<table>
<thead>
<tr>
<th>Page</th>
<th>Topic</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Variance</td>
<td>07a_variance_i</td>
</tr>
<tr>
<td>10</td>
<td>Properties of variance</td>
<td>07b_variance_ii</td>
</tr>
<tr>
<td>17</td>
<td>Bernoulli RV</td>
<td>07c_bernoulli</td>
</tr>
<tr>
<td>22</td>
<td>Binomial RV</td>
<td>07d_binomial</td>
</tr>
<tr>
<td>34</td>
<td>Exercises</td>
<td>LIVE</td>
</tr>
</tbody>
</table>
Variance
Average annual weather

Stanford, CA
$E[\text{high}] = 68^\circ F$
$E[\text{low}] = 52^\circ F$

Washington, DC
$E[\text{high}] = 67^\circ F$
$E[\text{low}] = 51^\circ F$

Is $E[X]$ enough?
Average annual weather

Stanford, CA

\[ E[\text{high}] = 68^\circ F \]
\[ E[\text{low}] = 52^\circ F \]

Washington, DC

\[ E[\text{high}] = 67^\circ F \]
\[ E[\text{low}] = 51^\circ F \]

Normalized histograms are approximations of PMFs.
Variance = “spread”

Consider the following three distributions (PMFs):

- Expectation: $E[X] = 3$ for all distributions
- But the “spread” in the distributions is different!
- **Variance**, $\text{Var}(X)$: a formal quantification of “spread”
Variance

The variance of a random variable $X$ with mean $E[X] = \mu$ is

$$\text{Var}(X) = E[(X - \mu)^2]$$

- Also written as: $E[(X - \frac{1}{n} E[X])^2]$
- Note: $\text{Var}(X) \geq 0$
- Other names: 2nd central moment, or square of the standard deviation

**def** standard deviation

$$\text{SD}(X) = \sqrt{\text{Var}(X)}$$

Units of $X^2$ Units of $X$
Variance of Stanford weather

Stanford, CA

\[ E[\text{high}] = 68 \, ^\circ F \]
\[ E[\text{low}] = 52 \, ^\circ F \]

Stanford high temps

\[ E[X] = \mu = 68 \, ^\circ F \]

<table>
<thead>
<tr>
<th>( X )</th>
<th>((X - \mu)^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>57 , ^\circ F</td>
<td>124 (°F)^2</td>
</tr>
<tr>
<td>71 , ^\circ F</td>
<td>9 (°F)^2</td>
</tr>
<tr>
<td>75 , ^\circ F</td>
<td>49 (°F)^2</td>
</tr>
<tr>
<td>69 , ^\circ F</td>
<td>1 (°F)^2</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

Variance \( E[(X - \mu)^2] = 39 \, (°F)^2 \)
Standard deviation \( = 6.2 \, ^\circ F \)
Comparing variance

Stanford, CA

\[ E[\text{high}] = 68\, ^\circ\text{F} \]

Washington, DC

\[ E[\text{high}] = 67\, ^\circ\text{F} \]

Variances:

\[ \text{Var}(X) = E[(X - E[X])^2] \]

Stanford high temps

\[ \text{Var}(X) = 39\, (\, ^\circ\text{F}\,)^2 \]

Washington high temps

\[ \text{Var}(X) = 248\, (\, ^\circ\text{F}\,)^2 \]
Properties of Variance
# Properties of variance

<table>
<thead>
<tr>
<th>Definition</th>
<th>Var($X$) $=$ $E[(X - E[X])^2]$</th>
<th>Units of $X^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>def</strong> standard deviation</td>
<td>SD($X$) $=$ $\sqrt{\text{Var}(X)}$</td>
<td>Units of $X$</td>
</tr>
</tbody>
</table>

| Property 1 | Var($X$) $=$ $E[X^2]$ $-$ $(E[X])^2$ |
| Property 2 | Var($aX + b$) $=$ $a^2 \text{Var}(X)$ |

- Property 1 is often easier to compute than the definition
- Unlike expectation, variance is not linear
# Properties of variance

**Definition**

\[
\text{Var}(X) = E[(X - E[X])^2]
\]

**def standard deviation**

\[
\text{SD}(X) = \sqrt{\text{Var}(X)}
\]

<table>
<thead>
<tr>
<th>Property</th>
<th>Equation</th>
</tr>
</thead>
</table>
| 1        | \[
\text{Var}(X) = E[X^2] - (E[X])^2
\] |
| 2        | \[
\text{Var}(aX + b) = a^2\text{Var}(X)
\] |

