CS 106B, Lecture 25
Sorting

Based on slides created by Marty Stepp, Chris Gregg, Keith Schwarz, Julie Zelenski, Jerry Cain, Eric Roberts, Mehran Sahami, Stuart Reges, Cynthia Lee, and others

Plan for Today

- Analyze several algorithms to do the same task: sorting
  - Big-Oh in the real world

Sorting

- **sorting**: Rearranging the values in a collection into a specific order.
  - *can be solved in many ways:*

Algorithm	Description
bogo sort	shuffle and pray
bubble sort	swap adjacent pairs that are out of order
selection sort	look for the smallest element, move to front
insertion sort	build an increasingly large sorted front portion
merge sort	recursively divide the data in half and sort it
heap sort	place the values into a binary heap then dequeue
quick sort	recursively "partition" data based on a pivot value
bucket sort	cluster elements into smaller groups, sort the groups
radix sort	sort integers by last digit, then 2nd to last, then ...

Bogo sort

- **bogo sort**: Orders a list of values by repetitively shuffling them and checking if they are sorted.
  - name comes from the word "bogus"; a.k.a. "bogus sort"

  The algorithm:
  - Scan the list, seeing if it is sorted. If so, stop.
  - Else, shuffle the values in the list and repeat.

- This sorting algorithm (obviously) has terrible performance!
  - What is its runtime?

Bogo sort code

```
// Places the elements of v into sorted order.
void bogoSort(Vector<int>& v) {
    while (!isSorted(v)) {
        shuffle(v);         // from shuffle.h
    }
}

// Returns true if v's elements are in sorted order.
bool isSorted(Vector<int>& v) {
    for (int i = 0; i < v.size() - 1; i++) {
        if (v[i] > v[i + 1]) {
            return false;
        }
    }
    return true;
}
```

Bogo sort runtime

- How long should we expect bogo sort to take?
  - related to probability of shuffling into sorted order
  - assuming shuffling code is fair, probability equals 1 / (number of permutations of *N* elements) = 1/N!
  - average case performance: O($N * N!$)
  - worst case performance: O(∞)
  - What is the best case performance?

Selection sort example

- **selection sort**: Repeatedly swap smallest unplaced value to front.

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	22	18	12	-4	27	30	36	50	7	68	91	56	2	85	42	98	25

- After 1st, 2nd, and 3rd passes:

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	**-4**	18	12	**22**	27	30	36	50	7	68	91	56	2	85	42	98	25

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	-4	**2**	12	22	27	30	36	50	7	68	91	56	**18**	85	42	98	25

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	-4	2	**7**	22	27	30	36	50	**12**	68	91	56	18	85	42	98	25

Selection sort code

```
// Rearranges elements of v into sorted order.
void selectionSort(Vector<int>& v) {
    for (int i = 0; i < v.size() - 1; i++) {
        // find index of smallest remaining value
        int min = i;
        for (int j = i + 1; j < v.size(); j++) {
            if (v[j] < v[min]) {
                min = j;
            }
        }
        // swap smallest value to proper place, v[i]
        if (i != min) {
            int temp = v[i];
            v[i] = v[min];
            v[min] = temp;
        }
    }
}
```

Insertion sort

- **insertion sort**: orders a list of values by repetitively inserting a particular value into a sorted subset of the list

- more specifically:
  - consider the first item to be a sorted sublist of length 1
  - insert second item into sorted sublist, shifting first item if needed
  - insert third item into sorted sublist, shifting items 1-2 as needed
  - ...
  - repeat until all values have been inserted into their proper positions
  - How people line up when they have different arrival times!

- Runtime: **$O(N^2)$.**
  - Generally somewhat faster than selection sort for most inputs.

