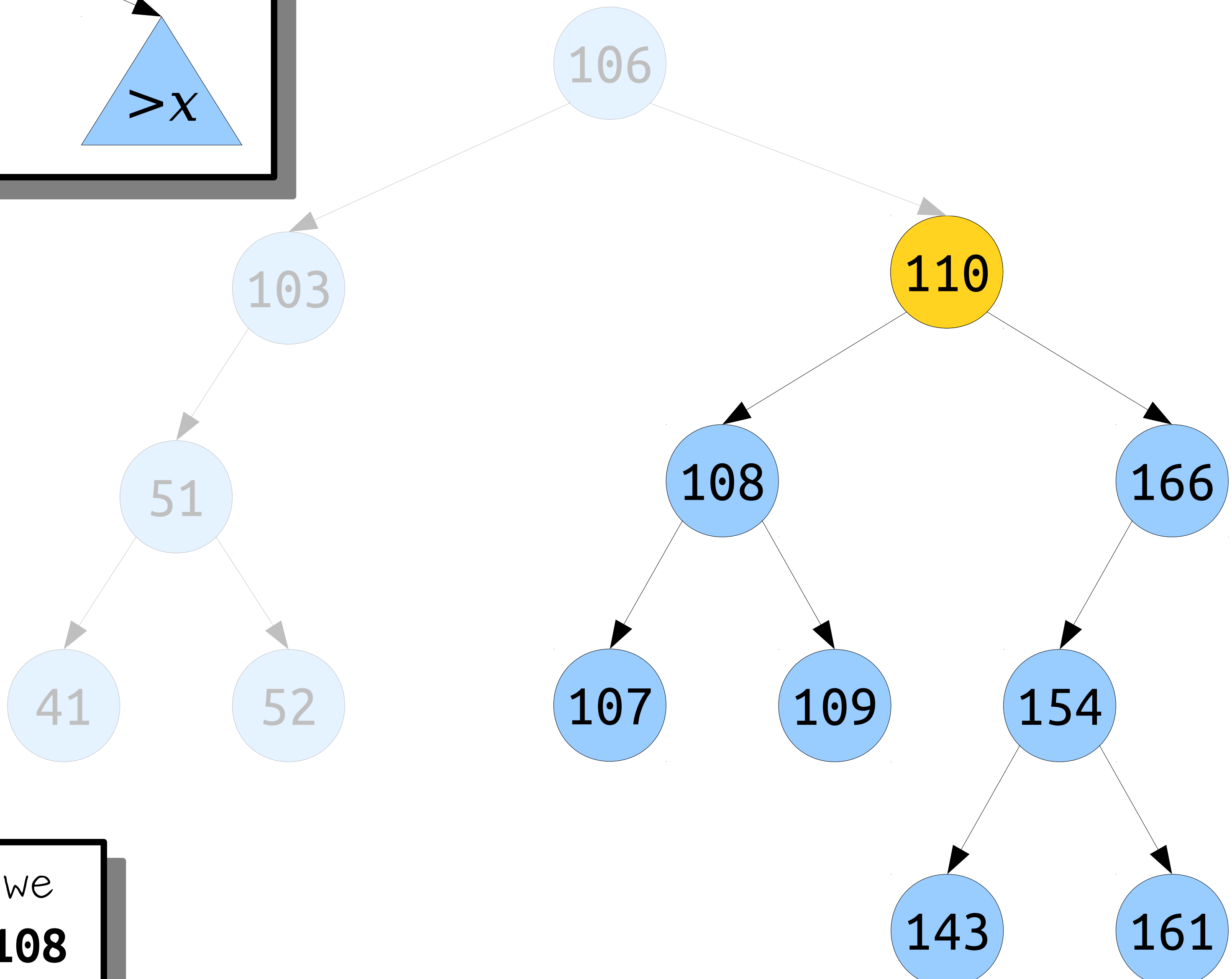




How can we check if **108** is in this tree?





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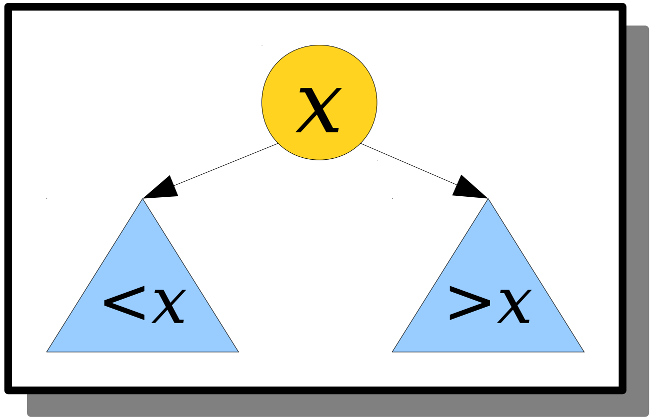

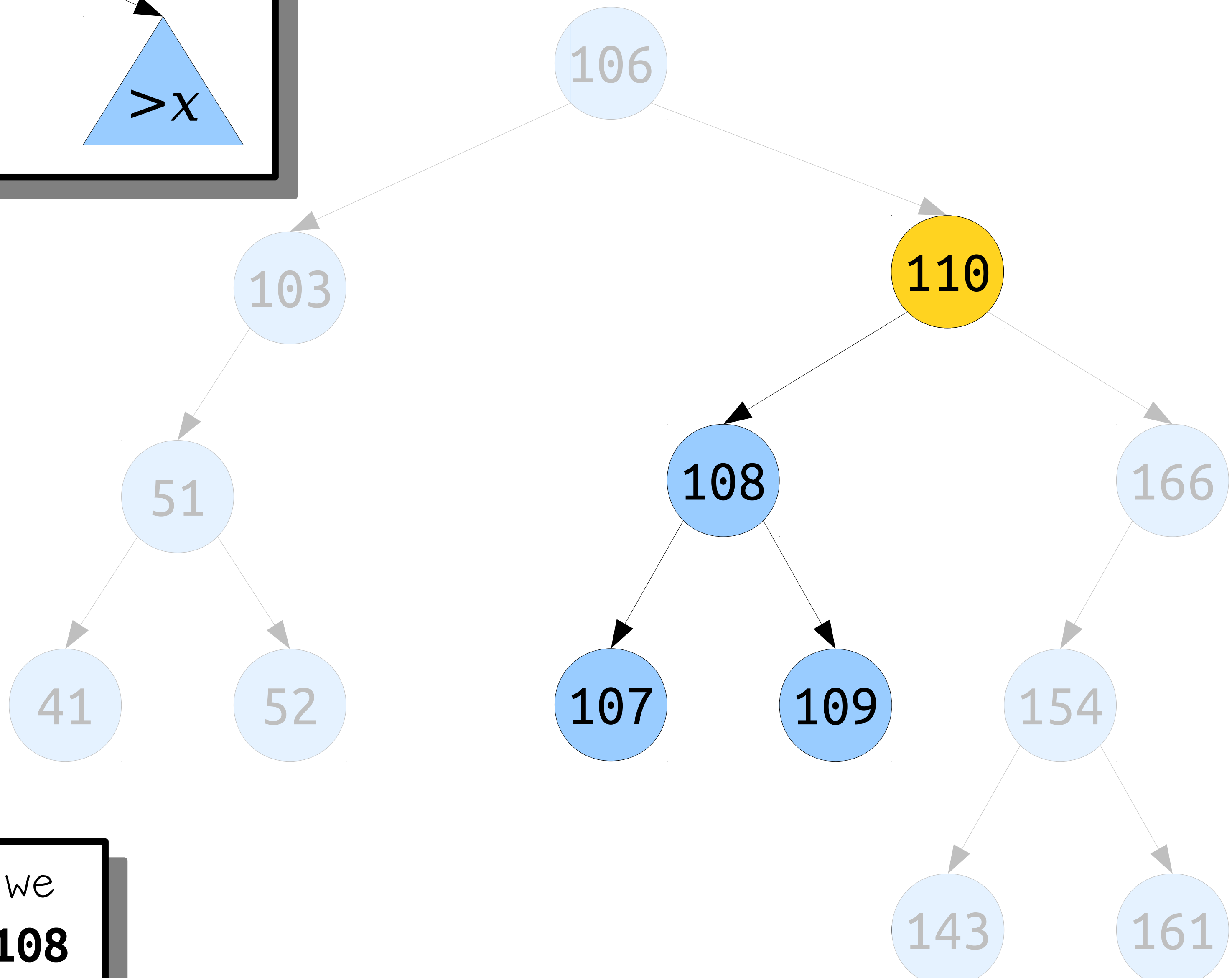

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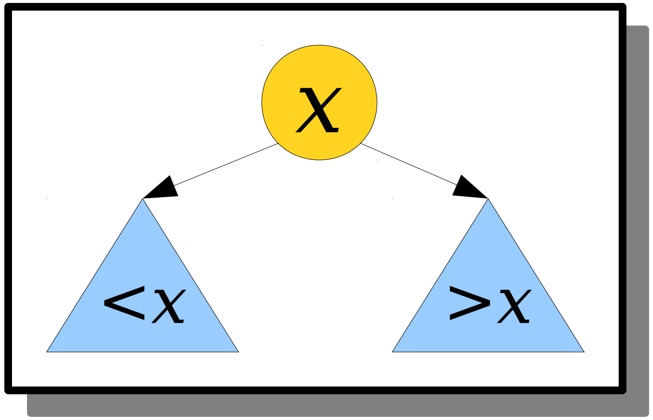

