1. **Random Number of Random Variables**: *law of total expectation*

Let $N$ be a non-negative integer-valued random variable; that is, takes values in $\{0, 1, 2, \ldots \}$. Let $X_1, X_2, X_3, \ldots$ be an infinite sequence of iid random variables (independent of $N$), each with mean $\mu$, and $X = \sum_{i=1}^{N} X_i$ be the sum of the first $N$ of them. Before doing any work, what do you think $E[X]$ will turn out to be? Show it mathematically.

2. **Beta Sum**: *beta distribution and sum of RVs*

What is the distribution of the sum of 100 IID Betas? Let $X$ be the sum

$$X = \sum_{i=1}^{100} X_i$$

Where each $X_i \sim \text{Beta}(a = 3, b = 4)$

Note the variance of a Beta:

$$\text{Var}(X_i) = \frac{ab}{(a + b)^2(a + b + 1)}$$

Where $X_i \sim \text{Beta}(a, b)$

3. **Medicine Doses**:

Megha has a health condition that requires unpredictable amounts of medication. Every day, there is a 20% chance that she feels perfectly fine and requires no medicine. Otherwise, she needs to take a dose of medication. The necessary dose is equally likely to be any value in the continuous range 1 to 5 ounces. How much medicine she needs on any given day is independent of all other days.

Megha’s insurance will fully cover 90 ounces of medicine for each 30-day period. What is the probability that 90 ounces will be enough for the next 30 days? Make your life easier by using Central Limit Theorem.