Insertion sort example

- Makes *N*-1 passes over the array.
- At the end of pass *i*, the elements that occupied *A*[0]…*A*[*i*] originally are still in those spots and in sorted order.

index	0	1	2	3	4	5	6	7
value	15	2	8	1	17	10	12	5
pass 1	**2**	**15**	8	1	17	10	12	5
pass 2	**2**	**8**	**15**	1	17	10	12	5
pass 3	**1**	**2**	**8**	**15**	17	10	12	5
pass 4	**1**	**2**	**8**	**15**	**17**	10	12	5
pass 5	**1**	**2**	**8**	**10**	**15**	**17**	12	5
pass 6	**1**	**2**	**8**	**10**	**12**	**15**	**17**	5
pass 7	**1**	**2**	**5**	**8**	**10**	**12**	**15**	**17**

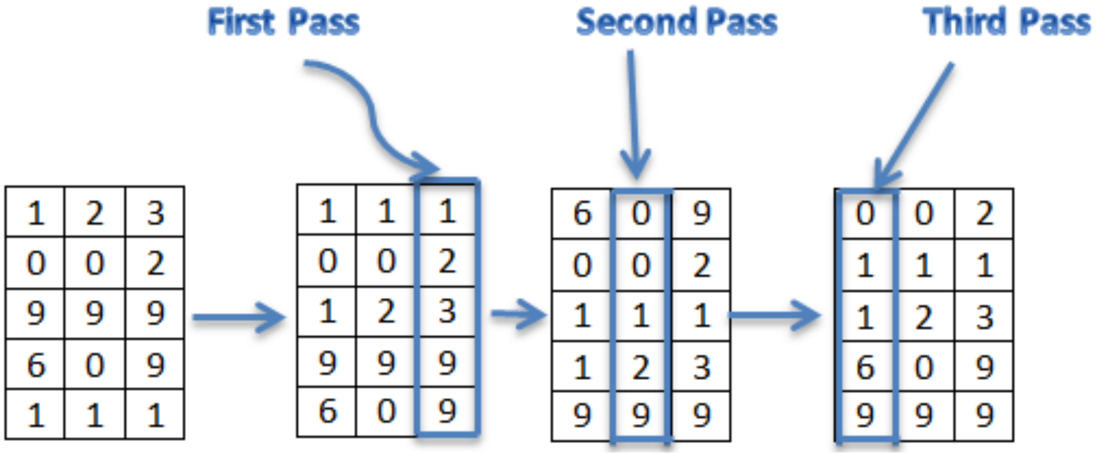
Insertion sort code

```
// Rearranges the elements of v into sorted order.
void insertionSort(Vector<int>& v) {
    for (int i = 1; i < v.size(); i++) {
        int temp = v[i];

        // slide elements right to make room for v[i]
        int j = i;
        while (j >= 1 && v[j - 1] > temp) {
            v[j] = v[j - 1];
            j--;
        }
        v[j] = temp;
    }
}
```

Bucket/radix sort

- **bucket sort**: arrange items into buckets or bins repeatedly
- **radix sort**: sort integers by 1s, then 10s, then 100s, ...
  - O(*N*) when used with data in a known fixed range (!)