- Property 1 is often easier to compute than the definition
- Unlike expectation, variance is not linear
Computing variance, a proof

$$\text{Var}(X) = E[(X - E[X])^2] = E[(X - \mu)^2]$$

Let $E[X] = \mu$

$$= \sum_x (x - \mu)^2 p(x)$$

$$= \sum_x (x^2 - 2\mu x + \mu^2) p(x)$$

$$= \left[ \sum_x x^2 p(x) \right] - 2\mu \sum_x xp(x) + \mu^2 \sum_x p(x)$$

$$= E[X^2] - 2\mu E[X] + \mu^2 \cdot 1$$

$$= E[X^2] - 2\mu^2 + \mu^2$$

$$= E[X^2] - \mu^2$$

$$= E[X^2] - (E[X])^2$$

Variance of $X$
Variance of a 6-sided die

Let $Y = \text{outcome of a single die roll}$. Recall $E[Y] = 7/2$. Calculate the variance of $Y$.

1. **Approach #1: Definition**

   \[
   \text{Var}(Y) = \sum \frac{1}{6} \left( y_i - \frac{7}{2} \right)^2 \]

   \[
   = \frac{1}{6} \left( 1 - \frac{7}{2} \right)^2 + \frac{1}{6} \left( 2 - \frac{7}{2} \right)^2 + \frac{1}{6} \left( 3 - \frac{7}{2} \right)^2 + \frac{1}{6} \left( 4 - \frac{7}{2} \right)^2 + \frac{1}{6} \left( 5 - \frac{7}{2} \right)^2 + \frac{1}{6} \left( 6 - \frac{7}{2} \right)^2
   \]

   \[
   = 35/12
   \]

2. **Approach #2: A property**

   \[
   E[Y^2] = \frac{1}{6} \left[ 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 \right] = 91/6
   \]

   \[
   \text{Var}(Y) = E[Y^2] - (E[Y])^2 = 91/6 - (7/2)^2 = 35/12
   \]
# Properties of variance

<table>
<thead>
<tr>
<th>Definition</th>
<th>Var(X) = E[(X - E[X])^2]</th>
<th>Units of (X^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>def standard deviation</td>
<td>SD(X) = \sqrt{\text{Var}(X)}</td>
<td>Units of (X)</td>
</tr>
</tbody>
</table>

### Property 1

\[ \text{Var}(X) = E[X^2] - (E[X])^2 \]

### Property 2

\[ \text{Var}(aX + b) = a^2 \text{Var}(X) \]

- Property 1 is often easier to compute than the definition
- Unlike expectation, variance is not linear
Property 2: A proof

Property 2  \( \text{Var}(aX + b) = a^2\text{Var}(X) \)

Proof:  \( \text{Var}(aX + b) \)

\[
= E[(aX + b)^2] - (E[aX + b])^2 \\
= E[a^2X^2 + 2abX + b^2] - (aE[X] + b)^2 \\
= a^2E[X^2] + 2abE[X] + b^2 - (a^2(E[X])^2 + 2abE[X] + b^2) \\
= a^2E[X^2] - a^2(E[X])^2 \\
= a^2(E[X^2] - (E[X])^2) \\
= a^2\text{Var}(X) \\

\text{Var}(aX + b) \times a \rightarrow \text{Var}(X) + b \]

Property 1

Factoring/Linearity of Expectation
Bernoulli RV

\[
\begin{array}{c|c}
0 & 3 \\
1 & -1 \\
\end{array}
\]

linear => mathematical

quadratic => step
Consider an experiment with two outcomes: “success” and “failure.”

A **Bernoulli** random variable $X$ maps “success” to 1 and “failure” to 0. Other names: **indicator** random variable, **boolean** random variable.

**Bernoulli Random Variable**

$X \sim \text{Ber}(p)$

<table>
<thead>
<tr>
<th>PMF</th>
<th>Expectation</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X = 1) = p(1) = p$</td>
<td>$E[X] = p$</td>
<td>$Var(X) = p(1 - p)$</td>
</tr>
<tr>
<td>$P(X = 0) = p(0) = 1 - p$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Support: $\{0, 1\}$

**Examples:**
- Coin flip
- Random binary digit
- Whether a disk drive crashed

Remember this nice property of expectation. It will come back!
Jacob Bernoulli (1654-1705), also known as “James”, was a Swiss mathematician

One of many mathematicians in Bernoulli family
The Bernoulli Random Variable is named for him
My academic great grandfather
Defining Bernoulli RVs