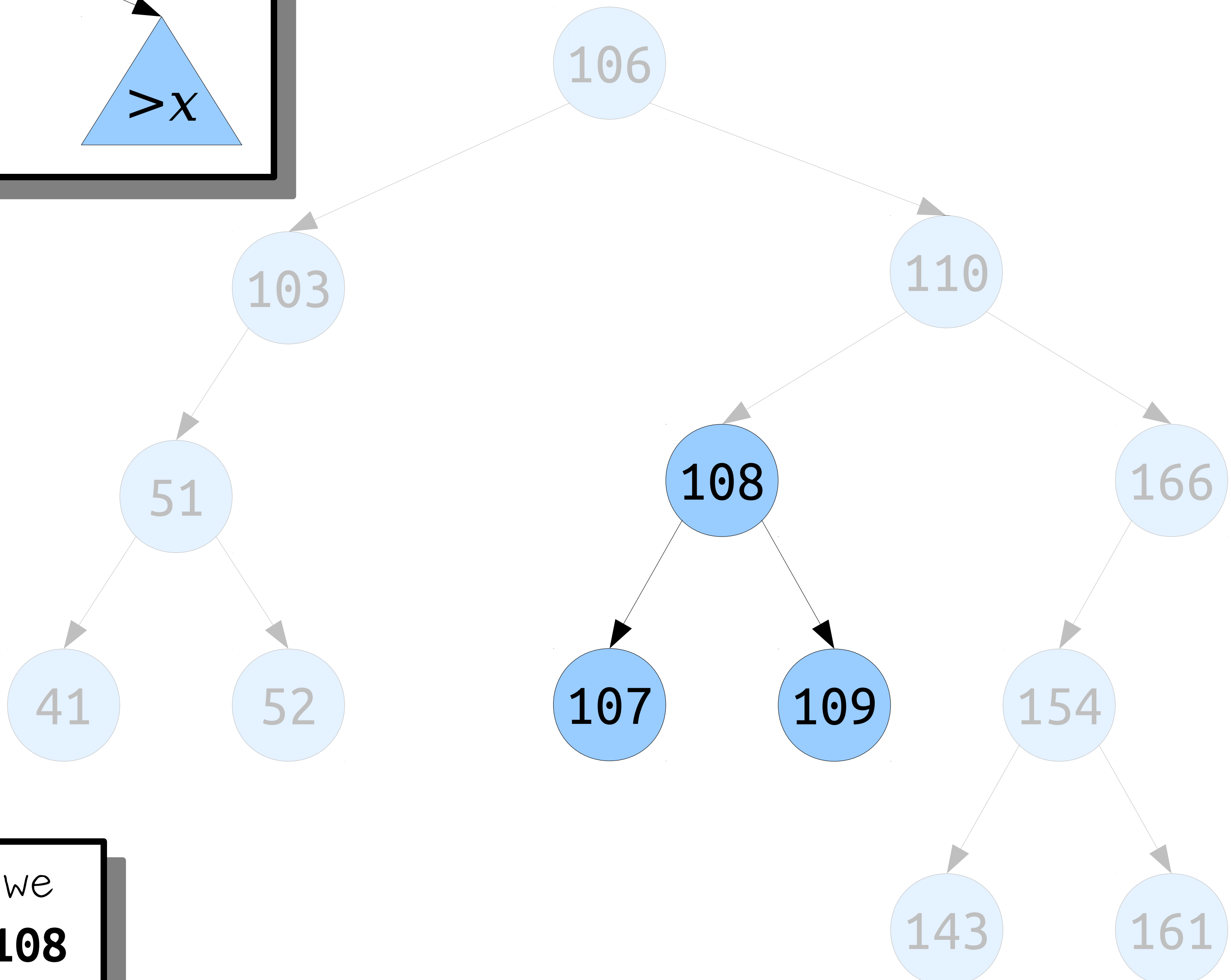

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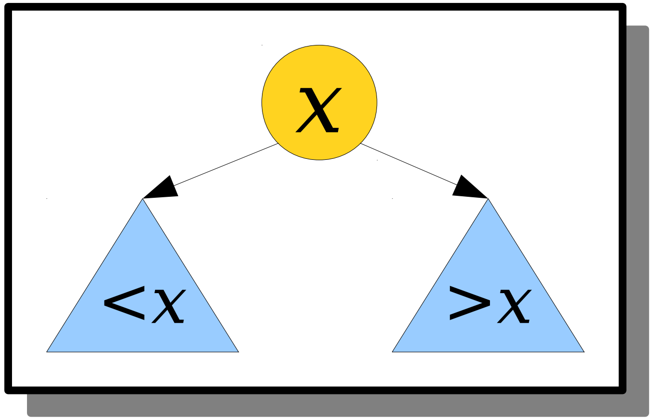
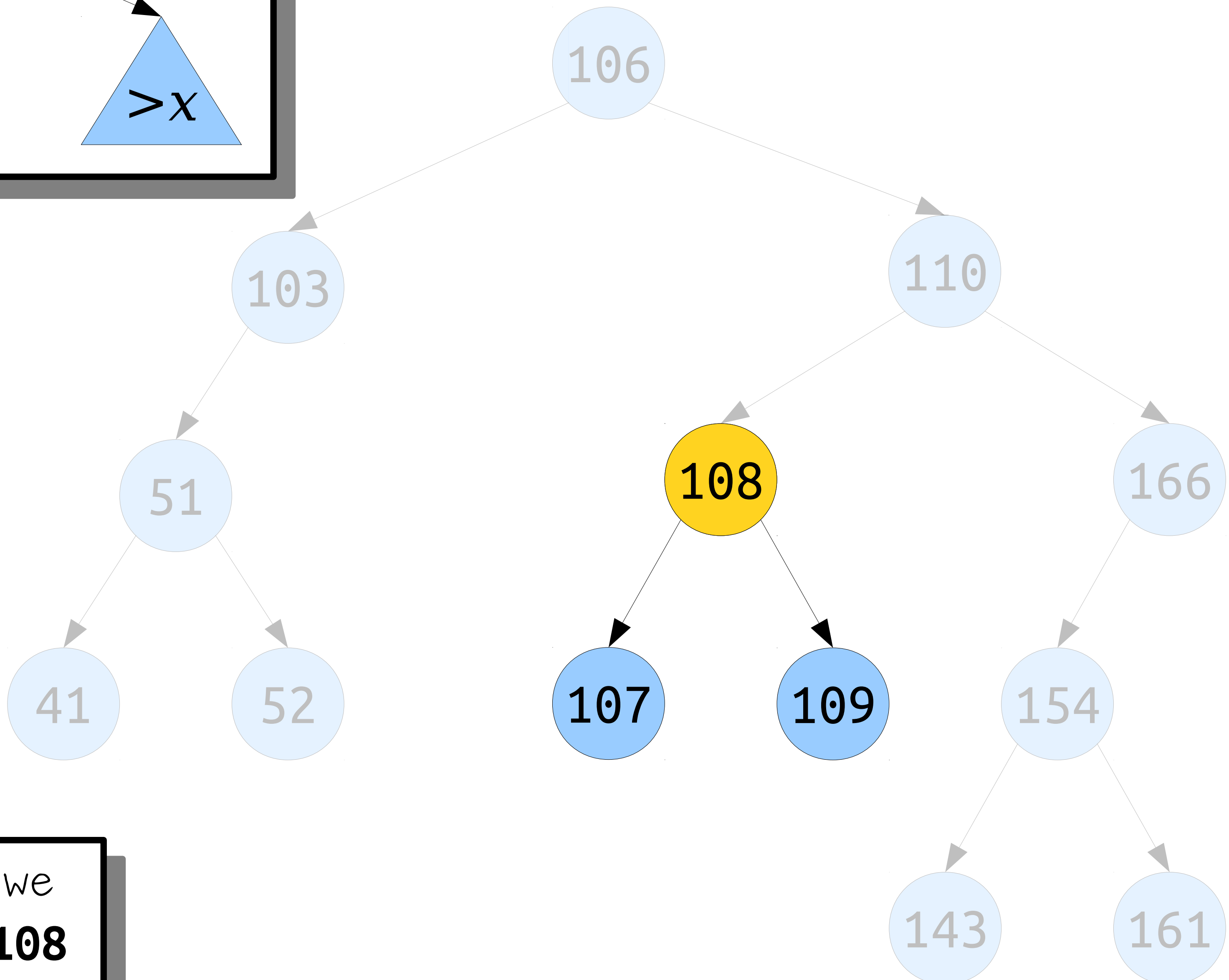
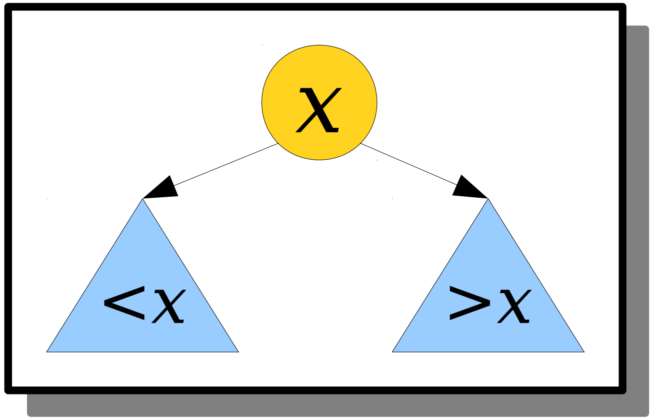
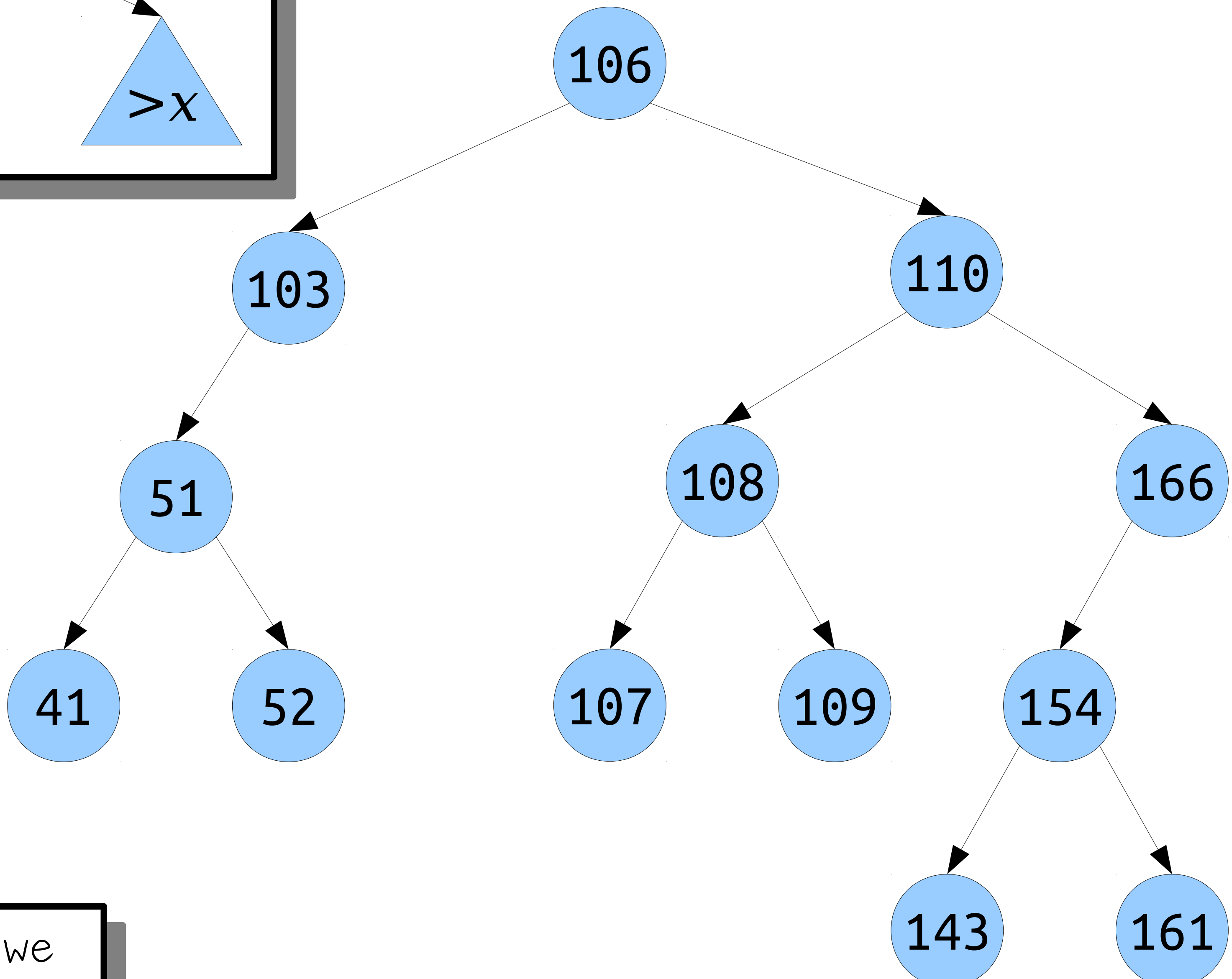




How can we check if **108** is in this tree?





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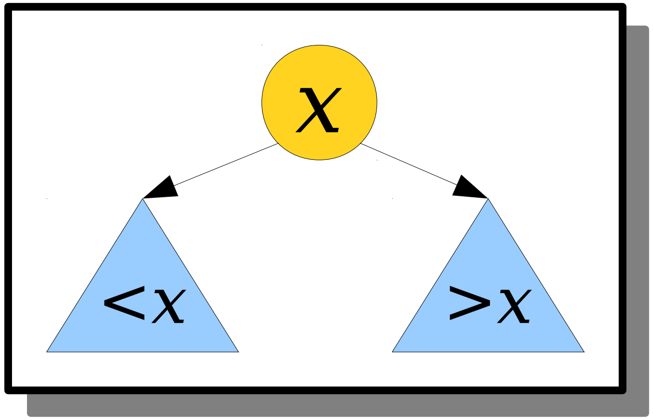
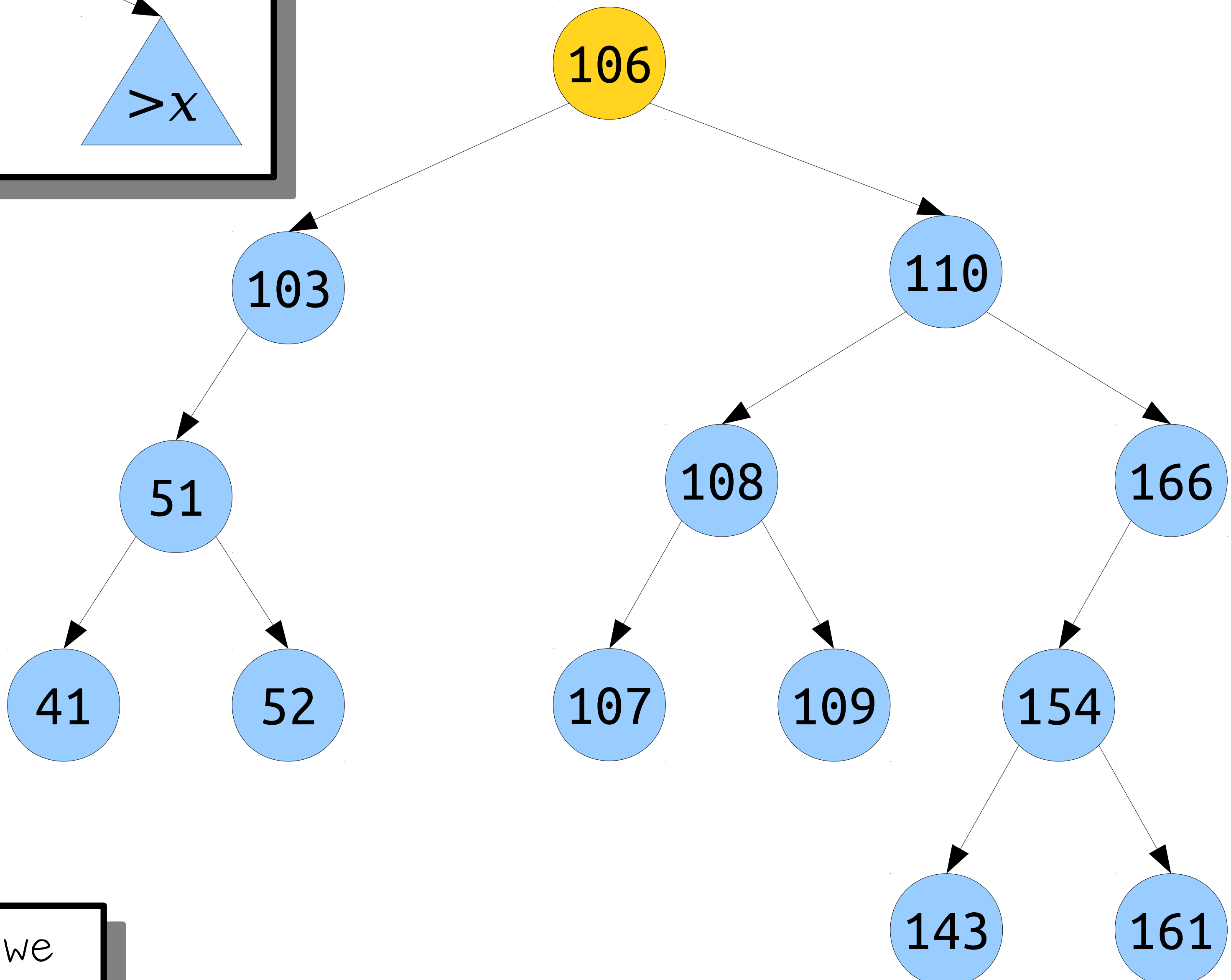
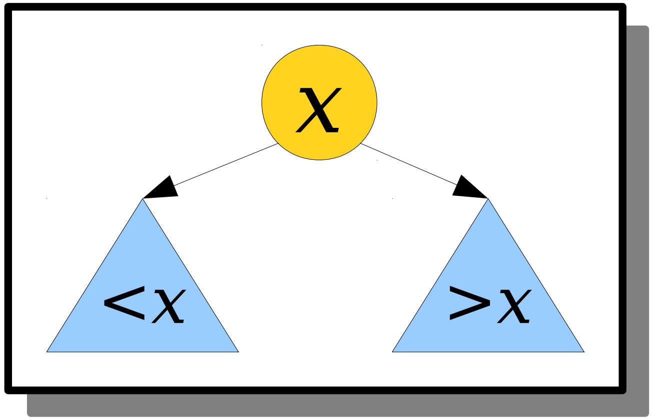
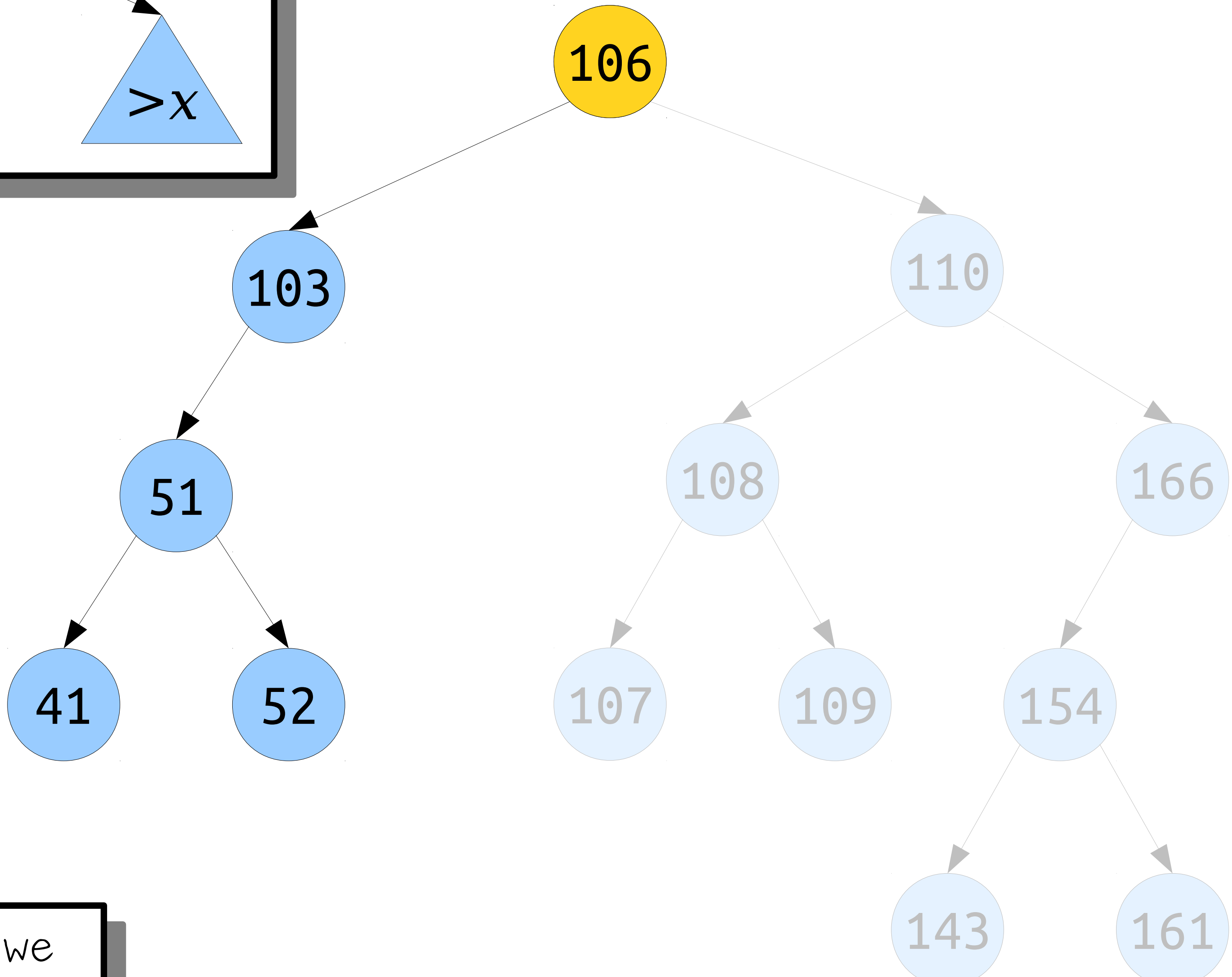




How can we check if **83** is in this tree?





