Lecture Topics: 3.1 Discrete Random Variables Basics

[Tags: Independence, Random Variables, PMFs, Expectation, PSet2 Q8 (Similar)]

1. There are 3 people in Alex’s family; his mom, dad, and sister. Each family member decides whether or not they want to come to lunch in his social-distancing home restaurant, independently of the others.
   - Mom wants to come with probability 0.8.
   - Dad wants to come with probability 0.6.
   - Sister wants to come with probability 0.1.

Unfortunately, if all 3 of them want to come, he must turn one of them away since the restaurant capacity is 2 guests. Otherwise, he will take everyone that comes. Let \( X \) be the number of customers that Alex serves at lunch.

a. What is the range \( \Omega_X \), the PMF \( p_X(k) \), and the expectation \( E[X] \)?

b. If he charges everyone who comes $10, but it costs him $50 to make all the food, what is his expected profit?

Solution:

a. The range is \( \Omega_X = \{0,1,2\} \) since we can have anywhere from 0 to 2 people. By independence,

\[
P(X = 0) = P(M^C, D^C, S^C) = P(M^C)P(D^C)P(S^C) = 0.2 \cdot 0.4 \cdot 0.9 = 0.072
\]

\[
P(X = 1) = P(M, D^C, S^C) + P(M^C, D, S^C) + P(M^C, D^C, S)
\]

\[
= 0.8 \cdot 0.4 \cdot 0.9 + 0.2 \cdot 0.6 \cdot 0.9 + 0.2 \cdot 0.4 \cdot 0.1 = 0.404
\]

\[
P(X = 2) = 1 - P(X = 0) - P(X = 1) = 0.524
\]

\[
p_X(k) = \begin{cases} 
0.072, & k = 0 \\
0.404, & k = 1 \\
0.524, & k = 2 
\end{cases}
\]

\[
E[X] = \sum_{k \in \Omega_X} k \cdot p_X(k) = 0 \cdot 0.072 + 1 \cdot 0.404 + 2 \cdot 0.524 = 1.452
\]