Run a program
- Crashes w.p. $p$
- Works w.p. $1 - p$

Let $X$: 1 if crash

$$X \sim \text{Ber}(p)$$

$$P(X = 1) = p$$
$$P(X = 0) = 1 - p$$

Serve an ad.
- User clicks w.p. 0.2
- Ignores otherwise

Let $X$: 1 if clicked

$$X \sim \text{Ber}(\__\__)$$

$$P(X = 1) = \__\__$$
$$P(X = 0) = \__\__$$

Roll two dice.
- Success: roll two 6’s
- Failure: anything else

Let $X$: 1 if success

$$X \sim \text{Ber}(\__\__)$$

$$E[X] = \__\__$$
## Defining Bernoulli RVs

### Run a program
- Crashes w.p. \( p \)
- Works w.p. \( 1 - p \)

Let \( X \): 1 if crash

\[
X \sim \text{Ber}(p) \\
P(X = 1) = p \\
P(X = 0) = 1 - p
\]

### Serve an ad.
- User clicks w.p. 0.2
- Ignores otherwise

Let \( X \): 1 if clicked

\[
X \sim \text{Ber}(0.2) \\
P(X = 1) = 0.2 \\
P(X = 0) = 0.8
\]

### Roll two dice.
- Success: roll two 6’s \( \frac{1}{36} \)
- Failure: anything else

Let \( X \): 1 if success

\[
X \sim \text{Ber}(\frac{1}{36}) \\
E[X] = \frac{1}{36}
\]
Binomial RV
Consider an experiment: $n$ independent trials of $\text{Ber}(p)$ random variables.

**Binomial** random variable $X$ is the number of successes in $n$ trials.

### Binomial Random Variable

- **PMF**
  
  $P(X = k) = p(k) = \binom{n}{k} p^k (1 - p)^{n-k}$

- **Support**: $\{0, 1, \ldots, n\}$

- **Expectation**
  
  $E[X] = np$

- **Variance**
  
  $\text{Var}(X) = np(1 - p)$

### Examples:

- # heads in $n$ coin flips
- # of 1’s in randomly generated length $n$ bit string
- # of disk drives crashed in 1000 computer cluster (assuming disks crash independently)
Reiterating notation

The parameters of a Binomial random variable:

- \( n \): number of independent trials
- \( p \): probability of success on each trial

\[ X \sim \text{Bin}(n, p) \]
Reiterating notation

If $X$ is a binomial with parameters $n$ and $p$, the PMF of $X$ is

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

- **Probability that $X$ takes on the value $k$**
- **Probability Mass Function for a Binomial**
Three coin flips

Three fair (“heads” with \( p = 0.5 \)) coins are flipped.
- \( X \) is number of heads
- \( X \sim \text{Bin}(3, 0.5) \)

Compute the following event probabilities:

\[
\begin{align*}
P(X = 0) \\
P(X = 1) \\
P(X = 2) \\
P(X = 3) \\
P(X = 7) \\
P(\text{event})
\end{align*}
\]
Three coin flips

Three fair (“heads” with $p = 0.5$) coins are flipped.

- $X$ is number of heads
- $X \sim \text{Bin}(3, 0.5)$

Compute the following event probabilities:

\[
\begin{align*}
P(X = 0) &= p(0) = \binom{3}{0} p^0 (1 - p)^3 = \frac{1}{8} \\
P(X = 1) &= p(1) = \binom{3}{1} p^1 (1 - p)^2 = \frac{3}{8} \\
P(X = 2) &= p(2) = \binom{3}{2} p^2 (1 - p)^1 = \frac{3}{8} \\
P(X = 3) &= p(3) = \binom{3}{3} p^3 (1 - p)^0 = \frac{1}{8}
\end{align*}
\]

P(event) PMF

Extra math note:
By Binomial Theorem, we can prove
\[
\sum_{k=0}^{n} P(X = k) = 1
\]
Consider an experiment: \( n \) independent trials of \( \text{Ber}(p) \) random variables.