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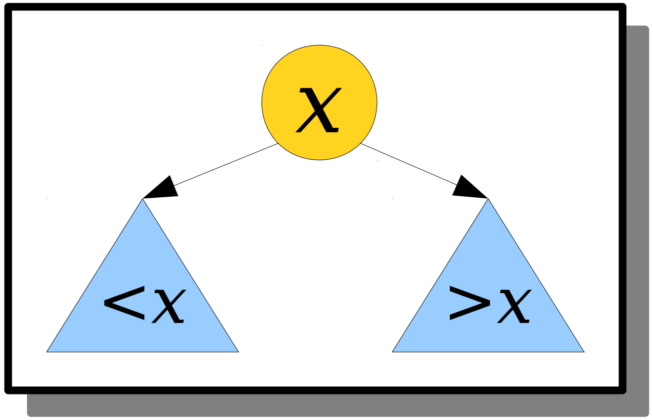

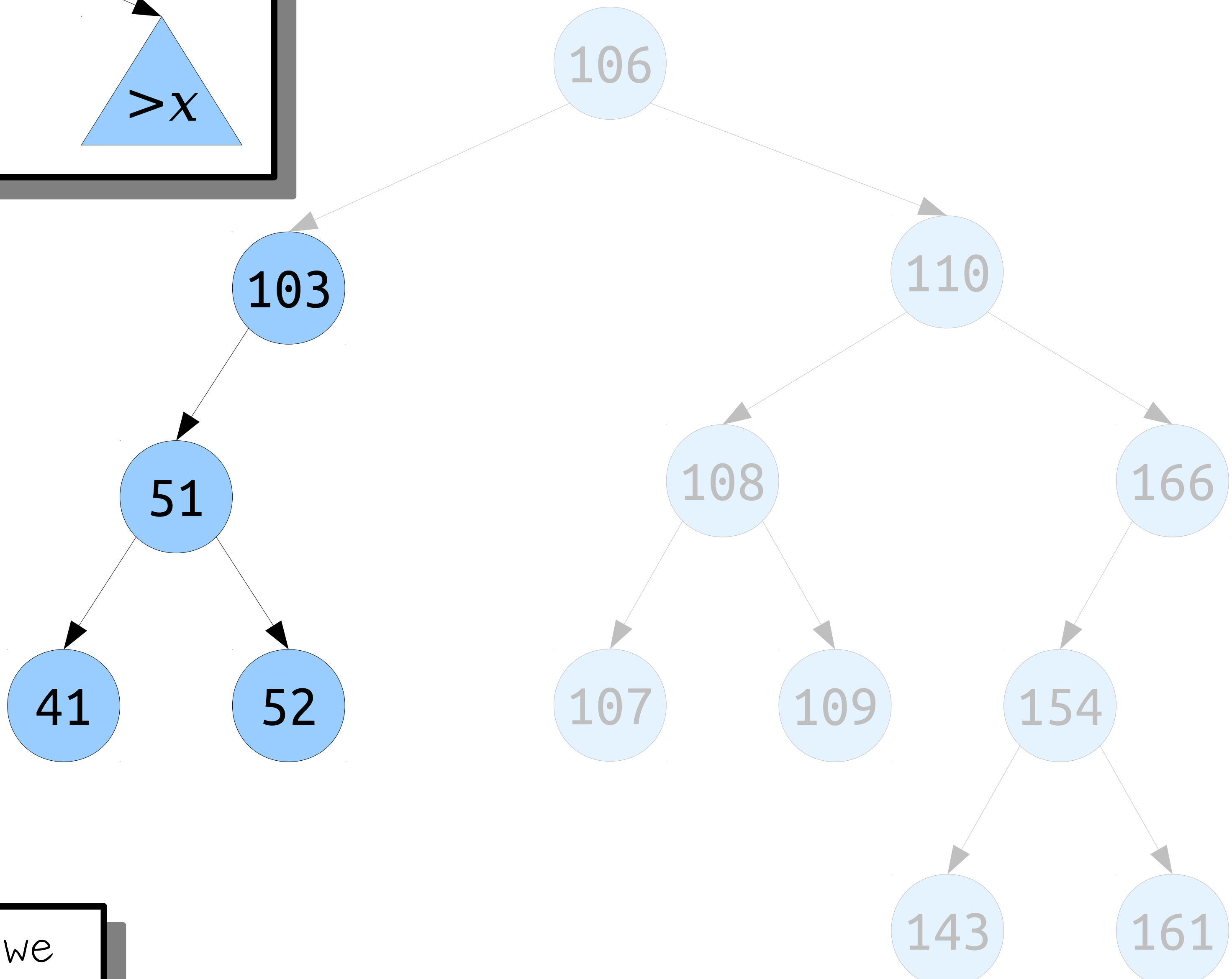

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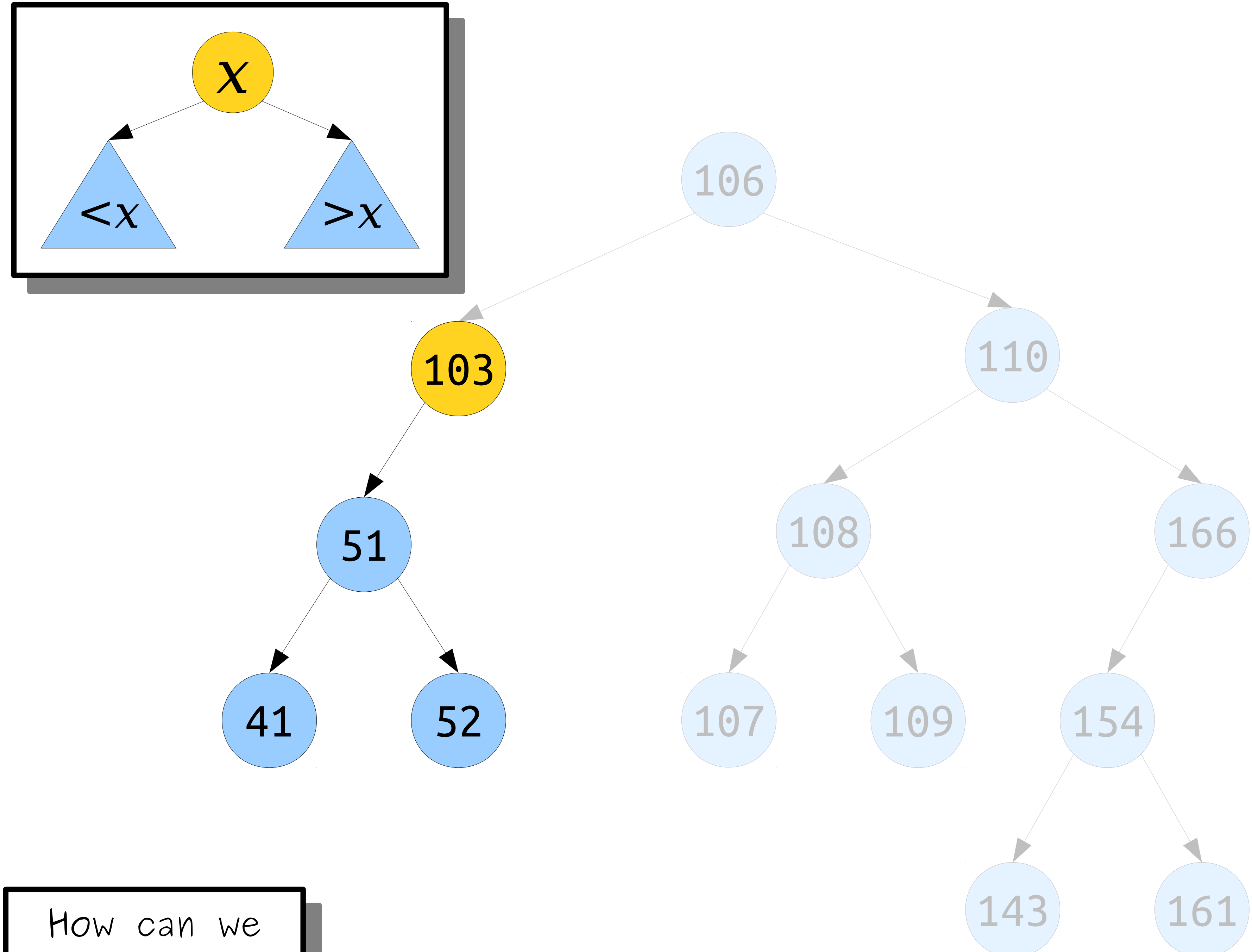

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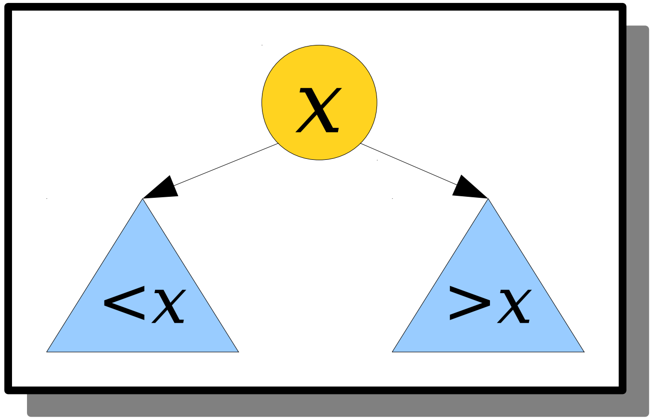
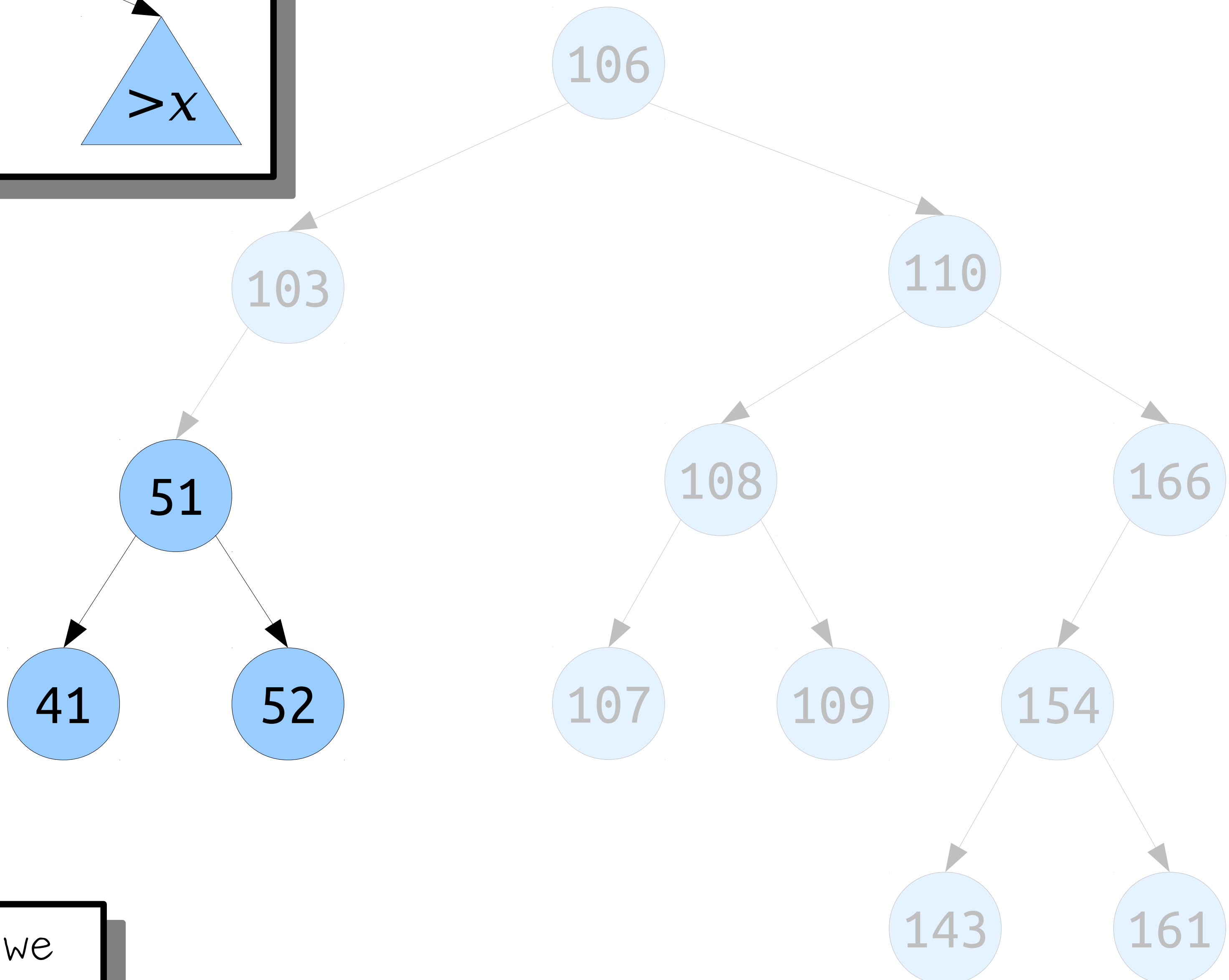
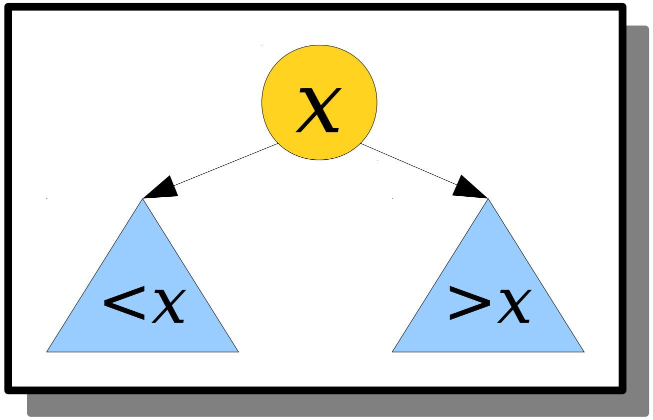
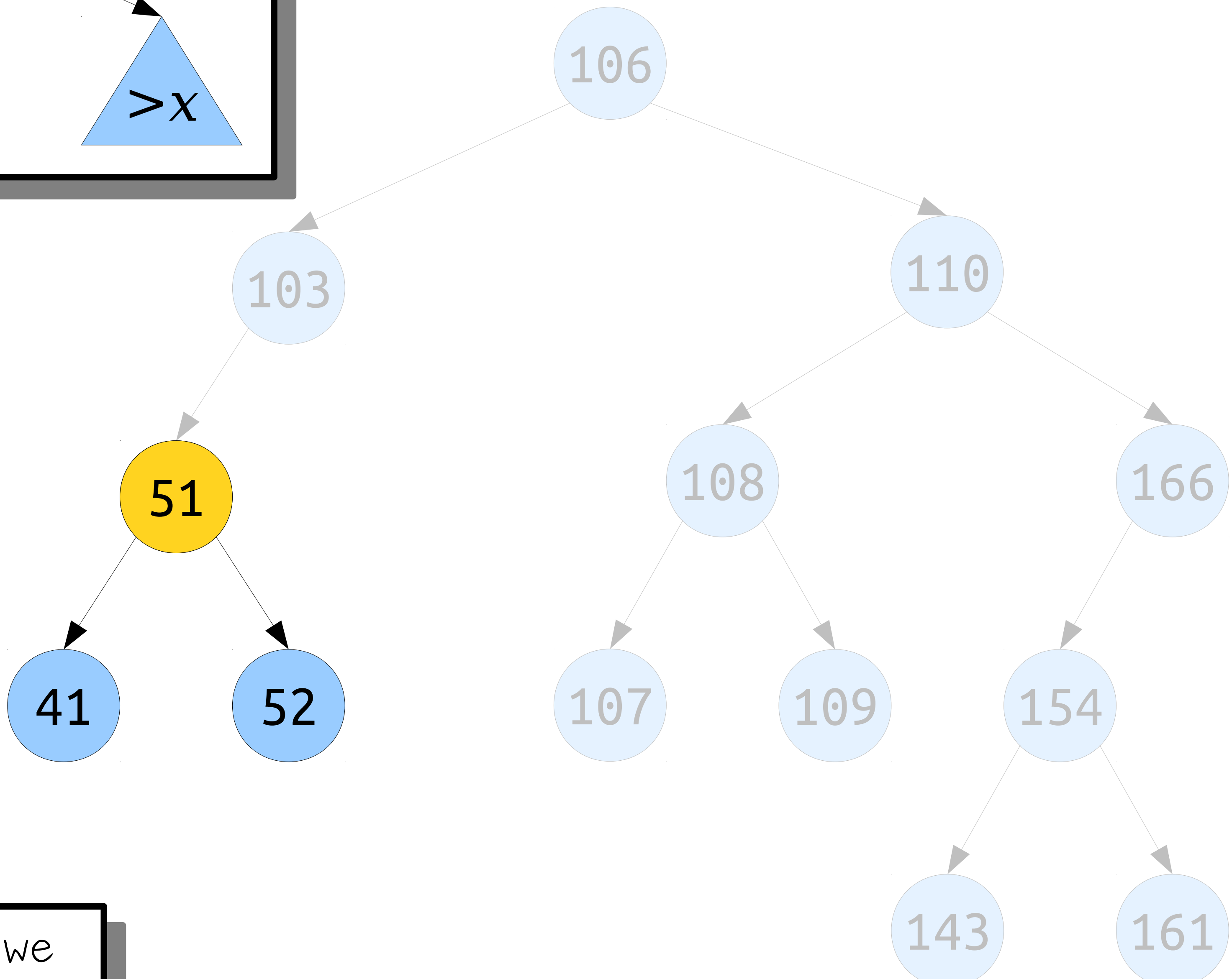