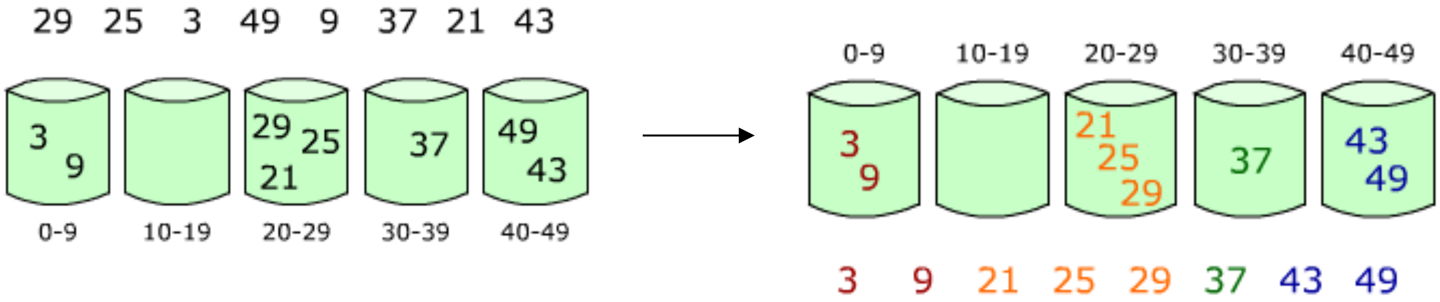


1	2	3
0	0	2
9	9	9
6	0	9
1	1	1

1	1	1
0	0	2
1	2	3
9	9	9
6	0	9

6	0	9
0	0	2
1	1	1
1	2	3
9	9	9

0	0	2
1	1	1
1	2	3
6	0	9
9	9	9



Announcements

- MiniBrowser is due today, Calligraphy will be released later today
  - Multiple parts, please start early (2nd and 3rd parts are harder than the 1st part)
- Final is a **week from Saturday**, at 8:30AM
  - Practice exam will be released in the next few days