**Def** A **Binomial** random variable \( X \) is the number of successes in \( n \) trials.

\[
X \sim \text{Bin}(n, p)
\]

- **PMF**
  \[
P(X = k) = p(k) = \binom{n}{k} p^k (1 - p)^{n-k}
\]

- **Expectation**
  \[
  E[X] = np
  \]

- **Variance**
  \[
  \text{Var}(X) = np(1 - p)
  \]

**Examples:**
- # heads in \( n \) coin flips
- # of 1’s in randomly generated length \( n \) bit string
- # of disk drives crashed in 1000 computer cluster (assuming disks crash independently)
Binomial RV is sum of Bernoulli RVs

Bernoulli
- \( X \sim \text{Ber}(p) \)

Binomial
- \( Y \sim \text{Bin}(n, p) \)
- The sum of \( n \) independent Bernoulli RVs

\[
Y = \sum_{i=1}^{n} X_i, \quad X_i \sim \text{Ber}(p)
\]

\( \text{Ber}(p) = \text{Bin}(1, p) \)
Consider an experiment: \( n \) independent trials of \( \text{Ber}(p) \) random variables.  

**def** A **Binomial** random variable \( X \) is the number of successes in \( n \) trials.

### Binomial Random Variable

\[
X \sim \text{Bin}(n, p)
\]

- **PMF**
  \[
P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}
\]

- **Range:** \( \{0, 1, \ldots, n\} \)

- **Expectation**
  \[
  E[X] = np
  \]

- **Variance**
  \[
  \text{Var}(X) = np(1 - p)
  \]

### Examples:
- \# heads in \( n \) coin flips
- \# of 1’s in randomly generated length \( n \) bit string
- \# of disk drives crashed in 1000 computer cluster (assuming disks crash independently)

### Proof:

\[
X = \sum_{i=1}^{n} X_i \quad , \quad X_i \sim \text{Ber}(p)
\]

\[
E[X] = E[\sum_{i=1}^{n} X_i] = \sum_{i=1}^{n} E[X_i]
\]

\[
= \sum_{i=1}^{n} p \quad , \quad E[X_i] = p
\]

\[
= np
\]
Consider an experiment: \( n \) independent trials of \( \text{Ber}(p) \) random variables. **Def**: A **Binomial** random variable \( X \) is the number of successes in \( n \) trials.

**PMF**

\[
P(X = k) = p(k) = \binom{n}{k} p^k (1 - p)^{n-k}
\]

**Expectation**

\[
E[X] = np
\]

**Variance**

\[
\text{Var}(X) = np(1 - p)
\]

**Examples:**

- # heads in \( n \) coin flips
- # of 1’s in randomly generated length \( n \) bit string
- # of disk drives crashed in 1000 computer cluster (assuming disks crash independently)

We’ll prove this later in the course.
To simplify the algebra a bit, let $q = 1 - p$, so $p + q = 1$.

So:

$$E(X^2) = \sum_{k=0}^{n} k^2 \binom{n}{k} p^k q^{n-k}$$

$$= \sum_{k=0}^{n} k n \binom{n-1}{k-1} p^k q^{n-k}$$

$$= np \sum_{k=1}^{n} \binom{n-1}{k-1} p^k q^{n-k}$$

$$= np \sum_{j=0}^{n} (j + 1) \binom{m}{j} p^j q^{m-j}$$

$$= np \left( \sum_{j=0}^{m} \binom{m-1}{j} p^j q^{m-j} + \sum_{j=0}^{m} \binom{m}{j} p^j q^{m-j} \right)$$

$$= np \left( \sum_{j=0}^{m} \binom{m-1}{j} p^j q^{m-j} + \sum_{j=0}^{m} \binom{m}{j} p^j q^{m-j} \right)$$

$$= np \left( (n-1)p \sum_{j=1}^{m-1} \binom{m-1}{j-1} p^{j-1} q^{(m-1)-(j-1)} + \sum_{j=0}^{m} \binom{m}{j} p^j q^{m-j} \right)$$

$$= np((n-1)p(p + q)^{m-1} + (p + q)^m)$$

$$= np(n-1)p + 1$$

$$= n^2 p^2 + np(1-p)$$

Then:

$$\text{var}(X) = E(X^2) - (E(X))^2$$

$$= np(1-p) + n^2 p^2 - (np)^2$$

$$= np(1-p)$$

as required.
Reminders: Lecture with Zoom

- Turn on your camera if you are able, mute your mic in the big room
- Virtual backgrounds are encouraged (classroom-appropriate)

Breakout Rooms for meeting your classmates
  - Just like sitting next to someone new
  - This experience is optional: You should be comfortable leaving the room at any time.