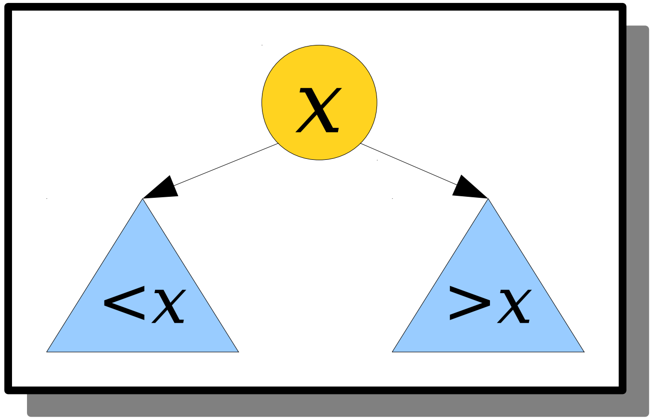


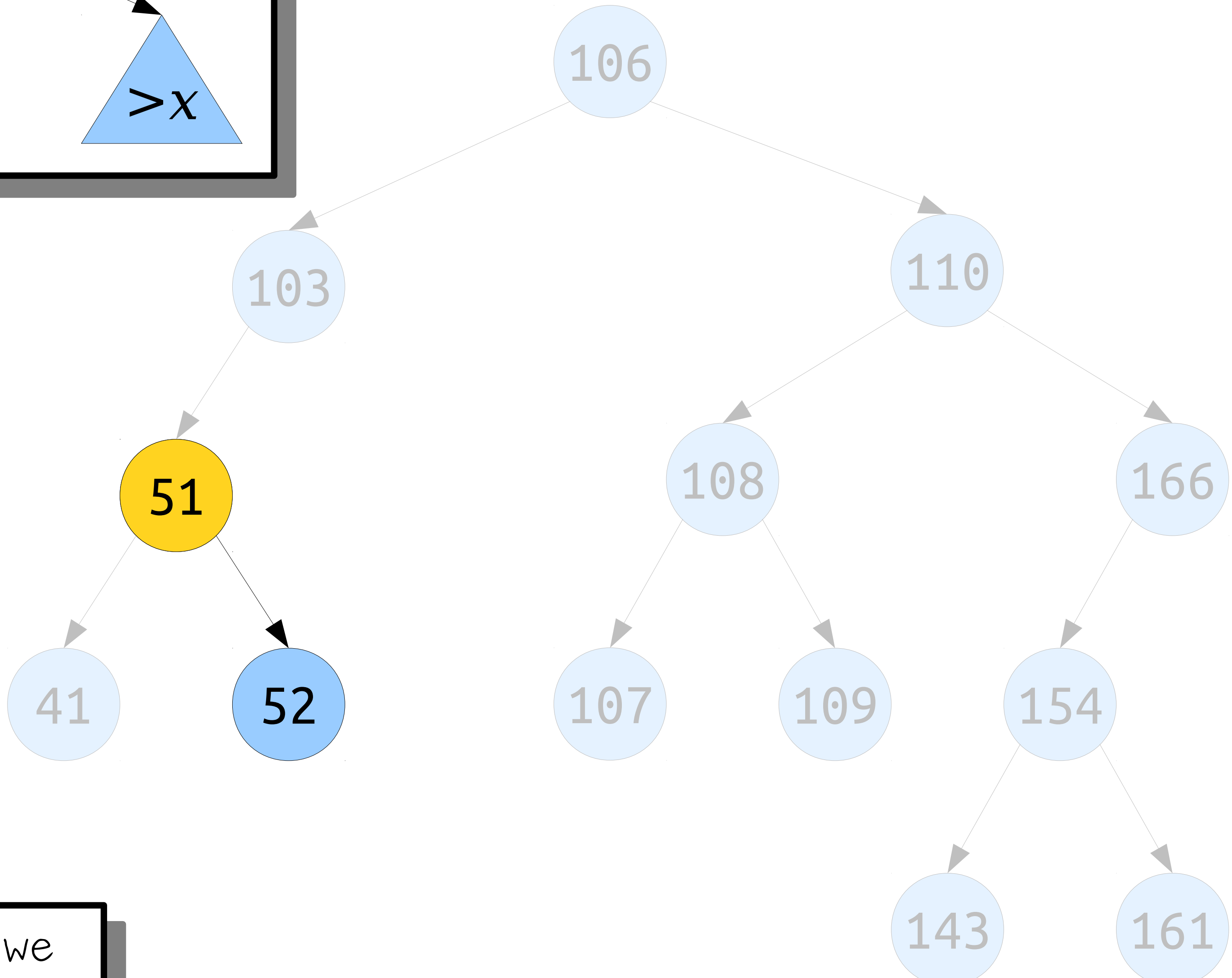
How can we check if **83** is in this tree?



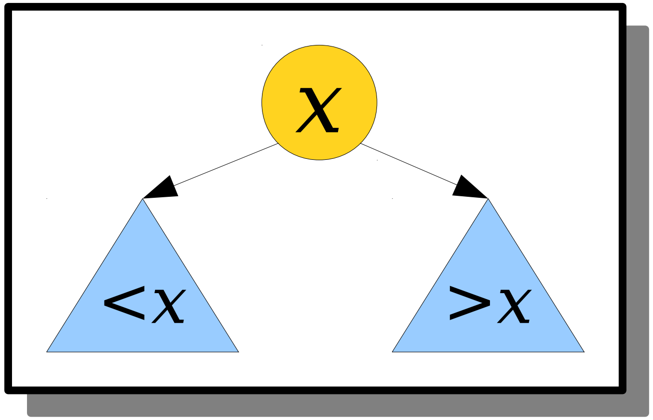


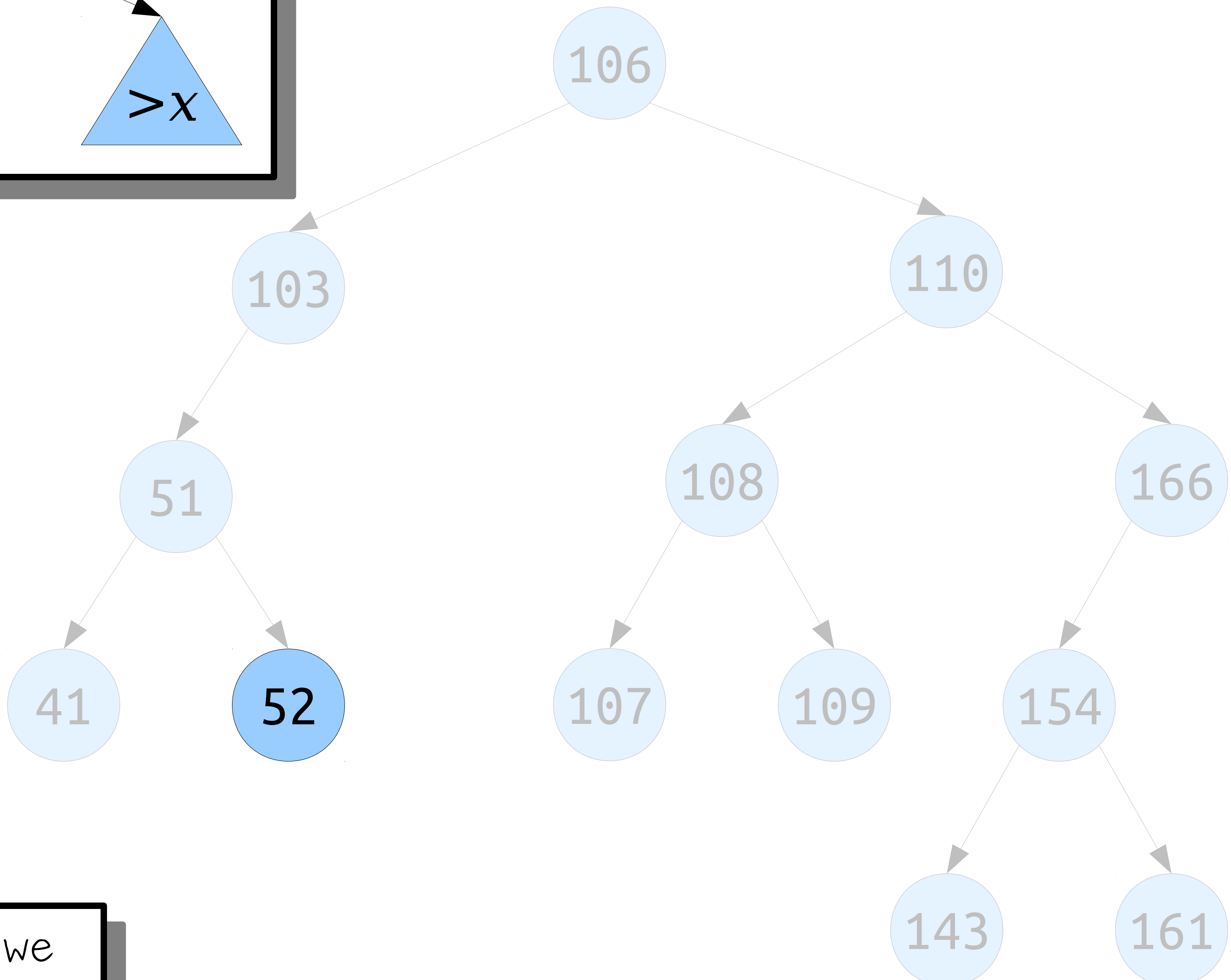
How can we check if **83** is in this tree?



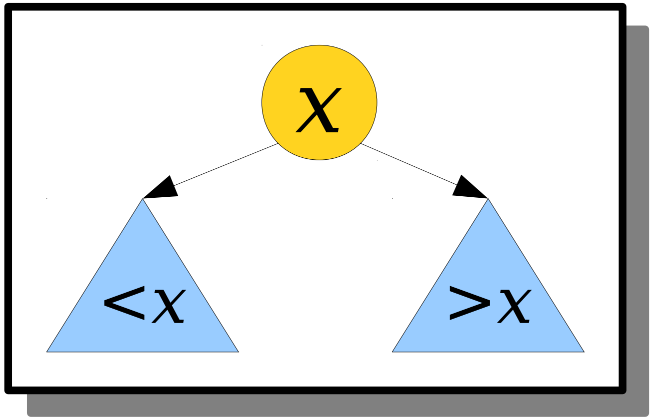


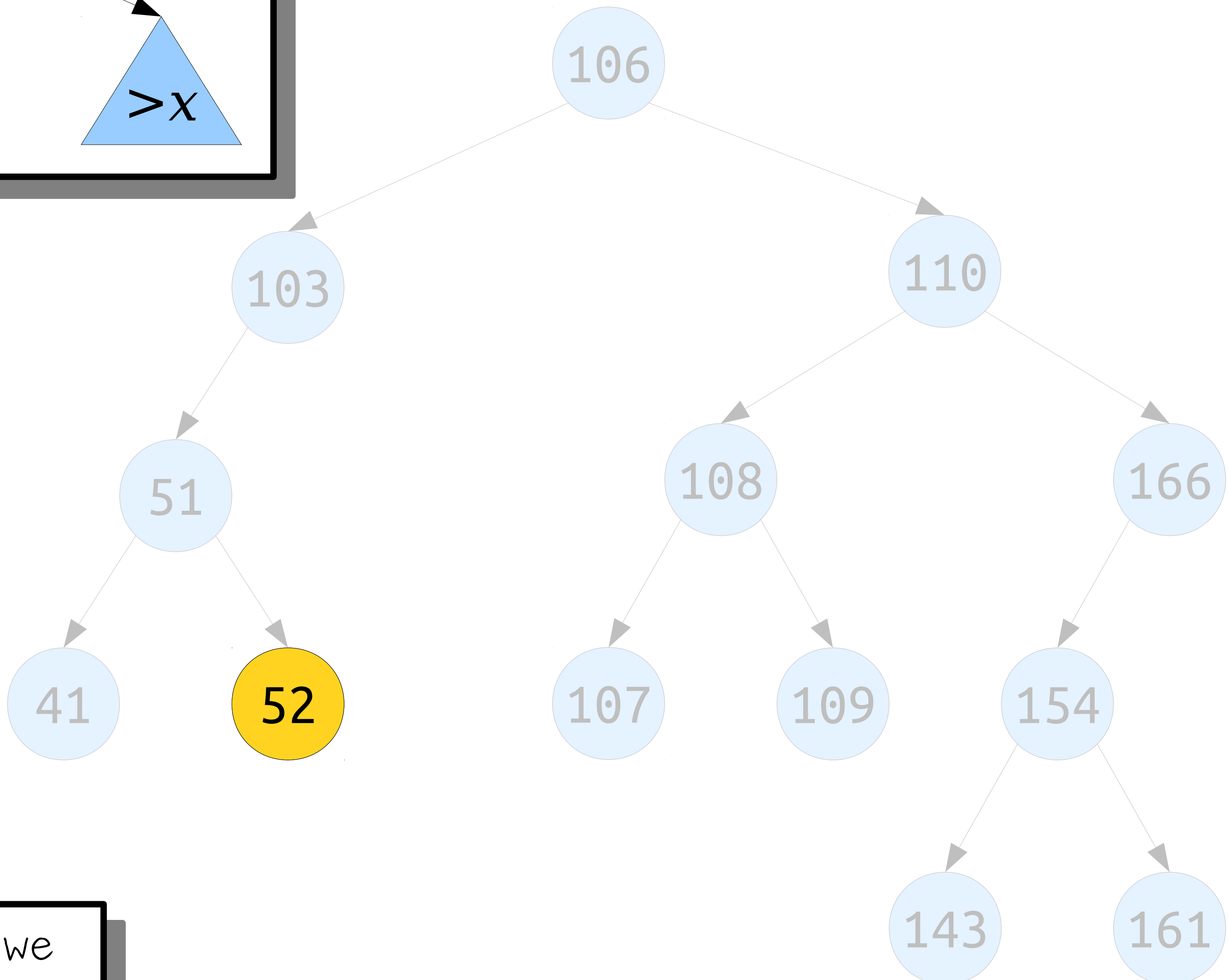
How can we check if **83** is in this tree?