- Please give us feedback! cs198.stanford.edu
- Course feedback:
  - A note on LaIR/Piazza

Merge sort

- **merge sort**: Repeatedly divides the data in half, sorts each half, and combines the sorted halves into a sorted whole.

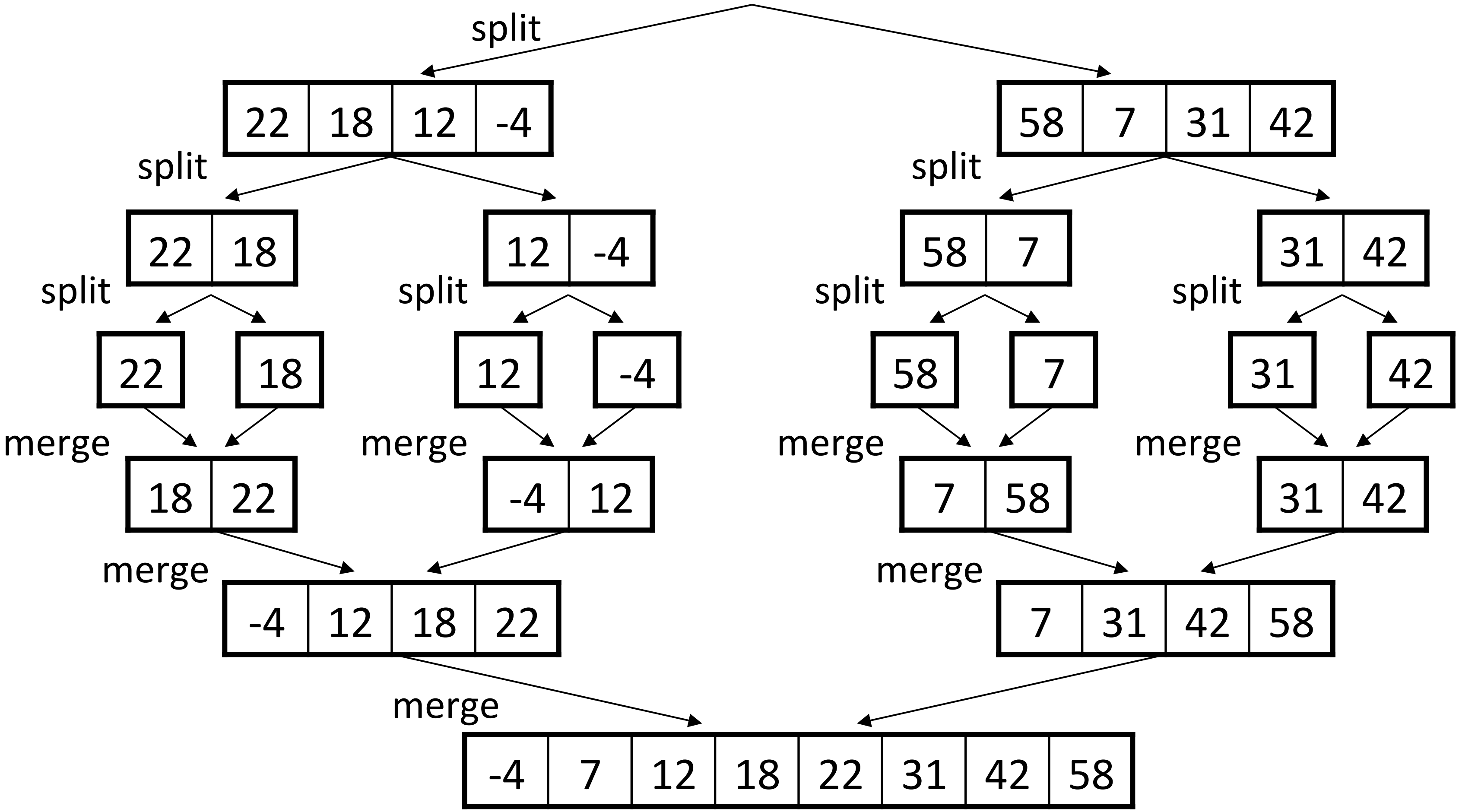
  The algorithm:
  - Divide the list into two roughly equal halves.
  - Sort the left half.
  - Sort the right half.
  - Merge the two sorted halves into one sorted list.