We will use Ed instead of Zoom chat

Today’s discussion thread: https://us.edstem.org/courses/109/discussion/39075
Our first common RVs

\[ X \sim \text{Ber}(p) \]

Example: Heads in one coin flip, \( P(\text{heads}) = 0.8 = p \)

\[ Y \sim \text{Bin}(n, p) \]

Example: # heads in 40 coin flips, \( P(\text{heads}) = 0.8 = p \)

1. The random variable
2. is distributed as a
3. Bernoulli
4. with parameter

otherwise

Identify PMF, or
identify as a function of an
existing random variable
Check out the questions on the next slide (Slide 38). Post any clarifications here!

https://us.edstem.org/courses/109/discussion/39075

Breakout rooms: 5 min. Introduce yourself!
Statistics: Expectation and variance

1. a. Let $X$ = the outcome of a 4-sided die roll. What is $E[X]$?
   b. Let $Y$ = the sum of three rolls of a 4-sided die. What is $E[Y]$?

2. a. Let $Z$ = # of tails on 10 flips of a biased coin (w.p. 0.4 of heads). What is $E[Z]$?
   b. What is Var($Z$)?

3. Compare the variances of $B_1 \sim \text{Ber}(0.1)$ and $B_2 \sim \text{Ber}(0.5)$. 
Statistics: Expectation and variance

1. a. Let $X = \text{the outcome of a 4-sided die roll}$. What is $E[X]$?

   b. Let $Y = \text{the sum of three rolls of a 4-sided die}$. What is $E[Y]$?

2. a. Let $Z = \# \text{ of tails on 10 flips of a biased coin (w.p. 0.3 of heads)}$. What is $E[Z]$?

   b. What is $\text{Var}(Z)$?

3. Compare the variances of $B_1 \sim \text{Ber}(0.1)$ and $B_2 \sim \text{Ber}(0.5)$.

If you can identify common RVs, just look up statistics instead of re-deriving from definitions.
Think

Slide 41 has a matching question to go over by yourself. We’ll go over it together afterwards.

Post any clarifications here!

https://us.edstem.org/courses/109/discussion/39075

Think by yourself: 2 min
Visualizing Binomial PMFs

Match the distribution to the graph:
1. Bin(10, 0.5)
2. Bin(10, 0.3)
3. Bin(10, 0.7)
4. Bin(5, 0.5)

\[ X \sim \text{Bin}(n, p) \quad p(i) = \binom{n}{k} p^k (1 - p)^{n-k} \]

\[ E[X] = np \]
Visualizing Binomial PMFs

Match the distribution to the graph:

1. Bin(10, 0.5) $\Rightarrow E[X] = 5$
2. Bin(10, 0.3) $\Rightarrow E[X] = 3$
3. Bin(10, 0.7) $\Rightarrow E[X] = 7$
4. Bin(5, 0.5)

$E[X] = np$

$X \sim \text{Bin}(n, p)$

$p(i) = \binom{n}{k} p^k (1 - p)^{n-k}$
Binomial RV is sum of Bernoulli RVs

Bernoulli
- $X \sim \text{Ber}(p)$

Binomial
- $Y \sim \text{Bin}(n, p)$
- The sum of $n$ independent Bernoulli RVs
$$Y = \sum_{i=1}^{n} X_i, \quad X_i \sim \text{Ber}(p)$$
Galton Board

http://web.stanford.edu/class/cs109/demos/galton.html
Think

Slide 46 has a question to go over by yourself.

Post any clarifications here!

https://us.edstem.org/courses/109/discussion/39075

Think by yourself: 2 min
When a marble hits a pin, it has an equal chance of going left or right. Let $B$ = the bucket index a ball drops into. What is the distribution of $B$?

(Interpret: If $B$ is a common random variable, report it, otherwise report PMF)
When a marble hits a pin, it has an equal chance of going left or right. Let $B =$ the bucket index a ball drops into. What is the distribution of $B$?

- Each pin is an independent trial
- One decision made for level $i = 1, 2, \ldots, 5$
- Consider a Bernoulli RV with success $R_i$ if ball went right on level $i$
- Bucket index $B = \sum_{i=1}^{5} R_i$

$$B \sim \text{Bin}(n = 5, p = 0.5)$$
When a marble hits a pin, it has an equal chance of going left or right. Let $B = \text{the bucket index a ball drops into}$. $B$ is distributed as a Binomial RV, $B \sim \text{Bin}(n = 5, p = 0.5)$.