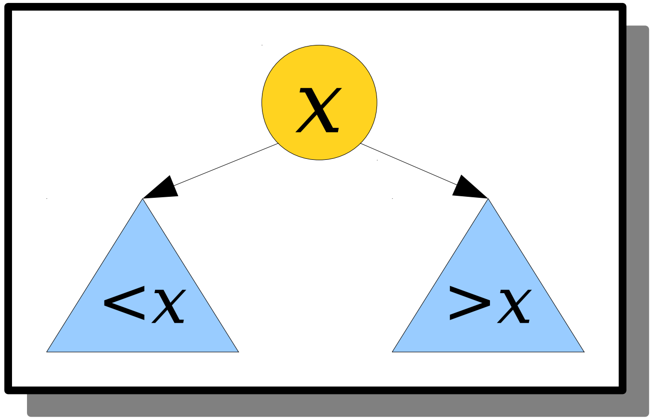


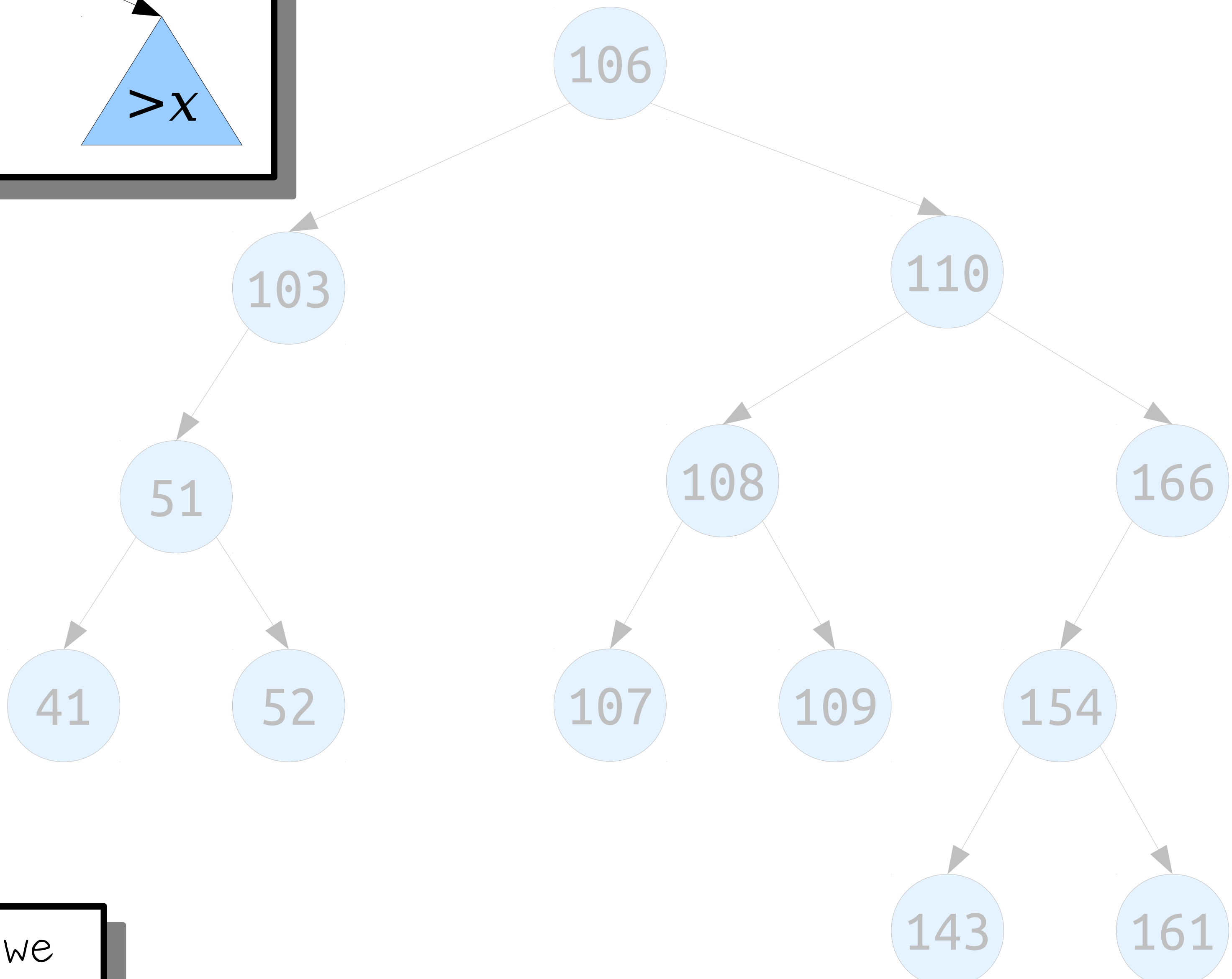
How can we check if **83** is in this tree?





How can we check if **83** is in this tree?

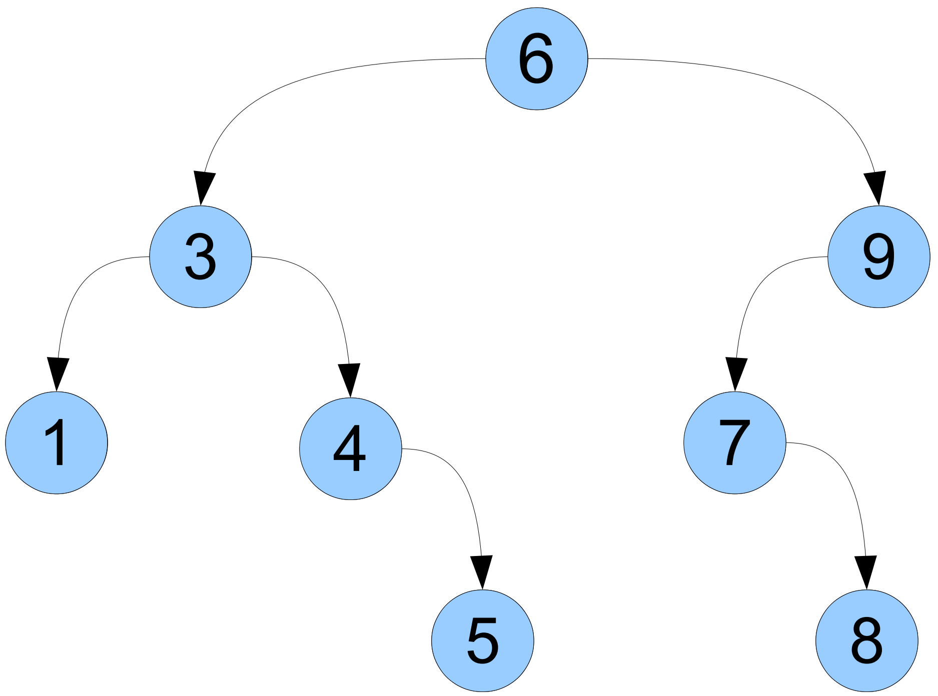




How can we check if **83** is in this tree?

Binary Search Trees

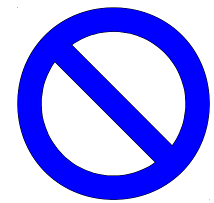
- The data structure we have just seen is called a ***binary search tree*** (or ***BST***).
- The tree consists of a number of ***nodes***, each of which stores a value and has zero, one, or two ***children***.
- All values in a node's left subtree are ***smaller*** than the node's value, and all values in a node's right subtree are ***greater*** than the node's value.



A Binary Search Tree Is Either…

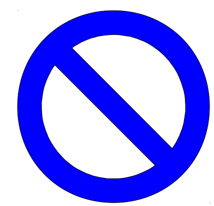
an empty tree,
represented by

`nullptr`

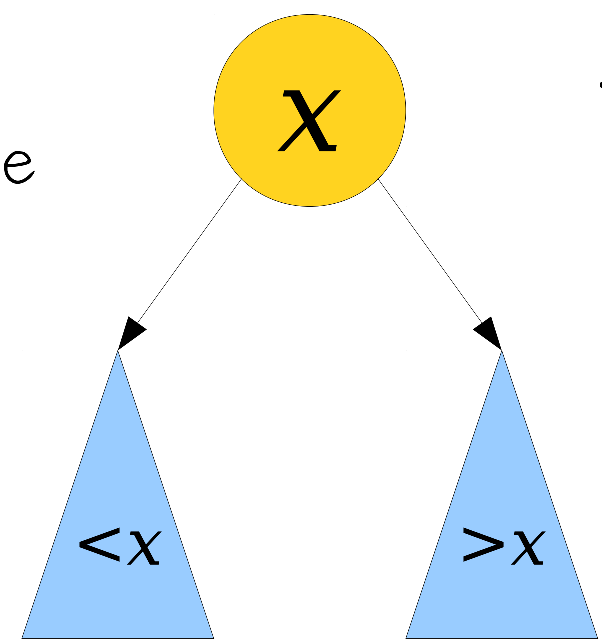


A Binary Search Tree Is Either…

an empty tree,
represented by
nullptr, or…



… a single node,
whose left subtree
is a BST of
smaller values …



… and whose right
subtree is a BST
of larger values.

Binary Search Tree Nodes

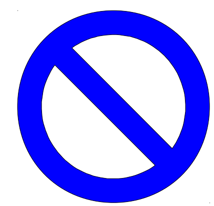
```
struct Node {
    Type value;
    Node* left;  // Smaller values
    Node* right; // Bigger values
};
```

Kinda like a linked list, but with two pointers instead of just one!

Searching Trees

A Binary Search Tree Is Either…

an empty tree,
represented by
`nullptr`



A Binary Search Tree Is Either…

an empty tree,
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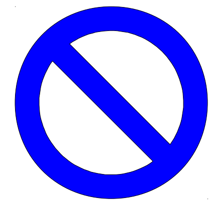
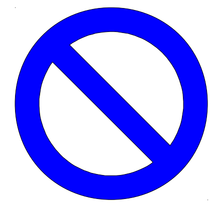
`nullptr`



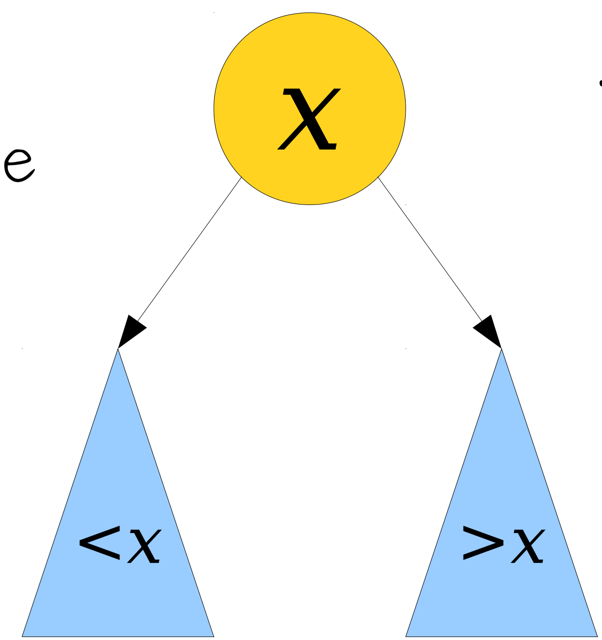
If you're looking for something in an empty BST, it's not there! Sorry.

A Binary Search Tree Is Either…

an empty tree,
represented by
`nullptr`, or…



… a single node,
whose left subtree
is a BST of
smaller values …



… and whose right
subtree is a BST
of larger values.

Good exercise:

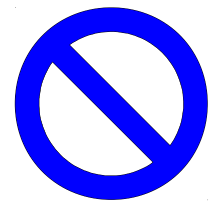
Rewrite this function iteratively!

Walking Trees

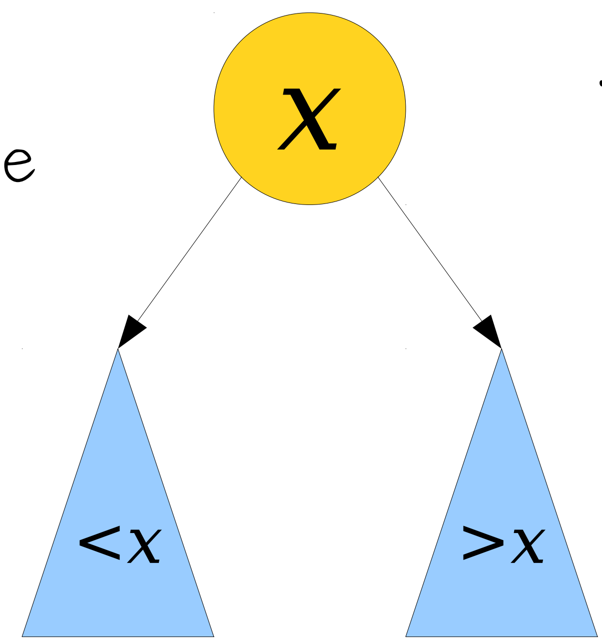
Print all the values in a BST,
in sorted order.

A Binary Search Tree Is Either…

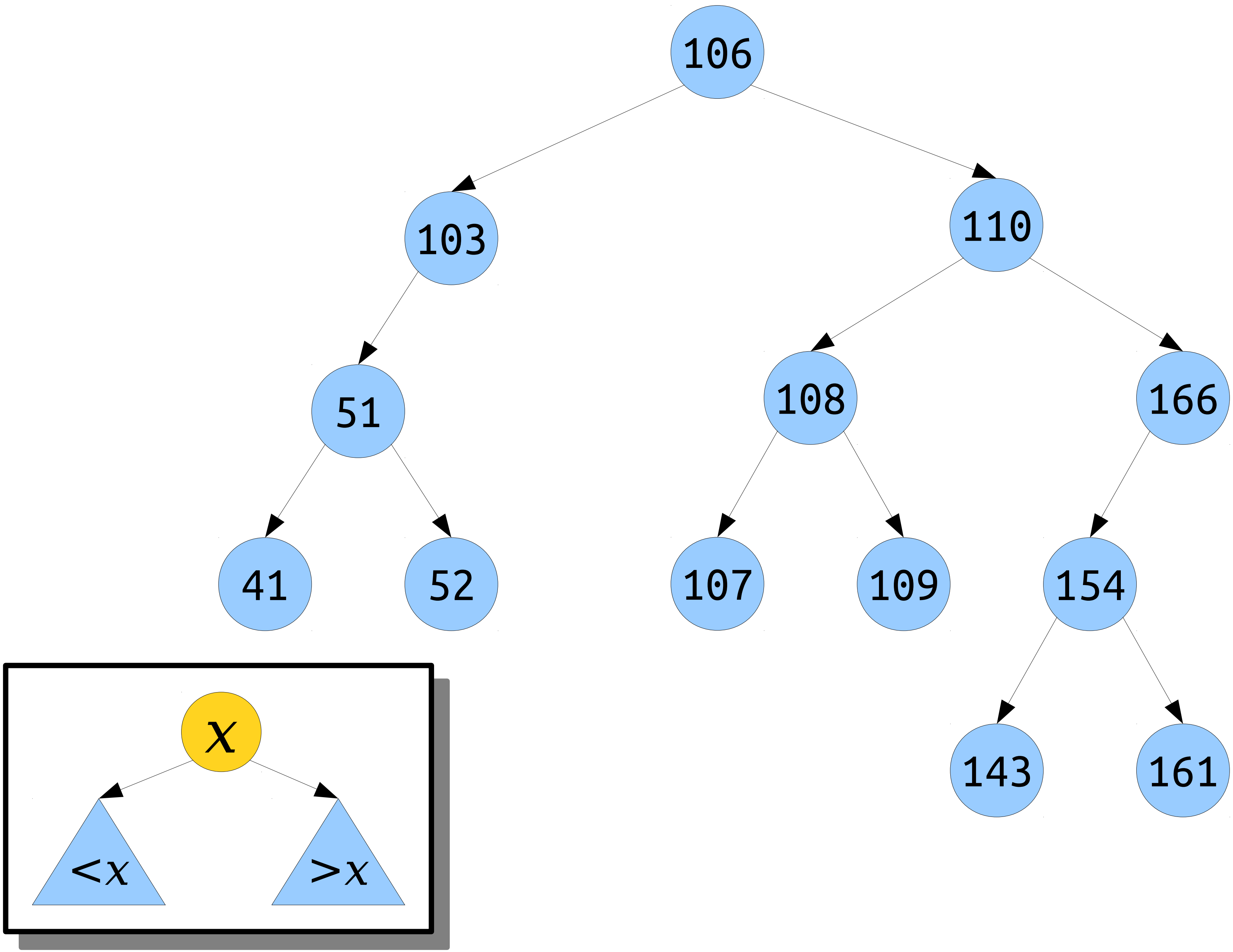
an empty tree, represented by `nullptr`, or…

