CS 106B, Lecture 22
Graphs
• Arguably the single most useful abstraction computer science: the graph
  – How to model problems using a graph
• Today and some of next week is algorithms to answer common graph questions
  – Learning these algorithms will help you solve very different problems more quickly
Google Maps

Source: https://www.google.com/maps
ADT Flowchart

Start

How many dimensions of data do I have?

Two

Grid

One

Is my data in pairs?

Yes

Map

No

Do I only care about membership?

Yes

Set

No

Which elements do I need to access?

Frequent looping or middle elements

Vector

First element

Queue

Last element

Stack
Molecules
Introducing: The Graph

• A **graph** is a mathematical structure for representing relationships

• Consists of **nodes** (aka vertices) and **edges** (aka arcs)
  – **edges** are the relationships, **nodes** are the items that have the relationship

• Examples:
  – Map: cities (nodes) are connected by roads (edges)
  – Flowchart: questions and recommendations (nodes) are connected by answers (edges)
  – Molecules: atoms (nodes) are connected by bonds (edges)
Graph examples

• For each, what are the nodes and what are the edges?
  – Web pages with links
  – Functions in a program that call each other
  – Airline routes
  – Facebook friends
  – Course pre-requisites
  – Family trees
  – Paths through a maze
Boggle as a graph

• Q: If a Boggle board is a graph, what is a node? What is an edge?
  A. Node = letter cube, Edge = Dictionary (lexicon)
  B. Node = dictionary word; Edge = letter cube
  C. Node = letter; Edge = between each letter that is part of a word
  D. Node = letter cube; Edge = connection to neighboring cube
  E. None of the above
Undirected vs. Directed

- Some relationships are mutual
  - Facebook

- Some are one-way
  - Twitter
  - Doesn't mean that all relationships are non-mutual
Representing Graphs

• Two main ways:
  – Have each node store the nodes it's connected to (adjacency list)
    • Enemies in problem 4 of the midterm
    • Ngrams
    • Doctors without Orders
  – Have a list of all the edges/edges (edge list)
    • Similar to Marbles

• The choice depends on the problem you're trying to solve
• You can sometimes represent graphs implicitly instead of explicitly storing the edges and nodes
  – e.g. Boggle, WordLadder
  – draw a picture to see the graph more clearly!
Adjacency List

- Map<Node, Vector<Node>>
- or Map<Node, Set<Node>>
Adjacency Matrix

• Store a boolean grid, rows/columns correspond to nodes
  – Alternative to Adjacency List
• Store a Vector<\textbf{Edge}> (or Set<\textbf{Edge}>)
  – \textbf{Edge} struct would have the two nodes

\textbf{Vector<\textbf{Edge}>}

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Edge Properties

• Not all edges are created equally
  – Some have greater **weight**

• Real life examples:
  – Flight costs
  – Miles on a road
  – Time spent on a road

• Store a number with each edge corresponding to its weight

Source: [https://www.google.com/maps](https://www.google.com/maps)
• I want a job at Google. Do I know anyone who works there? What about someone who knows someone?
• I want to find this word on a board made of letters "next to" each other (Boggle)
• A path is a sequence of nodes with edges between them connecting two nodes
  – Could store edges instead of nodes (why?)
  – You know Jane. Jane knows Sally. Sally knows knows Sergey Brin, the founder of Google, so the path is:
    You->Jane->Sally->Sergey
Other graph properties

• **reachable**: Vertex \( u \) is reachable from \( v \) if a path exists from \( u \) to \( v \).

• **connected**: A graph is connected if every vertex is reachable from every other.

• **complete**: If every vertex has a direct edge to every other.
• **cycle**: A path that begins and ends at the same node.
  – example: \{b, g, f, c, a\} or \{V, X, Y, W, U, V\}.
  – example: \{c, d, a\} or \{U, W, V, U\}.

  – **acyclic graph**: One that does not contain any cycles.