  - Often implemented recursively.
  - An example of a "divide and conquer" algorithm.
    - Invented by John von Neumann in 1945

  - Runtime: **O(*N* log *N*)**. Somewhat faster for asc/descending input.

Merge sort example

index	0	1	2	3	4	5	6	7
value	22	18	12	-4	58	7	31	42



split

22 | 18 | 12 | -4   58 | 7 | 31 | 42

split   split

22 | 18   12 | -4   58 | 7   31 | 42

split   split   split   split

22   18   12   -4   58   7   31   42

merge   merge   merge   merge

18 | 22   -4 | 12   7 | 58   31 | 42

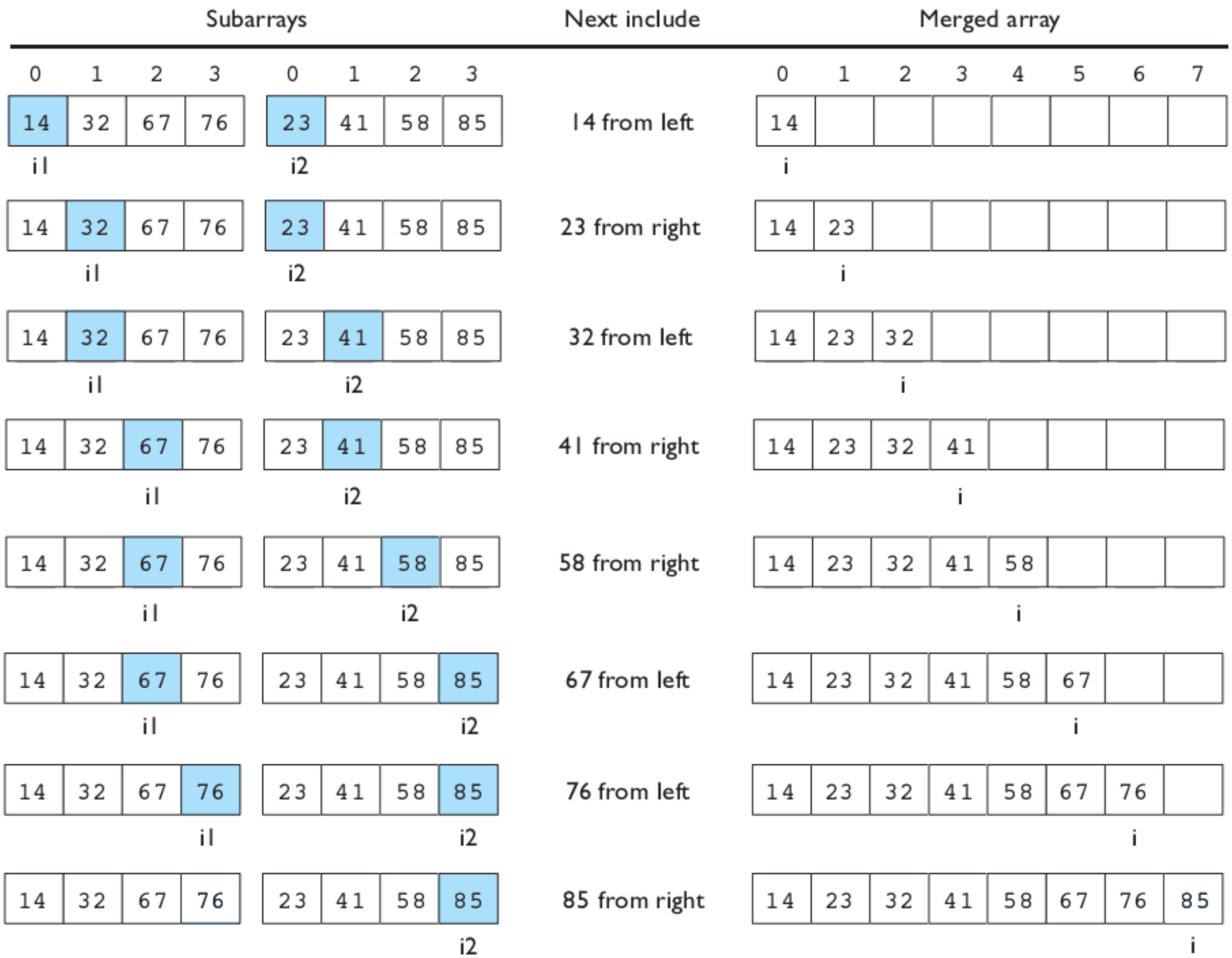
merge   merge

-4 | 12 | 18 | 22   7 | 31 | 42 | 58

merge

-4 | 7 | 12 | 18 | 22 | 31 | 42 | 58

Merging sorted halves



Subarrays								Next include	Merged array							
0	1	2	3	0	1	2	3		0	1	2	3	4	5	6	7
14	32	67	76	23	41	58	85	14 from left	14							
i1				i2					i							
14	32	67	76	23	41	58	85	23 from right	14	23						
	i1			i2						i						
14	32	67	76	23	41	58	85	32 from left	14	23	32					
	i1				i2						i					
14	32	67	76	23	41	58	85	41 from right	14	23	32	41				
		i1			i2							i				
14	32	67	76	23	41	58	85	58 from right	14	23	32	41	58			
		i1				i2							i			
14	32	67	76	23	41	58	85	67 from left	14	23	32	41	58	67		
		i1					i2							i		
14	32	67	76	23	41	58	85	76 from left	14	23	32	41	58	67	76	
			i1				i2								i	
14	32	67	76	23	41	58	85	85 from right	14	23	32	41	58	67	76	85
							i2									i

Merge sort code

```
// Rearranges the elements of v into sorted order using
// the merge sort algorithm.
void mergeSort(Vector<int>& v) {
    if (v.size() >= 2) {
        // split vector into two halves
        Vector<int> left = v.subList(0, v.size() / 2);
        Vector<int> right =
          v.subList(v.size() / 2 + 1, (v.size() - 1) / 2);

        // recursively sort the two halves
        mergeSort(left);
        mergeSort(right);

        // merge the sorted halves into a sorted whole
        v.clear();
        merge(v, left, right);
    }
}
```