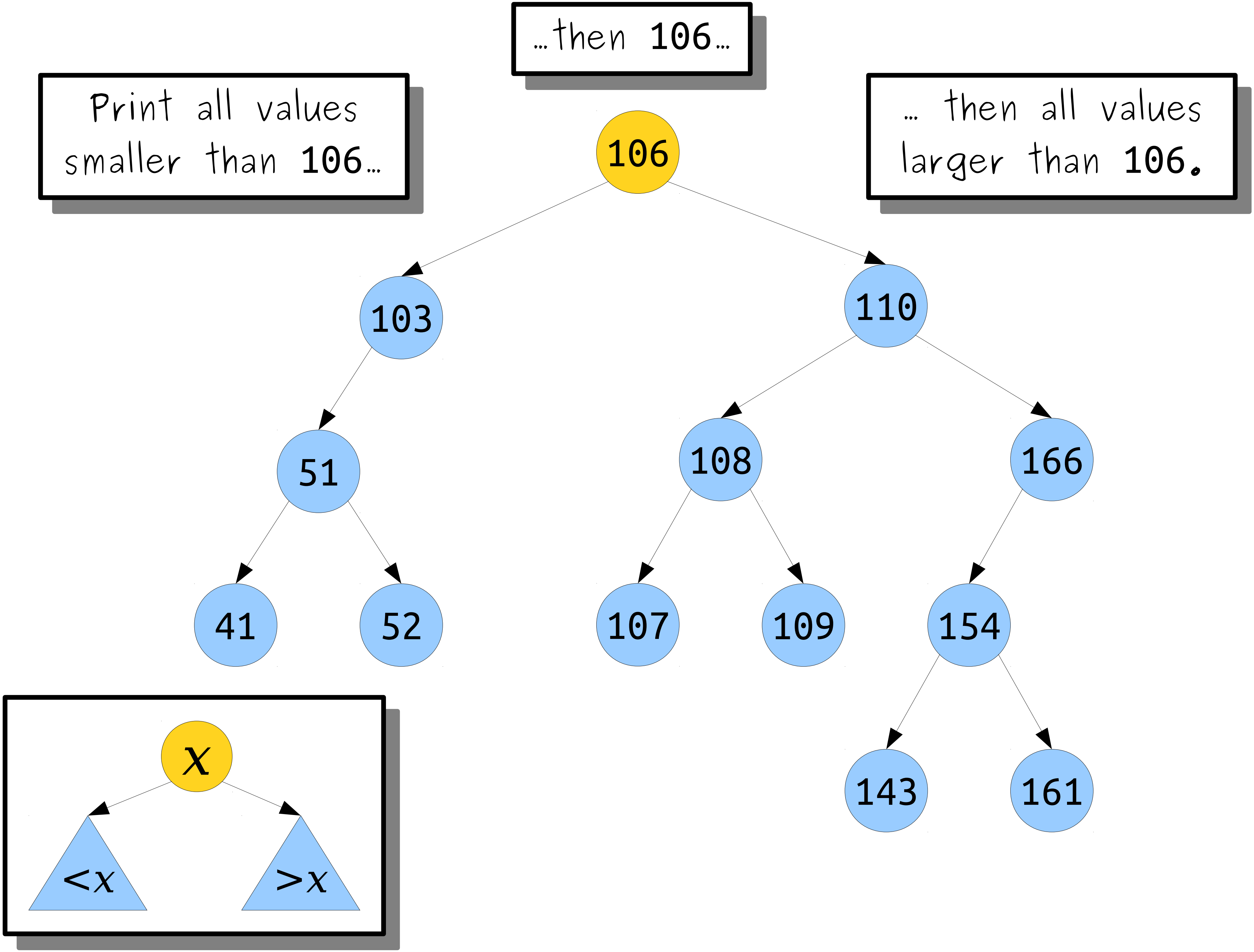


… a single node, whose left subtree is a BST of smaller values …



… and whose right subtree is a BST of larger values.


106
103
110
51
108
166
41
52
107
109
154
143
161
x
<x
>x



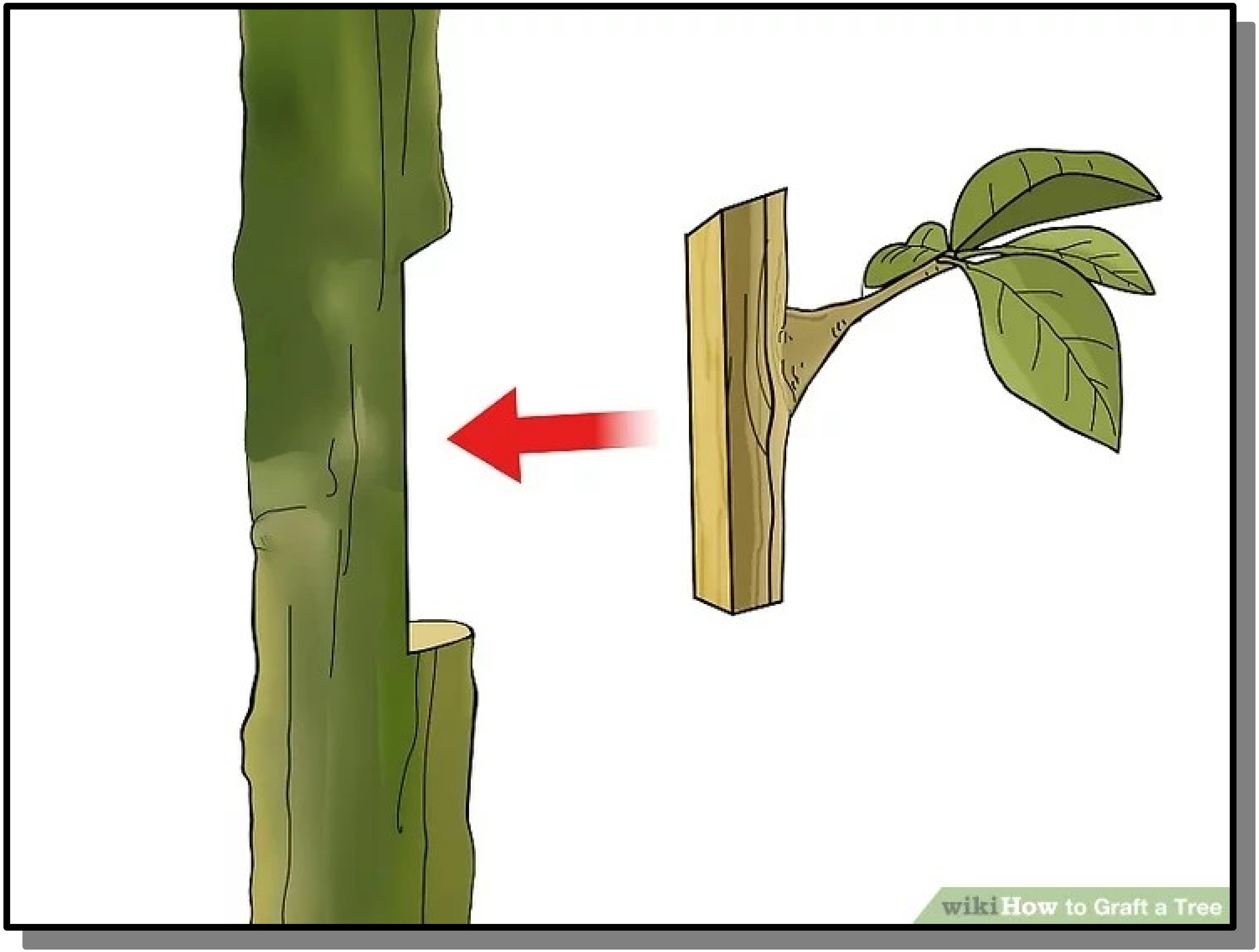
…then 106…
Print all values smaller than 106…
… then all values larger than 106.
106
103
110
51
108
166
41
52
107
109
154
143
161
x
<x
>x

Inorder Traversals

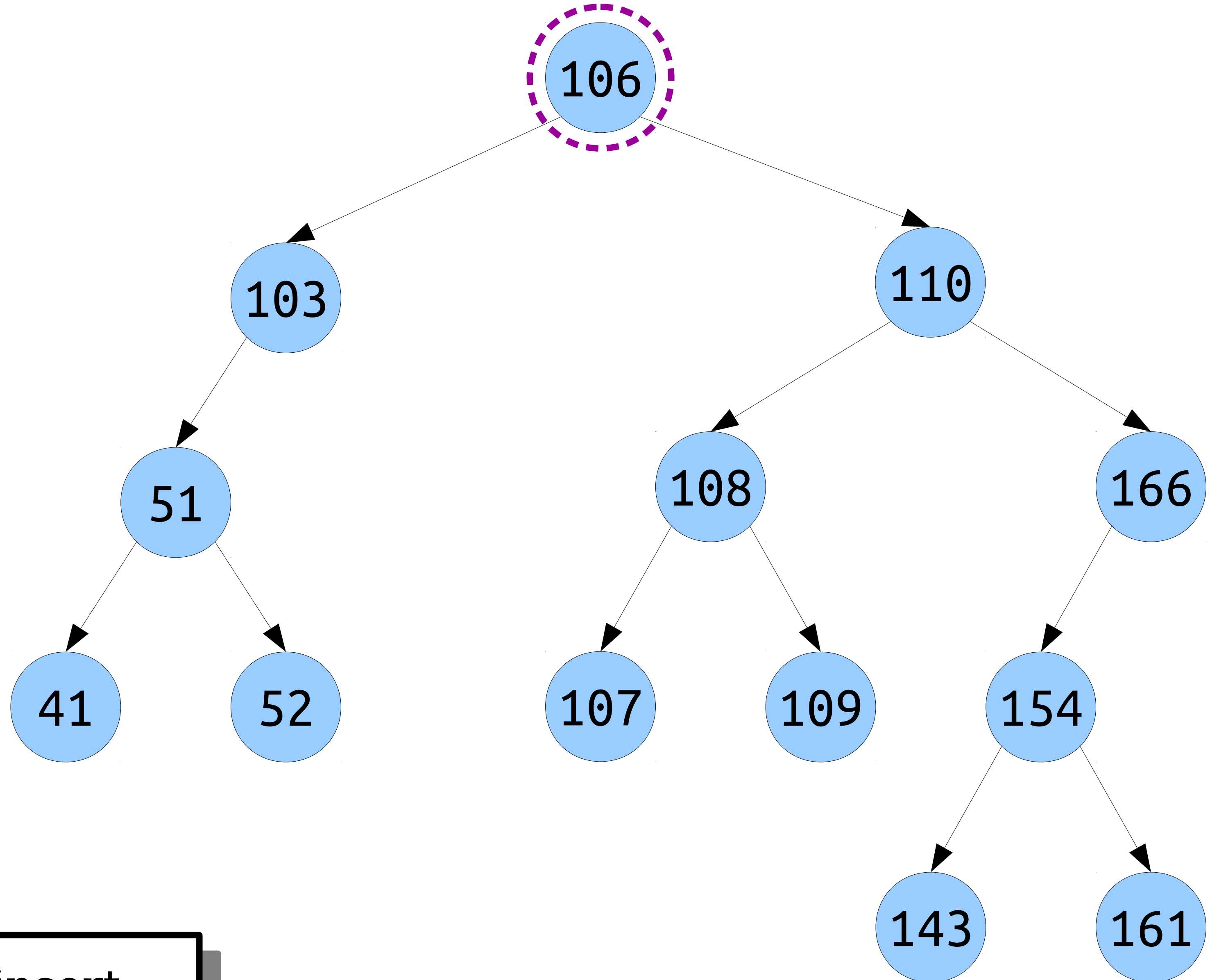
- The particular recursive pattern we just saw is called an ***inorder traversal*** of a binary tree.
- Specifically:
  - Recursively visit all the nodes in the left subtree.
  - Visit the node itself.
  - Recursively visit all the nodes in the right subtree.

Challenge problem:
Rewrite this function iteratively!

Adding to Trees



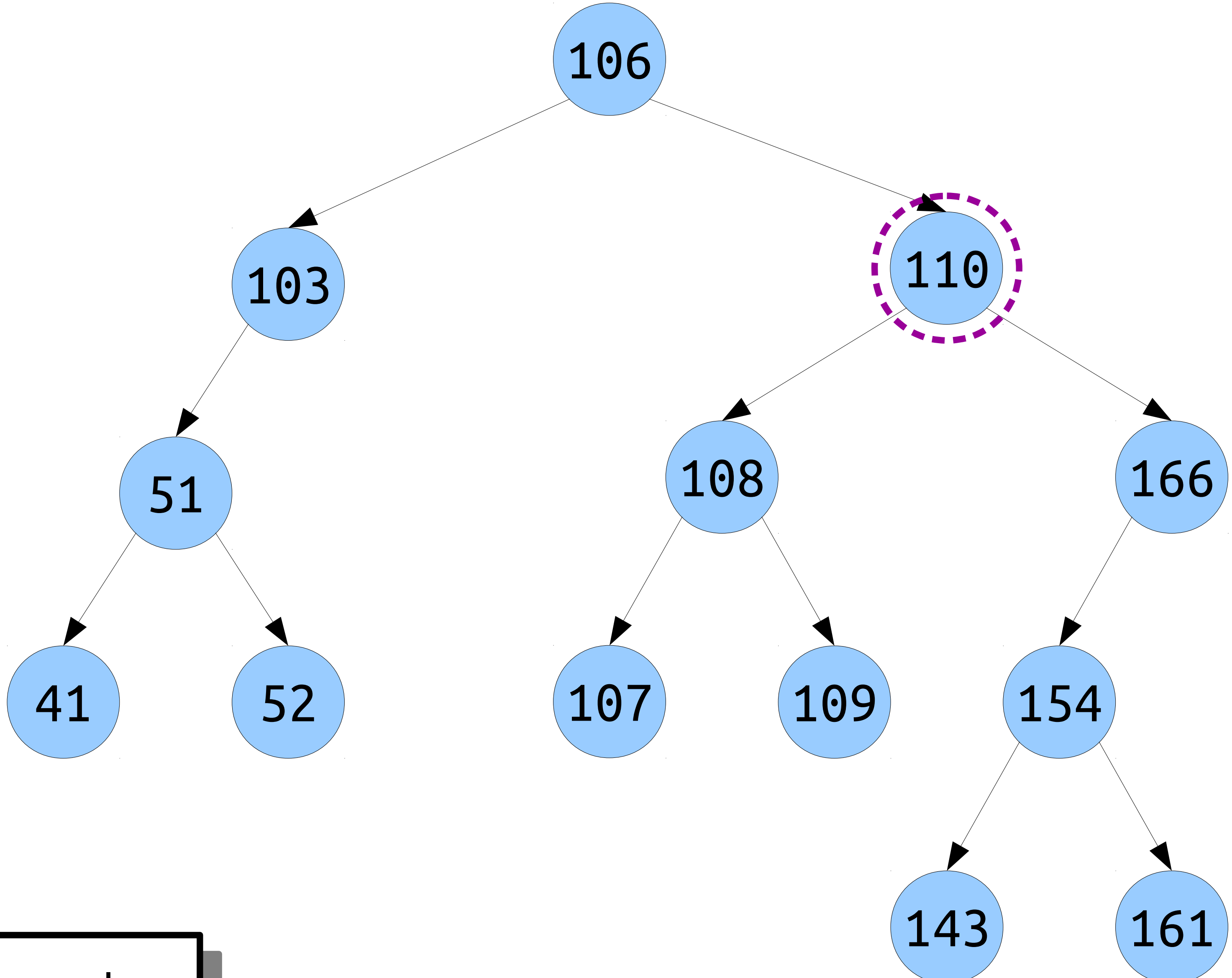
Thanks,
WikiHow!