• **loop**: An edge directly from a node to itself.
  – Many graphs don't allow loops.
Types of Graphs

• NGrams?
  – directed, weighted, cyclic, connected

• Boggle?
  – undirected, unweighted, cyclic, connected

• A molecule?
  – undirected, weighted, potentially cyclic, connected

• A map of flights?
  – directed, weighted, cyclic, perhaps not connected

• A tree?
  – directed, acyclic graph (not connected)
  – DAGs are especially important because of **topological sort**. More on that later!
Announcements

• You should be starting LineManager – it's hard.
• Please give us feedback! cs198.stanford.edu
• Feel free to use seepluspl.us to help you understand trees or pointers. It's still in development, so be patient with quirks
• Notes on course feedback:
  – If you have a question outside the scope of the class, please post on Piazza or come talk to me during OH! I don't want to stop your questions, but I sometimes have to make choices to ensure that I don't confuse other students or run out of time for material we need to cover.
Working with Graphs

• We've seen how to model data with a graph
• There are lots of cool graph algorithms that make it easy to solve certain problems
  – Goal: know how to apply a model a problem as a graph and apply the relevant graph algorithm to it
• We'll spend most of the rest of this unit learning about graph algorithms
Finding Paths

• Easiest way: Depth-First Search (DFS)
  – Recursive backtracking!

• Finds a path between two nodes if it exists
  – Or can find all the nodes **reachable** from a node
    • Where can I travel to starting in San Francisco?
    • If all my friends (and their friends, and so on) share my post, how many will eventually see it?
DFS

If we've seen the node before, stop
Otherwise, visit all the unvisited nodes from this node
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Otherwise, visit all the unvisited nodes from this node
• In an $n$-node, $m$-edge graph, takes $O(m + n)$ time with an adjacency list
  – Visit each edge once, visit each node at most once
• Pseudocode:
  \[
  \text{dfs from } v_1:
  \]
  mark $v_1$ as seen.
  for each of $v_1$'s unvisited neighbors $n$:
    \text{dfs}($n$)

• How could we modify the pseudocode to look for a specific path?
  – Recursive Backtracking
  – Look at maze example from week 4
Finding *Shortest* Paths

• We can find paths between two nodes, but how can we find the **shortest** path?
  – Fewest number of steps to complete a task?
  – Least amount of edits between two words?
• When have we solved this problem before?
Breadth-First Search (BFS)

• Idea: processing a node involves knowing we need to visit all its neighbors (just like DFS)
• Need to keep a TODO list of nodes to process
• Which node from our TODO list should we process first if we want the shortest path?
  – The first one we saw?
  – The last one we saw?
  – A random node?
Breadth-First Search (BFS)

- Keep a Queue of nodes as our TODO list
- Idea: dequeue a node, enqueue all its neighbors
- Still will return the same nodes as reachable, just might have shorter paths
Dequeue a node
Otherwise, add all its unseen neighbors to the queue

queue: a
BFS

Dequeue a node
Otherwise, add all its unseen neighbors to the queue

queue: e, g
BFS

Dequeue a node
Otherwise, add all its unseen neighbors to the queue

queue: e, g
BFS

queue: g, f

Dequeue a node
Otherwise, add all its unseen neighbors to the queue
BFS

Dequeue a node
Otherwise, add all its unseen neighbors to the queue

queue: g, f
BFS

Dequeue a node
Otherwise, add all its unseen neighbors to the queue

queue: f, h
BFS

Dequeue a node
Otherwise, add all its unseen neighbors to the queue

queue: f, h
BFS

Dequeue a node
Otherwise, add all its unseen neighbors to the queue

queue: h
BFS

Dequeue a node
Otherwise, add all its unseen neighbors to the queue

queue: h
BFS

Dequeue a node
Otherwise, add all its unseen neighbors to the queue

queue: i
Dequeue a node
Otherwise, add all its unseen neighbors to the queue

queue: i
Dequeue a node
Otherwise, add all its unseen neighbors to the queue

queue: c
BFS

Dequeue a node
Otherwise, add all its unseen neighbors to the queue

queue: c
BFS

Dequeue a node
Otherwise, add all its unseen neighbors to the queue

queue: c
BFS Details

• In an $n$-node, $m$-edge graph, takes $O(m + n)$ time with an adjacency list
  – Visit each edge once, visit each node at most once

• Pseudocode:
  ```
  bfs from $v_1$:
  add $v_1$ to the queue.
  while queue is not empty:
    dequeue a node $n$
    enqueue $n$'s unseen neighbors
  ```

• How could we modify the pseudocode to look for a specific path?