1. Are we due for an earthquake?:

a. What is the probability of no 8+ earthquakes in four years? Let $X$ be the time until an earthquake. $X \sim \text{Exp}(\lambda = 0.002)$.

$$P(X \geq 4) = 1 - P(X < 4)$$
$$= 1 - F_X(4)$$
$$= 1 - [1 - e^{-0.002 \cdot 4}]$$
$$= e^{-0.008} \approx 0.992$$

b. What is the probability of no 8+ earthquakes in the 113 years?

$$P(X \geq 113) = 1 - P(X < 113)$$
$$= 1 - F_X(113)$$
$$= 1 - [1 - e^{-0.002 \cdot 113}]$$
$$= e^{-0.226} \approx 0.798$$

c. What is $P(X > 113|X > 109)$?

$$P(X > 113|X > 109) = \frac{P(X > 113, X > 109)}{P(X > 109)}$$
$$= \frac{P(X > 113)}{P(X > 109)} = \frac{1 - F_X(113)}{1 - F_X(109)}$$
$$= \frac{e^{-0.002 \cdot 113}}{e^{-0.002 \cdot 109}} = e^{-0.008} \approx 0.992$$

d. It turns out that exponentials are what we call a “memoryless distribution.” If $X$ is an exponential random variable, it holds that $P(X > s + t|X > t) = P(X > s)$.

2. ReCaptcha

a. What the the probability density function of a robot clicking $X = x$ pixels from the left of the box and $Y = y$ pixels from the top of the box?

$$f_{X,Y}(x, y) = \begin{cases} \frac{1}{100^2} & \text{if } 0 < x, y < 10 \\ 0 & \text{else} \end{cases}$$
b. Let $D \sim \text{Rayleigh}(\theta = 2)$ be the distance a human clicks.

\[
P(D > 1.2) = 1 - P(D < 1.2) = 1 - F_D(1.2)
= 1 - [1 - e^{-1.2^2/2.2}] = e^{-1.2^2/4} \approx 0.698
\]

c. We can start by expanding Bayes theorem for the new belief of a Robot

\[P(\text{Robot}|D = 2) = \frac{f(D = 2|\text{Robot})P(\text{Robot})}{f(D = 2)}\]

The two terms on the top are both ones that we can calculate from formulas that we have. The denominator is more problematic: it asks, what is the density of a click two pixels away if we don’t know whether the user is a Robot or a Human. The answer is to use the law of total probability, just like in the past. Since all users are either humans or robots, $P(\text{Robot}|D = 2) + P(\text{Human}|D = 2) = 1$. As such:

\[
P(\text{Robot}|D = 2) = \frac{f(D = 2|\text{Robot})P(\text{Robot})}{f(D = 2|\text{Robot})P(\text{Robot}) + f(D = 2|\text{Human})P(\text{Human})}
= \frac{\frac{1}{100} \cdot 0.2}{\frac{1}{100} \cdot 0.2 + \frac{3}{2}e^{-2^2/2} \cdot 0.8 + 0.002}
\approx 0.006
\]

3. It’s Complicated

a. For each assignment to $R$, sum over all the values that $S$ can take on that is consistent with that assignment.

- Single = 0.44
- In a Relationship = 0.47
- It’s Complicated = 0.09

b. Single = 0.125

- In a Relationship = 0.875
- It’s Complicated = 0.00

c. Freshman = 0.35

- Sophomore = 0.39
- Junior = 0.42
- Senior = 0.90
- 5+ = 0.60