Let’s insert

147

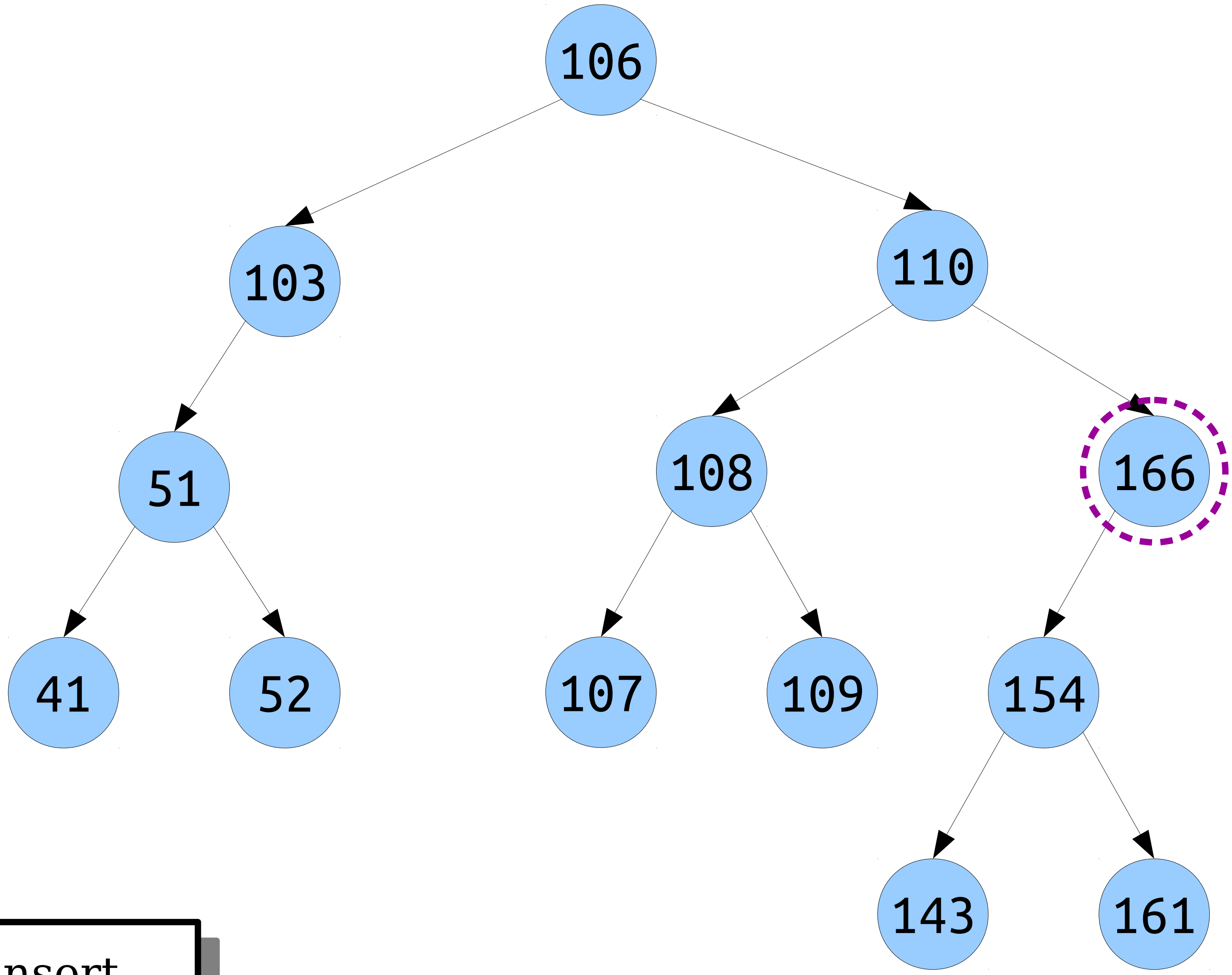
into this tree.



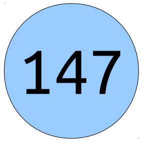
Let’s insert

147

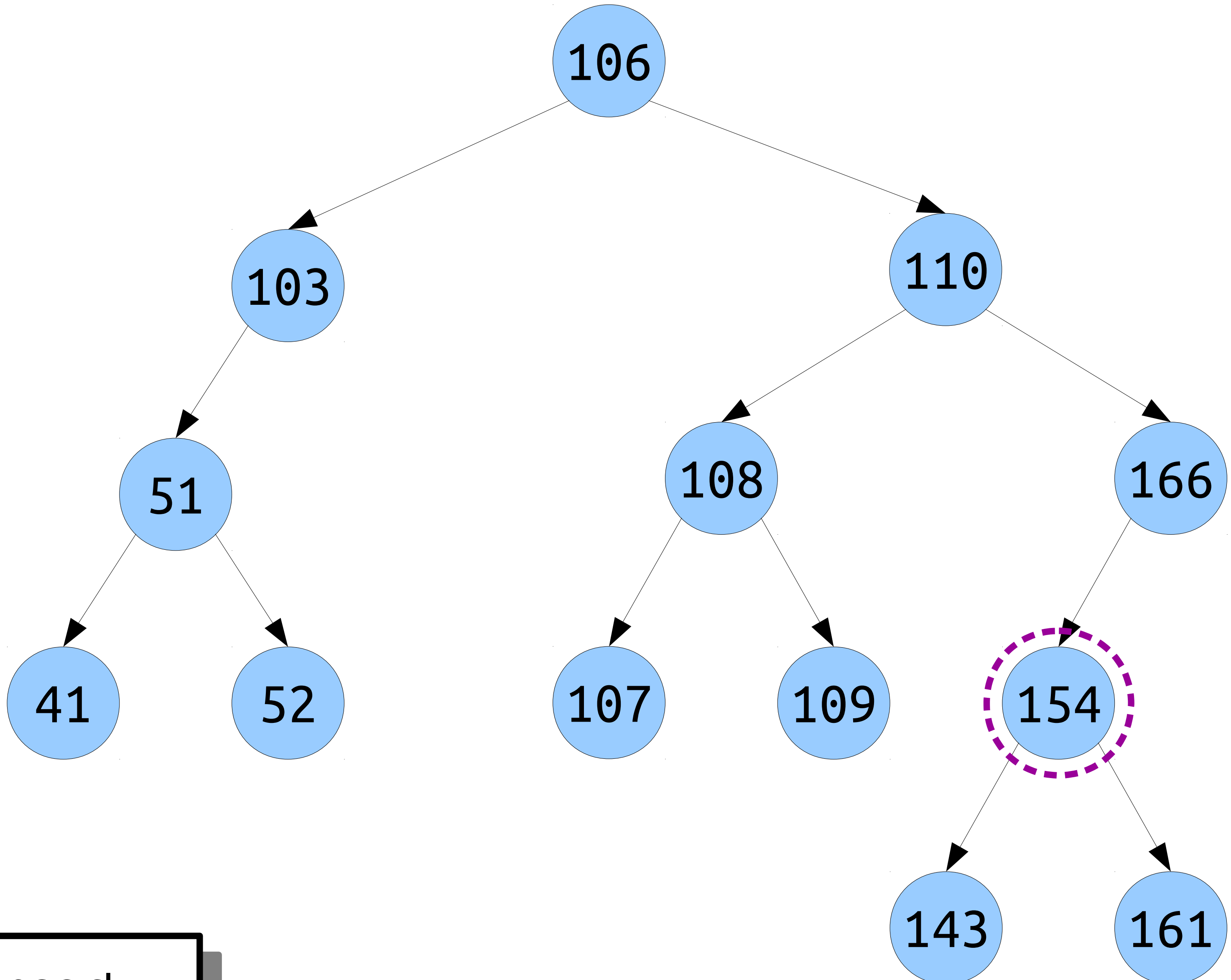
into this tree.



Let’s insert



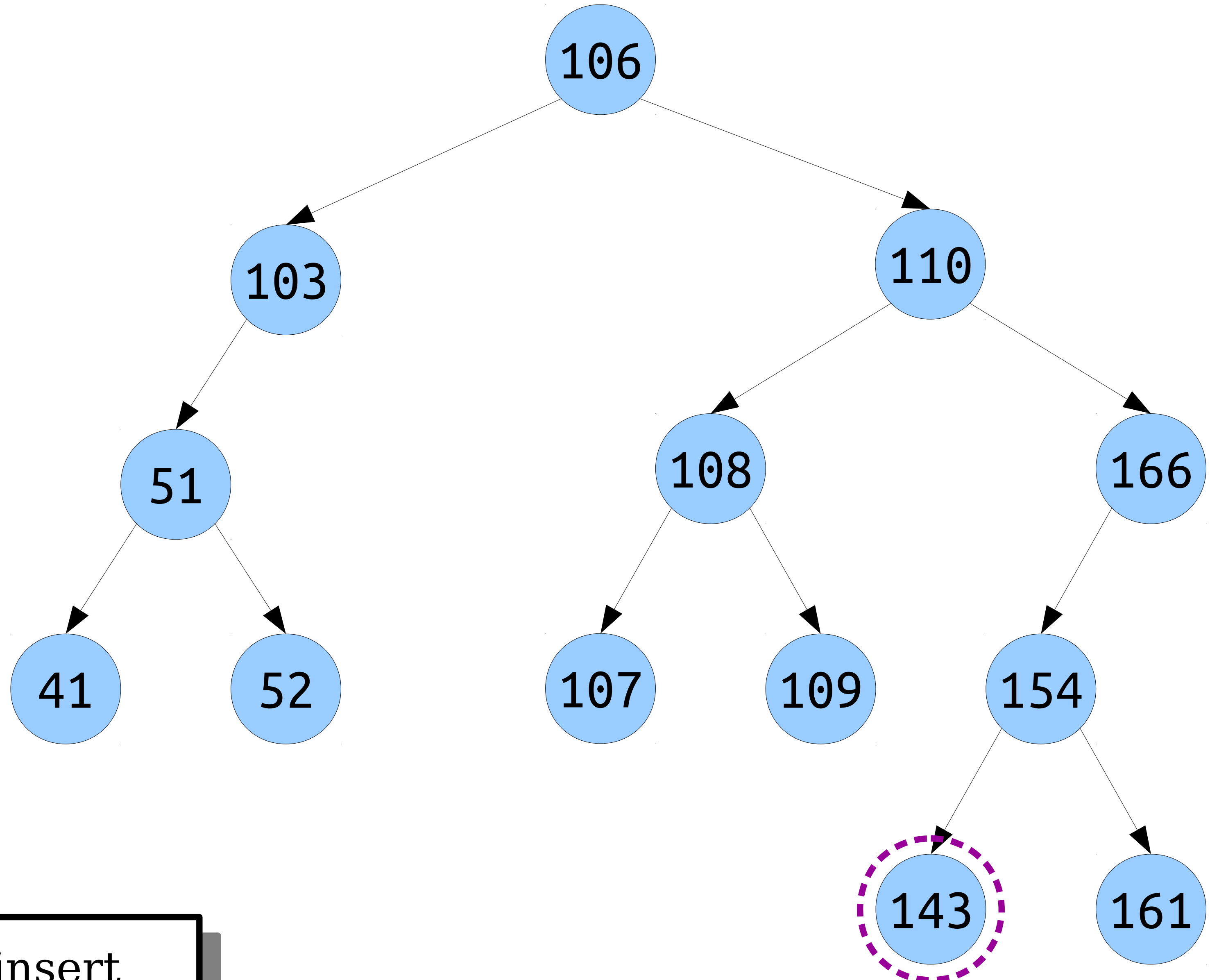
into this tree.



Let’s insert

147

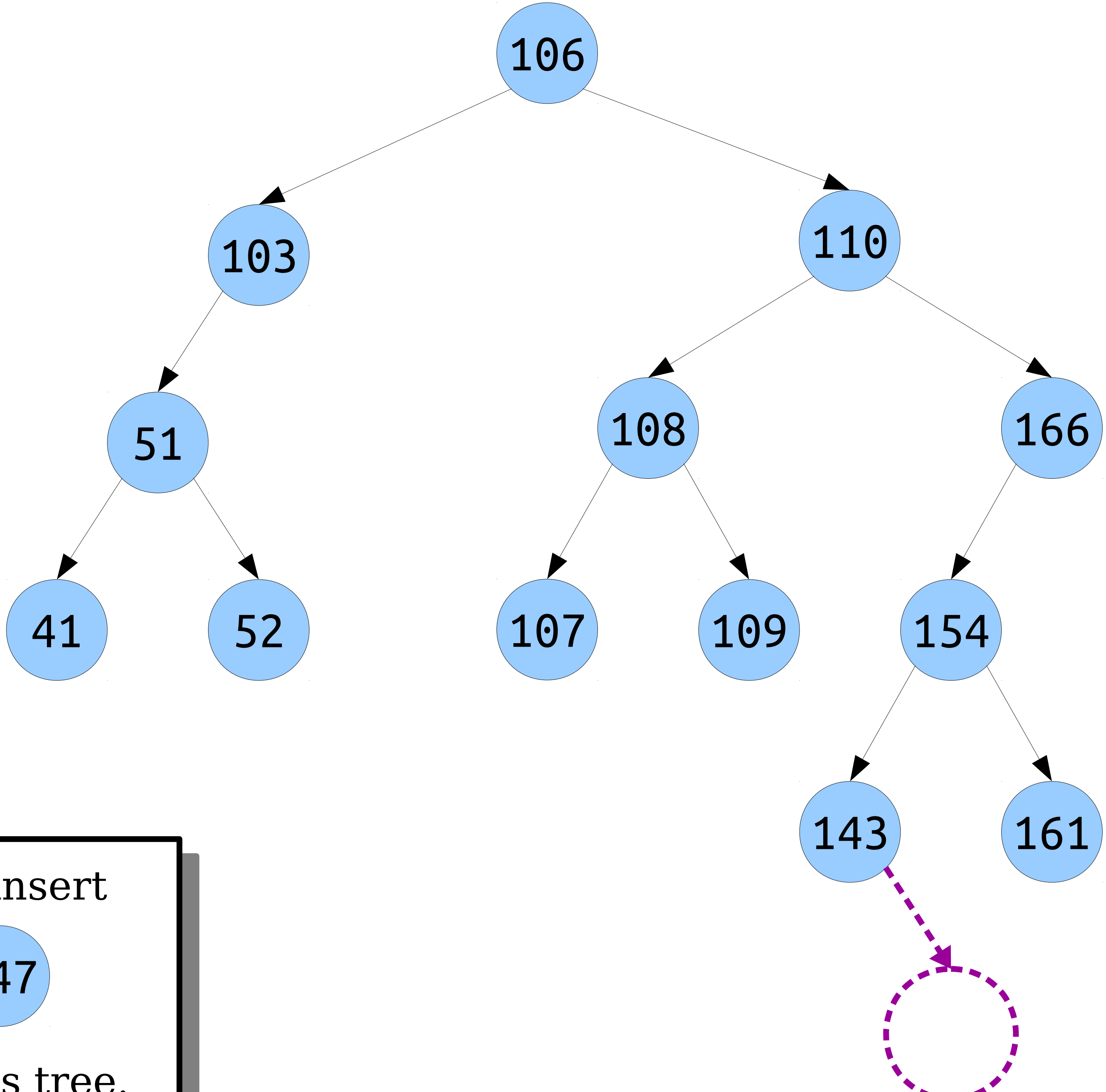
into this tree.



Let’s insert

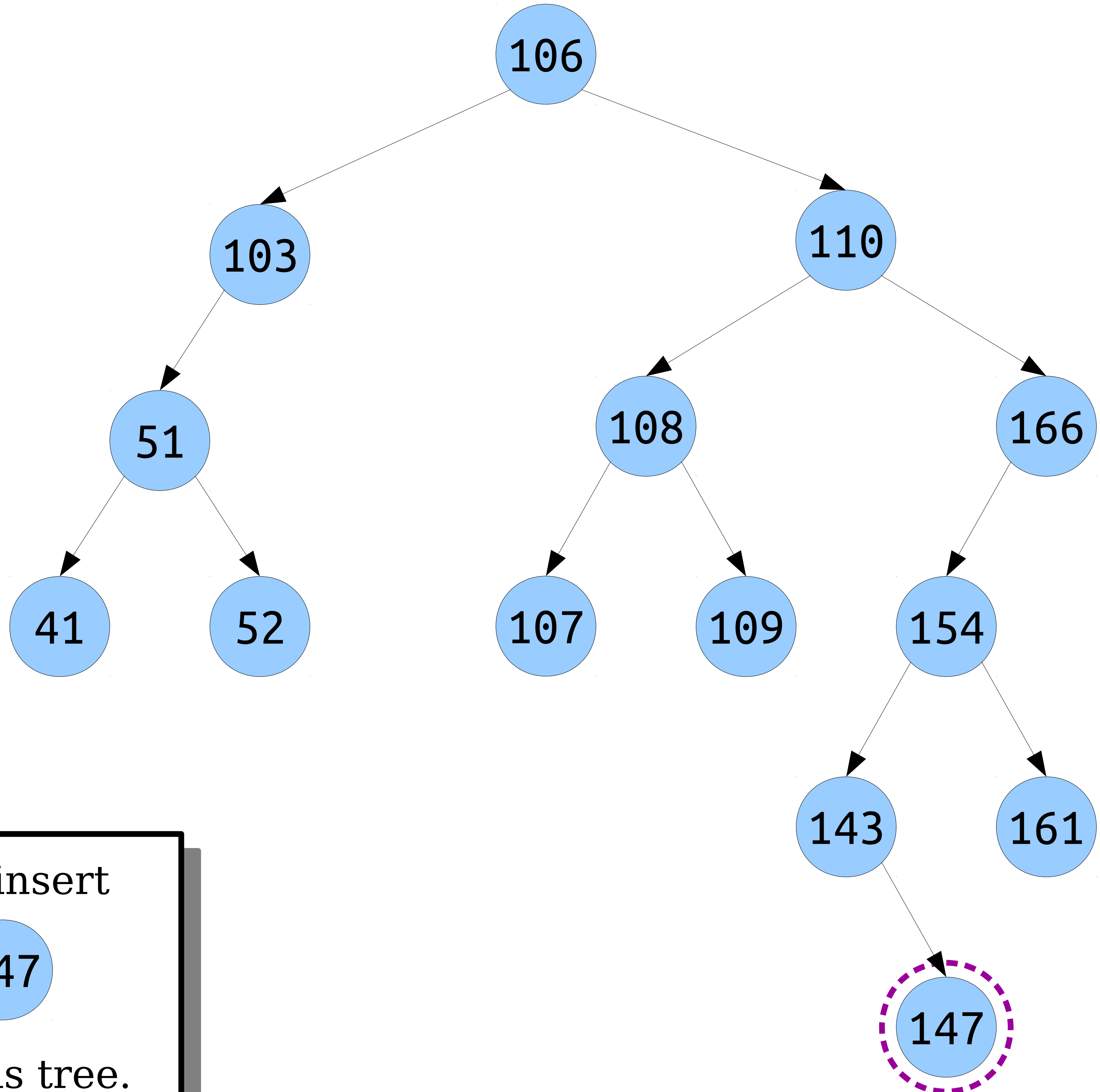
147

into this tree.



