Lecture Topics: 5.1 Joint Discrete Distributions

[Tags: Joint PMFs, Marginal PMFs, Expectation]

1. Suppose we flip a fair coin three times independently. Let $X$ be the number of heads in the first two flips, and $Y$ be the number of heads in the last two flips (there is overlap).
   a. What distribution do $X$ and $Y$ have marginally, and what are their ranges?
   b. What is $p_{X,Y}(x, y)$? Hint: Fill in the margins first representing the marginal distributions!
   c. What is $\Omega_{X,Y}$?
   d. Write a formula for $E[\cos(XY)]$.
   e. Are $X$ and $Y$ independent?

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2. Let $X$ be the roll of a fair 3-sided die. We then flip a fair coin $X$ times independently; let $Y$ be the number of heads.
   a. What are $\Omega_X$ and $\Omega_Y$? What is $\Omega_{X,Y}$? What is $X$’s marginal distribution?
   b. What is $p_{X,Y}(x, y)$? Hint: Fill in the margins for $X$!
   c. What is $p_Y(y)$?
   d. Write a formula for $E\left[\frac{X}{Y^2 + 1}\right]$.
   e. Are $X$ and $Y$ independent?