Calculate the probability of a ball landing in bucket $k$.

$$P(B = 0) = \binom{5}{0} 0.5^5 \approx 0.03$$

$$P(B = 1) = \binom{5}{1} 0.5^5 \approx 0.16$$

$$P(B = 2) = \binom{5}{2} 0.5^5 \approx 0.31$$
Let $B = \text{the bucket index} \text{ a ball drops into.}$

$B$ is distributed as a Binomial RV,

$\sim Bin(n = 5, p = 0.5)$

Calculate the probability of a ball landing in bucket $k$. 

\[ p(k) = \binom{n}{k} p^k (1 - p)^{n-k} \]
Interlude for jokes/announcements
Jokey time

Here's a fun one:

\[
\begin{array}{cccc}
0 & 1 & 2 & 3 \\
\vdots & \vdots & \vdots & \vdots \\
-3 & -2 & -1 & \vdots
\end{array}
\]

What do you call a sequence that goes like this? A FibonacciJOKE HERE
Interesting probability news

TikTok
Recommendation
Algorithm Optimizations

Empirical Evidence

Looking at my own videos, a simple power law distribution fits reasonably well. (These numbers are using the model $P(\text{Views} \geq 10^k) = 0.3^{k-2}$.)

<table>
<thead>
<tr>
<th>Number of videos</th>
<th>Percentage of videos</th>
<th>Expected percentage of videos</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt;10</td>
<td>12</td>
<td>100.00%</td>
</tr>
<tr>
<td>&gt;100</td>
<td>11</td>
<td>91.67%</td>
</tr>
<tr>
<td>&gt;1,000</td>
<td>4</td>
<td>33.33%</td>
</tr>
<tr>
<td>&gt;10,000</td>
<td>1</td>
<td>8.33%</td>
</tr>
<tr>
<td>&gt;100,000</td>
<td>1</td>
<td>8.33%</td>
</tr>
<tr>
<td>&gt;1,000,000</td>
<td>0</td>
<td>0.00%</td>
</tr>
</tbody>
</table>
NBA Finals (RIP) and genetics
Think, then Breakout Rooms

Check out the questions on the next slide (Slide 55). Post any clarifications here!

https://us.edstem.org/courses/109/discussion/39075

By yourself: 4 min

Breakout rooms: 5 min.
1. The Golden State Warriors are going to play the Toronto Raptors in a 7-game series during the 2019 NBA finals.
   • The Warriors have a probability of 58% of winning each game, independently.
   • A team wins the series if they win at least 4 games (we play all 7 games).

What is \( P(\text{Warriors winning}) \)?

2. Each person has 2 genes per trait (e.g., eye color).
   • Child receives 1 gene (equally likely) from each parent
   • **Brown** is “dominant”, **blue** is ”recessive”:
     • Child has brown eyes if either (or both) genes are brown
     • Blue eyes only if both genes are blue.
   • Parents each have 1 brown and 1 blue gene.

A family has 4 children. What is \( P(3 \text{ children with brown eyes}) \)?
NBA Finals

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What is $P$(Warriors winning)?

1. Define events/ RVs & state goal

   $X$: # games Warriors win
   $X \sim \text{Bin}(7, 0.58)$

   Want: Desired probability? (select all that apply)

   A. $P(X > 4)$
   B. $P(X \geq 4)$
   C. $P(X > 3)$
   D. $1 - P(X \leq 3)$
   E. $1 - P(X < 3)$
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2. Solve

\[
P( X \geq 4 ) = \sum_{k=4}^{7} P( X = k ) = \sum_{k=4}^{7} \binom{7}{k} 0.58^k (0.42)^{7-k}
\]

Cool Algebra/Probability Fact: this is identical to the probability of winning if we define winning = first to win 4 games
Genetic inheritance

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Subset of ideas:
- **A.** Product of 4 independent events
  - \( X \sim \text{Bin}(n, p) \) \( p(k) = \binom{n}{k} p^k (1 - p)^{n-k} \)
- **B.** Probability tree
- **C.** Bernoulli, success \( p = 3 \text{ children with brown eyes} \)
- **D.** Binomial, \( n = 3 \) trials, success \( p = \text{brown-eyed child} \)
- **E.** Binomial, \( n = 4 \) trials, success \( p = \text{brown-eyed child} \)
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A family has 4 children. What is $P(3$ children with brown eyes$)$?

1. Define events/RVs & state goal
2. Identify known probabilities
3. Solve

$X$: # brown-eyed children,
$X\sim\text{Bin}(4, p)\Rightarrow p = 0.75$
$p$: $P(\text{brown-eyed child})$
Want: $P(X = 3)$

$p(\text{brown}) = 1 - p(\text{blue}) = 1 - \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{4}$

$P(X = 3) = \binom{4}{3} (0.75)^3 (0.25)^1$
See you next time