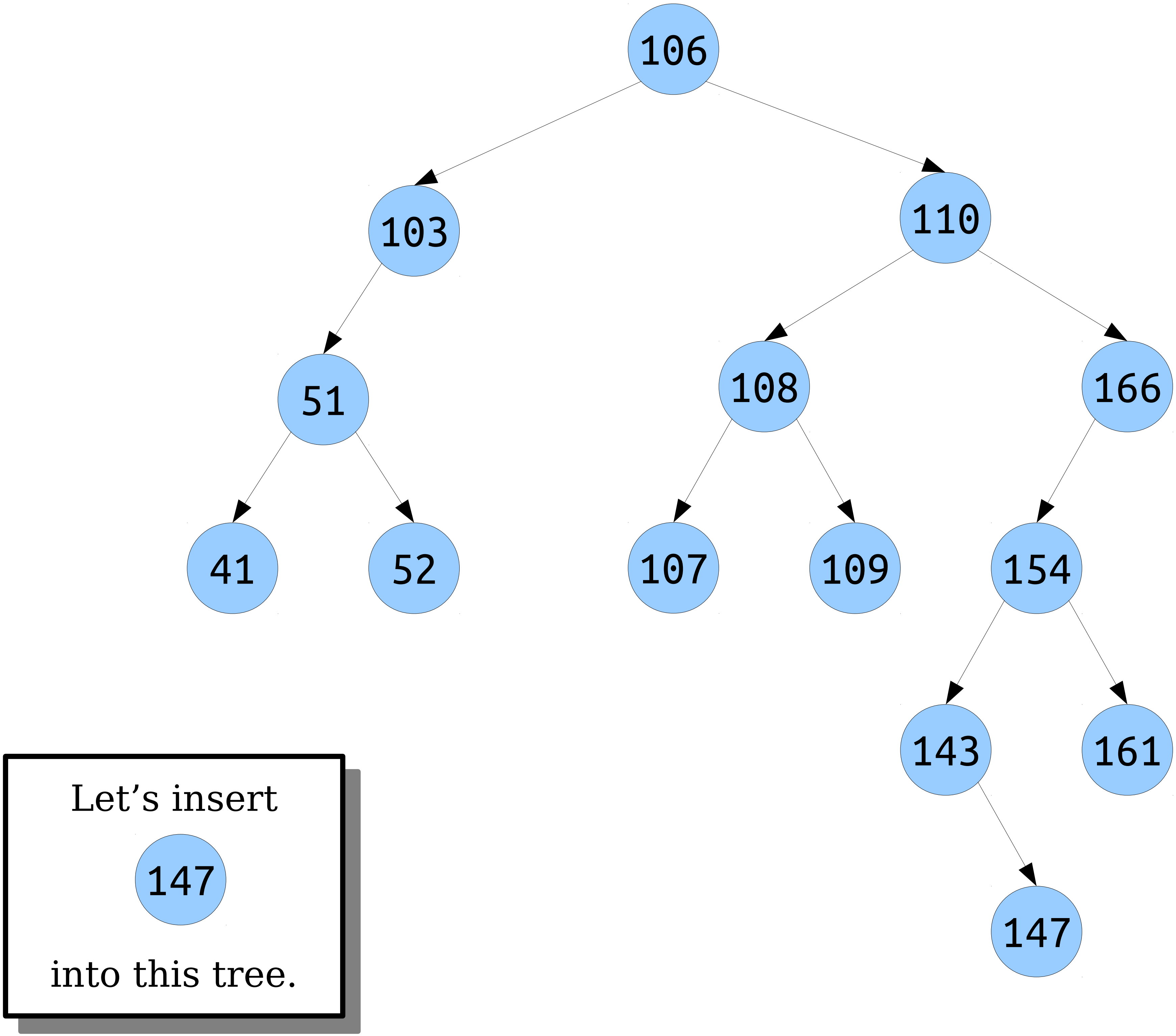
Let's insert

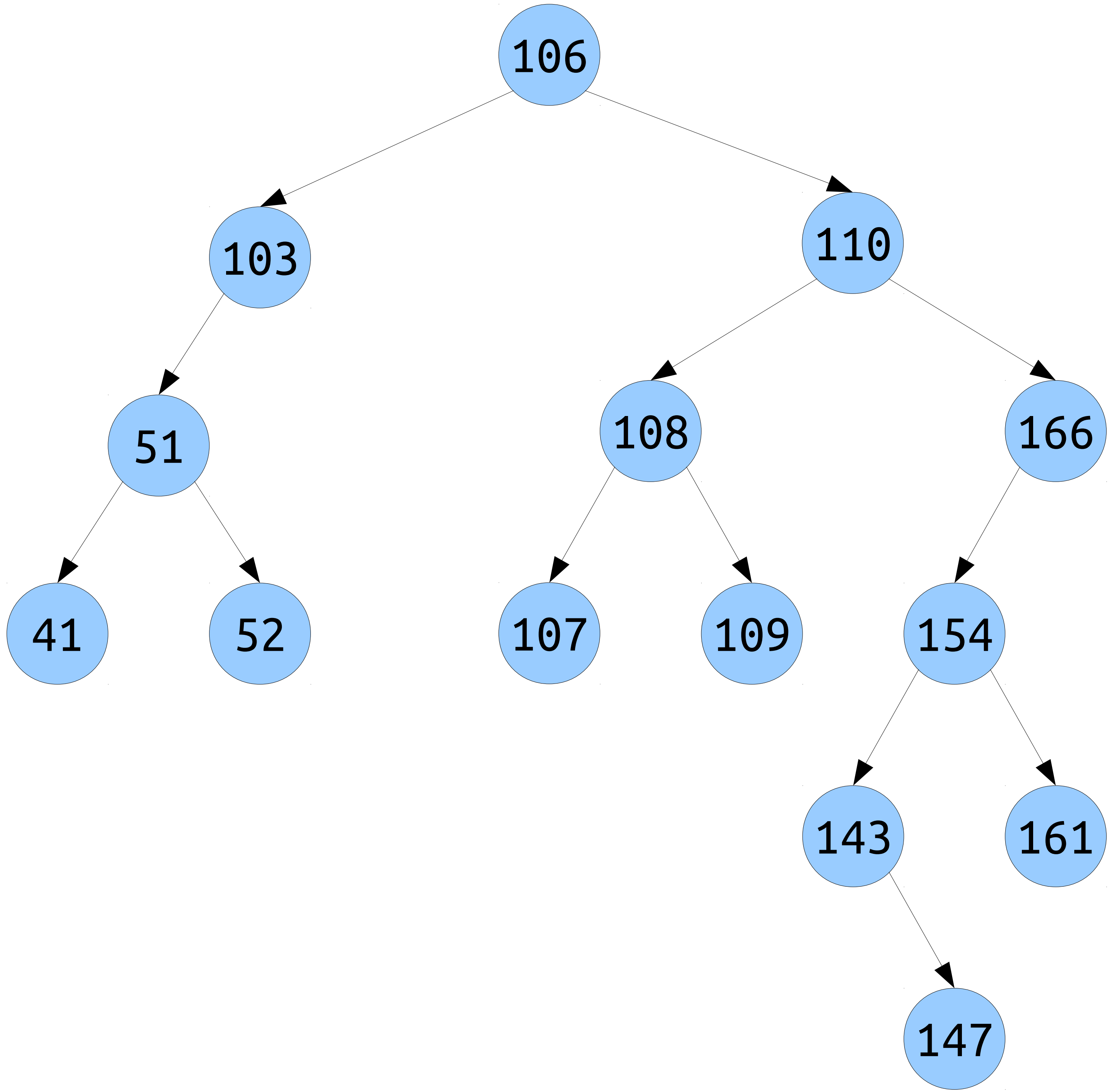
into this tree.



Let's insert

into this tree.


106
103
110
51
108
166
41
52
107
109
154
143
161
147
Let’s insert
147
into this tree.


106
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108
166
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107
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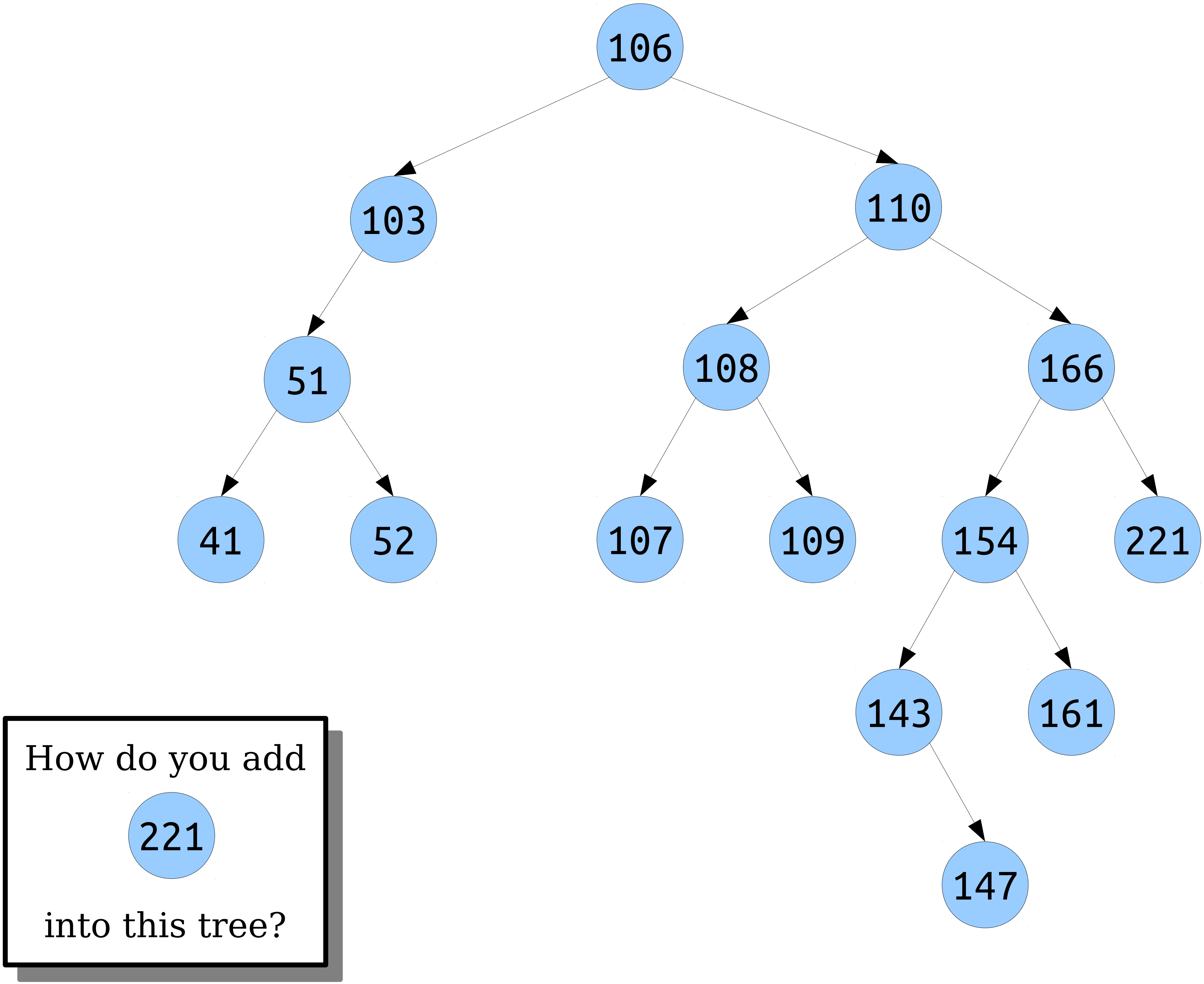
154
143
161
147


106
103
110
51
108
166
41
52
107
109
154
221
143
161
147
How do you add
221
into this tree?

Let's Code it Up!

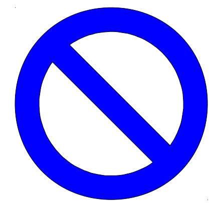
A Binary Search Tree Is Either…

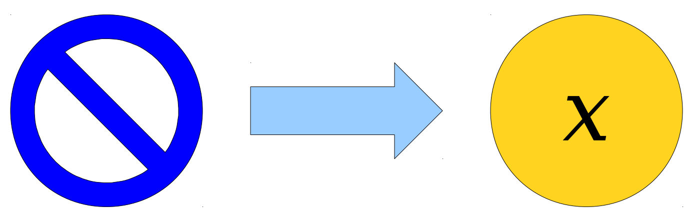
an empty tree,
represented by

`nullptr`

A Binary Search Tree Is Either…

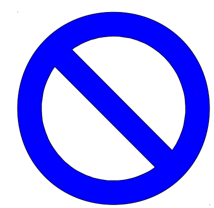
an empty tree,
represented by
nullptr



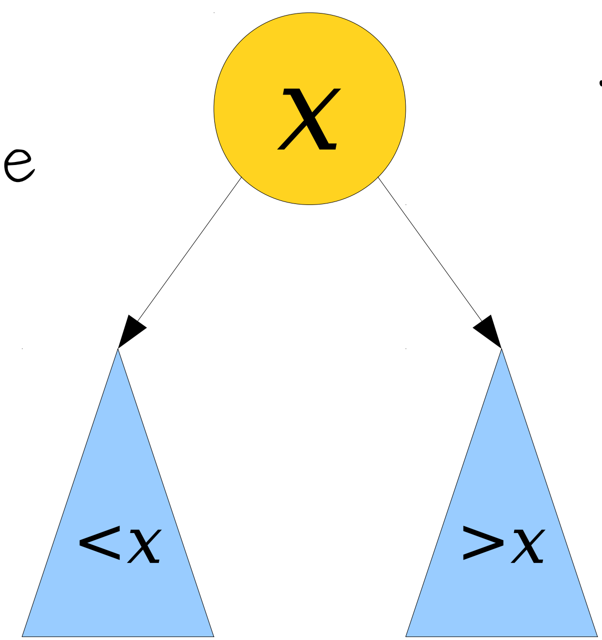


A Binary Search Tree Is Either…

an empty tree,
represented by
`nullptr`, or…



… a single node,
whose left subtree
is a BST of
smaller values …



… and whose right
subtree is a BST
of larger values.

Your Action Items

- ***Read Chapter 16.1 – 16.2.***
  - There's a bunch of BST topics in there, along with a different intuition for how they work.
- ***Keep working on Assignment 7.***
  - Hopefully you've escaped from your labyrinths! See if you can make progress on the Splicing and Dicing assignment next.

Next Time

- ***More BST Fun***
  - Some other cool tricks and techniques!