1. Binary Tree:

Let $X_1$ and $X_2$ be number of nodes the left and right calls to `randomTree`. 

$E[X_1] = E[X_2] = E[X]$.

\[
E[X] = p \cdot E[X | \text{true}] + (1 - p)E[X | \text{false}] \\
= p \cdot E[1 + X_1 + X_2] + (1 - p) \cdot 0 \\
= p \cdot (1 + E[X] + E[X]) \\
= p + 2pE[X] \\
(1 - 2p)E[X] = p \\
E[X] = \frac{p}{1 - 2p}
\]

2. Girl Scout Cookies

a. $X = Noa + Chaya \sim N(\mu + \mu = 2\mu, \sigma^2 + \sigma^2 = 2\sigma^2)$

b. $Y = 2 \cdot Maria \sim N(\mu \cdot 2 = 2\mu, (2\sigma)^2 = 4\sigma^2)$

3. Timing Attack

a. Let $Y$ be the amount of time to execute $k$ lines. $Y = \sum_{i=1}^{k} X_i$ where $X_i$ is the amount of time to execute line $i$. $X_i \sim N(\mu = 5, \sigma^2 = 0.5)$.

Since $Y$ is the sum of independent normals:

\[
Y \sim N(\mu = \sum_{i=1}^{k} 5, \sigma^2 = \sum_{i=1}^{k} 0.5) \\
\sim N(\mu = 5k, \sigma^2 = 0.5k)
\]

b. From last problem:

Time to run 6 lines of code $A \sim N(\mu = 30, \sigma^2 = 3)$

Time to run 4 lines of code $B \sim N(\mu = 20, \sigma^2 = 2)$

$-B \sim N(\mu = -20, \sigma^2 = 2)$

$A - B \sim N(\mu = 10, \sigma^2 = 5)$

$P(A > B) = P(A - B > 0)$

\[
= 1 - F_{A-B}(0) \\
= 1 - \Phi\left(\frac{0 - 10}{\sqrt{5}}\right) \\
\approx 1.0
\]
c. Let $M$ be the event that the first letter matched.

\[
\frac{P(M^C|T = 27)}{P(M|T = 27)} = \frac{f(T = 27|M^C)P(M^C)}{f(T = 27|M)P(M)} = \frac{f(T = 27|M^C) \frac{25}{26}}{f(T = 27|M) \frac{1}{26}} = \frac{25}{26} \cdot \frac{f(T = 27|M^C)}{f(T = 27|M)}
\]

\[
= \frac{25}{26} \cdot \frac{1}{\sqrt{6\pi}}e^{-(27-30)^2/6} \cdot \frac{1}{\sqrt{8\pi}}e^{-(27-40)^2/8} = \frac{25}{26} \cdot \frac{\sqrt{8}}{\sqrt{6}} \cdot \frac{e^{-9}}{e^{-169/8}} \approx 370 \text{ million}
\]

d. $7 \cdot 26 + 7 = 189$