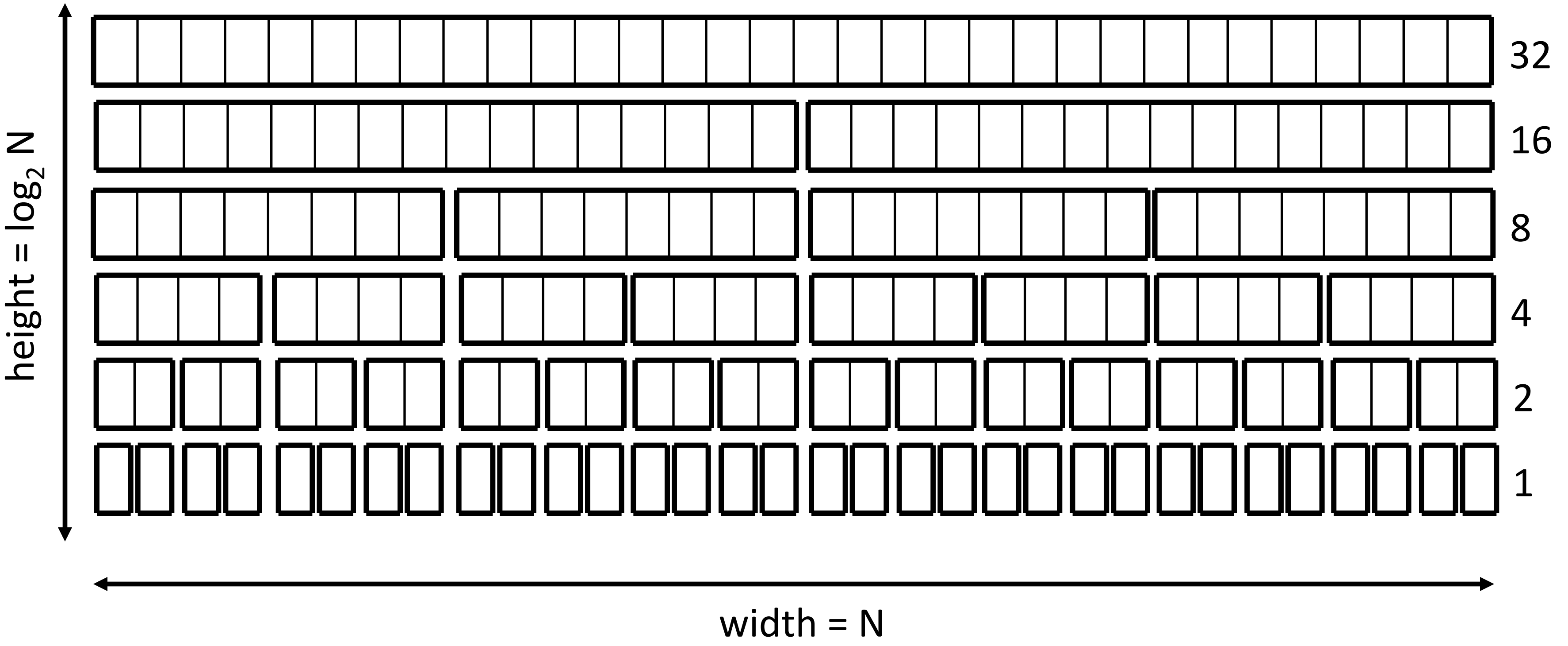
Merge halves code

```
// Merges the left/right elements into a sorted result.
// Precondition: left/right are sorted
void merge(Vector<int>& result,
           Vector<int>& left, Vector<int>& right) {
    int leftIndex = 0;
    int rightIndex = 0;

    for (int i = 0; i < left.size() + right.size(); i++) {
        if (rightIndex >= right.size() ||
           (leftIndex < left.size() &&
            left[leftIndex] <= right[rightIndex])) {
            result += left[leftIndex];    // take from left
            leftIndex++;
        } else {
            result += right[rightIndex];   // take from right
            rightIndex++;
        }
    }
}
```

Runtime intuition

- Merge sort performs O(N) operations on each level. *(width)*
  - Each level splits the data in 2, so there are $\log_2 N$ levels. *(height)*
  - Product of these = $N * \log_2 N$ = O($N \log N$). *(area)*
  - Example: $N = 32$. Performs ~ $\log_2 32 = 5$ levels of N operations each:



Quick sort

- **quick sort**: Orders a list of values by partitioning the list around one element called a *pivot*, then sorting each partition.
  - invented by British computer scientist C.A.R. Hoare in 1960

- Quick sort is another divide and conquer algorithm:
  - Choose one element in the list to be the pivot.
  - *Divide* the elements so that all elements less than the pivot are to its left and all greater (or equal) are to its right.
  - *Conquer* by applying quick sort (recursively) to both partitions.

- Runtime: **$O(N \log N)$** average, but $O(N^2)$ worst case.
  - Generally somewhat faster than merge sort.

Choosing a "pivot"

- The algorithm will work correctly no matter which element you choose as the pivot.
  - A simple implementation can just use the first element.

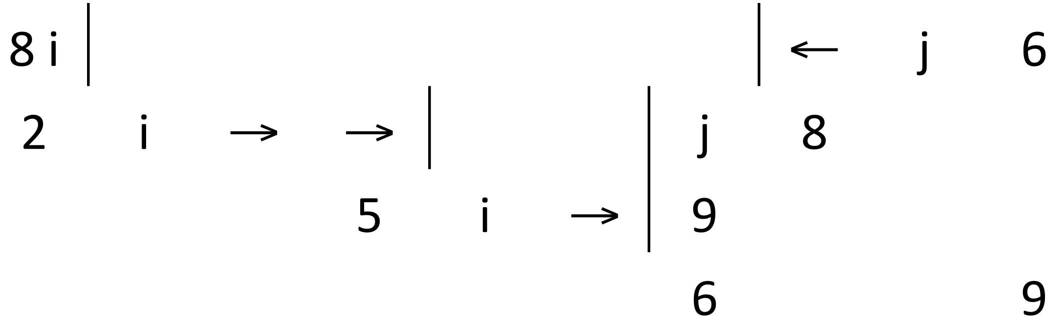
- But for efficiency, it is better if the pivot divides up the array into roughly equal partitions.
  - What kind of value would be a good pivot? A bad one?

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	8	18	12	-4	27	30	36	50	7	68	91	56	2	85	42	98	25

Partitioning an array

- Swap the pivot to the last array slot, temporarily.
- Repeat until done partitioning (until *i*,*j* meet):
  - Starting from *i* = 0, find an element a[*i*] ≥ pivot.
  - Starting from *j* = *N*-1, find an element a[*j*] ≤ pivot.
  - These elements are out of order, so swap a[*i*] and a[*j*].
- Swap the pivot back to index *i* to place it between the partitions.

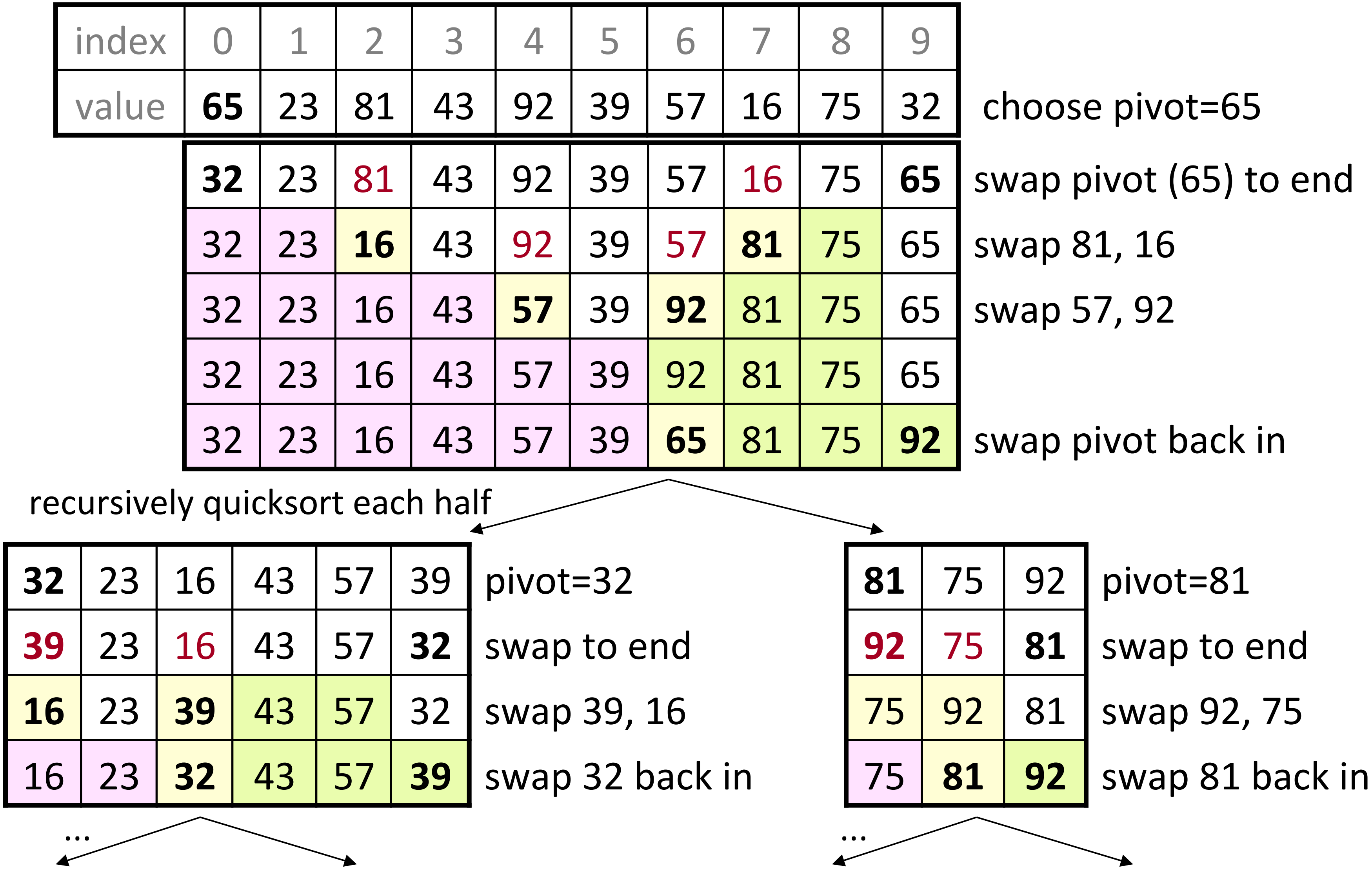
index	0	1	2	3	4	5	6	7	8	9
value	6	1	4	9	0	3	5	2	7	8



8 i							←	j	6
2	i	→	→			j	8		
			5	i	→	9			
						6			9

2	1	4	5	0	3	6	8	7	9

Quick sort example



index	0	1	2	3	4	5	6	7	8	9	
value	**65**	23	81	43	92	39	57	16	75	32	choose pivot=65
	32	23	81	43	92	39	57	16	75	**65**	swap pivot (65) to end
	32	23	**16**	43	92	39	57	**81**	75	65	swap 81, 16
	32	23	16	43	**57**	39	**92**	81	75	65	swap 57, 92
	32	23	16	43	57	39	92	81	75	65	
	32	23	16	43	57	39	**65**	81	75	**92**	swap pivot back in

recursively quicksort each half

32	23	16	43	57	39	pivot=32
39	23	16	43	57	**32**	swap to end
16	23	**39**	43	57	32	swap 39, 16
16	23	**32**	43	57	**39**	swap 32 back in

...

81	75	92	pivot=81
92	75	**81**	swap to end
75	92	81	swap 92, 75
75	**81**	**92**	swap 81 back in

...

Quick sort code

```
void quickSort(Vector<int>& v) {
    quickSortHelper(v, 0, v.size() - 1);
}

void quickSortHelper(Vector<int>& v, int min, int max) {
    if (min >= max) {  // base case; no need to sort
        return;
    }

    // choose pivot; we'll use the first element (might be bad!)
    int pivot = v[min];
    swap(v, min, max);       // move pivot to end

    // partition the two sides of the array
    int middle = partition(v, min, max - 1, pivot);

    swap(v, middle, max);    // restore pivot to proper location

    // recursively sort the left and right partitions
    quickSortHelper(v, min, middle - 1);
    quickSortHelper(v, middle + 1, max);
}
```

Partition code

```
// Partitions a with elements < pivot on left and
// elements > pivot on right;
// returns index of element that should be swapped with pivot
int partition(Vector<int>& v, int i, int j, int pivot) {
    while (i <= j) {
        // move index markers i,j toward center
        // until we find a pair of out-of-order elements
        while (i <= j && v[i] < pivot) { i++; }
        while (i <= j && v[j] > pivot) { j--; }

        if (i <= j) {
            swap(v, i++, j--);
        }
    }
    return i;
}

// Moves the value at index i to index j, and vice versa.
void swap(Vector<int>& v, int i, int j) {
    int temp = v[i];  v[i] = v[j];  v[j] = temp;
}
```

Choosing a better pivot

- Choosing the first element as the pivot leads to very poor performance on certain inputs (ascending, descending)
  - does not partition the array into roughly-equal size chunks

- Alternative methods of picking a pivot:
  - *random:* Pick a random index from [*min* .. *max*]
  - *median-of-3:* look at left/middle/right elements and pick the one with the medium value of the three:
    - `v[min]`, `v[(max+min)/2]`, and `v[max]`
    - better performance than picking random numbers every time
    - provides near-optimal runtime for almost all input orderings

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	8	18	91	-4	27	30	86	50	65	78	5	56	2	25	42	98	**31**